

The background of the slide features a close-up, slightly blurred image of a clock face with Roman numerals. Overlaid on the clock is a dark, coiled spiral spring, likely from a mechanical watch movement. The overall color palette is warm, with shades of orange, yellow, and brown.

10

PARAMETRIC EQUATIONS AND POLAR COORDINATES

PARAMETRIC EQUATIONS & POLAR COORDINATES

A coordinate system represents a point in the plane by an ordered pair of numbers called coordinates.

PARAMETRIC EQUATIONS & POLAR COORDINATES

Usually, we use Cartesian coordinates, which are directed distances from two perpendicular axes.

PARAMETRIC EQUATIONS & POLAR COORDINATES

Here, we describe a coordinate system introduced by Newton, called the polar coordinate system.

- It is more convenient for many purposes.

10.3

Polar Coordinates

In this section, we will learn:

How to represent points in polar coordinates.

POLE

We choose a point in the plane that is called the pole (or origin) and is labeled O .

POLAR AXIS

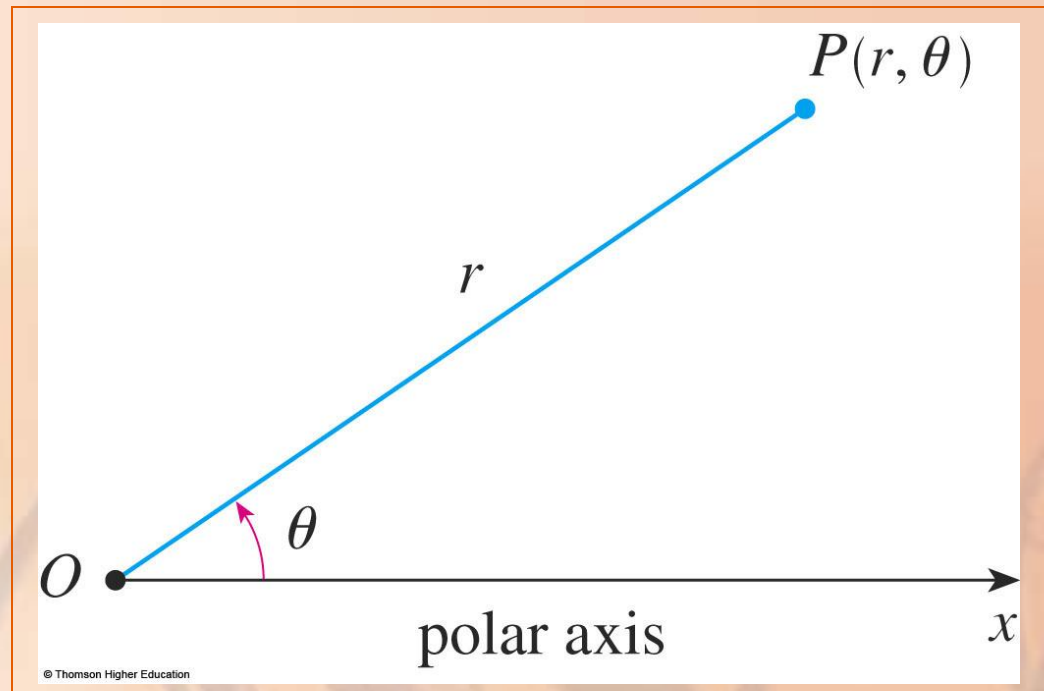
Then, we draw a ray (half-line) starting at O called the polar axis.

- This axis is usually drawn horizontally to the right corresponding to the positive x -axis in Cartesian coordinates.

ANOTHER POINT

If P is any other point in the plane, let:

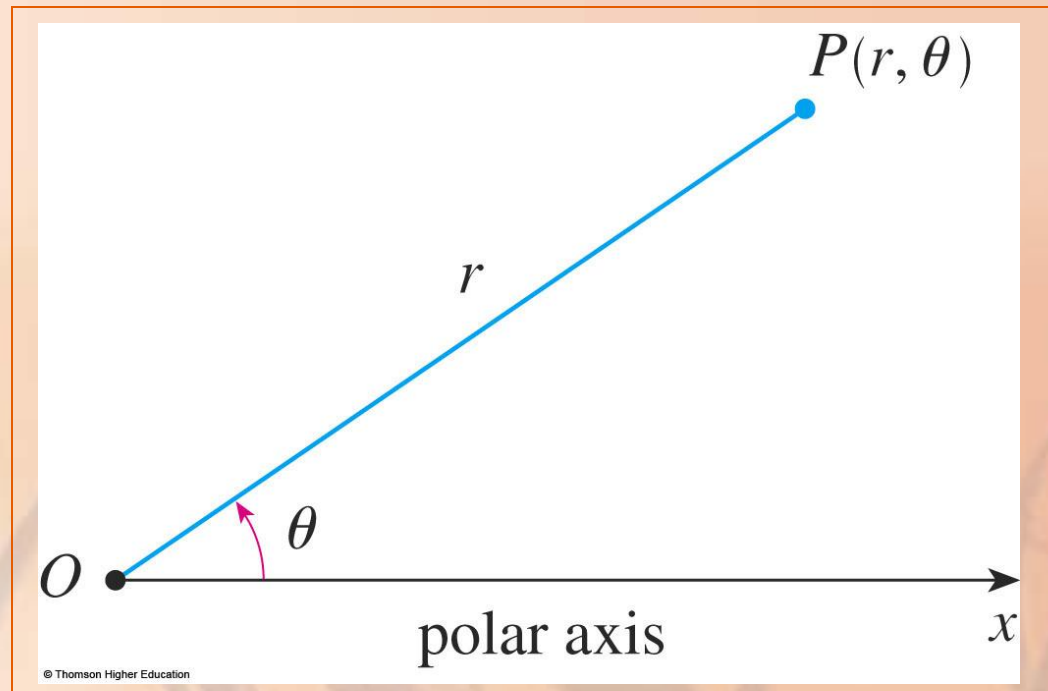
- r be the distance from O to P .
- θ be the angle (usually measured in radians) between the polar axis and the line OP .



POLAR COORDINATES

P is represented by the ordered pair (r, θ) .

r, θ are called polar coordinates of P .



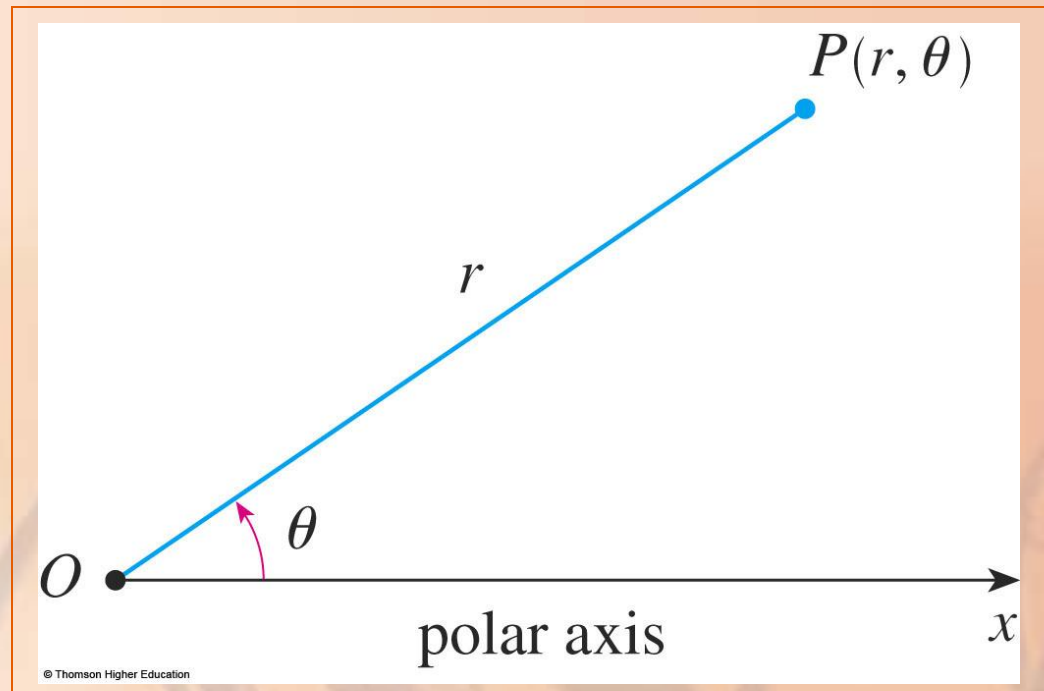
POLAR COORDINATES

We use the convention that an angle is:

- Positive—if measured in the counterclockwise direction from the polar axis.
- Negative—if measured in the clockwise direction from the polar axis.

POLAR COORDINATES

If $P = O$, then $r = 0$, and we agree that $(0, \theta)$ represents the pole for any value of θ .

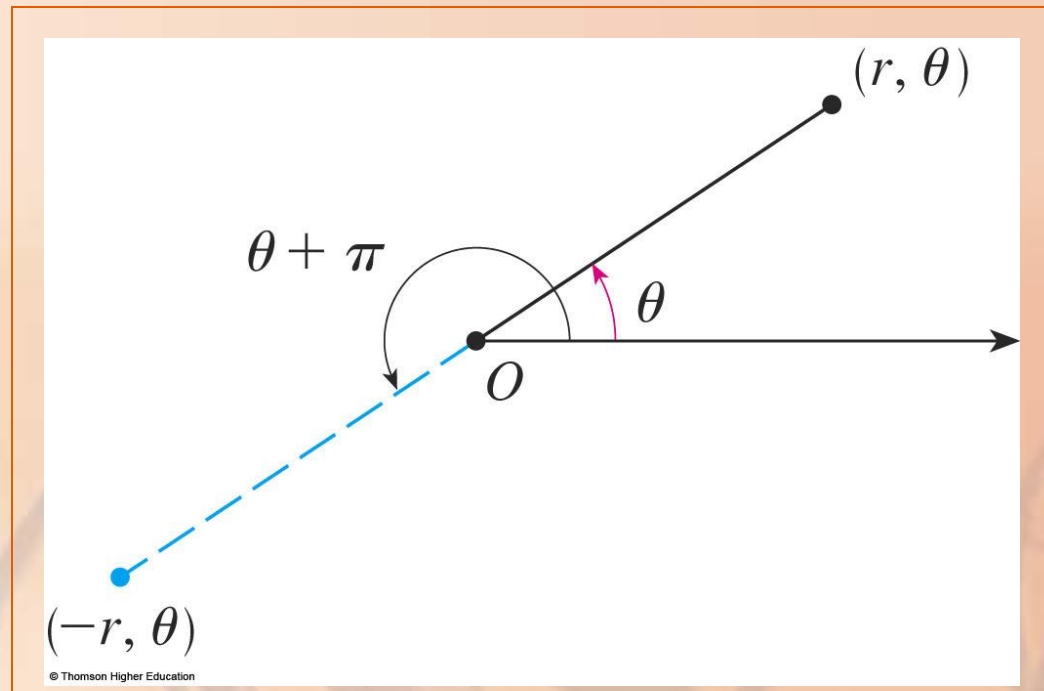


POLAR COORDINATES

We extend the meaning of polar coordinates (r, θ) to the case in which r is negative—as follows.

POLAR COORDINATES

We agree that, as shown, the points $(-r, \theta)$ and (r, θ) lie on the same line through O and at the same distance $|r|$ from O , but on opposite sides of O .

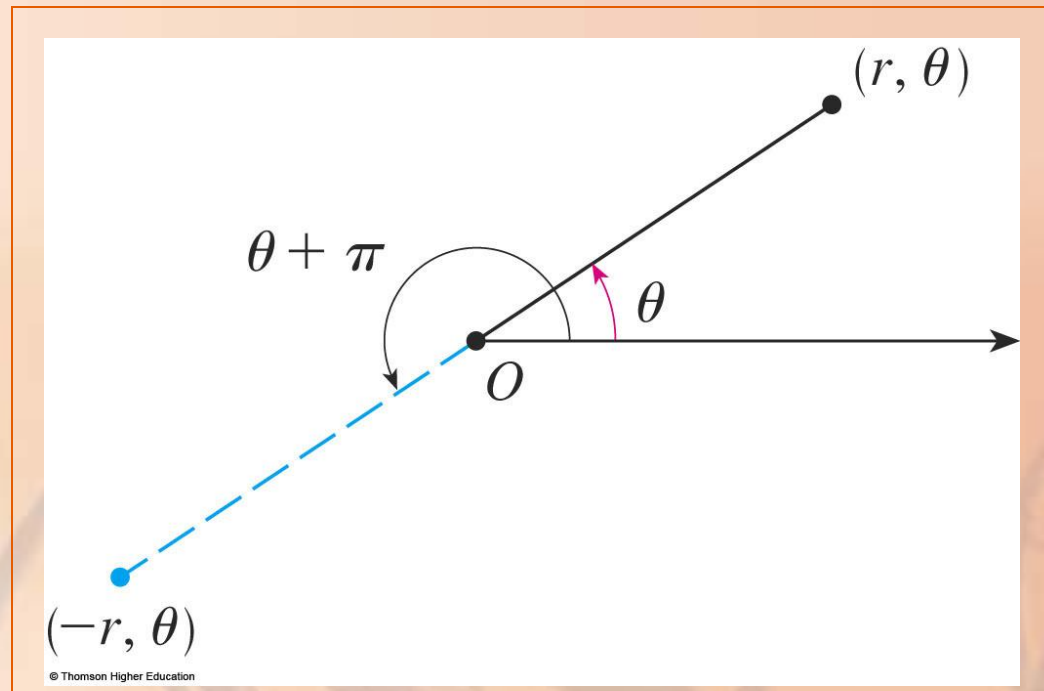


POLAR COORDINATES

If $r > 0$, the point (r, θ) lies in the same quadrant as θ .

If $r < 0$, it lies in the quadrant on the opposite side of the pole.

- Notice that $(-r, \theta)$ represents the same point as $(r, \theta + \pi)$.



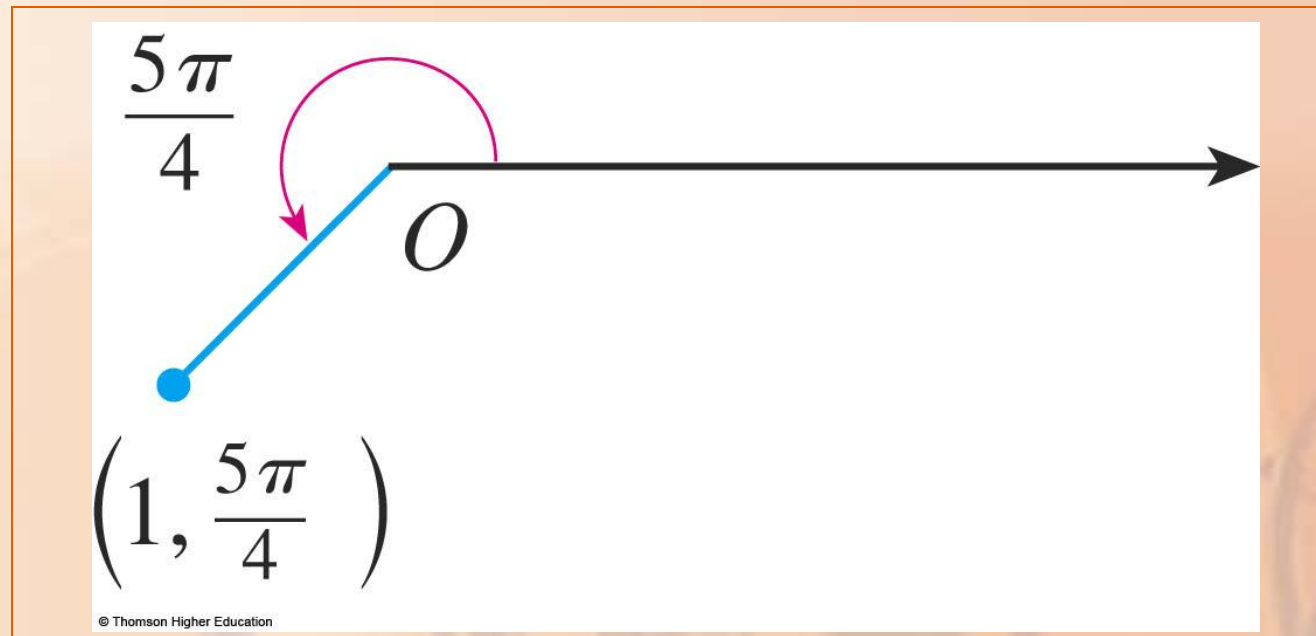
Plot the points whose polar coordinates are given.

- a. $(1, 5\pi/4)$
- b. $(2, 3\pi)$
- c. $(2, -2\pi/3)$
- d. $(-3, 3\pi/4)$

POLAR COORDINATES

Example 1 a

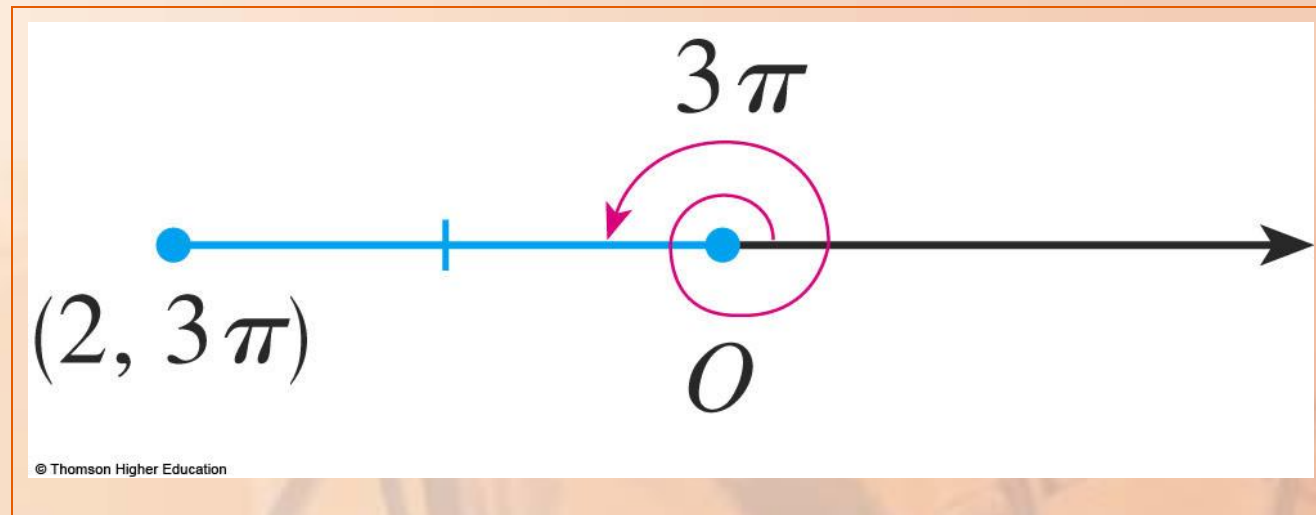
The point $(1, 5\pi/4)$ is plotted here.



POLAR COORDINATES

Example 1 b

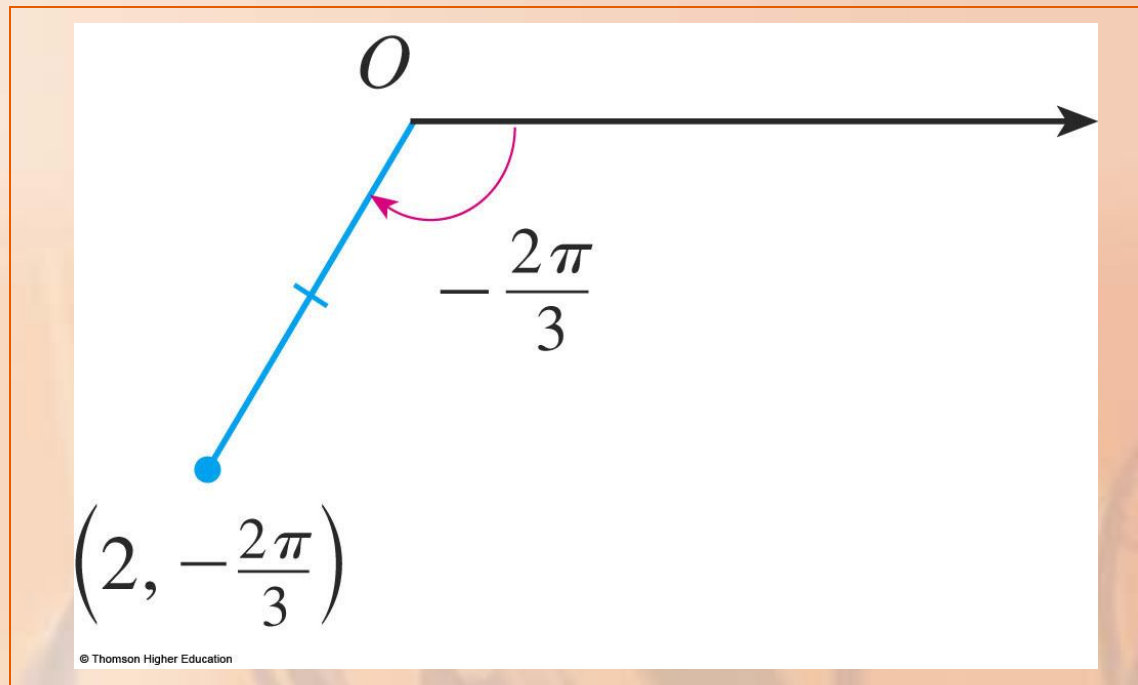
The point $(2, 3\pi)$ is plotted.



POLAR COORDINATES

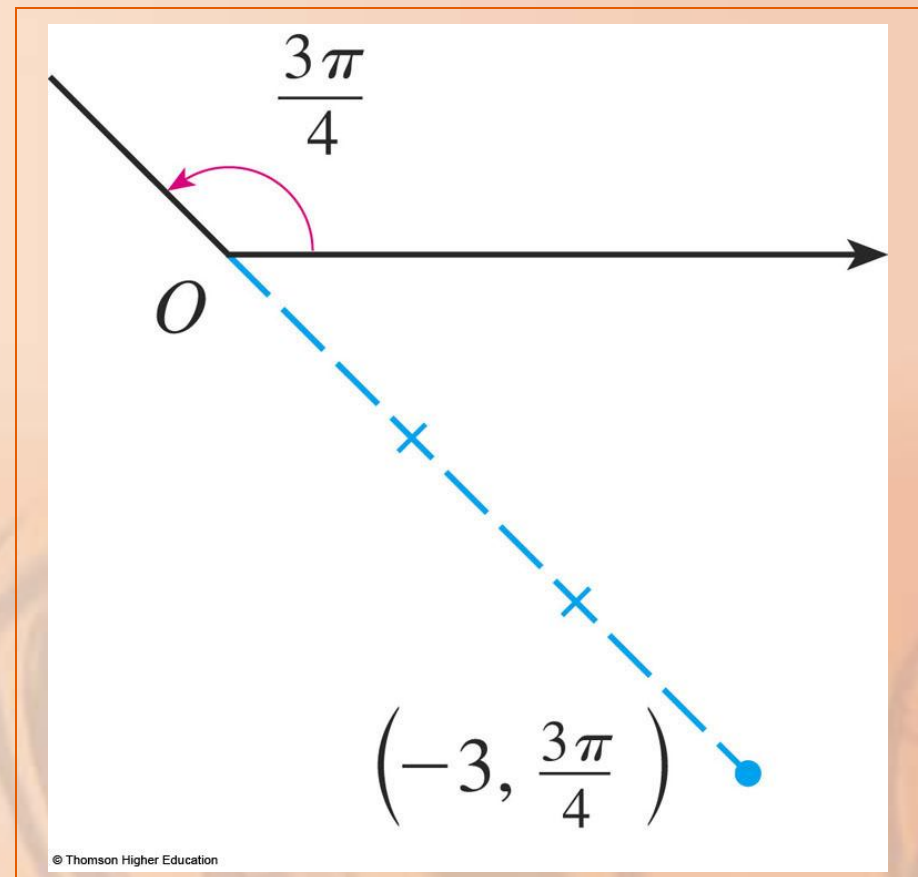
Example 1 c

The point $(2, -2\pi/3)$ is plotted.



The point $(-3, 3\pi/4)$ is plotted.

- It is located three units from the pole in the fourth quadrant.
- This is because the angle $3\pi/4$ is in the second quadrant and $r = -3$ is negative.



CARTESIAN VS. POLAR COORDINATES

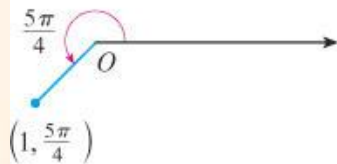
In the Cartesian coordinate system, every point has only one representation.

However, in the polar coordinate system, each point has many representations.

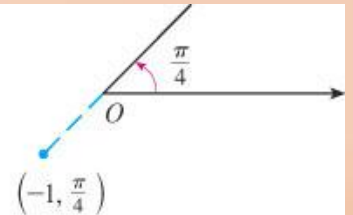
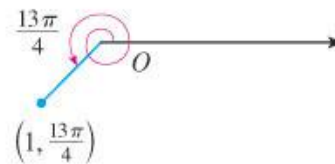
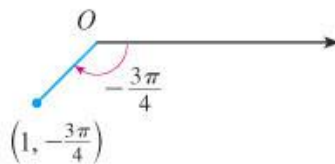
CARTESIAN VS. POLAR COORDINATES

For instance, the point $(1, 5\pi/4)$ in Example 1 a could be written as:

- $(1, -3\pi/4)$, $(1, 13\pi/4)$, or $(-1, \pi/4)$.



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CARTESIAN & POLAR COORDINATES

In fact, as a complete counterclockwise rotation is given by an angle 2π , the point represented by polar coordinates (r, θ) is also represented by

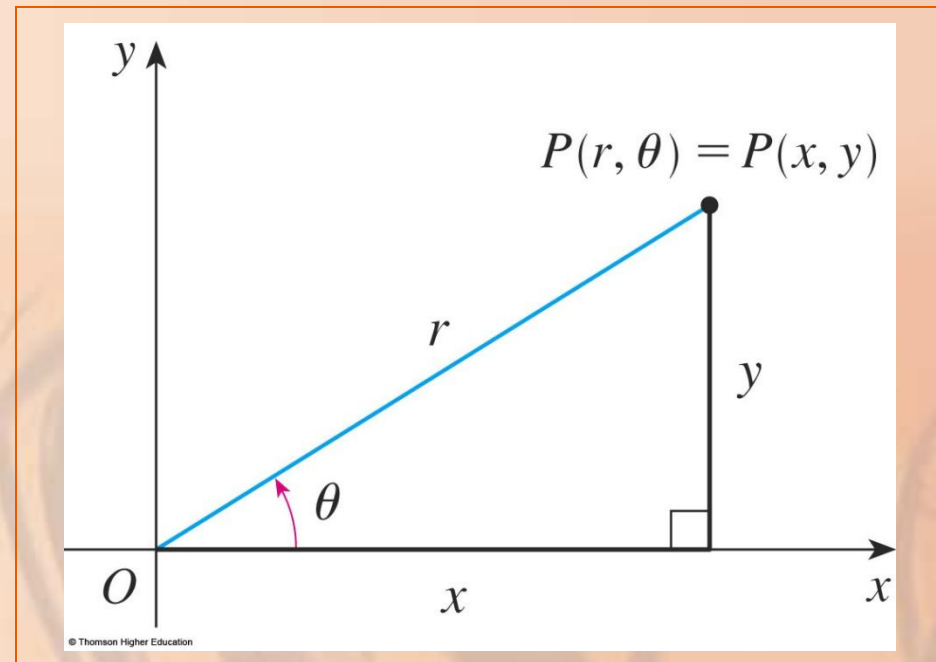
$$(r, \theta + 2n\pi) \quad \text{and} \quad (-r, \theta + (2n + 1)\pi)$$

where n is any integer.

CARTESIAN & POLAR COORDINATES

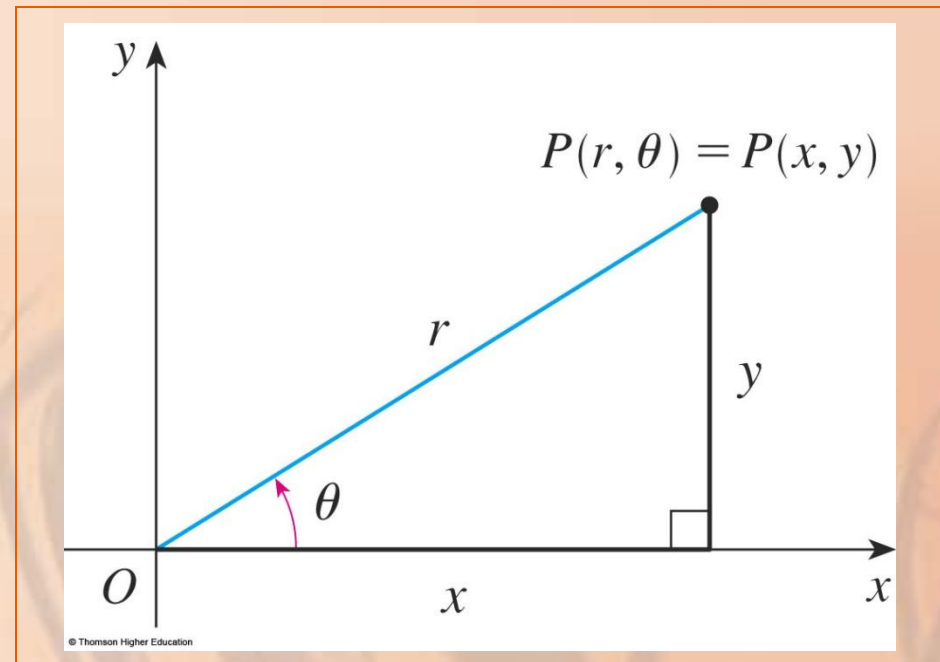
The connection between polar and Cartesian coordinates can be seen here.

- The pole corresponds to the origin.
- The polar axis coincides with the positive x -axis.



CARTESIAN & POLAR COORDINATES

If the point P has Cartesian coordinates (x, y) and polar coordinates (r, θ) , then, from the figure, we have:

$$\cos \theta = \frac{x}{r} \qquad \sin \theta = \frac{y}{r}$$


Therefore,

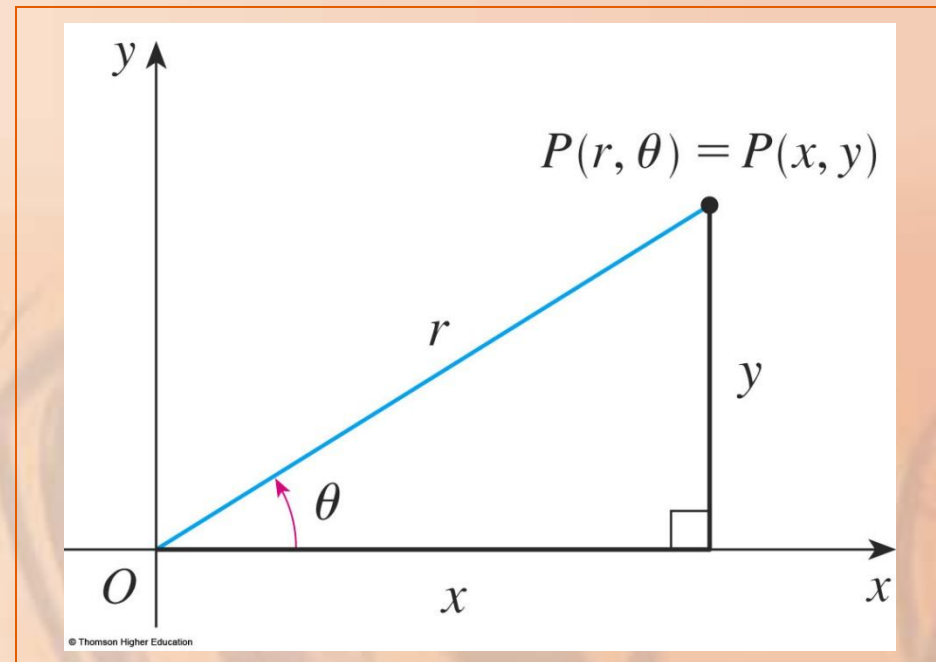
$$x = r \cos \theta$$

$$y = r \sin \theta$$

CARTESIAN & POLAR COORDS.

Although Equations 1 were deduced from the figure (which illustrates the case where $r > 0$ and $0 < \theta < \pi/2$), these equations are valid for all values of r and θ .

- See the general definition of $\sin \theta$ and $\cos \theta$ in Appendix D.



CARTESIAN & POLAR COORDS.

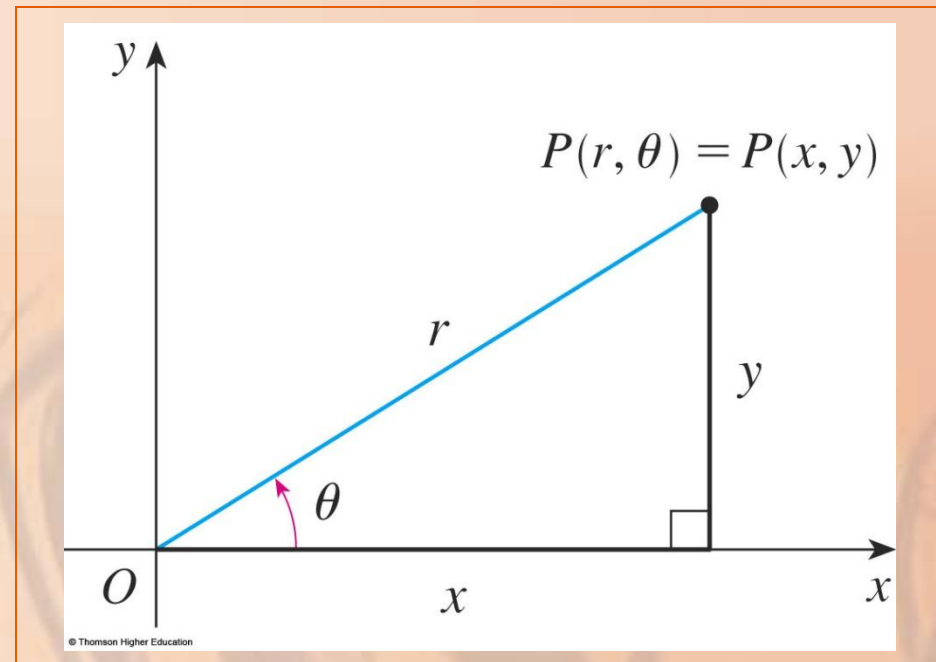
Equations 1 allow us to find the Cartesian coordinates of a point when the polar coordinates are known.

CARTESIAN & POLAR COORDS. Equations 2

To find r and θ when x and y are known, we use the equations

$$r^2 = x^2 + y^2 \qquad \tan \theta = \frac{y}{x}$$

- These can be deduced from Equations 1 or simply read from the figure.



CARTESIAN & POLAR COORDS. Example 2

Convert the point $(2, \pi/3)$ from polar to Cartesian coordinates.

- Since $r = 2$ and $\theta = \pi/3$,
Equations 1 give:

$$x = r \cos \theta = 2 \cos \frac{\pi}{3} = 2 \cdot \frac{1}{2} = 1$$

$$y = r \sin \theta = 2 \sin \frac{\pi}{3} = 2 \cdot \frac{\sqrt{3}}{2} = \sqrt{3}$$

- Thus, the point is $(1, \sqrt{3})$ in Cartesian coordinates.

CARTESIAN & POLAR COORDS. Example 3

Represent the point with Cartesian coordinates $(1, -1)$ in terms of polar coordinates.

CARTESIAN & POLAR COORDS. Example 3

If we choose r to be positive, then

Equations 2 give:

$$r = \sqrt{x^2 + y^2} = \sqrt{1^2 + (-1)^2} = \sqrt{2}$$

$$\tan \theta = \frac{y}{x} = -1$$

- As the point $(1, -1)$ lies in the fourth quadrant, we can choose $\theta = -\pi/4$ or $\theta = 7\pi/4$.

CARTESIAN & POLAR COORDS. Example 3

Thus, one possible answer is:

$$(\sqrt{2}, -\pi/4)$$

Another possible answer is:

$$(\sqrt{2}, 7\pi/4)$$

CARTESIAN & POLAR COORDS. Note

Equations 2 do not uniquely determine θ when x and y are given.

- This is because, as θ increases through the interval $0 \leq \theta \leq 2\pi$, each value of $\tan \theta$ occurs twice.

CARTESIAN & POLAR COORDS. Note

So, in converting from Cartesian to polar coordinates, it's not good enough just to find r and θ that satisfy Equations 2.

- As in Example 3, we must choose θ so that the point (r, θ) lies in the correct quadrant.

POLAR CURVES

The graph of a polar equation $r = f(\theta)$ [or, more generally, $F(r, \theta) = 0$] consists of all points that have at least one polar representation (r, θ) , whose coordinates satisfy the equation.

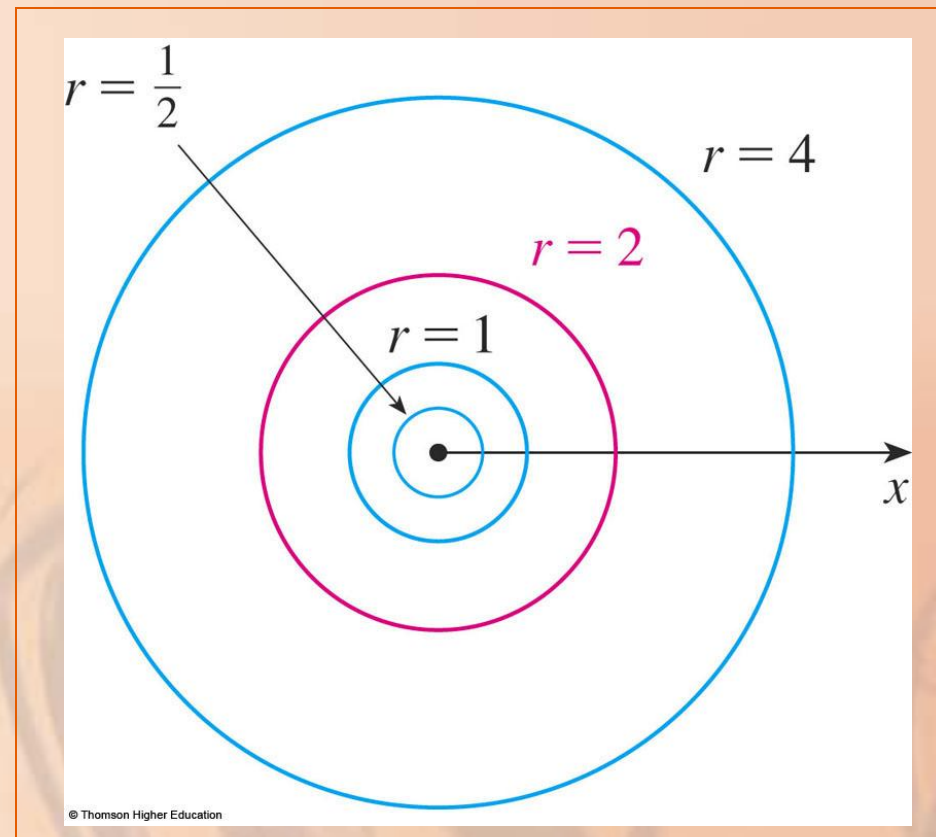
What curve is represented by the polar equation $r = 2$?

- The curve consists of all points (r, θ) with $r = 2$.
- r represents the distance from the point to the pole.

POLAR CURVES

Example 4

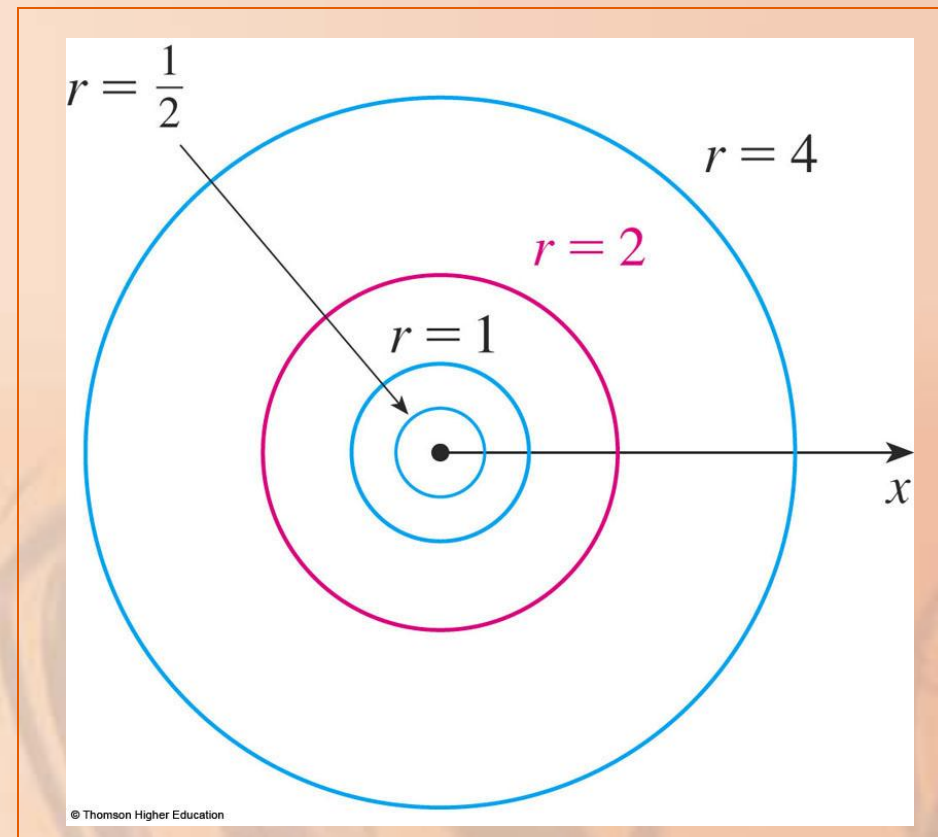
- Thus, the curve $r = 2$ represents the circle with center O and radius 2.



POLAR CURVES

Example 4

In general, the equation $r = a$ represents a circle O with center and radius $|a|$.



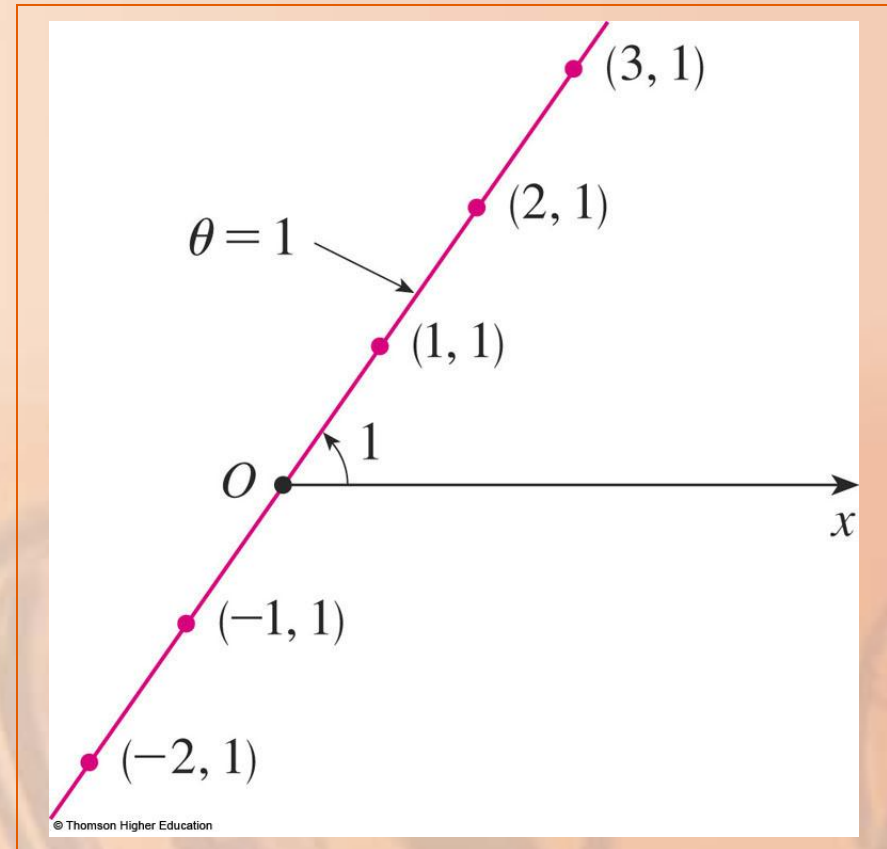
Sketch the polar curve $\theta = 1$.

- This curve consists of all points (r, θ) such that the polar angle θ is 1 radian.

POLAR CURVES

Example 5

It is the straight line that passes through O and makes an angle of 1 radian with the polar axis.

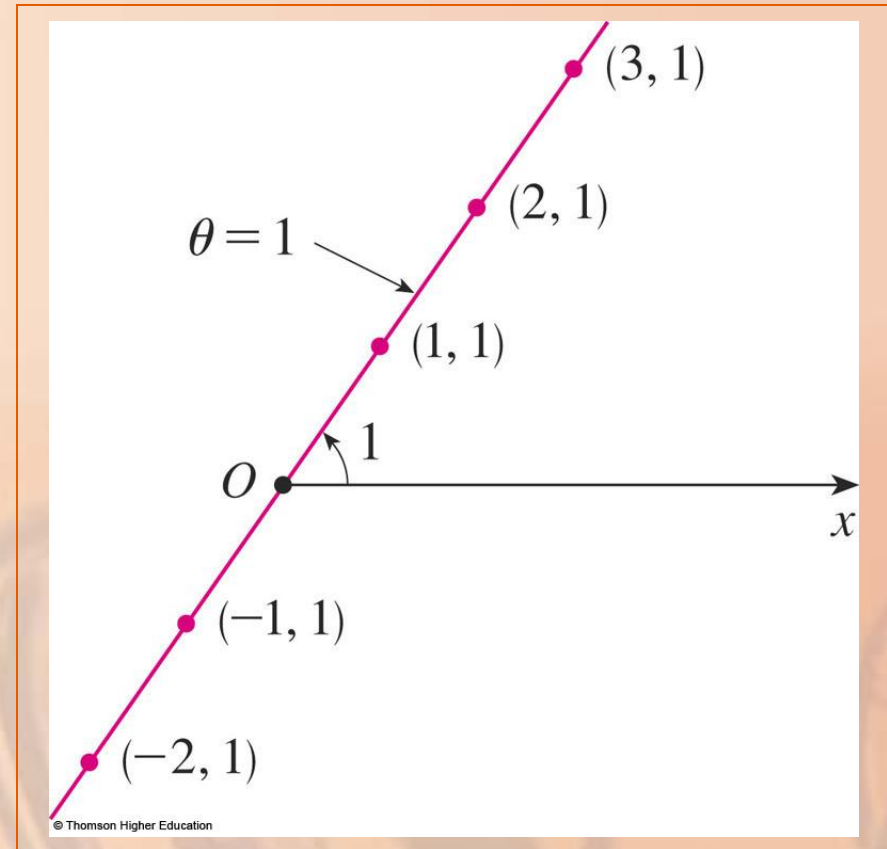


POLAR CURVES

Example 5

Notice that:

- The points $(r, 1)$ on the line with $r > 0$ are in the first quadrant.
- The points $(r, 1)$ on the line with $r < 0$ are in the third quadrant.



POLAR CURVES

Example 6

a. Sketch the curve with polar equation

$$r = 2 \cos \theta.$$

b. Find a Cartesian equation for this curve.

POLAR CURVES

Example 6 a

First, we find the values of r for some convenient values of θ .

θ	$r = 2 \cos \theta$
0	2
$\pi/6$	$\sqrt{3}$
$\pi/4$	$\sqrt{2}$
$\pi/3$	1
$\pi/2$	0
$2\pi/3$	-1
$3\pi/4$	$-\sqrt{2}$
$5\pi/6$	$-\sqrt{3}$
π	-2

POLAR CURVES

Example 6 a

We plot the corresponding points (r, θ) .

Then, we join these points to sketch the curve—as follows.

θ	$r = 2 \cos \theta$
0	2
$\pi/6$	$\sqrt{3}$
$\pi/4$	$\sqrt{2}$
$\pi/3$	1
$\pi/2$	0
$2\pi/3$	-1
$3\pi/4$	$-\sqrt{2}$
$5\pi/6$	$-\sqrt{3}$
π	-2

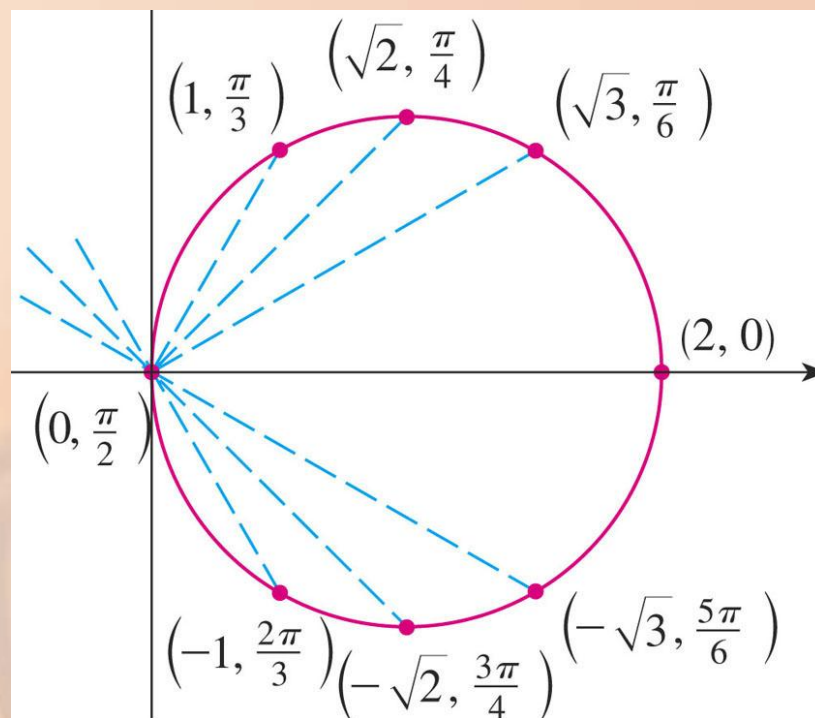
POLAR CURVES

Example 6 a

The curve appears to be a circle.

θ	$r = 2 \cos \theta$
0	2
$\pi/6$	$\sqrt{3}$
$\pi/4$	$\sqrt{2}$
$\pi/3$	1
$\pi/2$	0
$2\pi/3$	-1
$3\pi/4$	$-\sqrt{2}$
$5\pi/6$	$-\sqrt{3}$
π	-2

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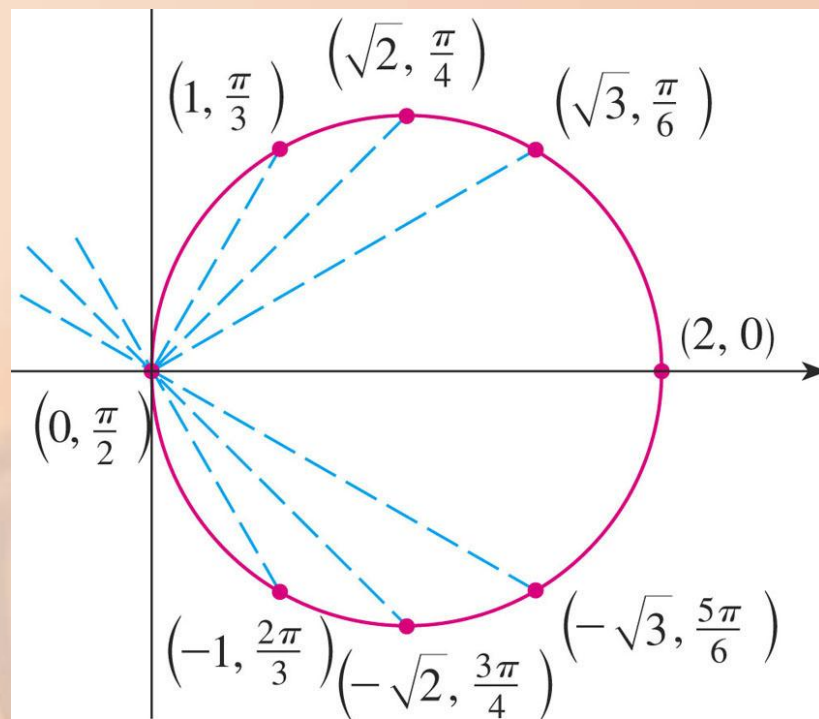
POLAR CURVES

Example 6 a

We have used only values of θ between 0 and π —since, if we let θ increase beyond π , we obtain the same points again.

θ	$r = 2 \cos \theta$
0	2
$\pi/6$	$\sqrt{3}$
$\pi/4$	$\sqrt{2}$
$\pi/3$	1
$\pi/2$	0
$2\pi/3$	-1
$3\pi/4$	$-\sqrt{2}$
$5\pi/6$	$-\sqrt{3}$
π	-2

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To convert the given equation to a Cartesian equation, we use Equations 1 and 2.

- From $x = r \cos \theta$, we have $\cos \theta = x/r$.
- So, the equation $r = 2 \cos \theta$ becomes $r = 2x/r$.
- This gives:
$$2x = r^2 = x^2 + y^2 \quad \text{or} \quad x^2 + y^2 - 2x = 0$$

Completing the square,
we obtain:

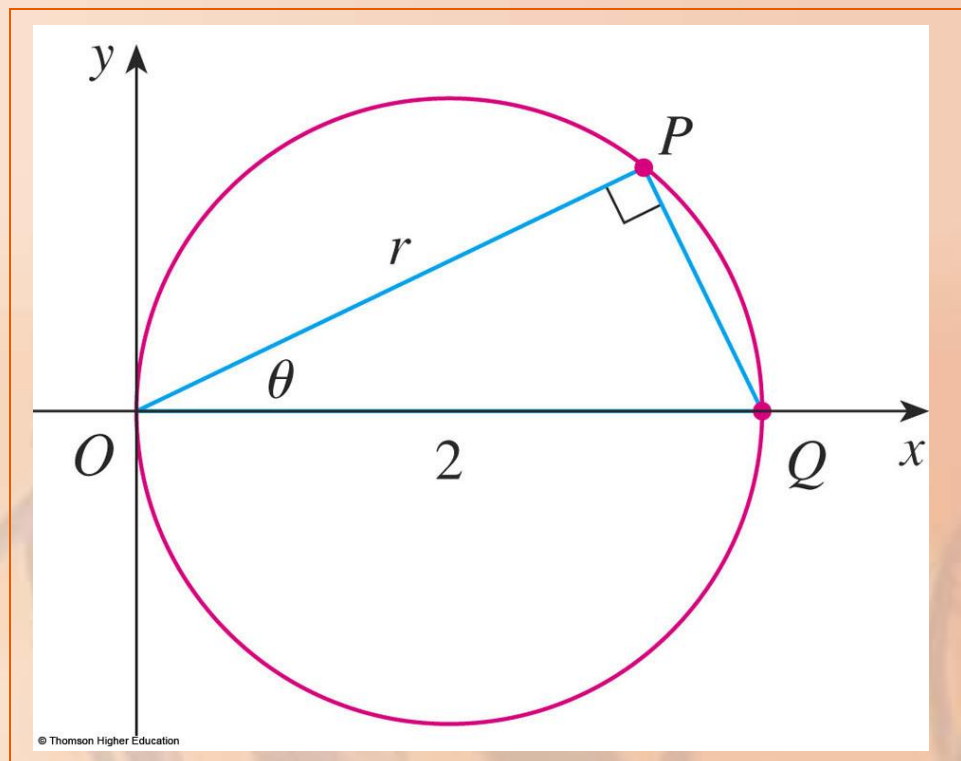
$$(x - 1)^2 + y^2 = 1$$

- The equation is of a circle with center $(1, 0)$ and radius 1.

POLAR CURVES

The figure shows a geometrical illustration that the circle in Example 6 has the equation $r = 2 \cos \theta$.

- The angle OPQ is a right angle, and so $r/2 = \cos \theta$.
- Why is OPQ a right angle?



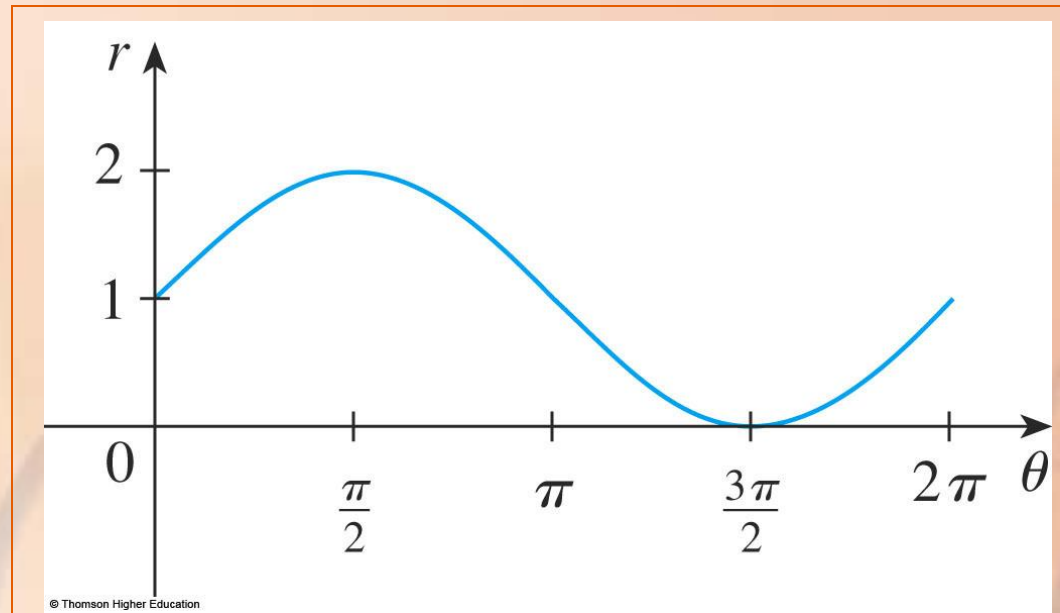
Sketch the curve $r = 1 + \sin \theta$.

- Here, we do not plot points as in Example 6.
- Rather, we first sketch the graph of $r = 1 + \sin \theta$ in Cartesian coordinates by shifting the sine curve up one unit—as follows.

POLAR CURVES

Example 7

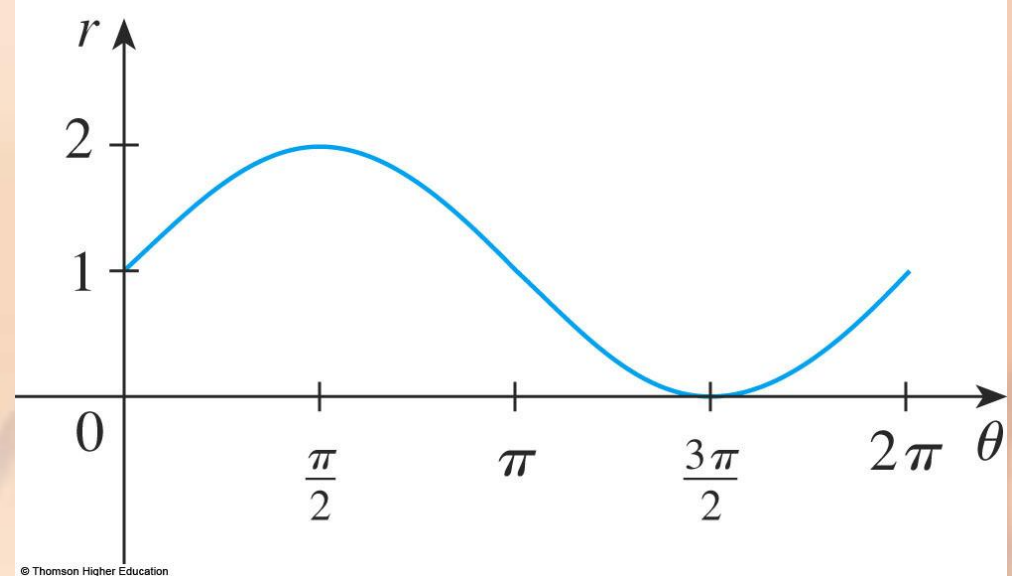
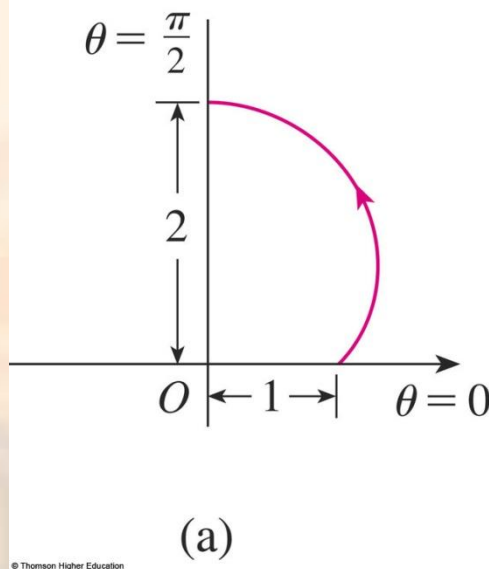
This enables us to read at a glance the values of r that correspond to increasing values of θ .



POLAR CURVES

Example 7

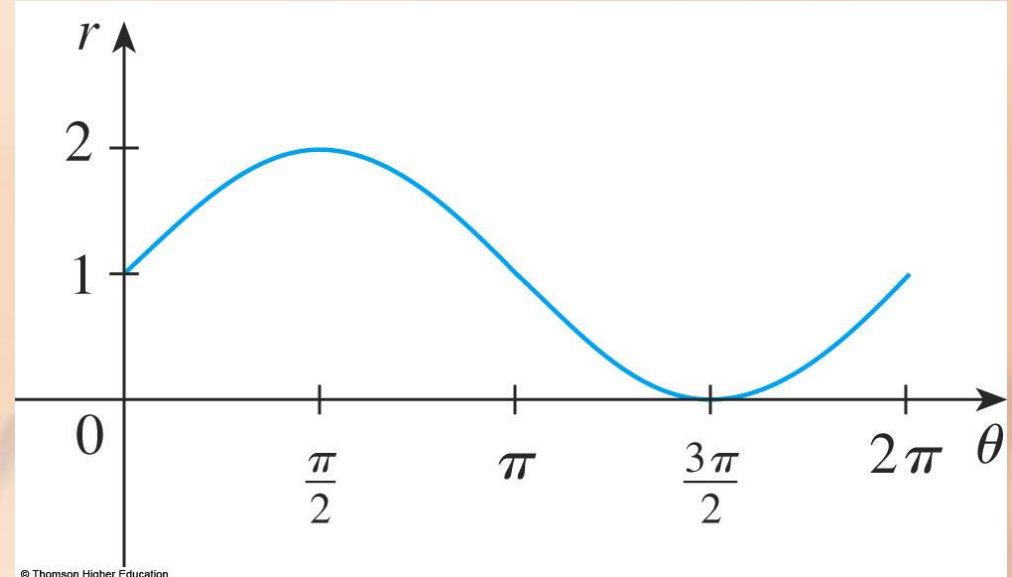
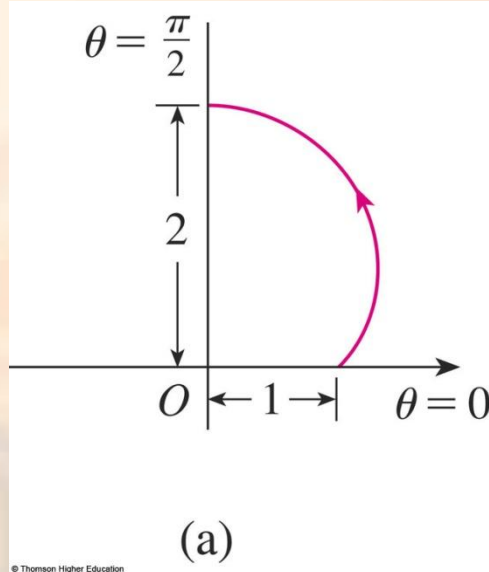
For instance, we see that, as θ increases from 0 to $\pi/2$, r (the distance from O) increases from 1 to 2.



POLAR CURVES

Example 7

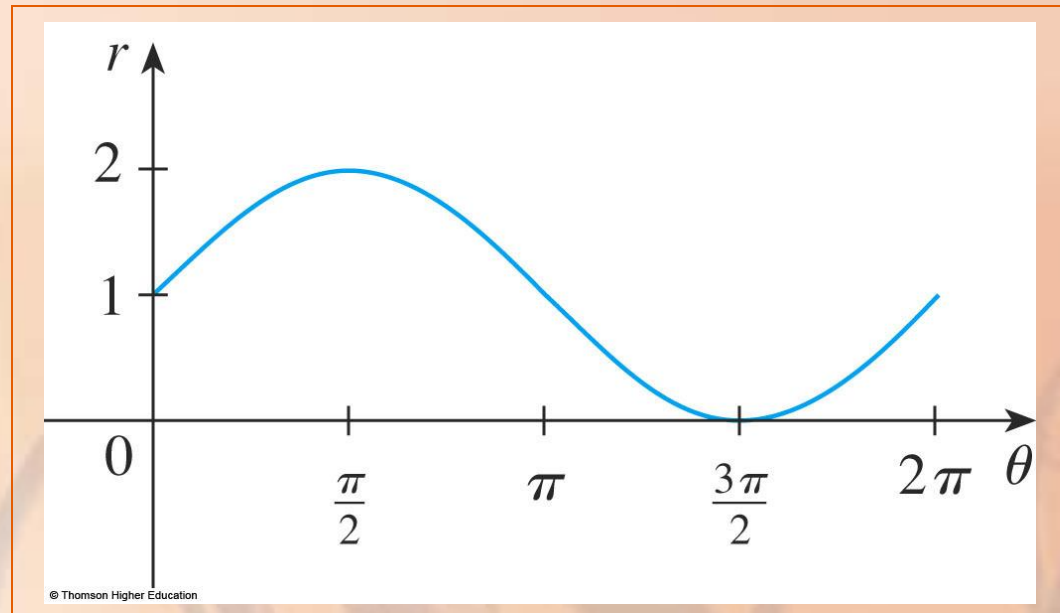
So, we sketch the corresponding part
of the polar curve.



POLAR CURVES

Example 7

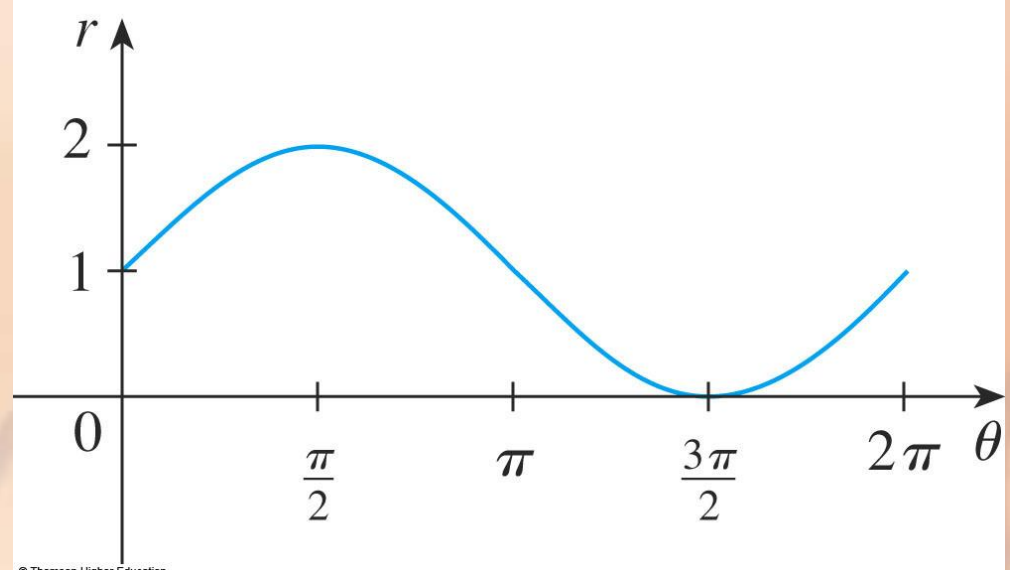
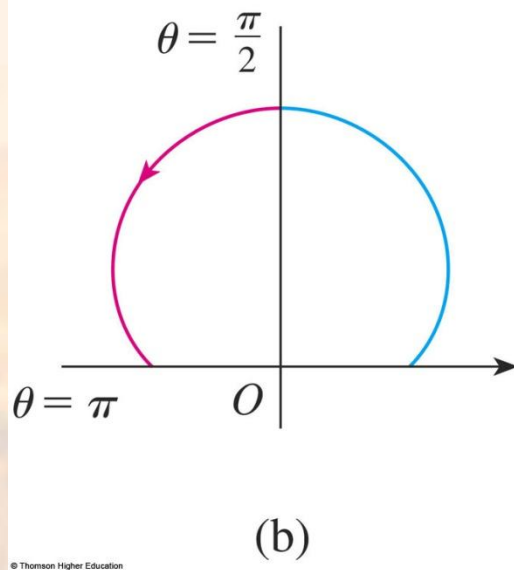
As θ increases from $\pi/2$ to π ,
the figure shows that r decreases
from 2 to 1.



POLAR CURVES

Example 7

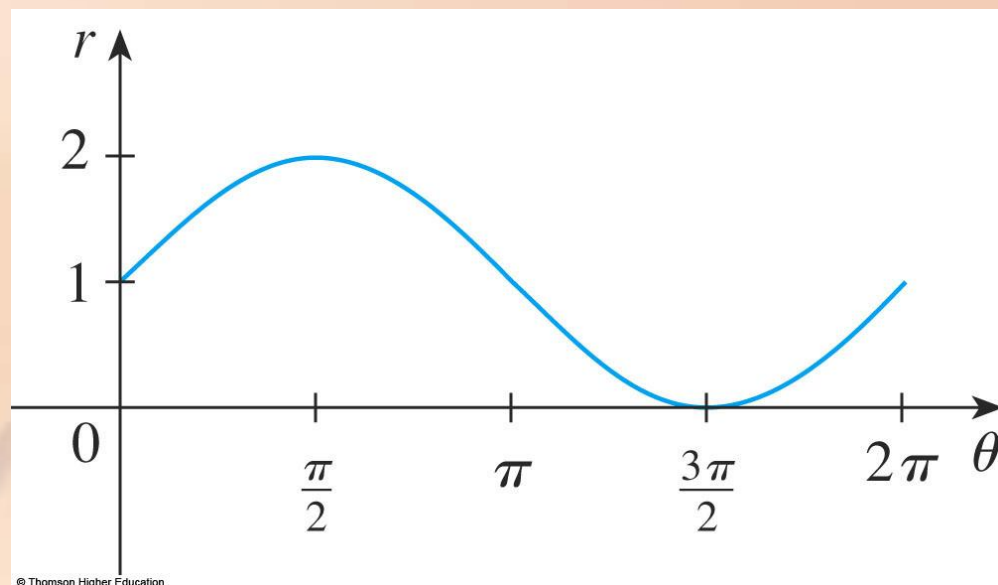
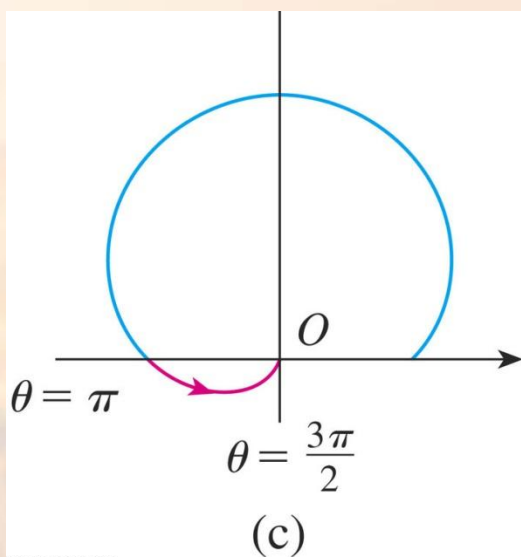
So, we sketch the next part of the curve.



POLAR CURVES

Example 7

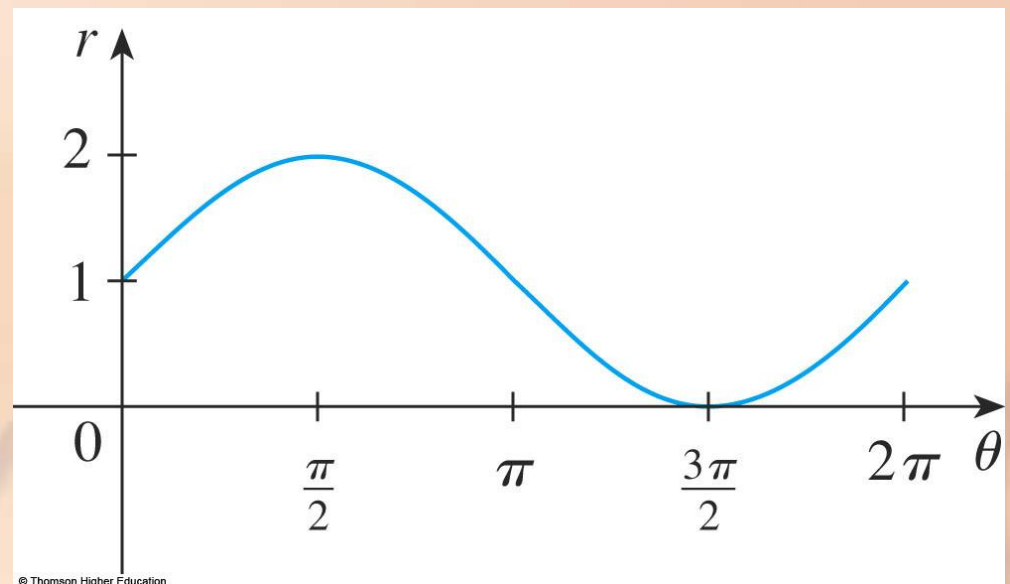
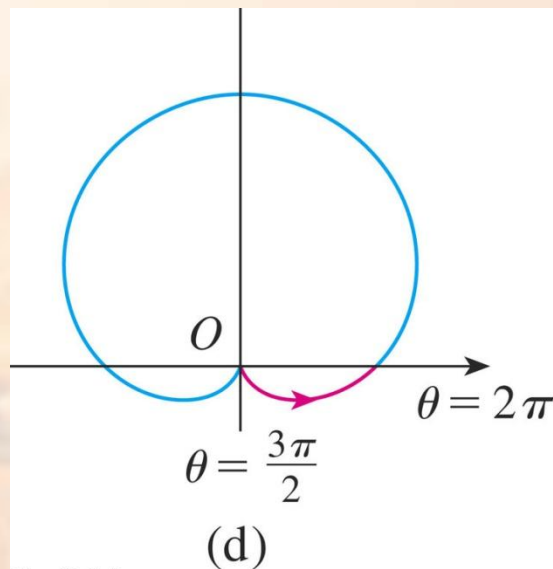
As θ increases from π to $3\pi/2$,
 r decreases from 1 to 0, as shown.



POLAR CURVES

Example 7

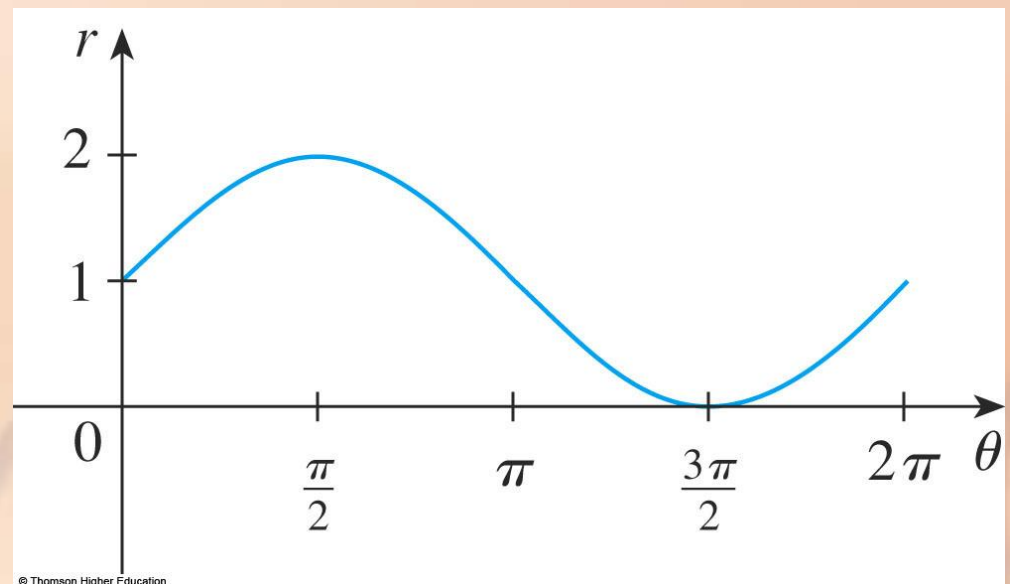
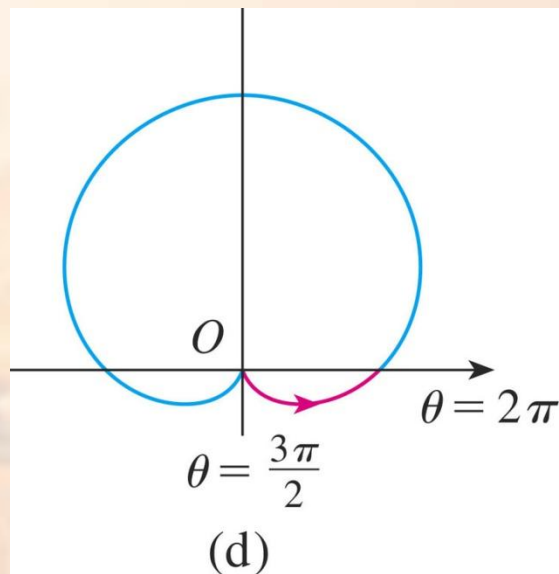
Finally, as θ increases from $3\pi/2$ to 2π , r increases from 0 to 1, as shown.



POLAR CURVES

Example 7

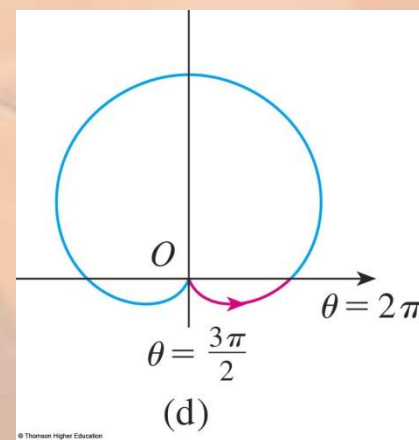
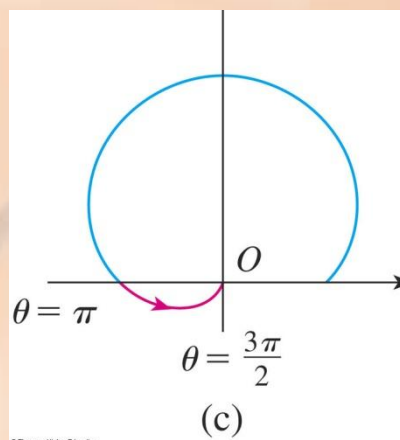
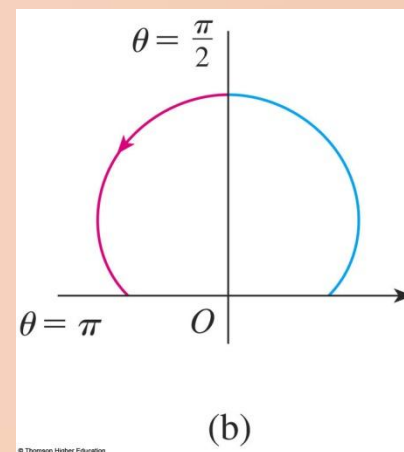
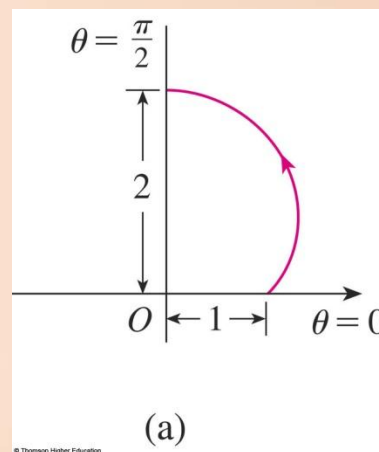
If we let θ increase beyond 2π or decrease beyond 0, we would simply retrace our path.



POLAR CURVES

Example 7

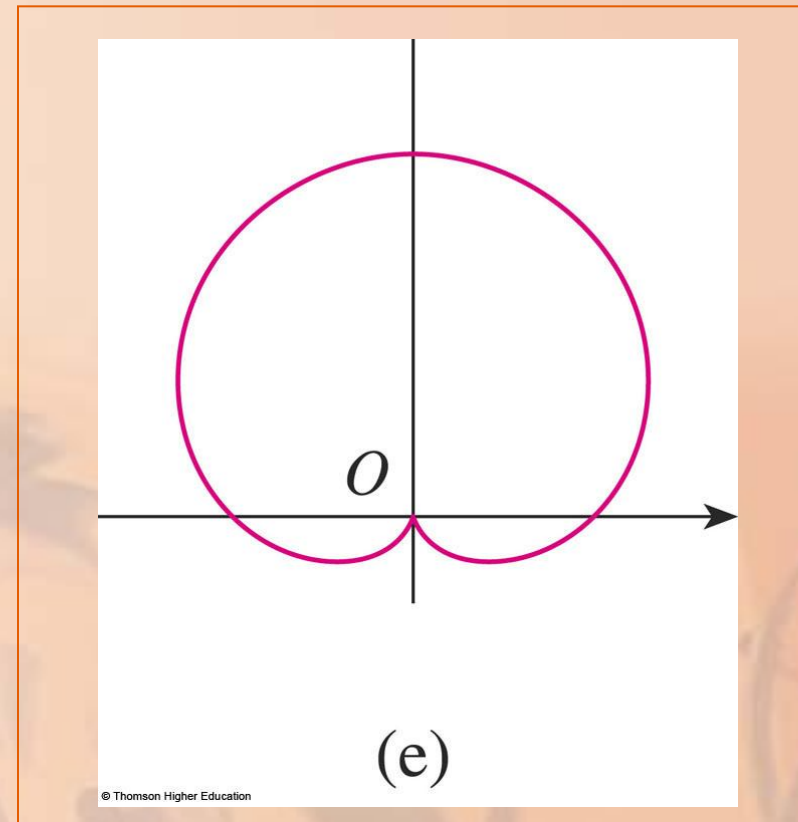
Putting together the various parts of the curve, we sketch the complete curve—as shown next.



CARDIOID

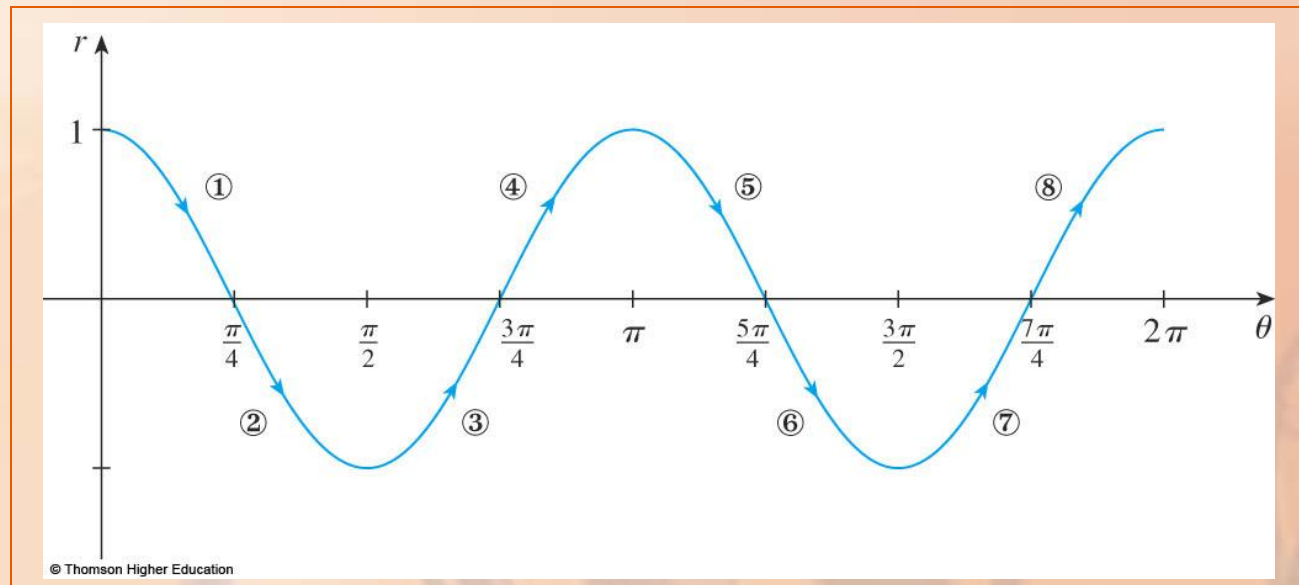
Example 7

It is called a cardioid—because it's shaped like a heart.



Sketch the curve $r = \cos 2\theta$.

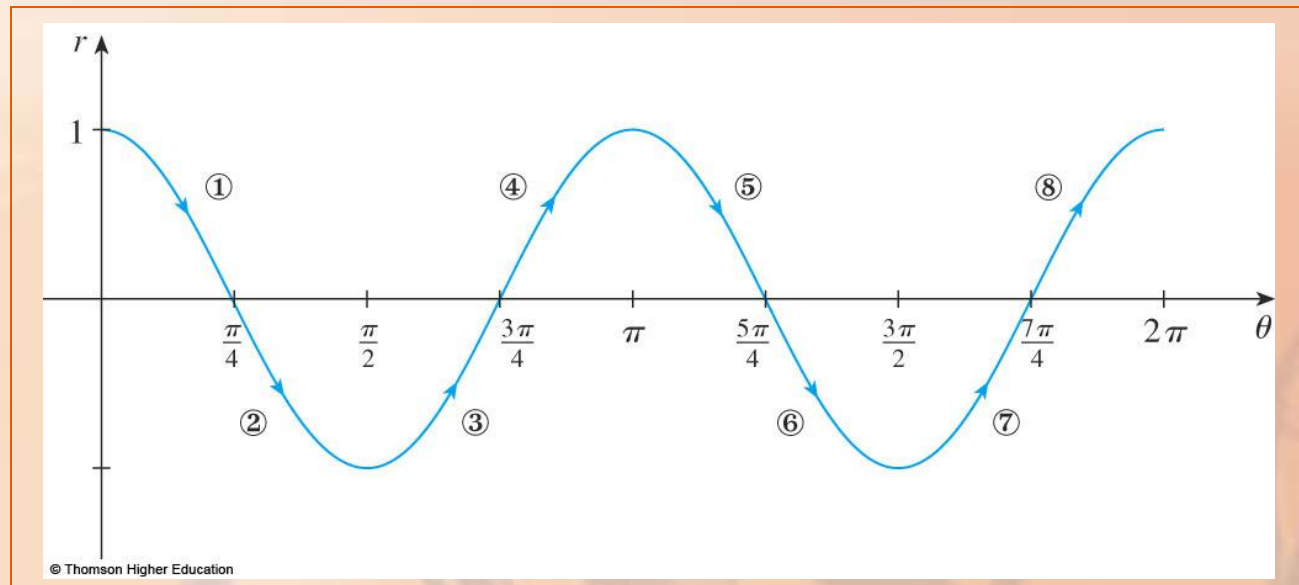
- As in Example 7, we first sketch $r = \cos 2\theta$, $0 \leq \theta \leq 2\pi$, in Cartesian coordinates.



POLAR CURVES

Example 8

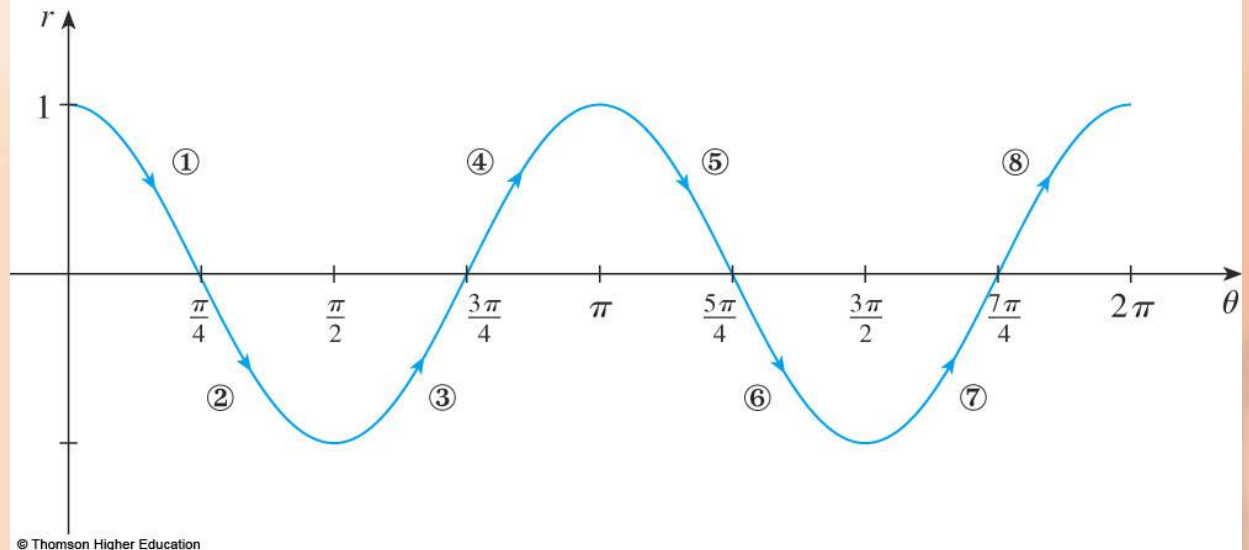
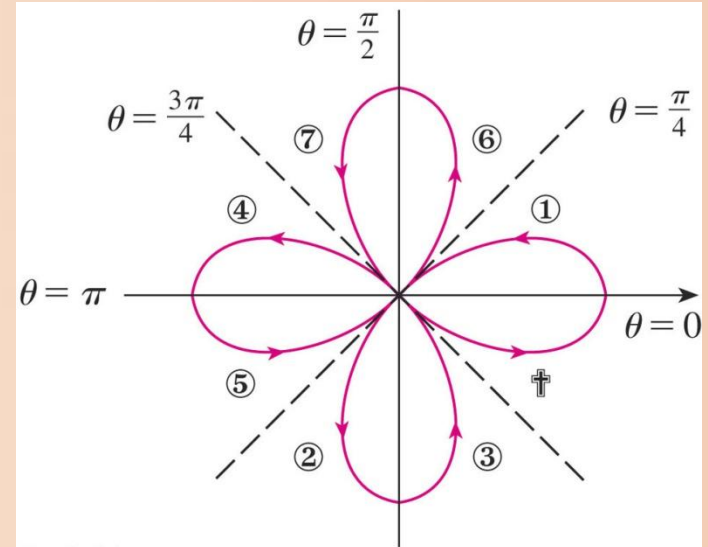
As θ increases from 0 to $\pi/4$,
the figure shows that r decreases
from 1 to 0.



POLAR CURVES

Example 8

So, we draw the corresponding portion of the polar curve (indicated by ①).

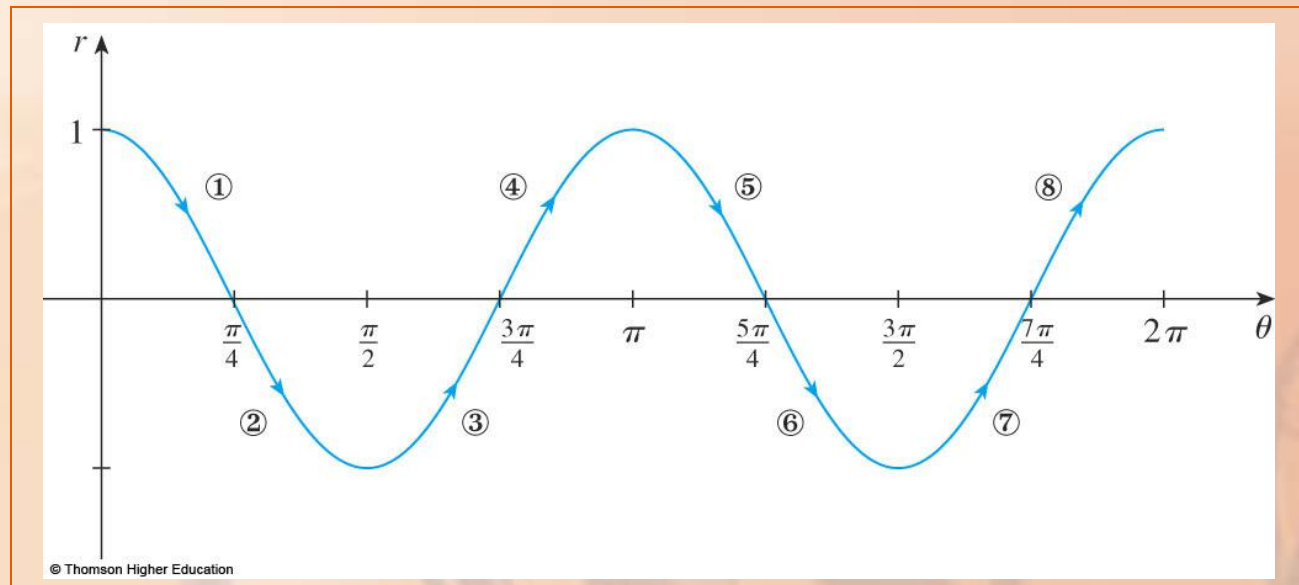


POLAR CURVES

Example 8

As θ increases from $\pi/4$ to $\pi/2$, r goes from 0 to -1 .

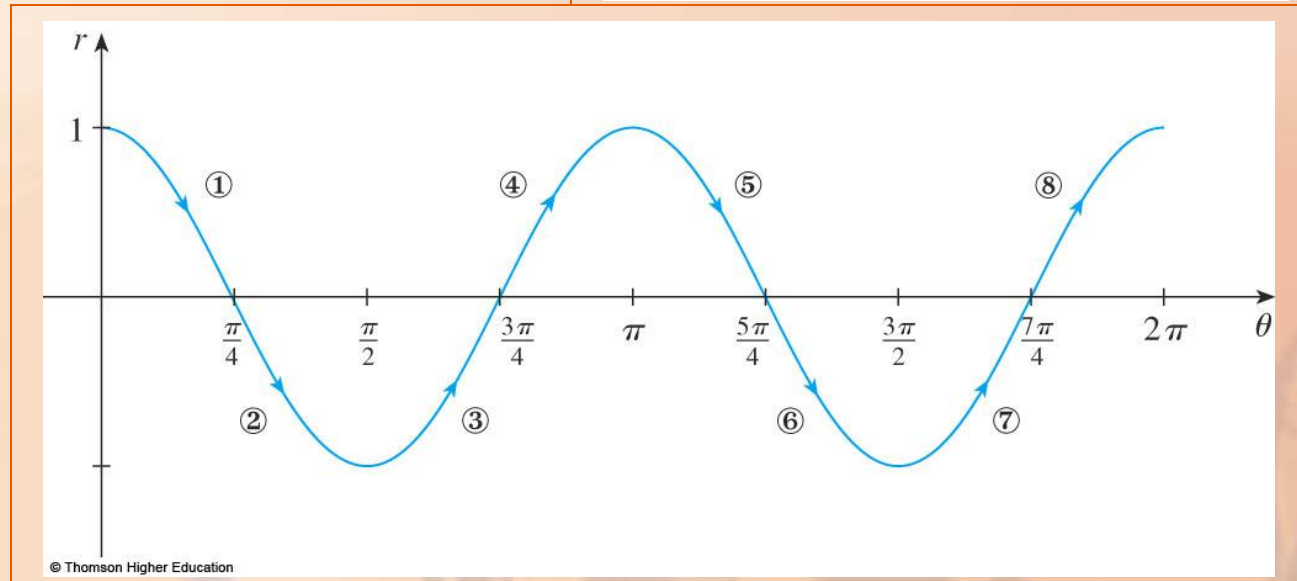
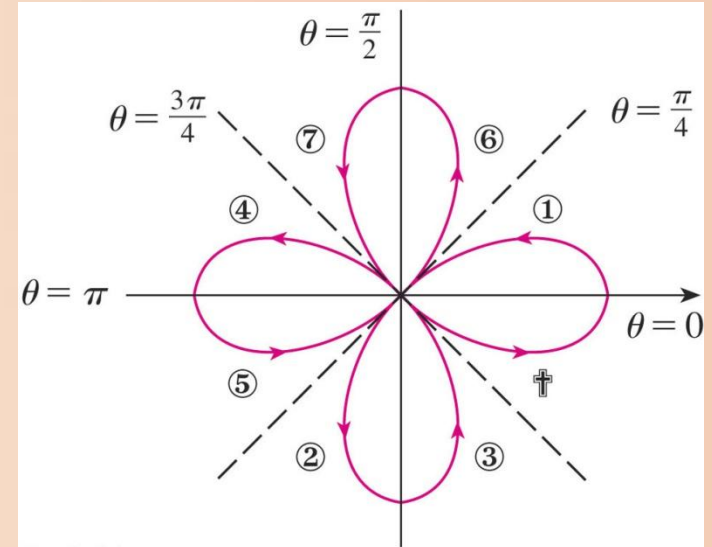
- This means that the distance from O increases from 0 to 1.



POLAR CURVES

Example 8

- However, instead of being in the first quadrant, this portion of the polar curve (indicated by ②) lies on the opposite side of the pole in the third quadrant.

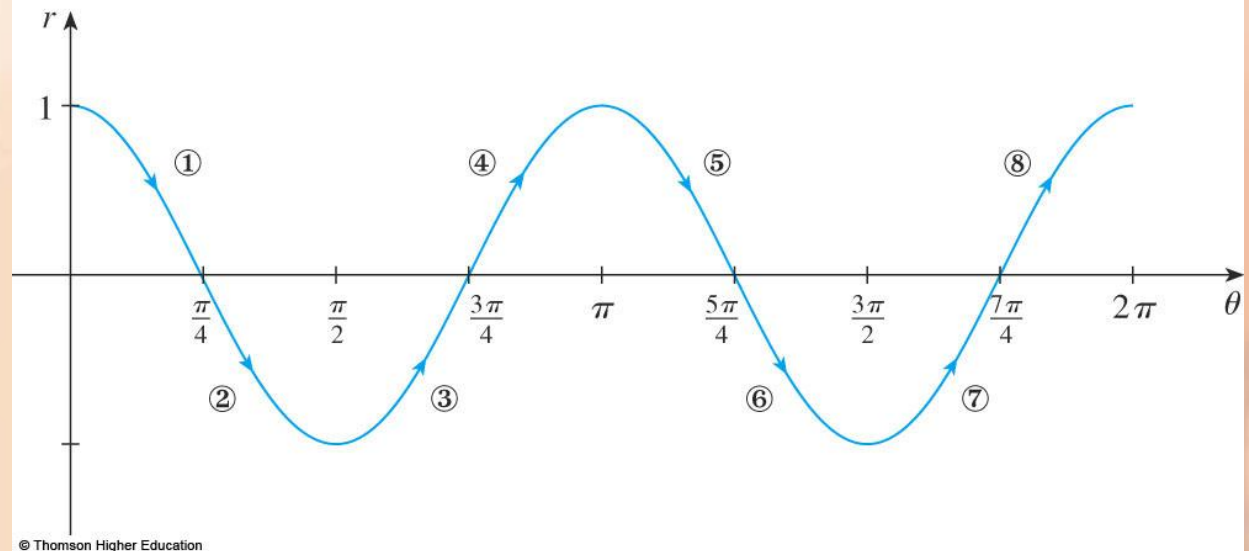
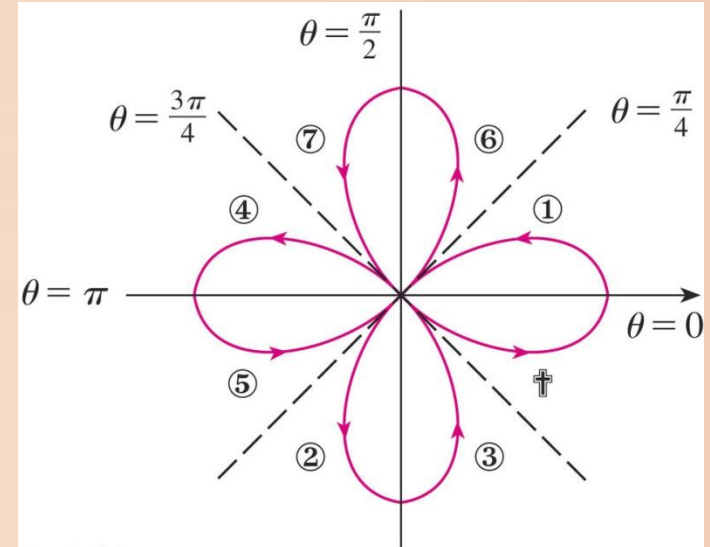


POLAR CURVES

Example 8

The rest of the curve is drawn in a similar fashion.

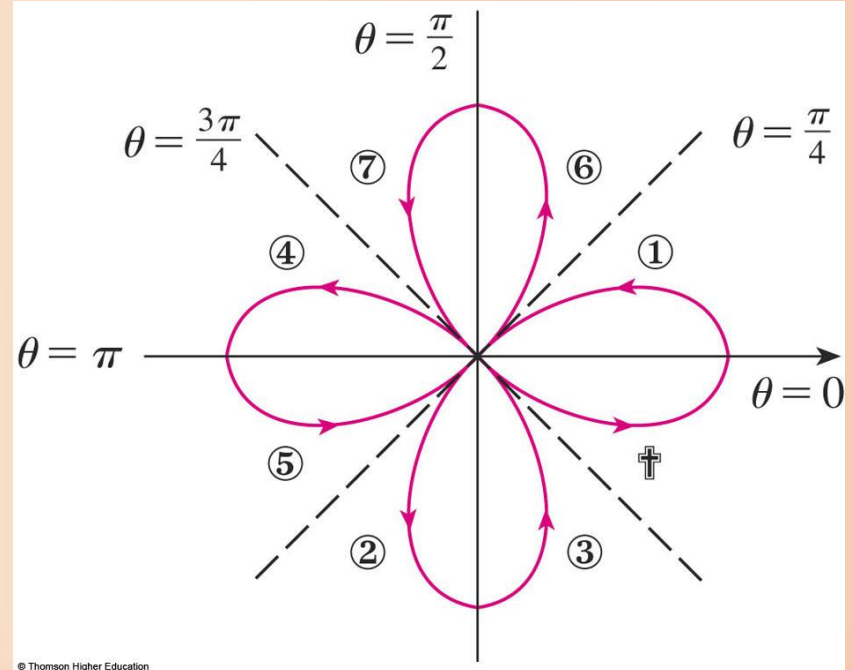
- The arrows and numbers indicate the order in which the portions are traced out.



POLAR CURVES

Example 8

The resulting curve has four loops and is called a four-leaved rose.



SYMMETRY

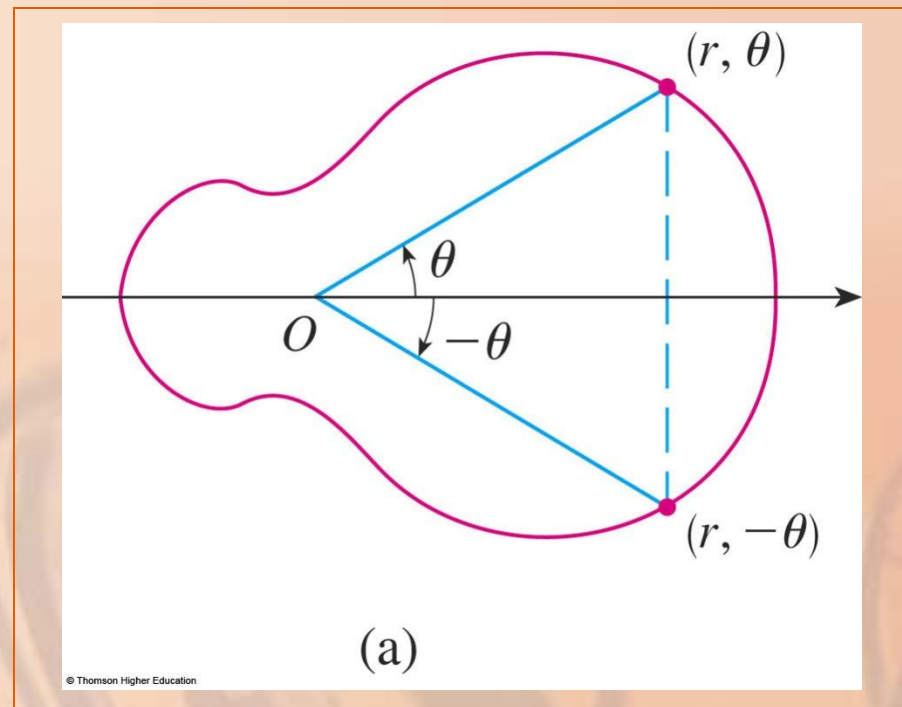
When we sketch polar curves, it is sometimes helpful to take advantage of symmetry.

RULES

The following three rules are explained by figures.

RULE 1

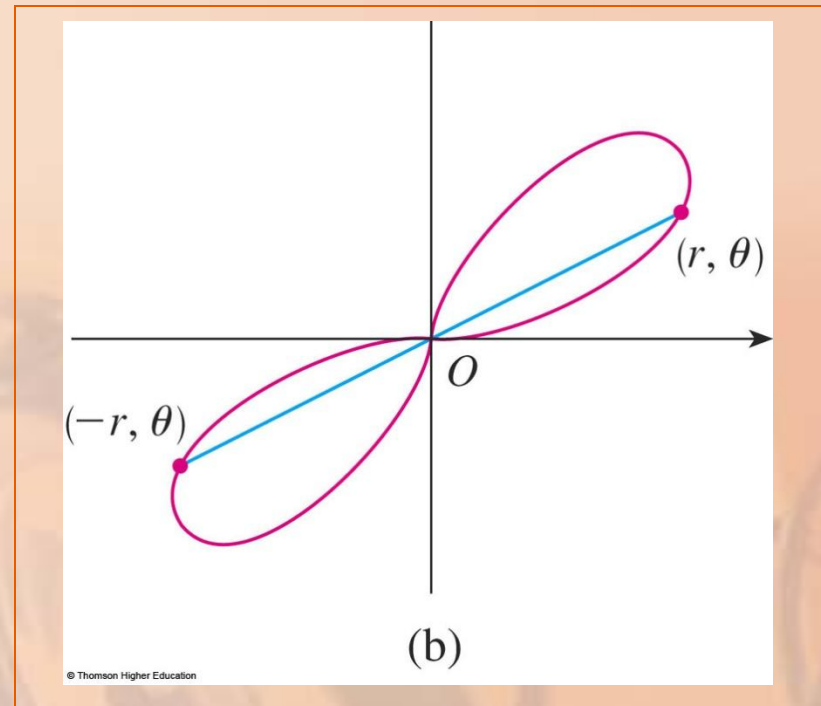
If a polar equation is unchanged when θ is replaced by $-\theta$, the curve is symmetric about the polar axis.



RULE 2

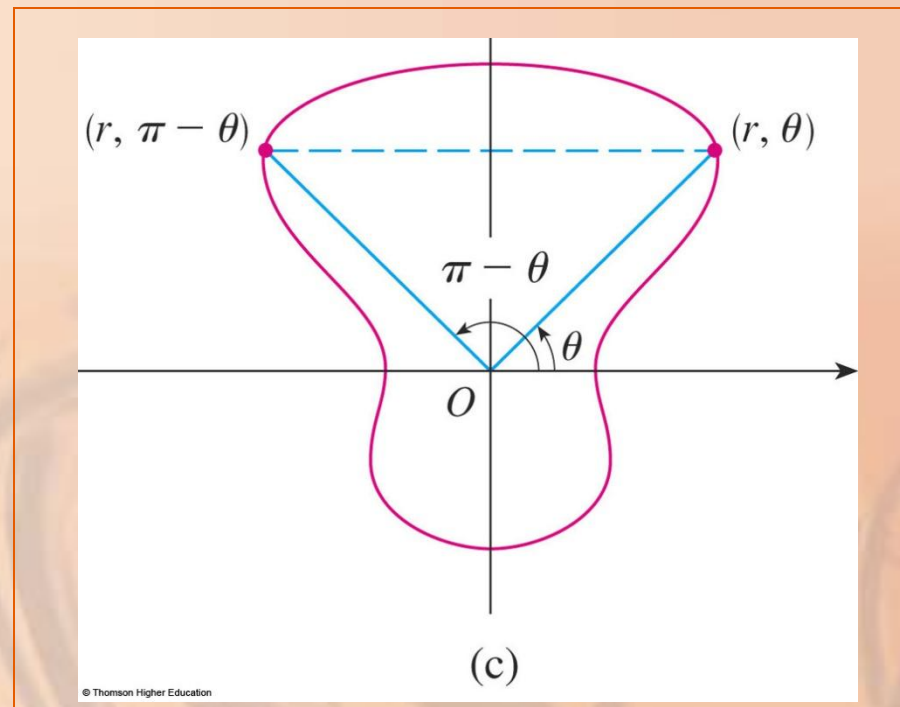
If the equation is unchanged when r is replaced by $-r$, or when θ is replaced by $\theta + \pi$, the curve is symmetric about the pole.

- This means that the curve remains unchanged if we rotate it through 180° about the origin.



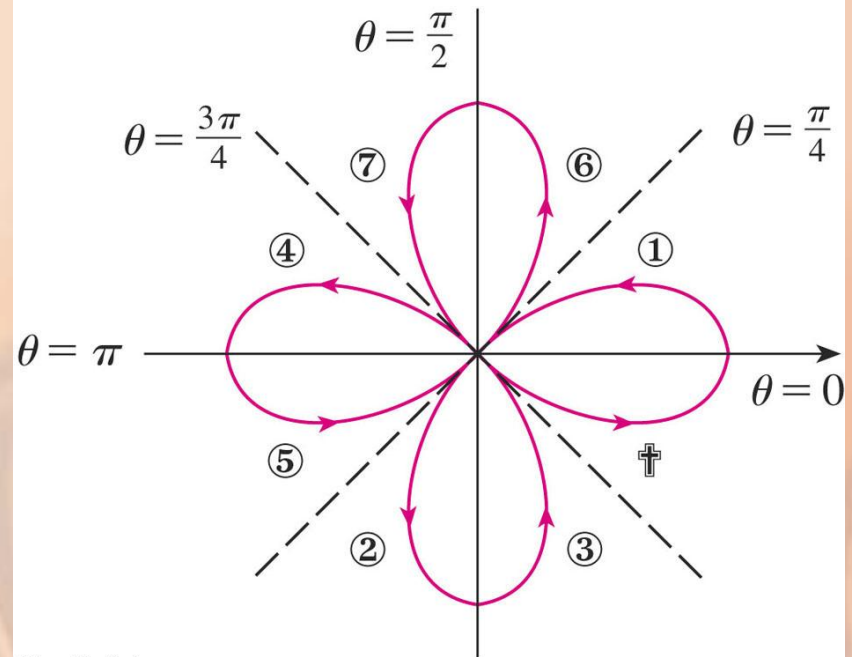
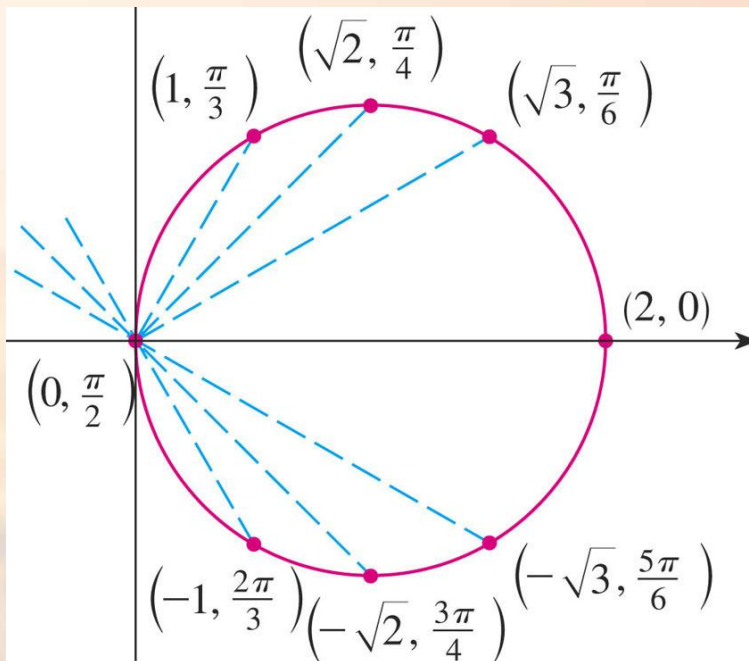
RULE 3

If the equation is unchanged when θ is replaced by $\pi - \theta$, the curve is symmetric about the vertical line $\theta = \pi/2$.



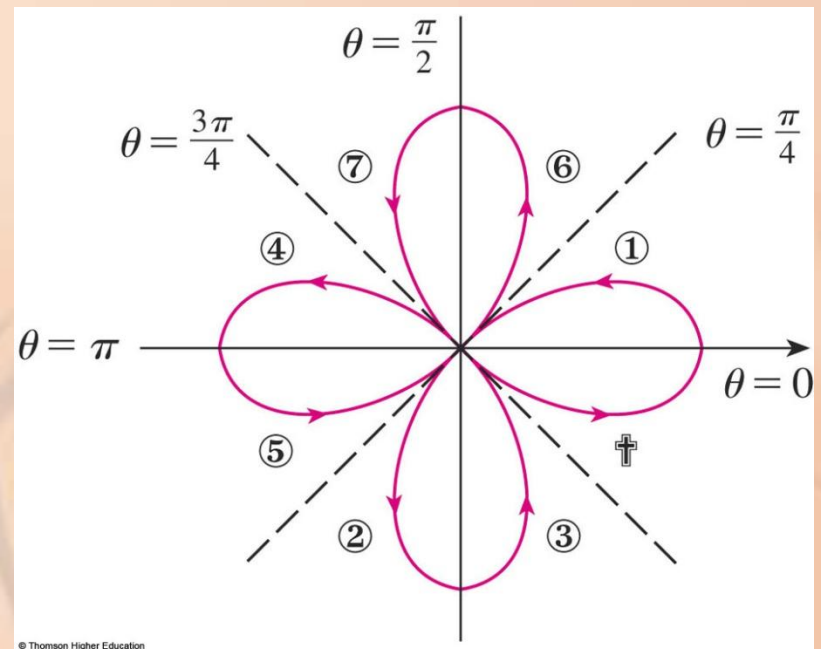
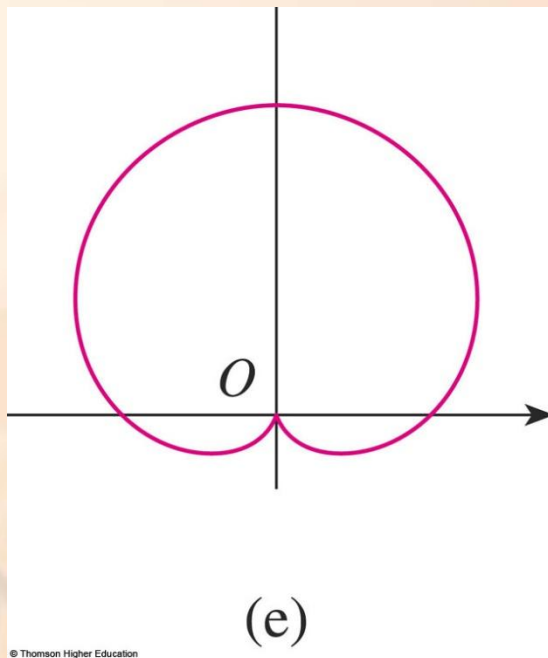
SYMMETRY

The curves sketched in Examples 6 and 8 are symmetric about the polar axis, since $\cos(-\theta) = \cos \theta$.



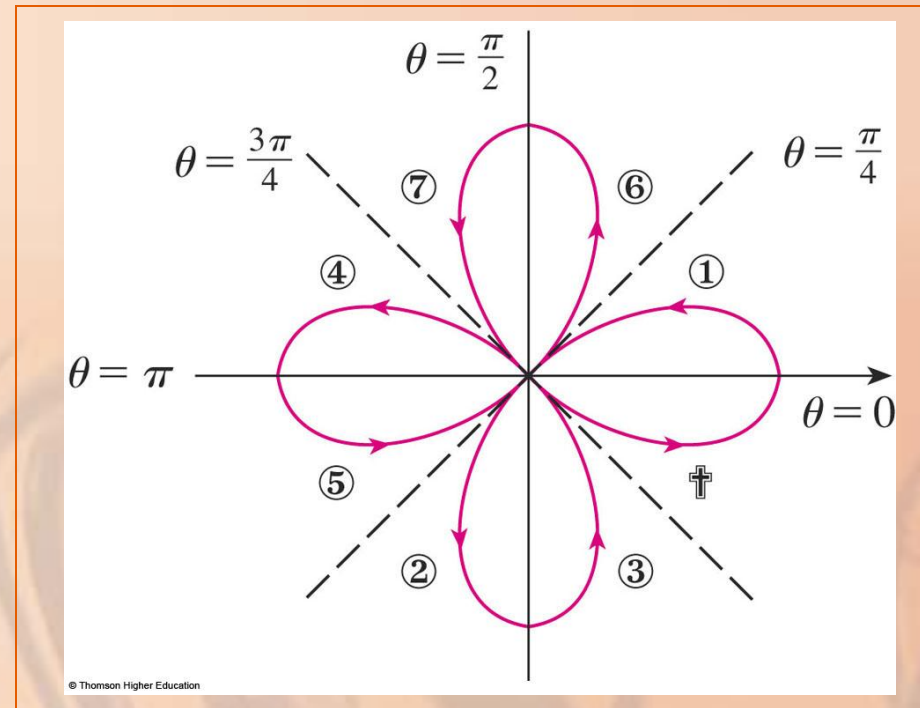
SYMMETRY

The curves in Examples 7 and 8 are symmetric about $\theta = \pi/2$, because $\sin(\pi - \theta) = \sin \theta$ and $\cos 2(\pi - \theta) = \cos 2\theta$.



SYMMETRY

The four-leaved rose is also symmetric about the pole.



SYMMETRY

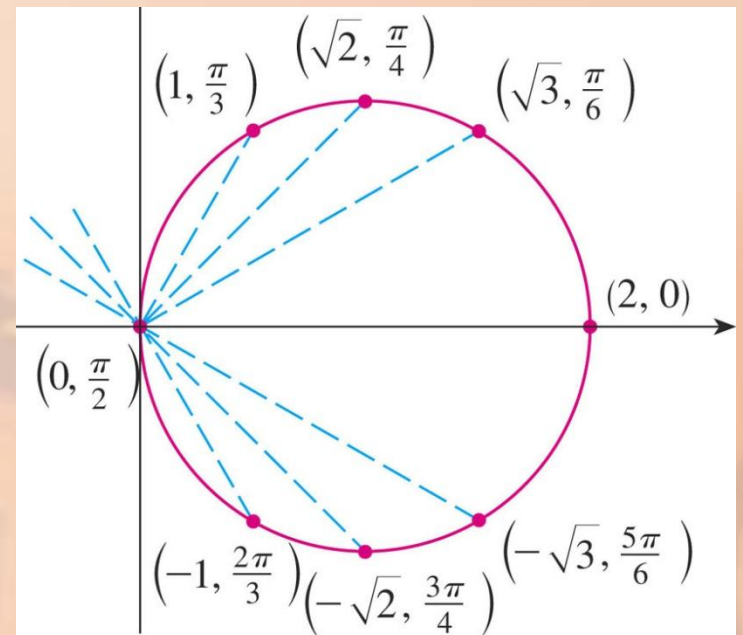
These symmetry properties
could have been used in sketching
the curves.

SYMMETRY

For instance, in Example 6, we need only have plotted points for $0 \leq \theta \leq \pi/2$ and then reflected about the polar axis to obtain the complete circle.

θ	$r = 2 \cos \theta$
0	2
$\pi/6$	$\sqrt{3}$
$\pi/4$	$\sqrt{2}$
$\pi/3$	1
$\pi/2$	0
$2\pi/3$	-1
$3\pi/4$	$-\sqrt{2}$
$5\pi/6$	$-\sqrt{3}$
π	-2

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TANGENTS TO POLAR CURVES

To find a tangent line to a polar curve $r = f(\theta)$, we regard θ as a parameter and write its parametric equations as:

$$x = r \cos \theta = f(\theta) \cos \theta$$

$$y = r \sin \theta = f(\theta) \sin \theta$$

TANGENTS TO POLAR CURVES

Equation 3

Then, using the method for finding slopes of parametric curves (Equation 2 in Section 10.2) and the Product Rule, we have:

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{\frac{dr}{d\theta} \sin \theta + r \cos \theta}{\frac{dr}{d\theta} \cos \theta - r \sin \theta}$$

TANGENTS TO POLAR CURVES

We locate horizontal tangents by finding the points where $dy/d\theta = 0$ (provided that $dx/d\theta \neq 0$).

Likewise, we locate vertical tangents at the points where $dx/d\theta = 0$ (provided that $dy/d\theta \neq 0$).

TANGENTS TO POLAR CURVES

Notice that, if we are looking for tangent lines at the pole, then $r = 0$ and Equation 3 simplifies to:

$$\frac{dy}{dx} = \tan \theta \quad \text{if} \quad \frac{dr}{d\theta} \neq 0$$

TANGENTS TO POLAR CURVES

For instance, in Example 8, we found that $r = \cos 2\theta = 0$ when $\theta = \pi/4$ or $3\pi/4$.

- This means that the lines $\theta = \pi/4$ and $\theta = 3\pi/4$ (or $y = x$ and $y = -x$) are tangent lines to $r = \cos 2\theta$ at the origin.

TANGENTS TO POLAR CURVES

Example 9

- a. For the cardioid $r = 1 + \sin \theta$ of Example 7, find the slope of the tangent line when $\theta = \pi/3$.
- b. Find the points on the cardioid where the tangent line is horizontal or vertical.

TANGENTS TO POLAR CURVES

Example 9

Using Equation 3 with $r = 1 + \sin \theta$, we have:

$$\begin{aligned}\frac{dy}{dx} &= \frac{\frac{dr}{d\theta} \sin \theta + r \cos \theta}{\frac{dr}{d\theta} \cos \theta - r \sin \theta} \\&= \frac{\cos \theta \sin \theta + (1 + \sin \theta) \cos \theta}{\cos \theta \cos \theta - (1 + \sin \theta) \sin \theta} \\&= \frac{\cos \theta (1 + 2 \sin \theta)}{1 - 2 \sin^2 \theta - \sin \theta} = \frac{\cos \theta (1 + 2 \sin \theta)}{(1 + \sin \theta)(1 - 2 \sin \theta)}\end{aligned}$$

TANGENTS TO POLAR CURVES

Example 9 a

The slope of the tangent at the point where $\theta = \pi/3$ is:

$$\begin{aligned}\left. \frac{dy}{dx} \right|_{\theta=\pi/3} &= \frac{\cos(\pi/3)(1+2\sin(\pi/3))}{(1+\sin(\pi/3))(1-2\sin(\pi/3))} \\ &= \frac{\frac{1}{2}(1+\sqrt{3})}{(1+\sqrt{3}/2)(1-\sqrt{3})} \\ &= \frac{1+\sqrt{3}}{(2+\sqrt{3})(1-\sqrt{3})} = \frac{1+\sqrt{3}}{-1-\sqrt{3}} = -1\end{aligned}$$

TANGENTS TO POLAR CURVES

Example 9 b

Observe that:

$$\frac{dy}{d\theta} = \cos \theta (1 + 2 \sin \theta) = 0 \quad \text{when } \theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

$$\frac{dx}{d\theta} = (1 + \sin \theta)(1 - 2 \sin \theta) = 0 \quad \text{when } \theta = \frac{3\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}$$

TANGENTS TO POLAR CURVES

Example 9 b

Hence, there are horizontal tangents at the points

$$(2, \pi/2), (\frac{1}{2}, 7\pi/6), (\frac{1}{2}, 11\pi/6)$$

and vertical tangents at

$$(3/2, \pi/6), (3/2, 5\pi/6)$$

- When $\theta = 3\pi/2$, both $dy/d\theta$ and $dx/d\theta$ are 0.
- So, we must be careful.

TANGENTS TO POLAR CURVES

Example 9 b

Using l'Hospital's Rule, we have:

$$\begin{aligned} & \lim_{\theta \rightarrow (3\pi/2)^-} \frac{dy}{dx} \\ &= \left(\lim_{\theta \rightarrow (3\pi/2)^-} \frac{1 + 2\sin \theta}{1 - 2\sin \theta} \right) \left(\lim_{\theta \rightarrow (3\pi/2)^-} \frac{\cos \theta}{1 + \sin \theta} \right) \\ &= -\frac{1}{3} \lim_{\theta \rightarrow (3\pi/2)^-} \frac{\cos \theta}{1 + \sin \theta} \\ &= -\frac{1}{3} \lim_{\theta \rightarrow (3\pi/2)^-} \frac{-\sin \theta}{\cos \theta} = \infty \end{aligned}$$

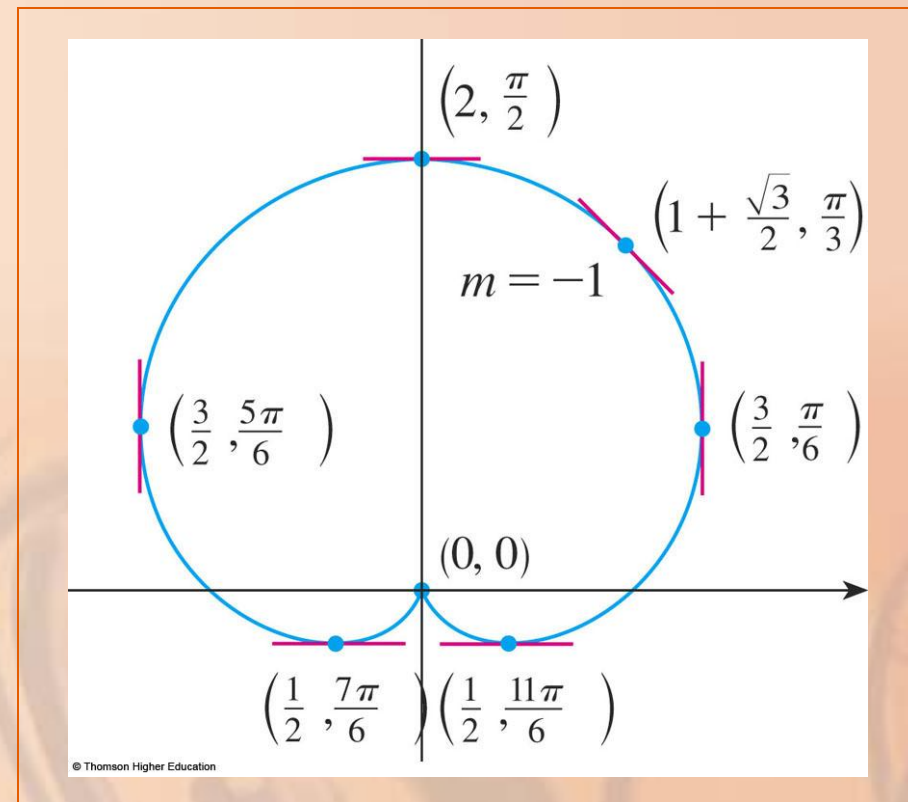
By symmetry,

$$\lim_{\theta \rightarrow (3\pi/2)^+} \frac{dy}{dx} = -\infty$$

TANGENTS TO POLAR CURVES

Example 9 b

Thus, there is a vertical tangent line at the pole.



TANGENTS TO POLAR CURVES

Note

Instead of having to remember Equation 3, we could employ the method used to derive it.

- For instance, in Example 9, we could have written:

$$x = r \cos \theta = (1 + \sin \theta) \cos \theta = \cos \theta + \frac{1}{2} \sin 2\theta$$

$$y = r \sin \theta = (1 + \sin \theta) \sin \theta = \sin \theta + \sin^2 \theta$$

TANGENTS TO POLAR CURVES

Note

- Then, we would have

$$\begin{aligned}\frac{dy}{dx} &= \frac{dy / d\theta}{dx / d\theta} = \frac{\cos \theta + 2 \sin \theta \cos \theta}{-\sin \theta + \cos 2\theta} \\ &= \frac{\cos \theta + \sin 2\theta}{-\sin \theta + \cos 2\theta}\end{aligned}$$

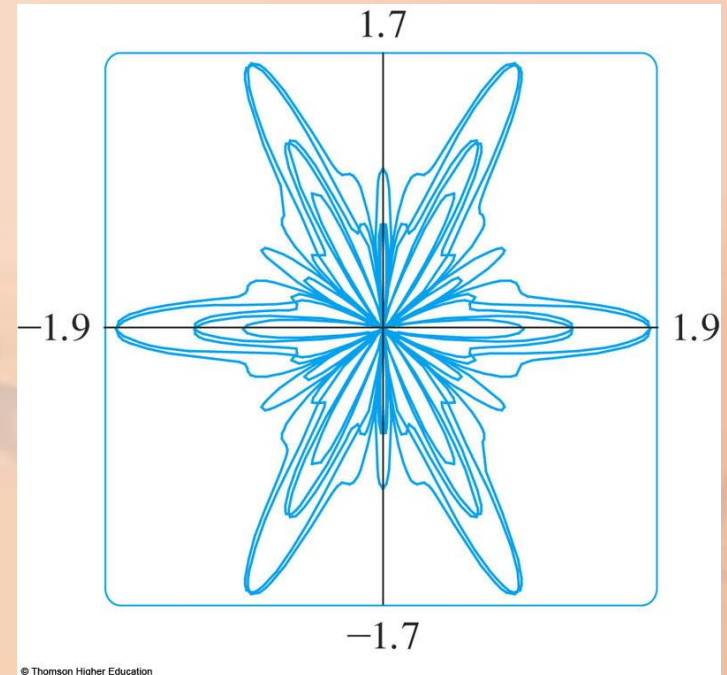
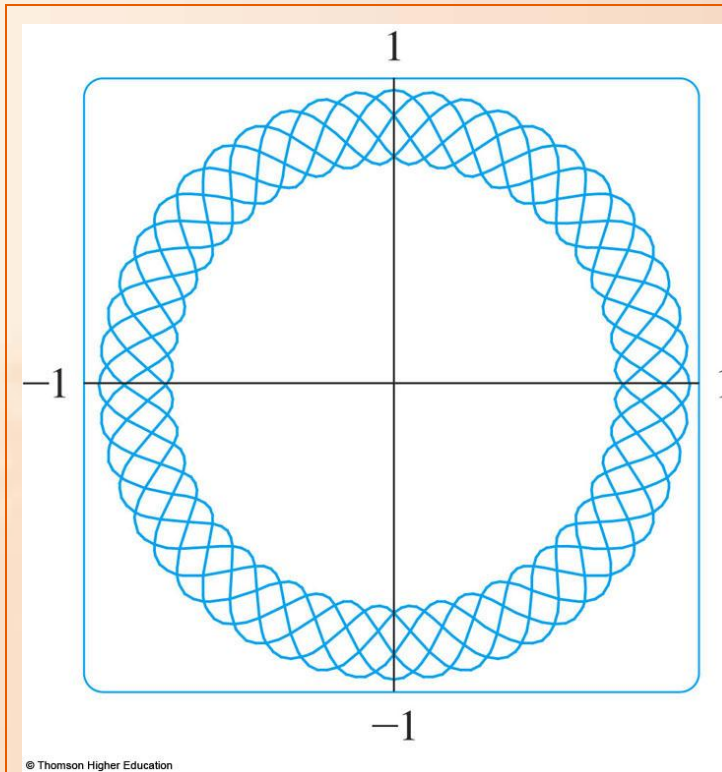
which is equivalent to our previous expression.

GRAPHING POLAR CURVES

It's useful to be able to sketch simple polar curves by hand.

GRAPHING POLAR CURVES

However, we need to use a graphing calculator or computer when faced with curves as complicated as shown.



GRAPHING POLAR CURVES WITH GRAPHING DEVICES

Some graphing devices have commands that enable us to graph polar curves directly.

With other machines, we need to convert to parametric equations first.

GRAPHING WITH DEVICES

In this case, we take the polar equation $r = f(\theta)$ and write its parametric equations as:

$$x = r \cos \theta = f(\theta) \cos \theta$$

$$y = r \sin \theta = f(\theta) \sin \theta$$

- Some machines require that the parameter be called t rather than θ .

Graph the curve $r = \sin(8\theta / 5)$.

- Let's assume that our graphing device doesn't have a built-in polar graphing command.

In this case, we need to work with the corresponding parametric equations, which are:

$$x = r \cos \theta = \sin(8\theta / 5) \cos \theta$$

$$y = r \sin \theta = \sin(8\theta / 5) \sin \theta$$

- In any case, we need to determine the domain for θ .

So, we ask ourselves:

- How many complete rotations are required until the curve starts to repeat itself ?

If the answer is n , then

$$\begin{aligned}\sin \frac{8(\theta + 2n\pi)}{5} &= \sin \left(\frac{8\theta}{5} + \frac{16n\pi}{5} \right) \\ &= \sin \frac{8\theta}{5}\end{aligned}$$

- So, we require that $16n\pi/5$ be an even multiple of π .

This will first occur when $n = 5$.

- Hence, we will graph the entire curve if we specify that $0 \leq \theta \leq 10\pi$.

Switching from θ to t ,
we have the equations

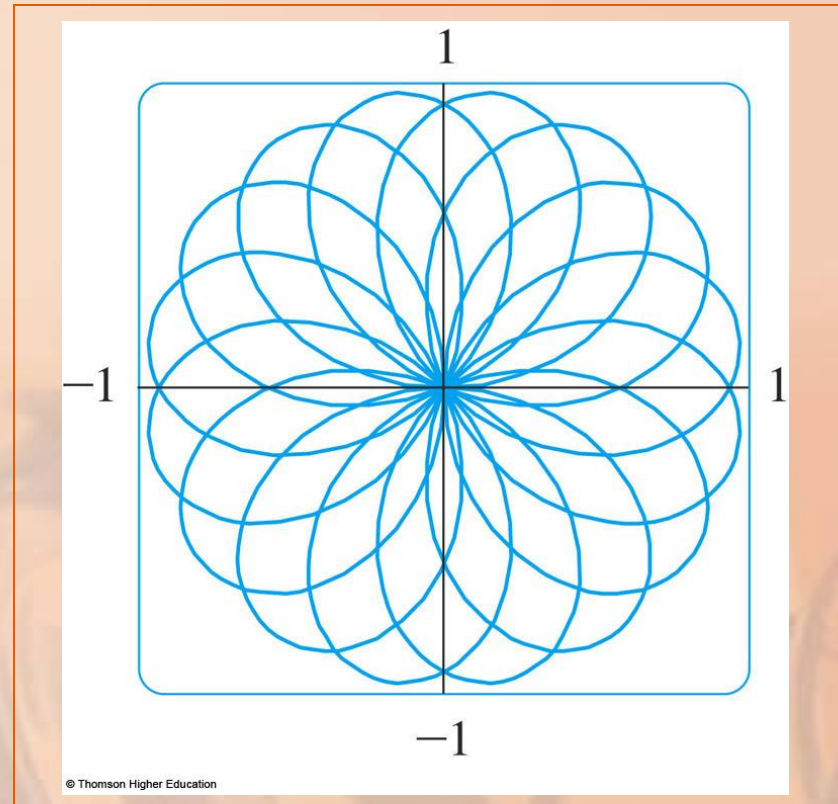
$$x = \sin(8t / 5) \cos t$$

$$y = \sin(8t / 5) \sin t$$

$$0 \leq t \leq 10\pi$$

This is the resulting curve.

- Notice that this rose has 16 loops.

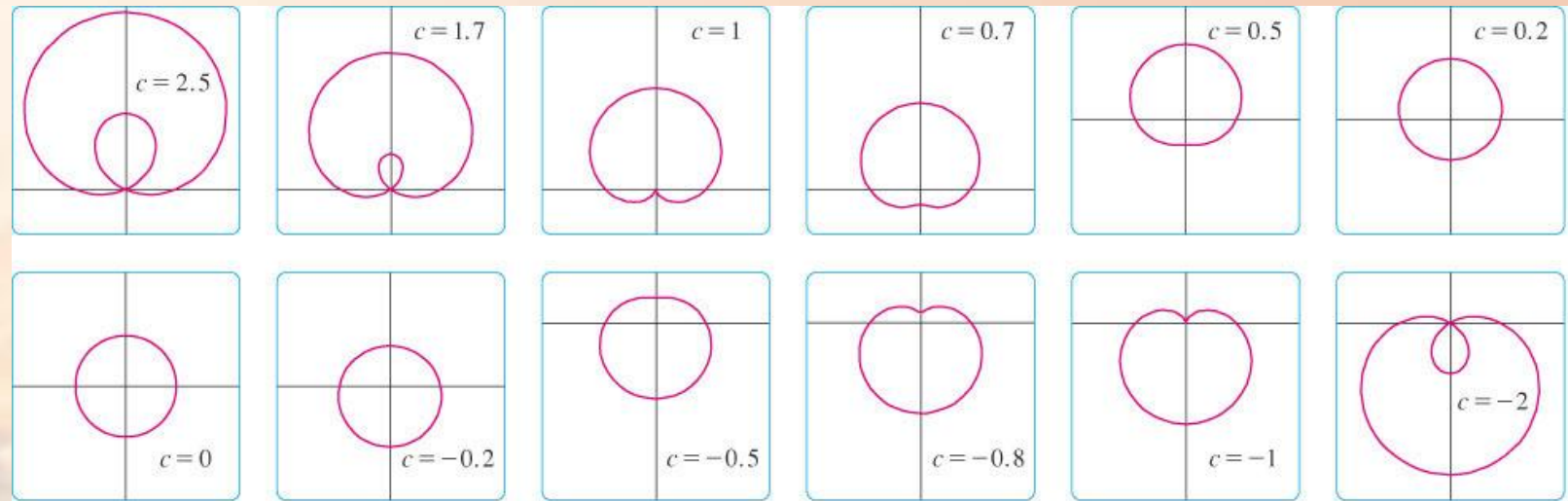


Investigate the family of polar curves given by $r = 1 + c \sin \theta$.

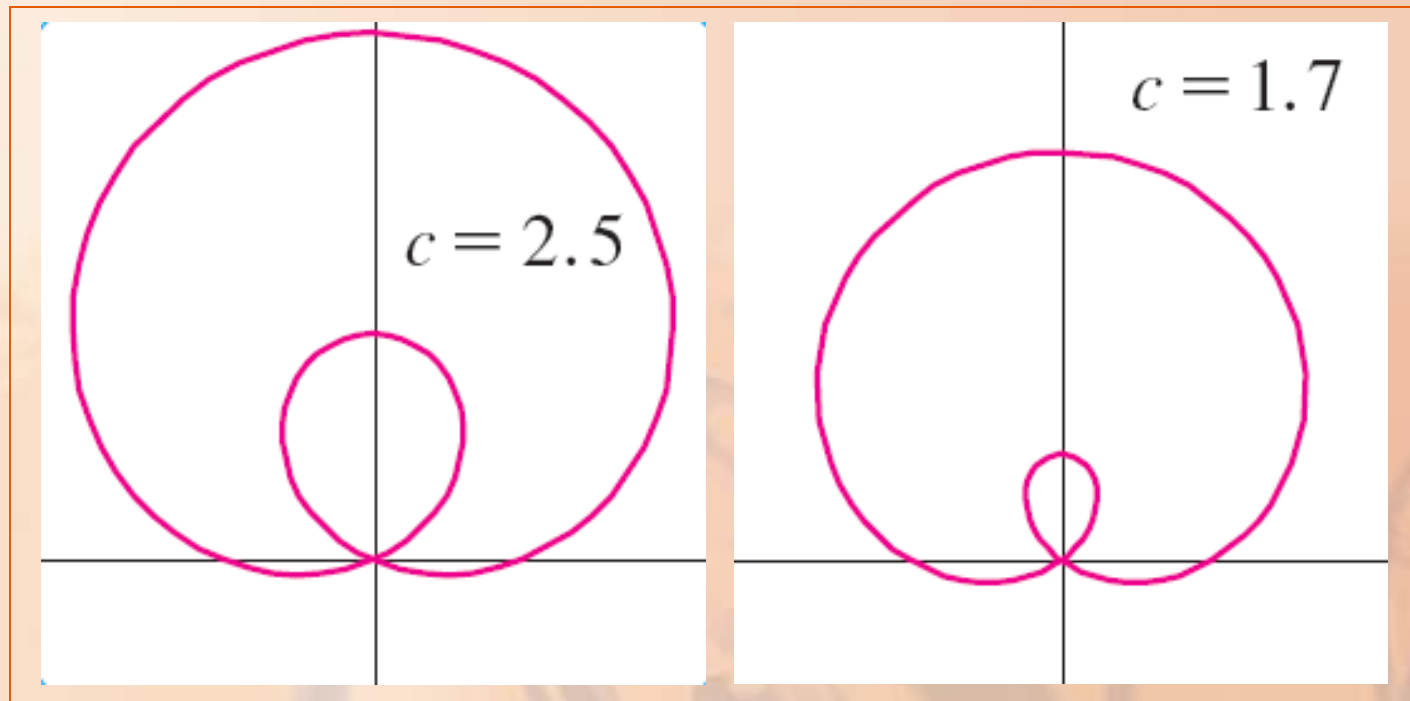
How does the shape change as c changes?

- These curves are called limaçons—after a French word for snail, because of the shape of the curves for certain values of c .

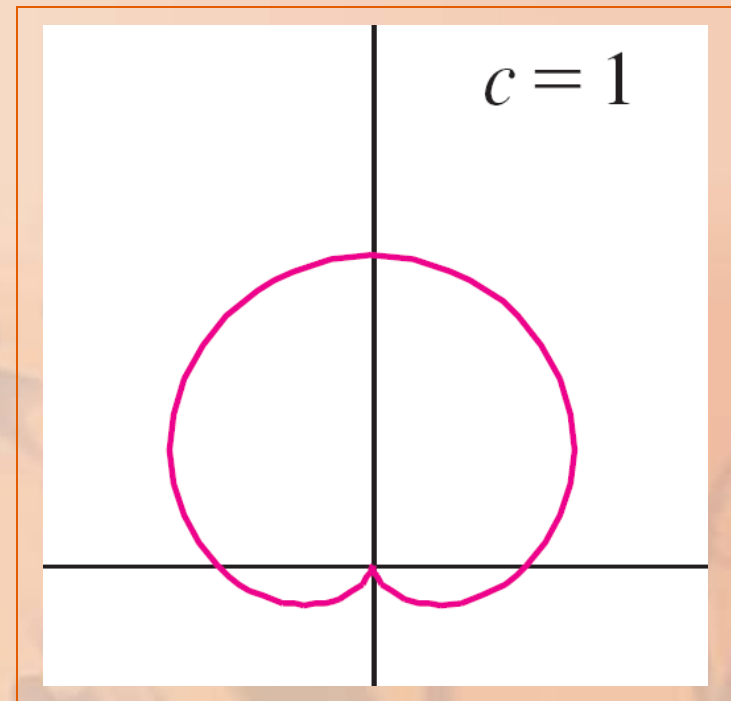
The figures show computer-drawn graphs for various values of c .



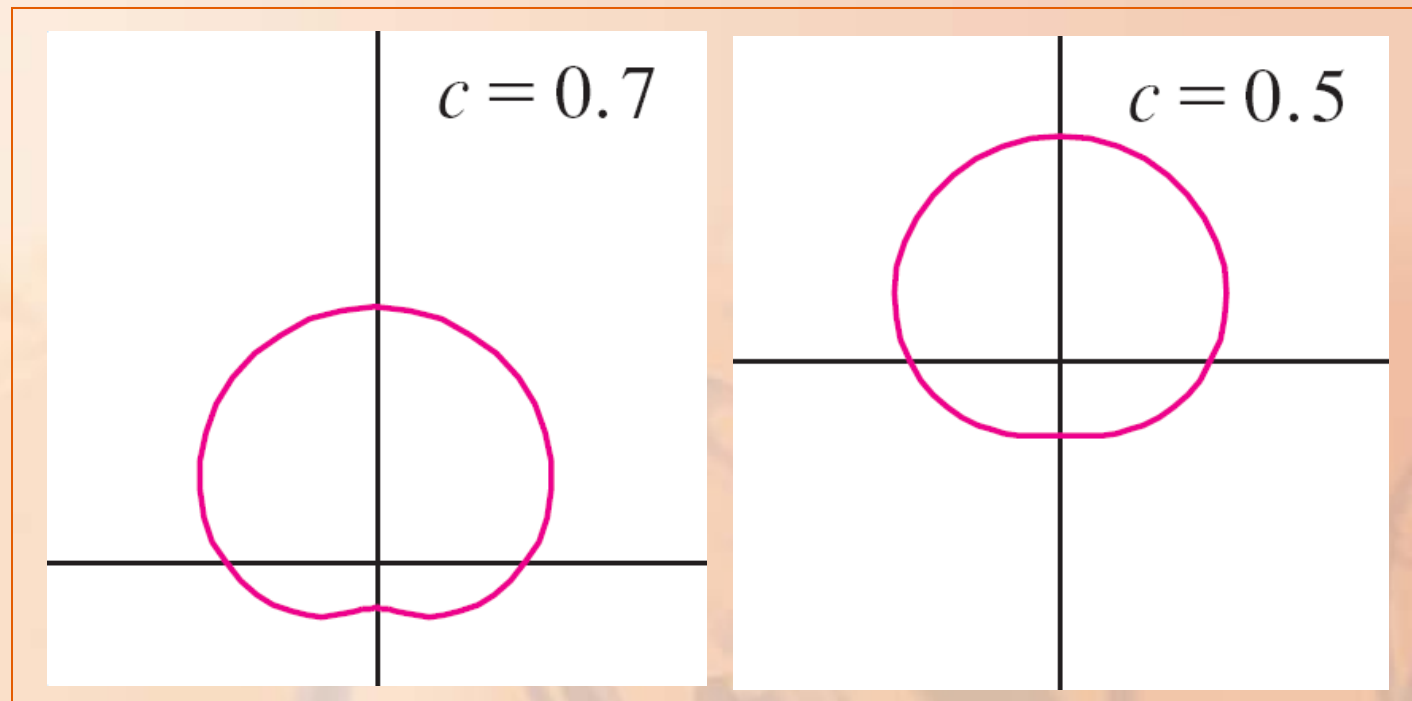
For $c > 1$, there is a loop that decreases in size as c decreases.



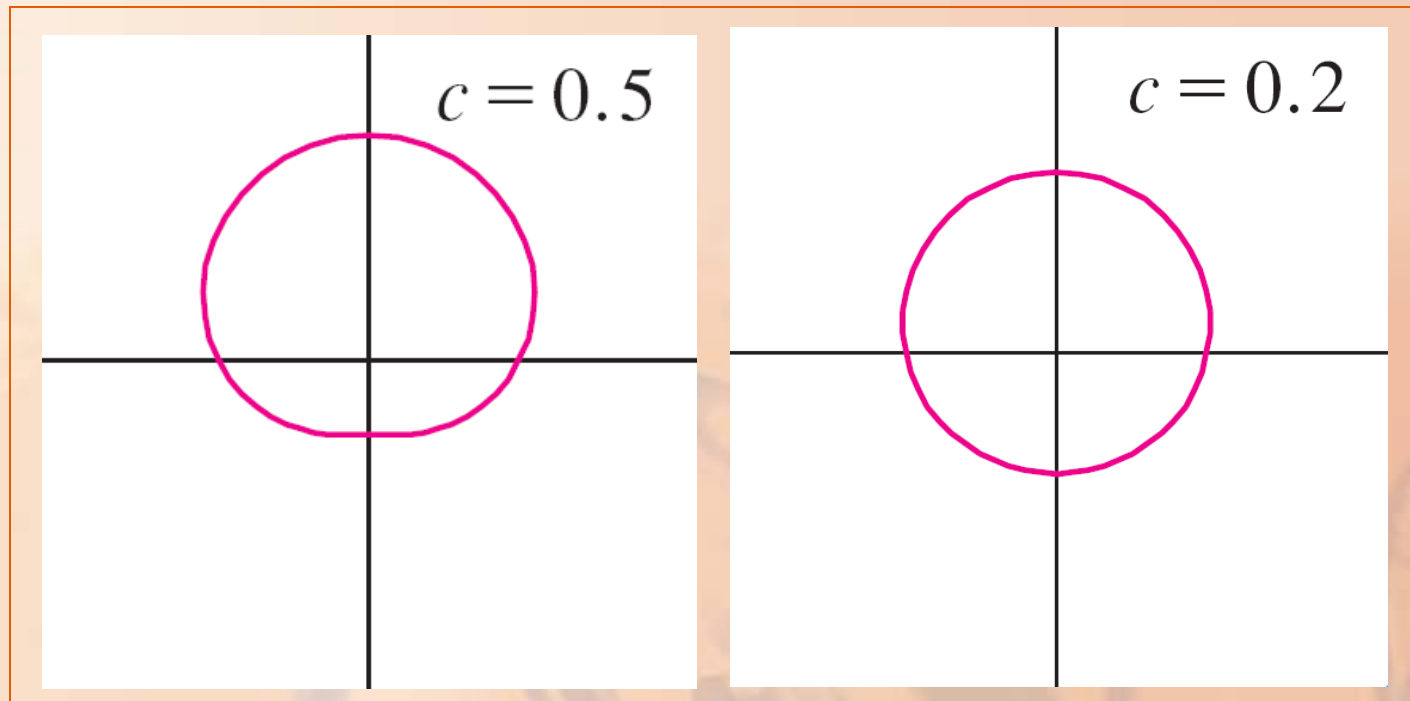
When $c = 1$, the loop disappears and the curve becomes the cardioid that we sketched in Example 7.



For c between 1 and $\frac{1}{2}$, the cardioid's cusp is smoothed out and becomes a “dimple.”



When c decreases from $\frac{1}{2}$ to 0, the limaçon is shaped like an oval.

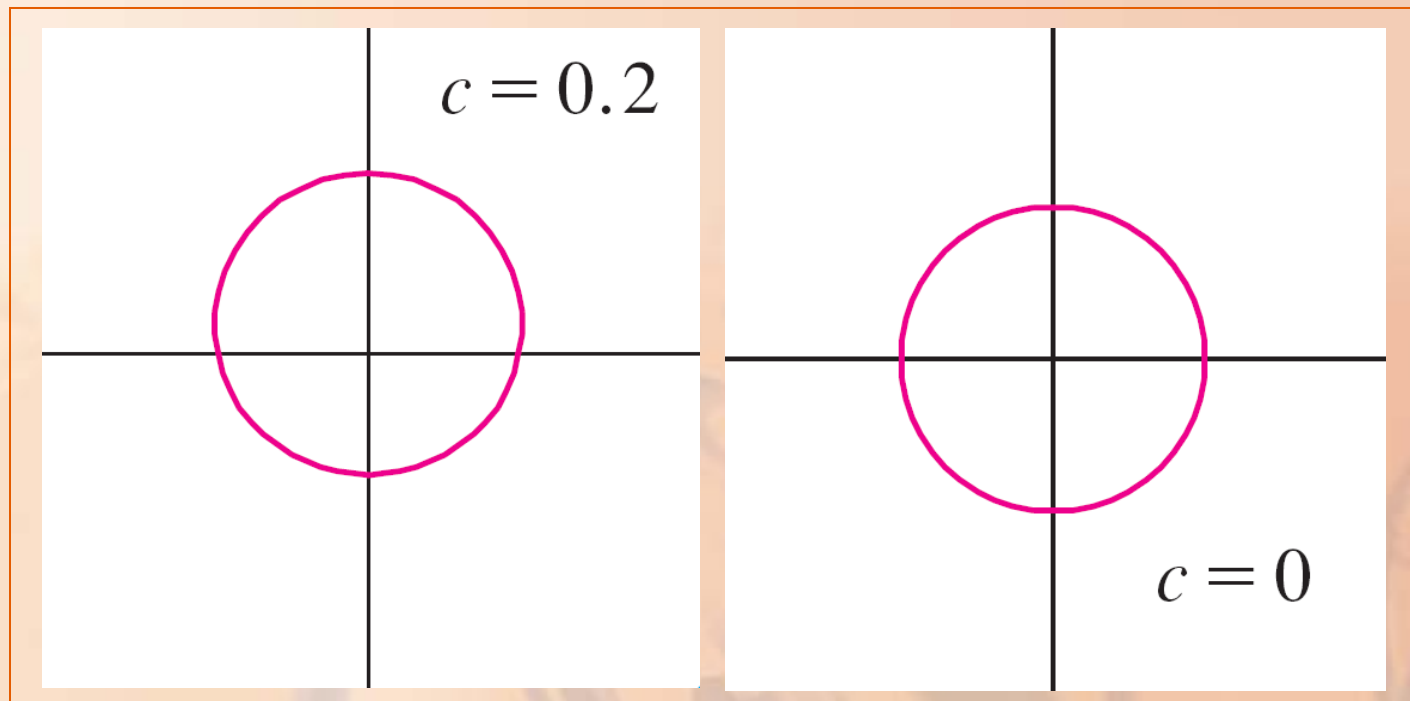


GRAPHING WITH DEVICES

Example 11

This oval becomes more circular as $c \rightarrow 0$.

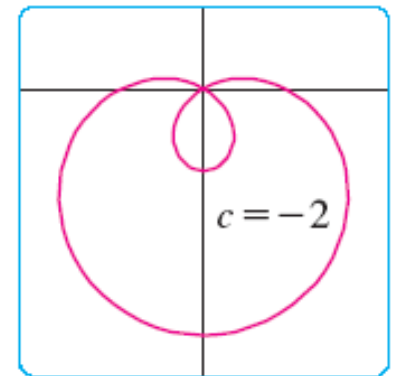
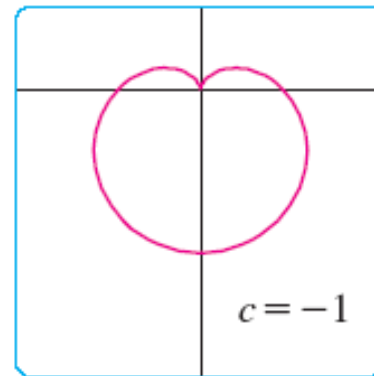
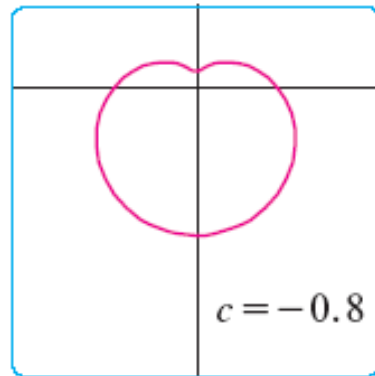
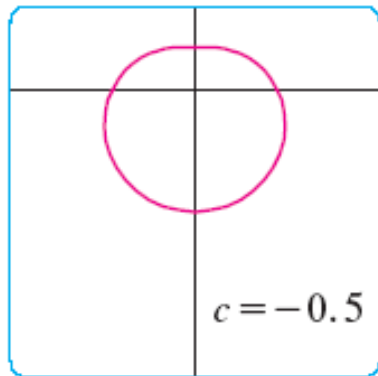
When $c = 0$, the curve is just the circle $r = 1$.



GRAPHING WITH DEVICES

Example 11

The remaining parts show that, as c becomes negative, the shapes change in reverse order.



GRAPHING WITH DEVICES

Example 11

In fact, these curves are reflections about the horizontal axis of the corresponding curves with positive c .

