Direct Current (DC) Circuits

Lecture Contents

- Electromotive Force (emf "$\mathcal{E}$").

- Combination of resistors.

- Kirchhoff's Rules.

Introduction

•Analysis of simple circuits (batteries, resistors, and capacitors).

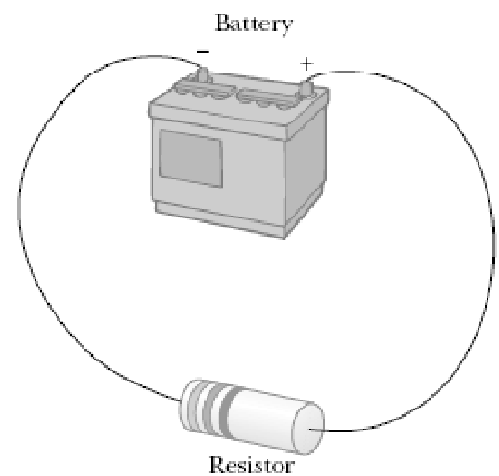
•Complicated circuits will be simplified using Kirchhoff's Rules.

•The current in this chapter is assumed to be in steady state condition (const in magnitude and direction) and it called ***direct current*** (DC).

•Alternating current will be investigated in a later chapter. This current changes its direction periodically.

Electromotive Force (emf "ε")

- Will use a battery in a closed circuit.
- Battery produced Current (const in mag and dir ➔ DC)
- The battery is called a source of electromotive force, or source of emf.
- emf is called a force but it is actually a potential difference that cause electrical force.
- emf is the max possible voltage that the battery can provide across its terminals.



Electromotive Force (emf "ε")

•Zero resistance for the wires.

•The +ve terminal has a higher voltage than the –ve terminal.

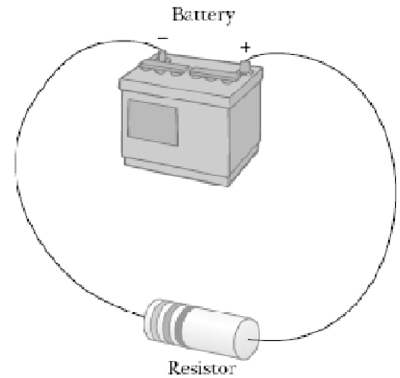
$$\Delta V = \mathcal{E} - Ir$$

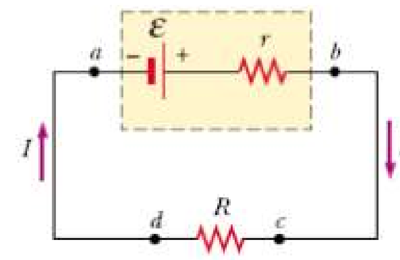
•Resistance of the battery due to its matter contents ➔ called internal resistance (*r*).

•*ε* is equal to the open circuit voltage (i.e. when the current is zero).

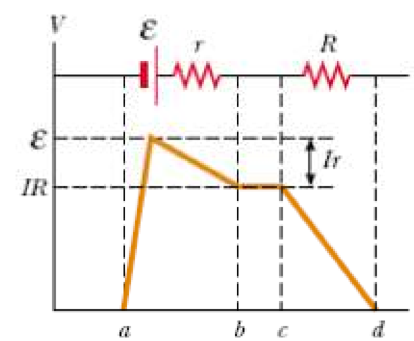
•Terminal voltage must equal to the voltage difference across the external resistance (Load resistance).

$$\Delta V = IR$$



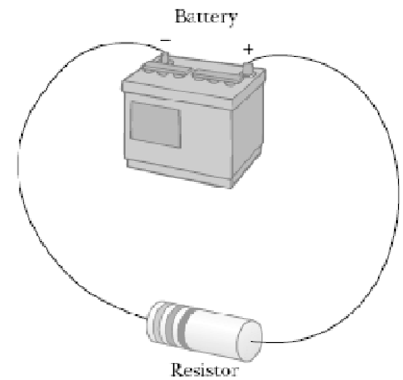


(a)



(b)

Electromotive Force (emf "ε")

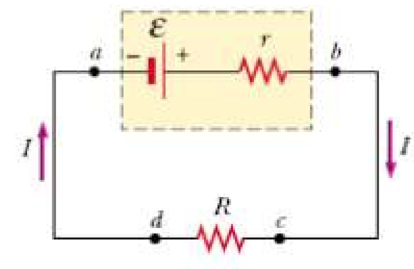


- Combining the two potential difference equations gives

$$\Delta V = \mathcal{E} - Ir \qquad \Delta V = IR \quad \Longrightarrow \quad \mathcal{E} = IR + Ir$$

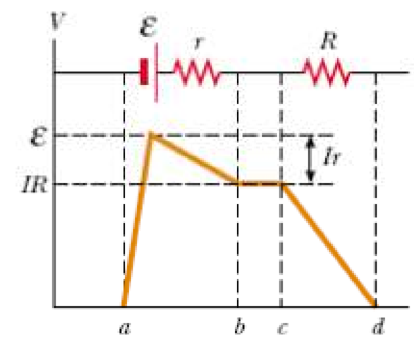
- Or the current (*I*) can be calculated as

$$I = \frac{\mathcal{E}}{R + r}$$



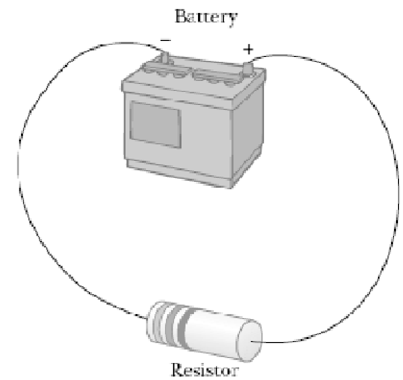
(a)

- In many real life applications, R is much greater than r, then we can neglect r. (*But if its value mentioned we must use it*).



(b)

Electromotive Force (emf “ε”)



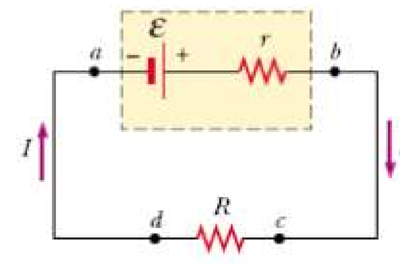
- If we multiply I by

$$\mathcal{E} = IR + Ir$$

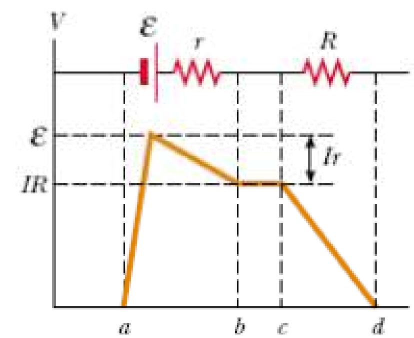
- It gives,

$$I\mathcal{E} = I^2R + I^2r$$

- $P = I\ \Delta V$, then the total power output ($I\varepsilon$) of the battery is delivered to the external load resistance and internal resistance.

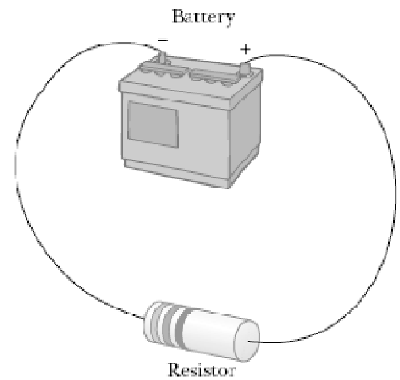


(a)

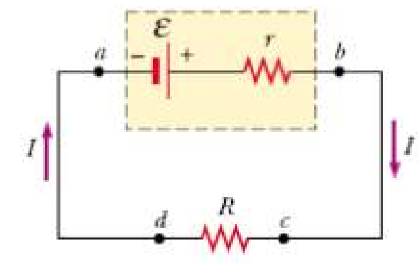


(b)

Quiz

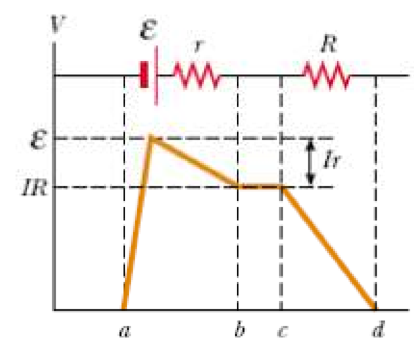


In order to maximise the percentage of the power that is delivered from a battery to a device, the internal resistance of the battery should be
(a) as low as possible
(b) as high as possible
(c) the percentage does not depend on the internal resistance.



(a)

$$I\mathcal{E} = I^2R + I^2r$$



(b)

Example 1

A battery has an emf of 12.0 V and an internal resistance of 0.05 Ω. Its terminals are connected to a load resistance of 3.00 Ω.

(A) Find the current in the circuit and the terminal voltage of the battery.

(B) Calculate the power delivered to the load resistor, the power delivered to the internal resistance of the battery, and the power delivered by the battery.

$$I = \frac{\mathcal{E}}{R + r}$$

$$= \frac{12.0\text{ V}}{3.05\ \Omega} = \boxed{3.93\text{ A}}$$

$$\Delta V = \mathcal{E} - Ir = 12.0\text{ V} - (3.93\text{ A})(0.05\ \Omega) = \boxed{11.8\text{ V}}$$

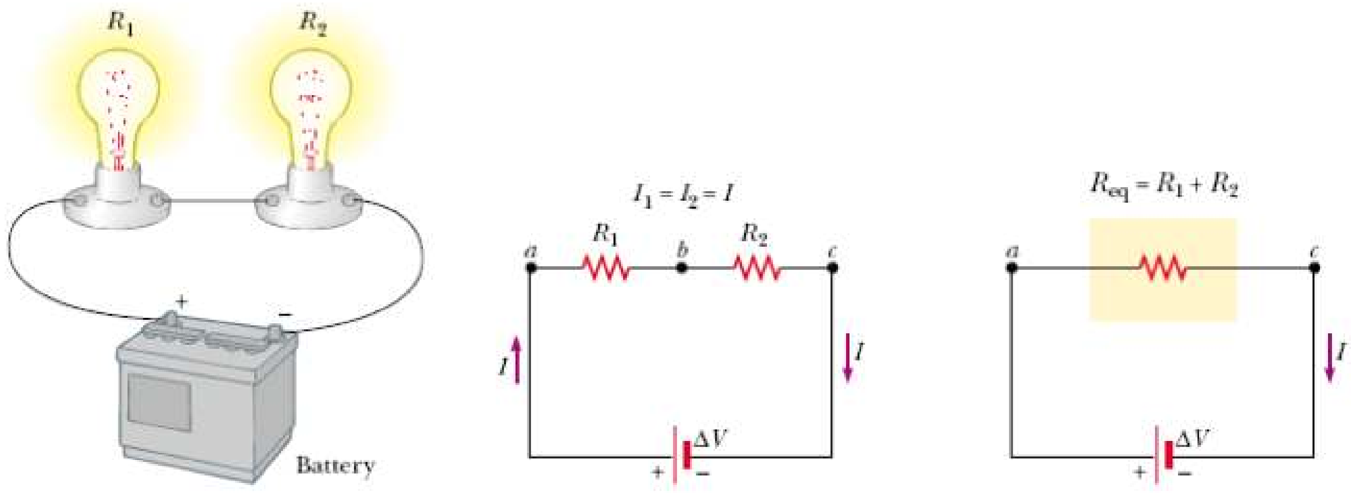
$$\mathcal{P}_R = I^2R = (3.93\text{ A})^2\,(3.00\ \Omega) = \boxed{46.3\text{ W}}$$

$$\mathcal{P}_r = I^2r = (3.93\text{ A})^2\,(0.05\ \Omega) = \boxed{0.772\text{ W}}$$

$$\mathcal{P} = I\mathcal{E} \Longrightarrow \mathcal{P}_R + \mathcal{P}_r = 47.072\text{ W}$$

Combination of Resistors

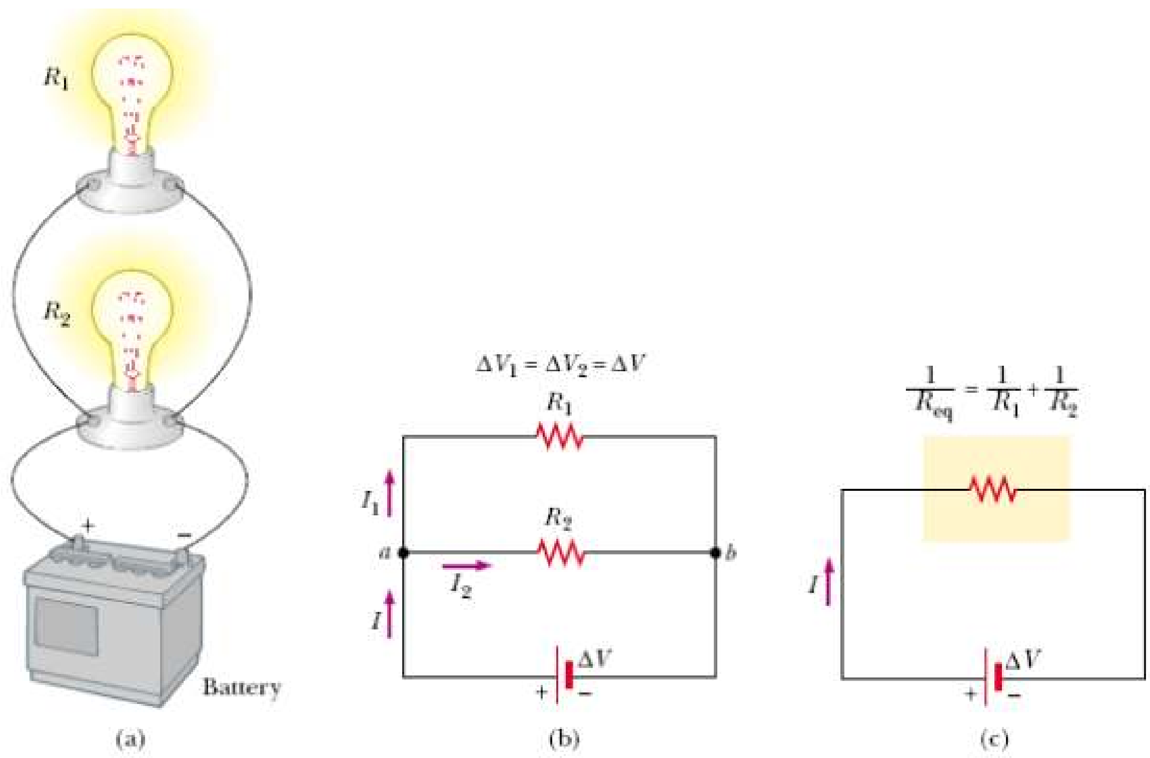
•Resistors in series:



(a) (b) (c)

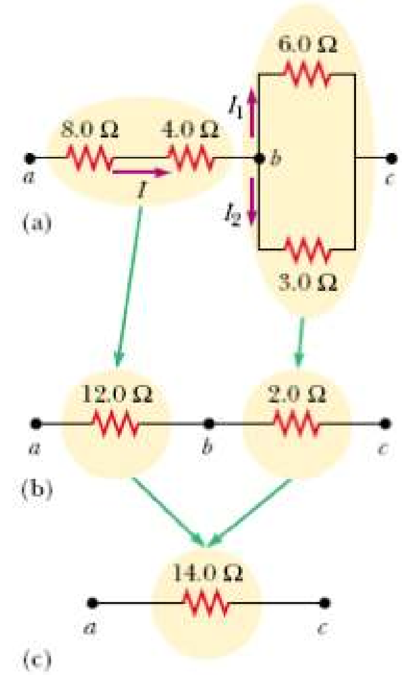
Combination of Resistors

- Resistors in parallel:



Example 2

Find the equivalent resistance between a and b. what is the current in each resistor if a potential difference of 42 V is maintained between a and c.



$$I = \frac{\Delta V_{ac}}{R_{eq}} = \frac{42\ \text{V}}{14.0\ \Omega} = 3.0\ \text{A}$$

The above current is passing through the 8 and 4 Ω resistors. This current will split when it reach the junction at b. $\Delta V_{bc} = I_1 * 6 = I_2 * 3$ or

$I_2 = 2 * I_1$, $I_1 + I_2 = I$, this gives $I_1 = 1$ A which passes through the resistance 6Ω, $I_2 = 2$ A which passes through the resistance 3Ω.

Example 3

Three resistors are connected in parallel as shown in Figure a. A potential difference of 18.0 V is maintained between points *a* and *b*.

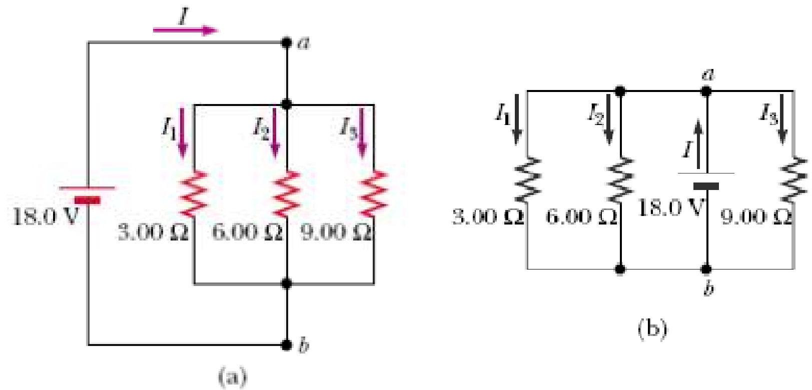
(A) Find the current in each resistor.

(B) Calculate the power delivered to each resistor and the total power delivered to the combination of resistors.

$$I_1 = \frac{\Delta V}{R_1} = \frac{18.0\ \text{V}}{3.00\ \Omega} = \boxed{6.00\ \text{A}}$$

$$I_2 = \frac{\Delta V}{R_2} = \frac{18.0\ \text{V}}{6.00\ \Omega} = \boxed{3.00\ \text{A}}$$

$$I_3 = \frac{\Delta V}{R_3} = \frac{18.0\ \text{V}}{9.00\ \Omega} = \boxed{2.00\ \text{A}}$$



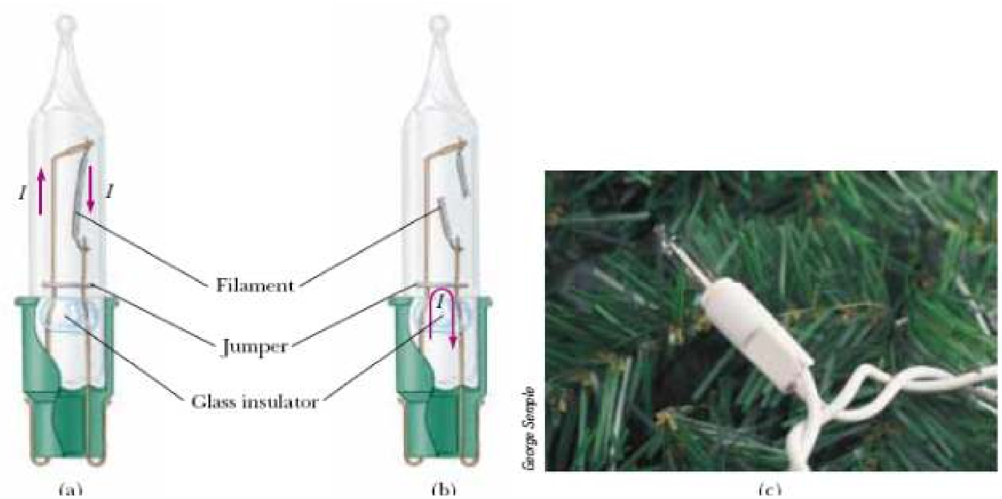
3.00-Ω: $\mathcal{P}_1 = I_1^2 R_1 = (6.00\ \text{A})^2(3.00\ \Omega) = \boxed{108\ \text{W}}$

6.00-Ω: $\mathcal{P}_2 = I_2^2 R_2 = (3.00\ \text{A})^2(6.00\ \Omega) = \boxed{54.0\ \text{W}}$

9.00-Ω: $\mathcal{P}_3 = I_3^2 R_3 = (2.00\ \text{A})^2(9.00\ \Omega) = \boxed{36.0\ \text{W}}$

String of Lights

Both parallel and series connections have been used for strings of lights. Series-wired bulbs are safer than parallel-wired bulbs for indoor use because series-wired bulbs operate with less energy per bulb and at a lower temperature. However, filament of a single bulb fails (or if the bulb is removed from its socket), all the lights on the string go out. the bulbs are brighter and hotter than those on a series-wired string. As a result, these bulbs are inherently more dangerous (more likely to start a fire, for instance), but if one bulb in a parallel-wired string fails or is removed, the rest of the bulbs continue to glow.



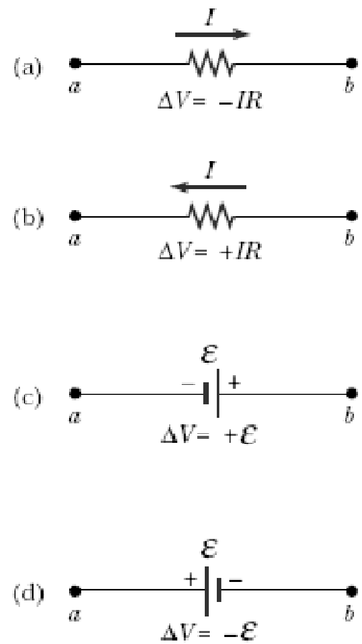
Kirchhoff’s Rules

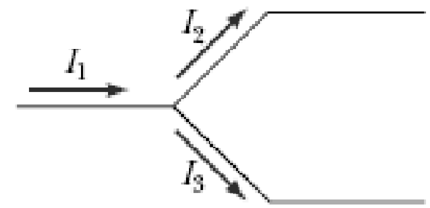
Two rules of Kirchhoff are greatly simplify the complexity of some circuits.

Junction Rule, Loop Rule.

$$\sum I_{\text{in}} = \sum I_{\text{out}}$$

$$\sum_{\substack{\text{closed}\\ \text{loop}}} \Delta V = 0$$



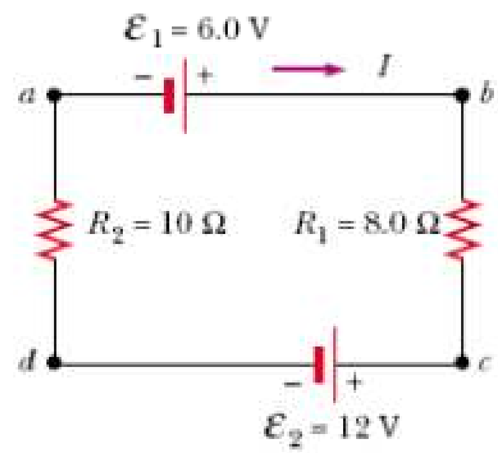


$$I_1 = I_2 + I_3$$

The following examples illustrate how to use Kirchhoff’s rules. In all cases, it is assumed that the circuits have reached steady-state conditions—that is, the currents in the various branches are constant. Any capacitor acts as an open branch in a circuit; that is, the current in the branch containing the capacitor is zero under steady-state conditions.

Example 4

A single-loop circuit contains two resistors and two batteries, as shown in the Figure. (Neglect the internal resistances of the batteries.) (A) find the current in the circuit, (B) What power to each resistor, and what power is delivered by the 12 V battery.



$$\sum \Delta V = 0$$

A

$$\mathcal{E}_1 - IR_1 - \mathcal{E}_2 - IR_2 = 0$$

$$I = \frac{\mathcal{E}_1 - \mathcal{E}_2}{R_1 + R_2} = \frac{6.0\text{ V} - 12\text{ V}}{8.0\ \Omega + 10\ \Omega} = \boxed{-0.33\text{ A}}$$

B

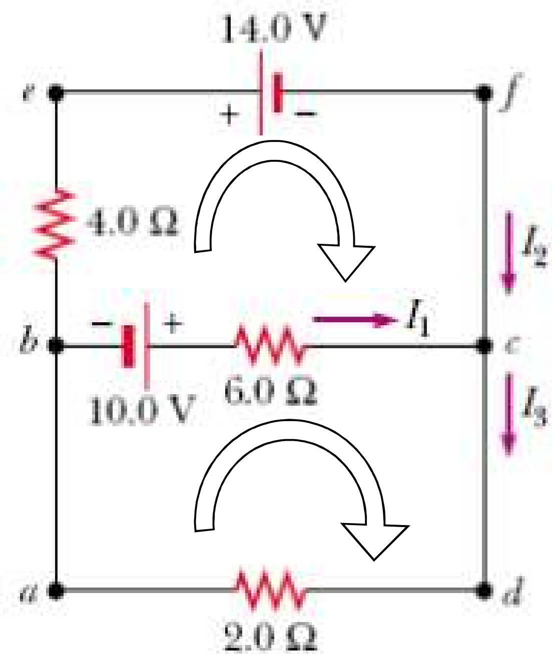
$$\mathcal{P}_1 = I^2 R_1 = (0.33\text{ A})^2(8.0\ \Omega) = \boxed{0.87\text{ W}}$$

$$\mathcal{P}_2 = I^2 R_2 = (0.33\text{ A})^2(10\ \Omega) = \boxed{1.1\text{ W}}$$

$$I\mathcal{E}_2 = 4.0\text{ W.}$$

Example 5

Find the Currents I_1 , I_2 , I_3 in the circuit below.



(1) $\quad I_1 + I_2 = I_3$

(2) $\quad$ *abcda* $\quad 10.0\text{ V} - (6.0\ \Omega)I_1 - (2.0\ \Omega)I_3 = 0$

(3) *befcb* $\quad -14.0\text{ V} + (6.0\ \Omega)I_1 - 10.0\text{ V} - (4.0\ \Omega)\ I_2 = 0$

Substituting Equation (1) into Equation (2) gives

$$10.0\text{ V} - (6.0\ \Omega)I_1 - (2.0\ \Omega)\ (I_1 + I_2) = 0$$

(4) $\quad 10.0\text{ V} = (8.0\ \Omega)I_1 + (2.0\ \Omega)I_2$

Dividing each term in Equation (3) by 2 and rearranging gives

(5) $\quad -12.0\text{ V} = -(3.0\ \Omega)I_1 + (2.0\ \Omega)I_2$

Subtracting Equation (5) from Equation (4) eliminates $I2$, giving

$$22.0\text{ V} = (11.0\ \Omega)I_1$$

$$I_1 = \boxed{2.0\text{ A}}$$

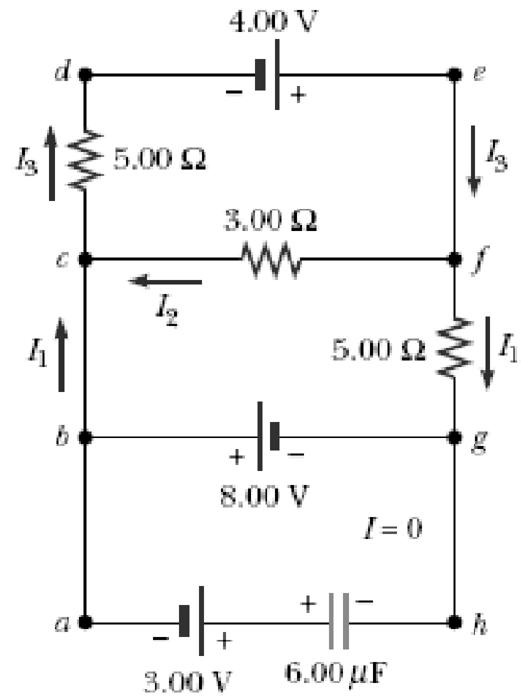
$$(2.0\ \Omega)I_2 = (3.0\ \Omega)I_1 - 12.0\text{ V}$$
$$= (3.0\ \Omega)(2.0\text{ A}) - 12.0\text{ V} = -6.0\text{ V}$$

$$I_2 = \boxed{-3.0\text{ A}}$$

$$I_3 = I_1 + I_2 = \boxed{-1.0\text{ A}}$$

Example 6

(A) Under steady-state conditions, find the unknown currents I_1, I_2, and I_3 in the multi-loop circuit shown in the Figure. (B) What is the charge on the capacitor.



(1) $I_1 + I_2 = I_3$

(2) *defcd* $4.00\ \text{V} - (3.00\ \Omega)I_2 - (5.00\ \Omega)I_3 = 0$

(3) *cfgbc* $(3.00\ \Omega)I_2 - (5.00\ \Omega)I_1 + 8.00\ \text{V} = 0$

From Equation (1) we see that $I_1 = I_3 - I_2$, which, when substituted into Equation (3), gives

(4) $(8.00\ \Omega)I_2 - (5.00\ \Omega)I_3 + 8.00\ \text{V} = 0$

Subtracting Equation (4) from Equation (2), we eliminate I_3 and find that

$$I_2 = -\frac{4.00\ \text{V}}{11.0\ \Omega} = \boxed{-0.364\ \text{A}}$$

Using I_2 = - 0.364 A in Equations (3) and (1) gives

$I_1 = \boxed{1.38\ \text{A}}$ $\quad I_3 = \boxed{1.02\ \text{A}}$

$$-8.00\ \text{V} + \Delta V_{\text{cap}} - 3.00\ \text{V} = 0$$

$$\Delta V_{\text{cap}} = 11.0\ \text{V}$$

$$Q = C\ \Delta V_{\text{cap}}$$

$$Q = (6.00\ \mu\text{F})(11.0\ \text{V}) = \boxed{66.0\ \mu\text{C}}$$