

Ch 6: Current and Resistance

Ch 27 on textbook



Lecture Contents

- Introduction and Current Definition.
- Resistance definition.
- Resistance colour coding
- Resistance and temperature
- Superconductors
- Electric Power

Introduction

•So far, we have considered in our study in this module the electrostatic of charges or charges at equilibrium. Here after, we will involve non-equilibrium situations involving electric charges. The flow of charge, or more precisely the rate of flow of charge is termed *electric current* or simply *current*.

Current Definition

Suppose that charges are moving perpendicular to a surface of area A . (cross-sectional area of a wire, for example.) The current is the rate at which charge flows through this surface. If ΔQ is the amount of charge that passes through this area in a time interval Δt , the average current I_{av} is equal to the charge that passes through A per unit time: the unit of current is Ampere (A).

$$I_{av} = \frac{\Delta Q}{\Delta t}$$

$$I \equiv \frac{dQ}{dt}$$

Resistance

Consider a conductor of cross-sectional area A carrying a current I . The current density J in the conductor is defined as the current per unit area. Because the current $I = nqv_dA$, the current density is

$$J \equiv \frac{I}{A} = nqv_d$$

$$\mathbf{J} = \sigma \mathbf{E}$$

$$\Delta V = E\ell$$

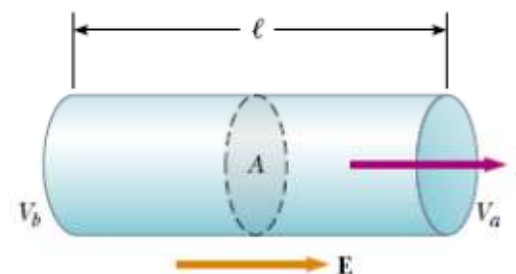
$$J = \sigma E = \sigma \frac{\Delta V}{\ell}$$

$$J = I/A$$

$$\Delta V = \frac{\ell}{\sigma} J = \left(\frac{\ell}{\sigma A} \right) I = RI$$

$$R \equiv \frac{\Delta V}{I}$$

$$R = \rho \frac{\ell}{A}$$



Resistivities and Temperature Coefficients of Resistivity for Various Materials

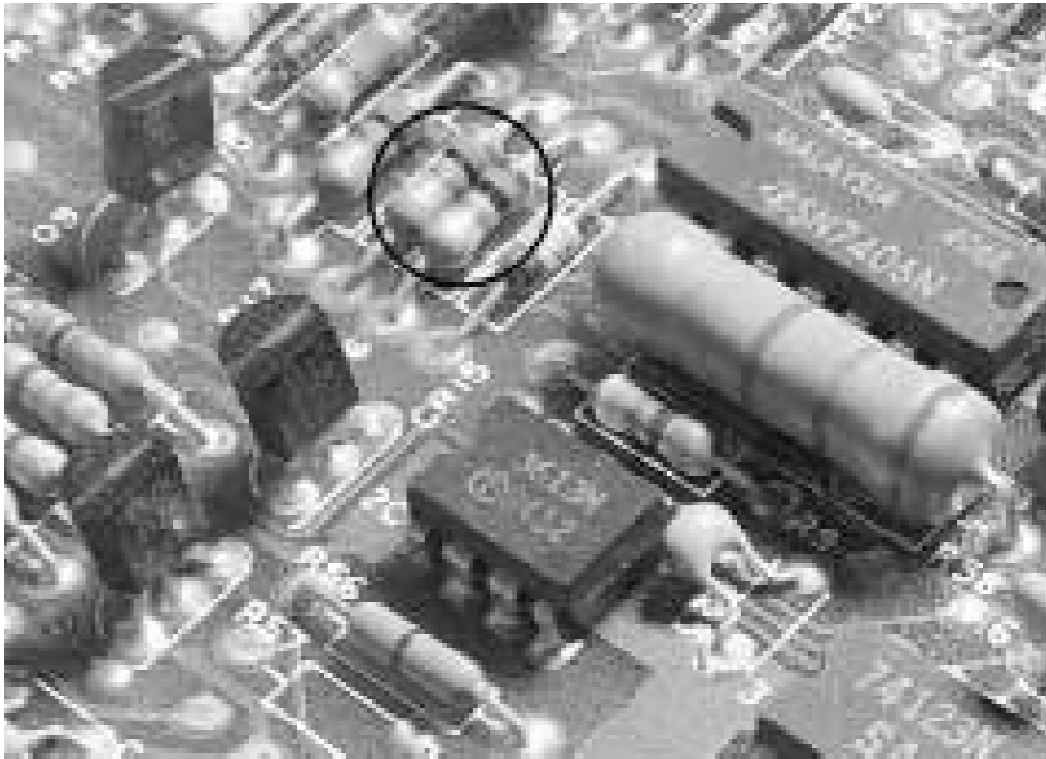
Material	Resistivity ^a ($\Omega \cdot \text{m}$)	Temperature Coefficient ^b $\alpha [(^{\circ}\text{C})^{-1}]$
Silver	1.59×10^{-8}	3.8×10^{-3}
Copper	1.7×10^{-8}	3.9×10^{-3}
Gold	2.44×10^{-8}	3.4×10^{-3}
Aluminum	2.82×10^{-8}	3.9×10^{-3}
Tungsten	5.6×10^{-8}	4.5×10^{-3}
Iron	10×10^{-8}	5.0×10^{-3}
Platinum	11×10^{-8}	3.92×10^{-3}
Lead	22×10^{-8}	3.9×10^{-3}
Nichrome ^c	1.50×10^{-6}	0.4×10^{-3}
Carbon	3.5×10^{-5}	-0.5×10^{-3}
Germanium	0.46	-48×10^{-3}
Silicon	640	-75×10^{-3}
Glass	10^{10} to 10^{14}	
Hard rubber	$\sim 10^{13}$	
Sulfur	10^{15}	
Quartz (fused)	75×10^{16}	



Resistance Colour coding system

The resistance are usually colour coded. That means the resistance has a colour bands printed on its surface to indicate and give its value. Sometimes, the colours are substituted by a numbers with the same functionality. Table 2 gives the values of the colour/number coding system for resistance.

The first two colours give the first two digits in the resistance value. The third colour represents the power of ten for the multiplier of the resistance value. The last colour is the tolerance of the resistance value. As an example, given the four colours on the resistors are red (=2), black (=0), orange ($=10^3$), and gold ($=5\%$), and so the resistance value is $20 * 10^3 \Omega = 20 \text{ k}\Omega$ with a tolerance value of $5\% = 1 \text{ k}\Omega$.



Color Coding for Resistors

Color	Number	Multiplier	Tolerance
Black	0	1	
Brown	1	10^1	
Red	2	10^2	
Orange	3	10^3	
Yellow	4	10^4	
Green	5	10^5	
Blue	6	10^6	
Violet	7	10^7	
Gray	8	10^8	
White	9	10^9	
Gold		10^{-1}	5%
Silver		10^{-2}	10%
Gold			20%

Example 1

Calculate the resistance of an aluminium cylinder (resistivity = 2.82×10^{-8}) that has a length of 10.0 cm and a cross-sectional area of $2.00 \times 10^{-4} \text{ m}^2$. Repeat the calculation for a cylinder of the same dimensions and made of glass having a resistivity of $3.0 \times 10^{10} \Omega\cdot\text{m}$.

$$\begin{aligned} R &= \rho \frac{\ell}{A} \\ &= (2.82 \times 10^{-8} \Omega\cdot\text{m}) \left(\frac{0.100 \text{ m}}{2.00 \times 10^{-4} \text{ m}^2} \right) \\ &= 1.41 \times 10^{-5} \Omega \end{aligned}$$

$$\begin{aligned} R &= \rho \frac{\ell}{A} = (3.0 \times 10^{10} \Omega\cdot\text{m}) \left(\frac{0.100 \text{ m}}{2.00 \times 10^{-4} \text{ m}^2} \right) \\ &= 1.5 \times 10^{13} \Omega \end{aligned}$$

Example 2

(A) Calculate the resistance per unit length of a 22-gauge Nichrome wire, which has a radius of 0.321 mm. The resistivity of Nichrome = $1.5 \times 10^{-6} \Omega \cdot \text{m}$.

(B) If a potential difference of 10 V is maintained across a 1.0-m length of the Nichrome wire, what is the current in the wire?

A

$$R = \rho \frac{\ell}{A}$$

$$A = \pi r^2 = \pi (0.321 \times 10^{-3} \text{ m})^2 = 3.24 \times 10^{-7} \text{ m}^2$$

$$\frac{R}{\ell} = \frac{\rho}{A} = \frac{1.5 \times 10^{-6} \Omega \cdot \text{m}}{3.24 \times 10^{-7} \text{ m}^2} = 4.6 \Omega/\text{m}$$

B

$$I = \frac{\Delta V}{R} = \frac{10 \text{ V}}{4.6 \Omega} = 2.2 \text{ A}$$

Resistance and Temperature

Over a limited range of temperature, the resistivity of material varies linearly with temperature.

$$\rho = \rho_0[1 + \alpha(T - T_0)]$$

Where ρ is the resistivity at some temperature, ρ_0 is the resistivity at some reference temperature ($T_0 = 20^\circ\text{C}$), α is the temperature coefficient of resistivity which can be expressed as:

$$\alpha = \frac{1}{\rho_0} \frac{\Delta\rho}{\Delta T}$$

Also the resistance can be related according to the same relation as

$$R = R_0[1 + \alpha(T - T_0)]$$

Example 3

A resistance thermometer, which measures temperature by measuring the change in resistance of a conductor, is made from platinum and has a resistance of $50.0\ \Omega$ at 20.0°C . When immersed in a vessel containing melting indium, its resistance increases to $76.8\ \Omega$. Calculate the melting point of the indium. Given the temperature coefficient of resistivity of platinum = $3.92 \times 10^{-3}\ \text{C}^{-1}$.

$$\Delta T = \frac{R - R_0}{\alpha R_0} = \frac{76.8\ \Omega - 50.0\ \Omega}{[3.92 \times 10^{-3}(\text{C})^{-1}](50.0\ \Omega)} = 137^\circ\text{C}$$

Then $T = 20 + 137 = 157^\circ\text{C}$

Superconductors

is a class of metals and compounds whose resistance decreases to zero when they are below a certain temperature T_c , known as the critical temperature.

The resistance–temperature graph for a superconductor follows that of a normal metal at temperatures above T_c the resistivities of superconductors below their T_c values are less than $4 \times 10^{-25} \Omega \cdot m$ —around 10^{17} times smaller than the resistivity of copper and in practice considered to be zero.

Two kinds of superconductors:

Ceramics (recently identified) with high critical temperatures,

Metal superconductors.

If a room-temperature superconductor is ever identified, its impact on technology could be tremendous.

The value of T_c is sensitive to chemical composition, pressure, and molecular structure. It is interesting to note that copper, silver, and gold, which are excellent conductors, do not exhibit superconductivity.

Superconductors

Critical Temperatures for Various Superconductors	
Material	T_c (K)
HgBa ₂ Ca ₂ Cu ₃ O ₈	134
Tl-Ba-Ca-Cu-O	125
Bi-Sr-Ca-Cu-O	105
YBa ₂ Cu ₃ O ₇	92
Nb ₃ Ge	23.2
Nb ₃ Sn	18.05
Nb	9.46
Pb	7.18
Hg	4.15
Sn	3.72
Al	1.19
Zn	0.88

Electrical Power of a Resistance

The electric power (P) can be calculated as

$$\mathcal{P} = I \Delta V$$

Where I is the current passing through the resistor, and ΔV is the voltage drop across the resistor. R is the value of the resistance.

$$\mathcal{P} = I^2 R = \frac{(\Delta V)^2}{R}$$

Electrical Power of a Resistance

An electric heater is constructed by applying a potential difference of 120 V to a Nichrome wire that has a total resistance of $8.00\ \Omega$. Find the current carried by the wire and the power rating of the heater.

Solution Because $\Delta V = IR$, we have

$$I = \frac{\Delta V}{R} = \frac{120\ \text{V}}{8.00\ \Omega} = 15.0\ \text{A}$$

We can find the power rating using the expression $\mathcal{P} = I^2 R$:

$$\mathcal{P} = I^2 R = (15.0\ \text{A})^2 (8.00\ \Omega) = 1.80 \times 10^3\ \text{W}$$