Ch 5: Capacitance and Dielectric

Ch 26 on textbook

Lecture Contents

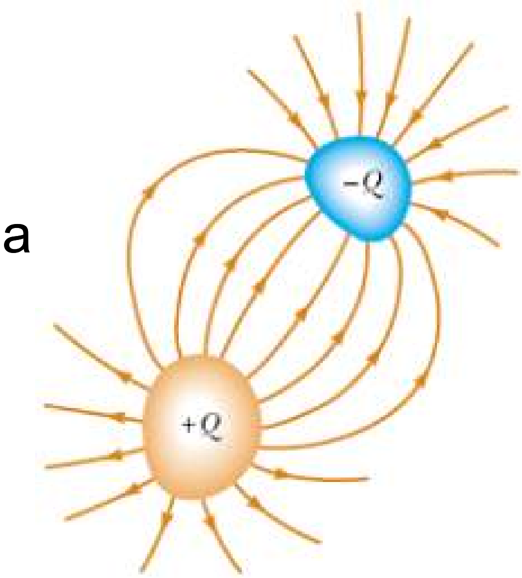
- Capacitance Definition.
- Calculating Capacitance.
- Combination of Capacitors.
- Energy stored in a Charged Capacitor.
- Capacitors with Dielectric material.

Introduction

- Capacitors are devices used to store electric charge.
- Used in variety of electric circuits (used to tune the frequency of radio receivers, filters in power supply, eliminate sparking in automobile ignition systems, energy storing in electronic flash devices.

Capacitance Definition

- Two conductors carry equal but opposite charge.
- This combination is called Capacitor.
- The conductors are called ***<u>Plates</u>***.
- Due to the presence charge on the plates, there is a potential difference between the plates (ΔV).
- Experiments show that the charge is linearly proportional to the potential difference across the plates. I.e. $Q \propto \Delta V$
- We can write this relation as $Q = C\Delta V$



Capacitance Definition

- Capacitance can be defined as the ratio of the charge on either of the capacitor plates (conductor) to the magnitude of the potential difference across the plates

$$C \equiv \frac{Q}{\Delta V}$$

- Unit is Coulomb / Volt or Farad (F). This unit is relatively large and usually the MicroFarads (μF)is used

Calculating Capacitance

•Capacitance of an isolated charged sphere

$$C = \frac{Q}{\Delta V} = \frac{Q}{k_e Q / R} = \frac{R}{k_e} = 4\pi\epsilon_0 R$$

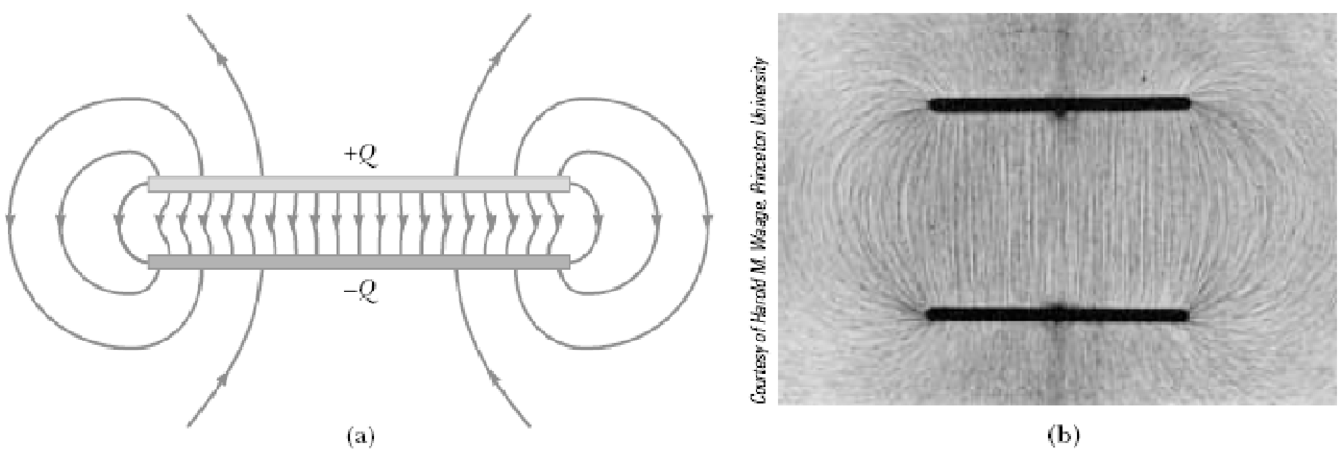
•Parallel plate capacitor

$$E = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 A} \qquad \Delta V = Ed = \frac{Qd}{\epsilon_0 A} \qquad C = \frac{Q}{\Delta V} = \frac{Q}{Qd/\epsilon_0 A}$$

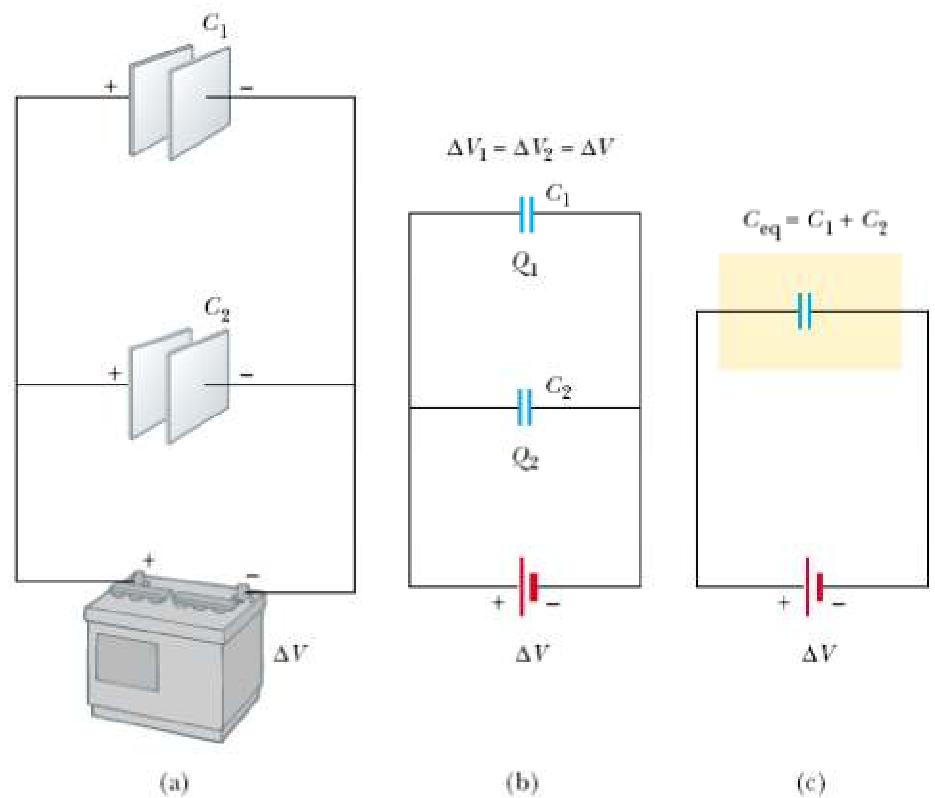
$$\boxed{C = \frac{\epsilon_0 A}{d}}$$

Calculating Capacitance

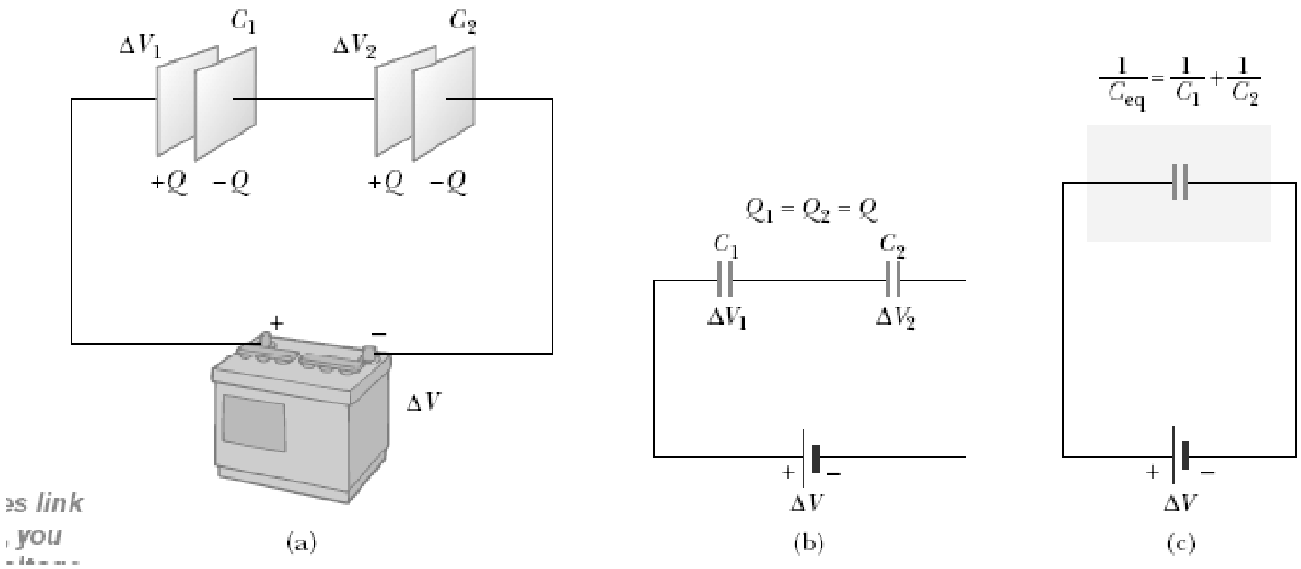
•Parallel plate capacitor



Combination Capacitance



Combination Capacitance



Example 1

A parallel-plate capacitor with air between the plates has an area A = 2.00 x 10^{-4} m^2 and a plate separation d = 1.00 mm. Find its capacitance.

$$C = \frac{\epsilon_0 A}{d} = \frac{(8.85 \times 10^{-12}\ \mathrm{C^2/N\cdot m^2})(2.00 \times 10^{-4}\ \mathrm{m^2})}{1.00 \times 10^{-3}\ \mathrm{m}}$$

$$= 1.77 \times 10^{-12}\ \mathrm{F} = \boxed{1.77\ \mathrm{pF}}$$

Example 2

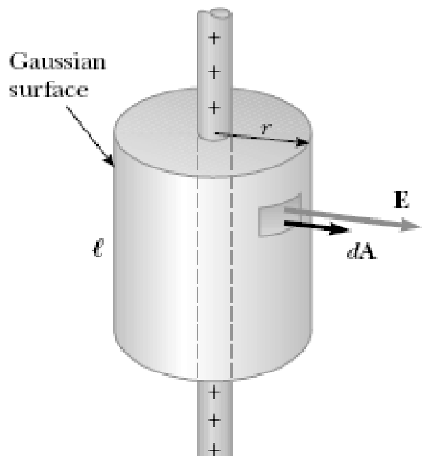
A solid cylindrical conductor of radius *a* and charge *Q* is coaxial with a cylindrical shell of negligible thickness, radius *b* > *a*, and charge -*Q* (see Figure). Find the capacitance of this cylindrical capacitor if its length is *l*.

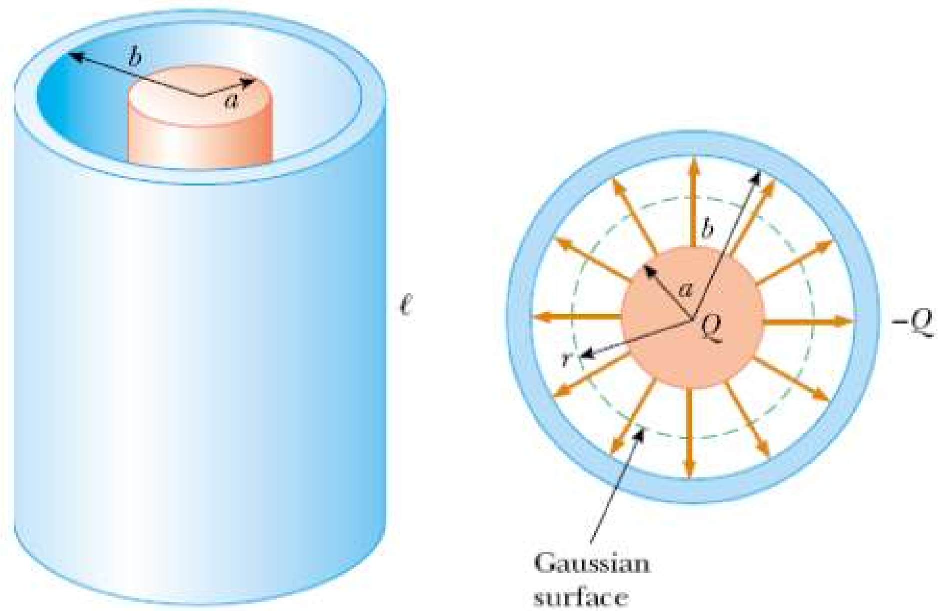
$$V_b - V_a = -\int_a^b \mathbf{E} \cdot d\mathbf{s}$$

$$\Phi_E = \oint \mathbf{E} \cdot d\mathbf{A} = E \oint dA = EA = \frac{q_{\text{in}}}{\epsilon_0} = \frac{\lambda \ell}{\epsilon_0}$$

$$E(2\pi r \ell) = \frac{\lambda \ell}{\epsilon_0}$$

$$E = 2k_e \lambda / r$$





(a) (b)

Example 2

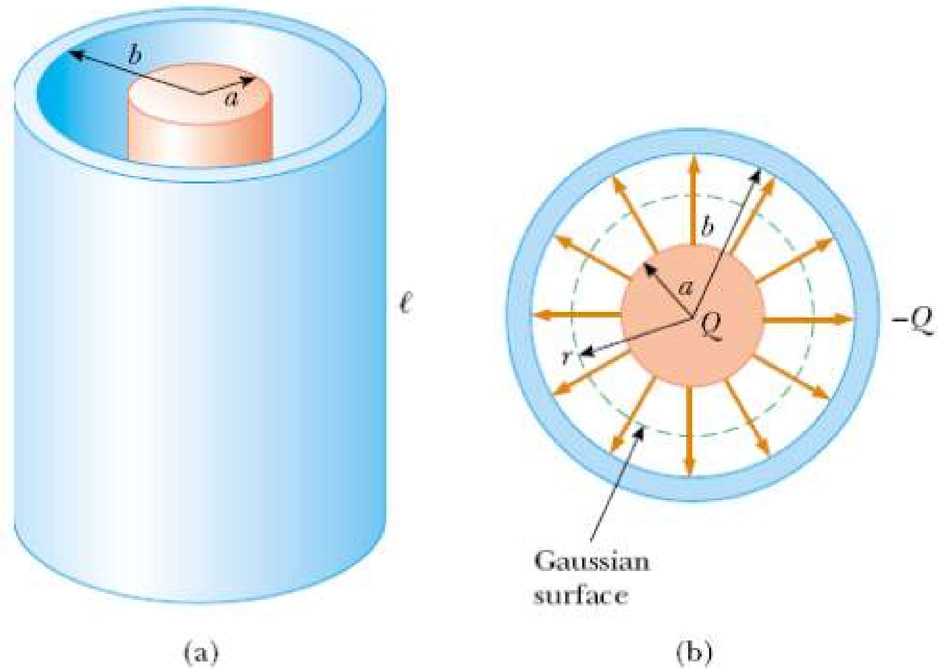
A solid cylindrical conductor of radius *a* and charge *Q* is coaxial with a cylindrical shell of negligible thickness, radius *b* > *a*, and charge -*Q* (see Figure). Find the capacitance of this cylindrical capacitor if its length is *l*.

$$V_b - V_a = -\int_a^b \mathbf{E}\cdot d\mathbf{s}$$

$$E = 2k_e\lambda/r$$

$$V_b - V_a = -\int_a^b E_r\,dr = -2k_e\lambda\int_a^b \frac{dr}{r} = -2k_e\lambda \ln\left(\frac{b}{a}\right)$$

$$C = \frac{Q}{\Delta V} = \frac{Q}{(2k_eQ/\ell)\ln(b/a)} = \boxed{\frac{\ell}{2k_e\ln(b/a)}}$$

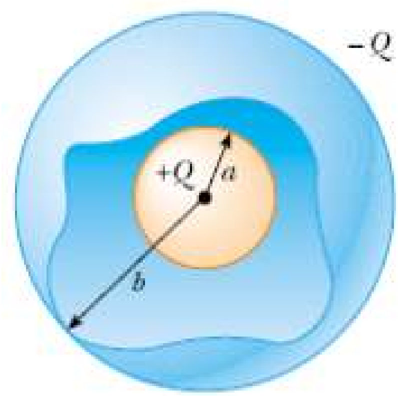


Example 3

A spherical capacitor consists of a spherical conducting shell of radius *b* and charge $\$Q$ concentric with a smaller conducting sphere of radius *a* and charge *Q* (see Figure). Find the capacitance of this device.

$$V_b - V_a = -\int_a^b E_r\,dr = -k_eQ\int_a^b \frac{dr}{r^2} = k_eQ\left[\frac{1}{r}\right]_a^b$$

$$= k_eQ\left(\frac{1}{b} - \frac{1}{a}\right)$$

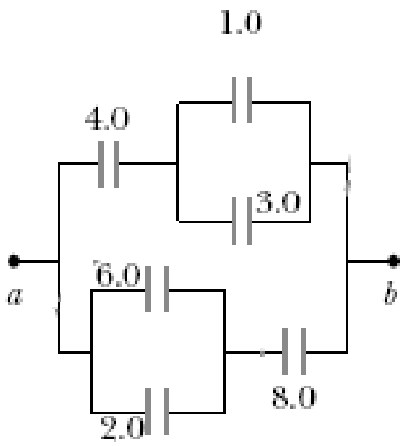


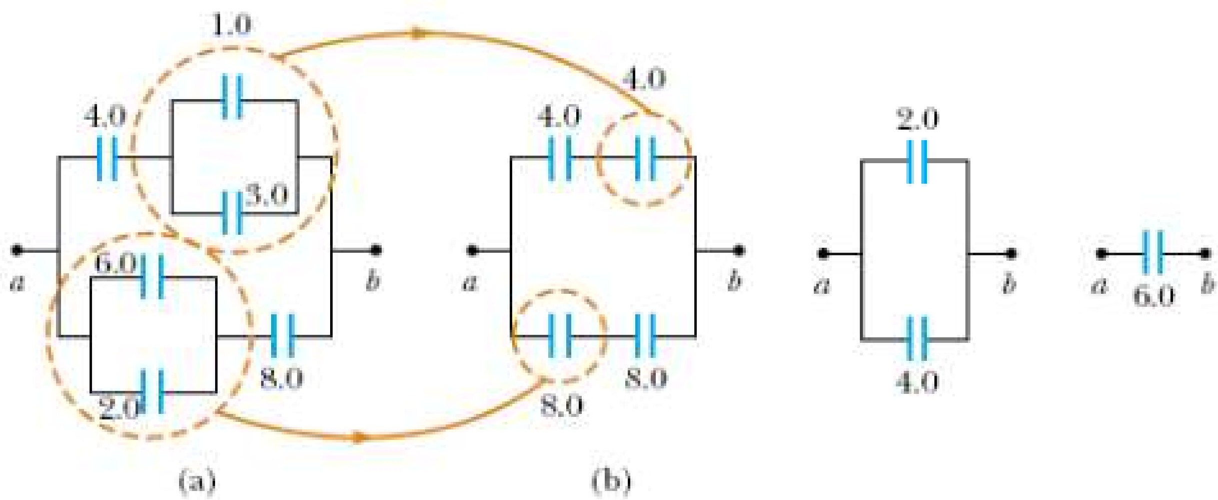
$$\Delta V = |V_b - V_a| = k_eQ\,\frac{(b-a)}{ab}$$

$$C = \frac{Q}{\Delta V} = \boxed{\frac{ab}{k_e(b-a)}}$$

Example 4

Find the equivalent capacitance between *a* and *b* for the combination of capacitors shown in the Figure. All capacitances are in microfarads.





Energy Stored in a Capacitor

The electric potential energy stored in a capacitor is

$$U = \frac{Q^2}{2C} = \tfrac{1}{2}Q\,\Delta V = \tfrac{1}{2}C(\Delta V)^2$$

For a parallel plate capacitor, U is given by:

$$U = \frac{1}{2}\frac{\epsilon_0 A}{d}(E^2 d^2) = \tfrac{1}{2}(\epsilon_0 A d)E^2$$

And because the energy occupied by the volume (*A d)*
Then the energy per unit volume is given by:

$$u_E = U/Ad, \qquad \boxed{u_E = \tfrac{1}{2}\epsilon_0 E^2}$$

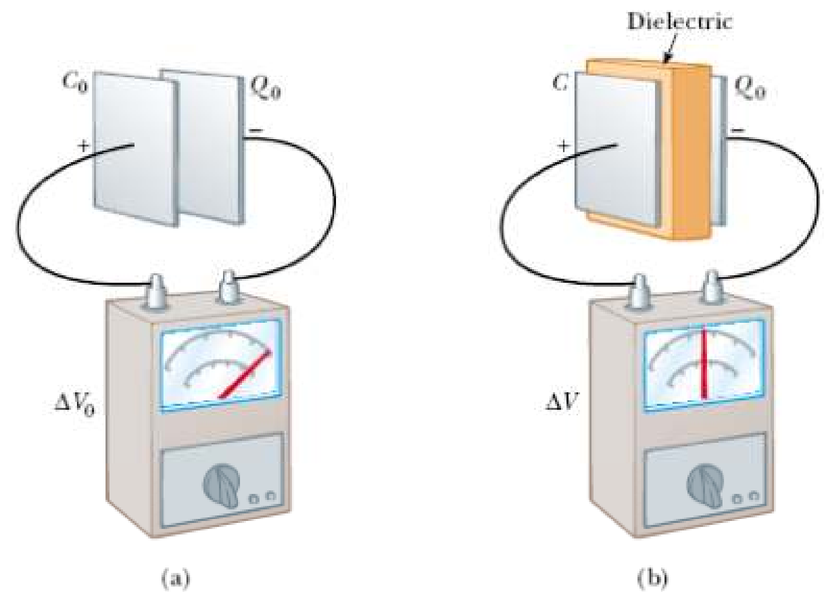
Capacitors with Dielectrics

A dielectric is a non-conducting material, such as rubber, glass, or waxed paper.
When dielectric material is inserted between the plates of a capacitor, the capacitance increases.
If the dielectric completely fill the gap, then the capacitance increase by a factor called Dielectric constant (κ). “Kappa”.

$$\Delta V = \frac{\Delta V_0}{\kappa}$$

$$C = \frac{Q_0}{\Delta V} = \frac{Q_0}{\Delta V_0/\kappa} = \kappa \frac{Q_0}{\Delta V_0}$$

$$\boxed{C = \kappa C_0}$$



Capacitors with Dielectrics

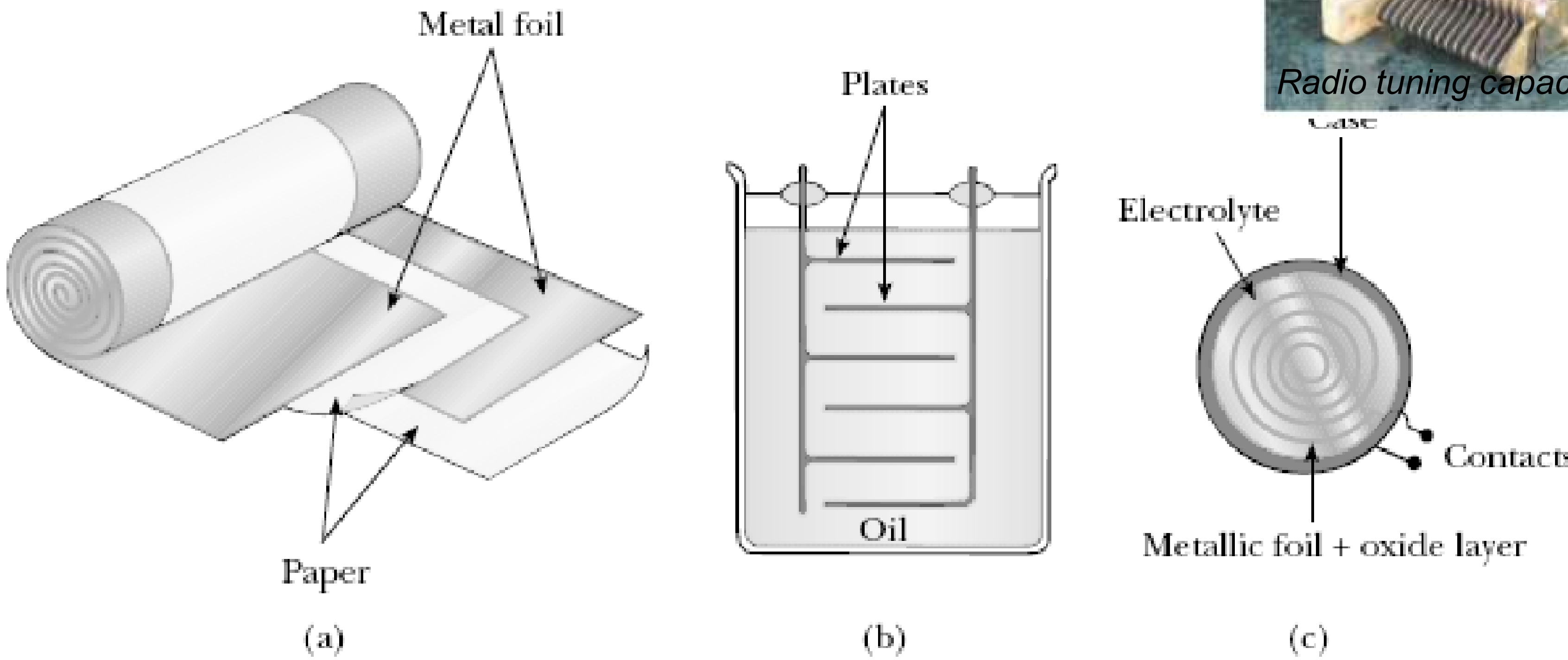
Approximate Dielectric Constants and Dielectric Strengths of Various Materials at Room Temperature

Material	Dielectric Constant κ	Dielectric Strength[a] (10^6 V/m)
Air (dry)	1.000 59	3
Bakelite	4.9	24
Fused quartz	3.78	8
Mylar	3.2	7
Neoprene rubber	6.7	12
Nylon	3.4	14
Paper	3.7	16
Paraffin-impregnated paper	3.5	11
Polystyrene	2.56	24
Polyvinyl chloride	3.4	40
Porcelain	6	12
Pyrex glass	5.6	14
Silicone oil	2.5	15
Strontium titanate	233	8
Teflon	2.1	60
Vacuum	1.000 00	—
Water	80	—

[a] The dielectric strength equals the maximum electric field that can exist in a dielectric without electrical breakdown. Note that these values depend strongly on the presence of impurities and flaws in the materials.

Capacitors with Dielectrics


Radio tuning capacitor



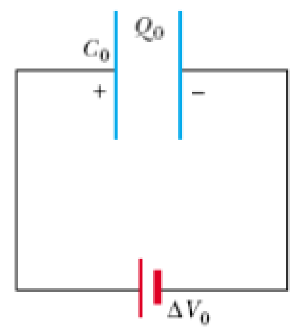
Commercial capacitors are often made from metallic foil interlaced with thin sheets of either paraffin-impregnated paper or Mylar as the dielectric material.

electrolytic capacitor is used to store large amounts of charge at relatively low voltages

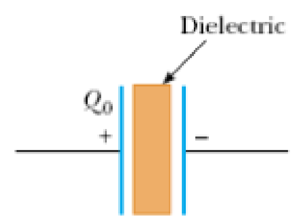
High-voltage capacitors commonly consist of a number of interwoven metallic plates immersed in silicone oil.
Small capacitors are often constructed from ceramic materials.

Example

A parallel-plate capacitor is charged with a battery to a charge Q_0, as shown in Figure a. The battery is then removed, and a slab of material that has a dielectric constant κ is inserted between the plates, as shown in Figure b. Find the energy stored in the capacitor before and after the dielectric is inserted.



(a)



(b)

$$U_0 = \boxed{\frac{Q_0^{\,2}}{2C_0}}$$

$$U = \frac{Q_0^{\,2}}{2C}$$

$$U = \frac{Q_0^{\,2}}{2\kappa C_0} = \boxed{\frac{U_0}{\kappa}}$$