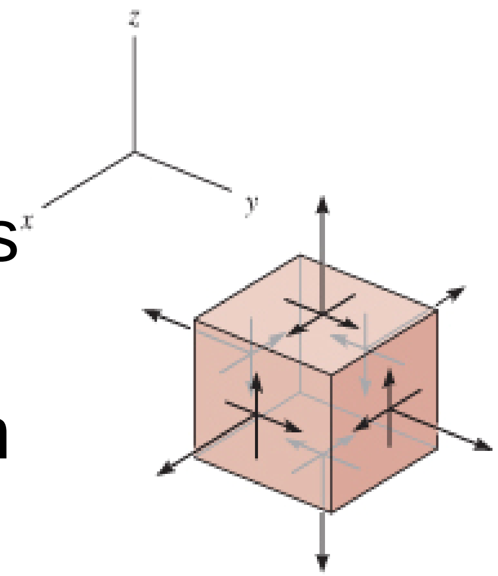
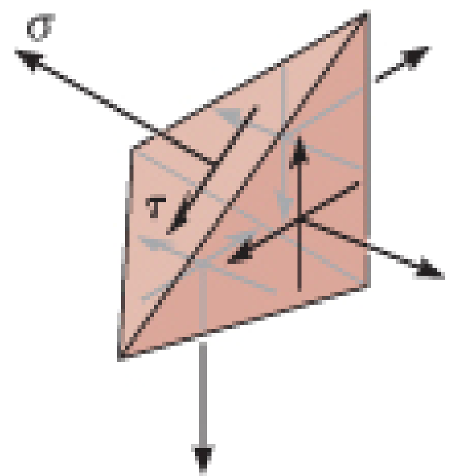
9.7 ABSOLUTE MAXIMUM SHEAR STRESS

- A pt in a body subjected to a general 3-D state of stress will have a normal stress and 2 shear-stress components acting on each of its faces.
- We can develop stress-transformation equations to determine the normal and shear stress components acting on ANY skewed plane of the element.



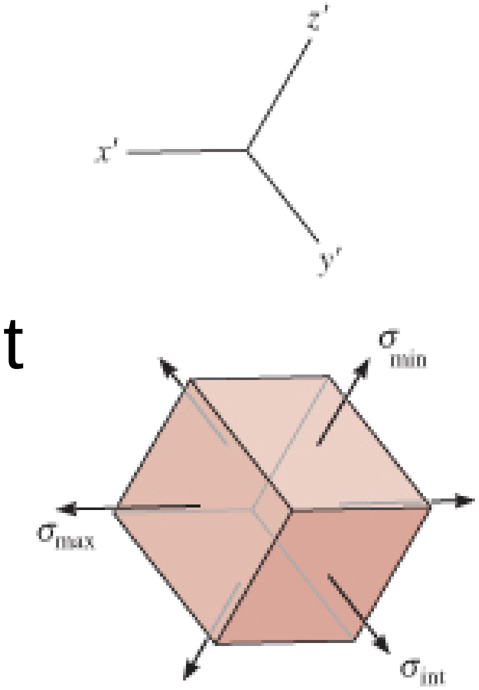
(a)



(b)

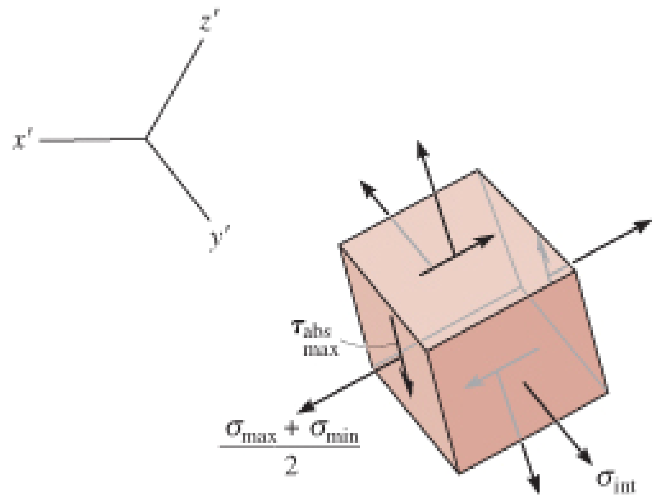
9.7 ABSOLUTE MAXIMUM SHEAR STRESS

- These principal stresses are assumed to have maximum, intermediate and minimum intensity: $\sigma_{max} \geq \sigma_{int} \geq \sigma_{min}$.
- Assume that orientation of the element and principal stress are known, thus we have a condition known as triaxial stress.



triaxial stress

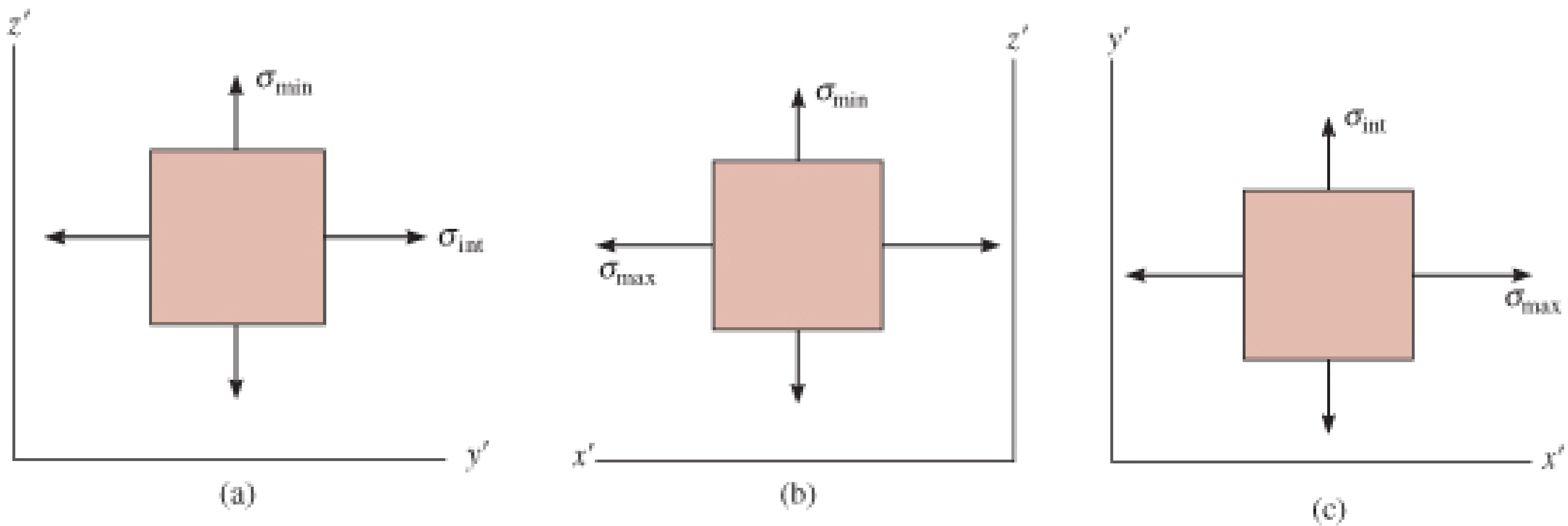
(c)



(d)

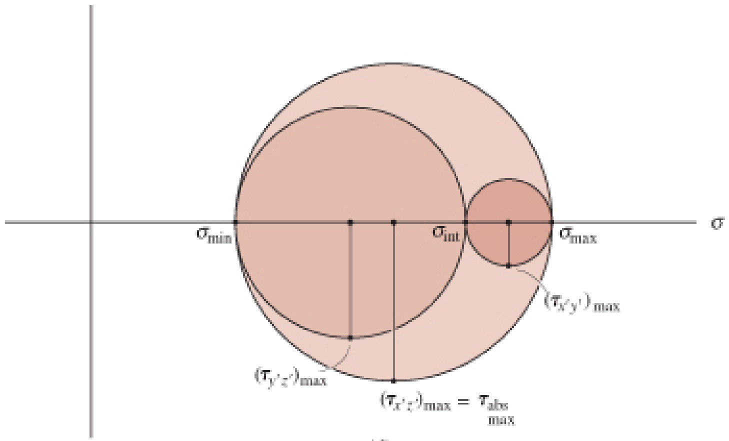
9.7 ABSOLUTE MAXIMUM SHEAR STRESS

- Viewing the element in 2D (y'-z', x'-z', x'-y') we then use Mohr's circle to determine the maximum in-plane shear stress for each case.

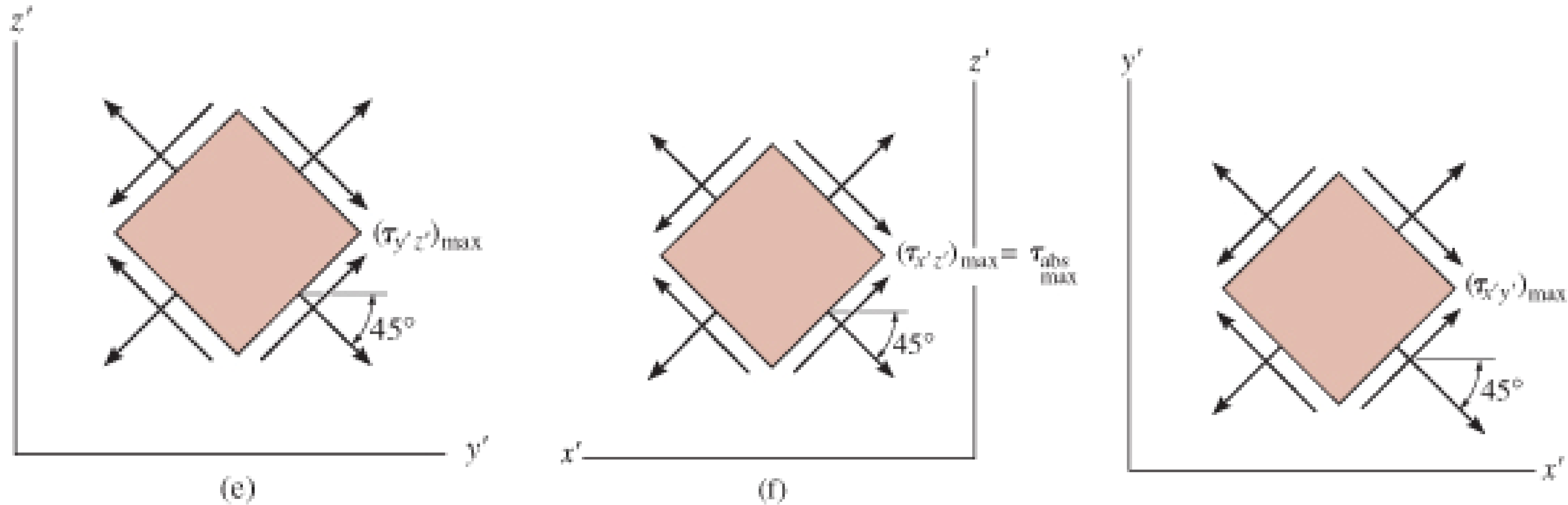


9.7 ABSOLUTE MAXIMUM SHEAR STRESS

- As shown, the element have a 45° orientation and is subjected to maximum in-plane shear and average normal stress components.



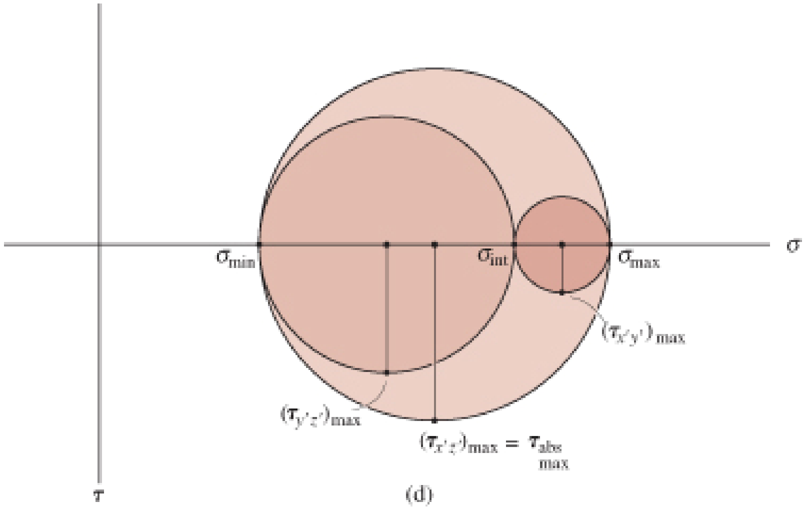
(d)



(e) (f) (g)

9.7 ABSOLUTE MAXIMUM SHEAR STRESS

- Comparing the 3 circles, we see that the absolute maximum shear stress $\tau_{\substack{abs \\ max}}$ is defined by the circle having the largest radius.



- This condition can also be determined directly by choosing the maximum and minimum principal stresses:

$$\tau_{\substack{abs \\ max}} = \frac{\sigma_{max} - \sigma_{min}}{2} \qquad (9\text{-}13)$$

9.7 ABSOLUTE MAXIMUM SHEAR STRESS

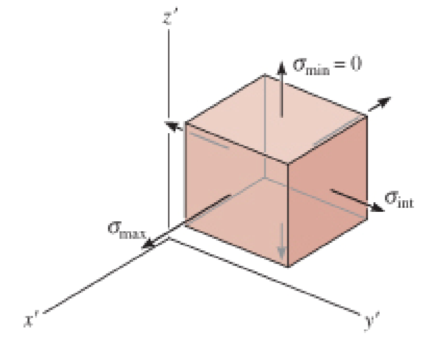
- Associated average normal stress

$$\sigma_{\text{avg}} = \frac{\sigma_{\max} + \sigma_{\min}}{2} \qquad (9\text{-}14)$$

- We can show that regardless of the orientation of the plane, specific values of shear stress τ on the plane is always less than absolute maximum shear stress found from Eqn 9-13.
- The normal stress acting on any plane will have a value lying between maximum and minimum principal stresses, $\sigma_{\max} \geq \sigma \geq \sigma_{\min}$.

9.7 ABSOLUTE MAXIMUM SHEAR STRESS

Plane stress

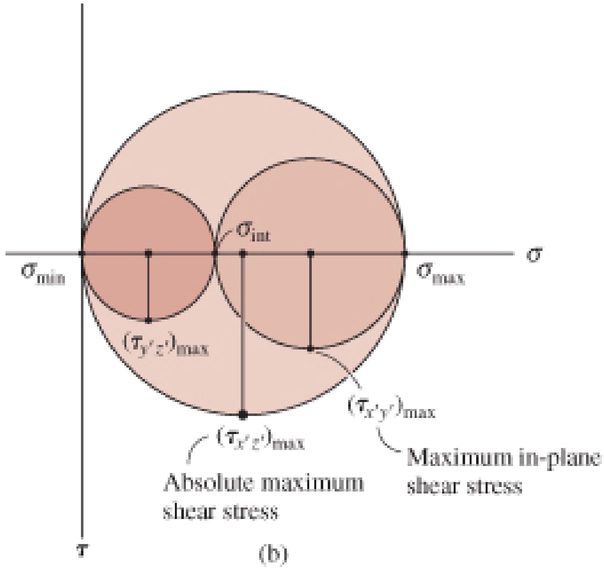


x'–y' plane stress

(a)

- Consider a material subjected to plane stress such that the in-plane principal stresses are represented as σ_{max} and σ_{int}, in the x' and y' directions respectively; while the out-of-plane principal stress in the z' direction is $\sigma_{min} = 0$.
- By Mohr's circle and Eqn. 9-13,

$$\tau_{\substack{abs \\ max}} = (\tau_{x'z'})_{max} = \frac{\sigma_{max}}{2} \qquad (9\text{-}15)$$



(b)

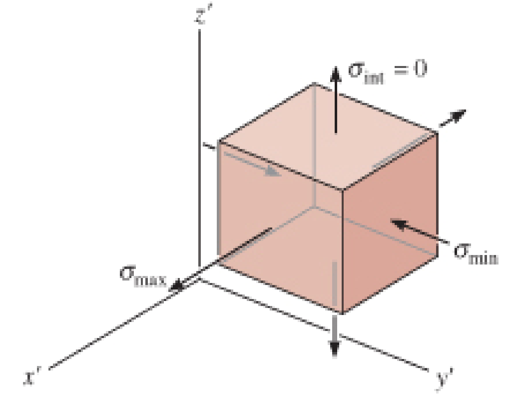
9.7 ABSOLUTE MAXIMUM SHEAR STRESS

Plane stress

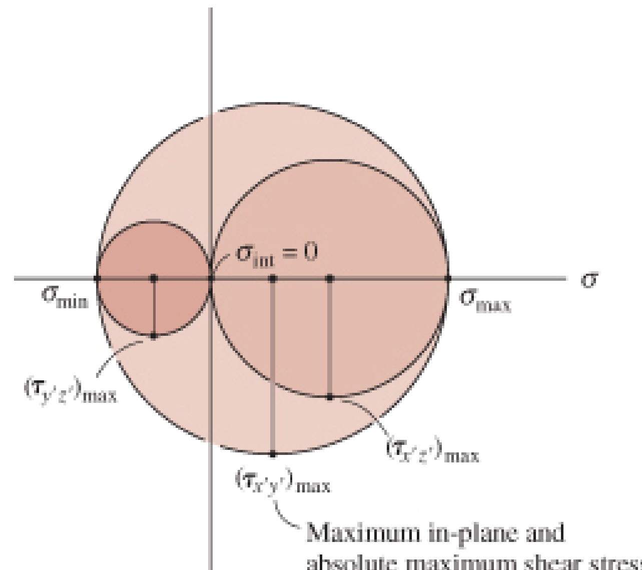
- If one of the principal stresses has an opposite sign of the other, then these stresses are represented as σ_{max} and σ_{min}, and out-of-plane principal stress $\sigma_{int} = 0$.
- By Mohr's circle and Eqn. 9-13,

$$\tau_{\substack{abs\\max}} = \left(\tau_{x'y'}\right)_{max}$$

$$= \frac{\sigma_{max} - \sigma_{min}}{2} \qquad (9\text{-}16)$$



x′–y′ plane stress



(b)

9.7 ABSOLUTE MAXIMUM SHEAR STRESS

IMPORTANT

- The general 3-D state of stress at a pt can be represented by an element oriented so that only three principal stresses act on it.
- From this orientation, orientation of element representing the absolute maximum shear stress can be obtained by rotating element 45° about the axis defining the direction of int.
- If in-plane principal stresses both have the same sign, the absolute maximum shear stress occurs out of the plane, and has a value of $\tau_{\text{abs max}} = \sigma_{\max}/2$

9.7 ABSOLUTE MAXIMUM SHEAR STRESS

IMPORTANT

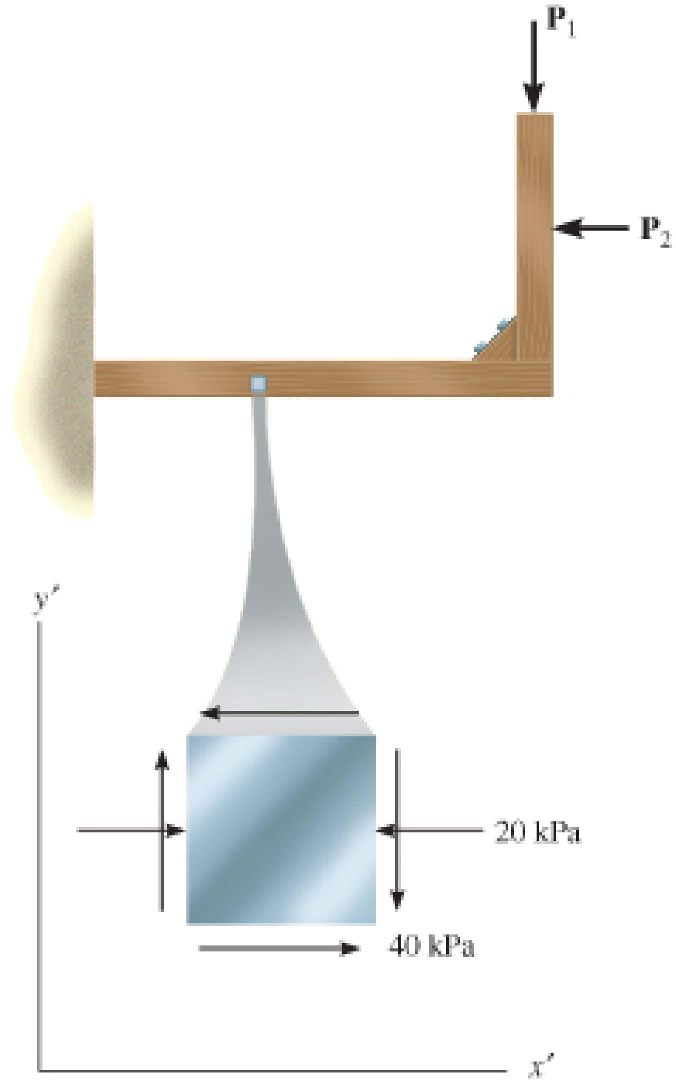
- If in-plane principal stresses are of opposite signs, the absolute maximum shear stress equals the maximum in-plane shear stress; that is

$$\tau_{\substack{\text{abs}\\ \text{max}}} = (\sigma_{\max} - \sigma_{\min})/2$$

EXAMPLE 9.14

Due to applied loading, element at the pt on the frame is subjected to the state of plane stress shown.

Determine the principal stresses and absolute maximum shear stress at the pt.



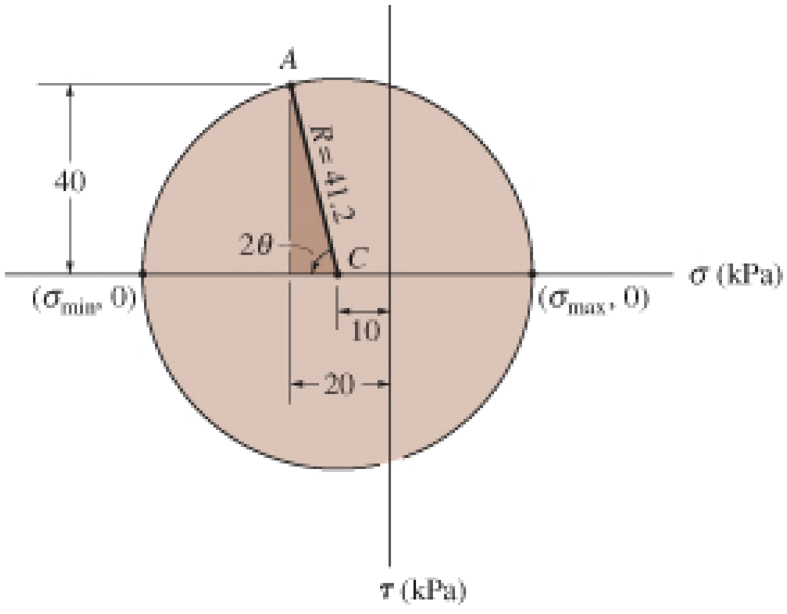
(a)

EXAMPLE 9.14 (SOLN)

Principal stresses

The in-plane principal stresses can be determined from Mohr's circle. Center of circle is on the axis at σ_{avg} = (–20 + 20)/2 = –10 kPa. Plotting controlling pt A (–20, –40), circle can be drawn as shown. The radius is

$$R = \sqrt{(20-10)^2 + (40)^2}$$
$$= 41.2 \text{ kPa}$$



(b)

EXAMPLE 9.14 (SOLN)

Principal stresses

The principal stresses at the pt where the circle intersects the σ-axis:

$$\sigma_{\max} = -10 + 41.2 = 31.2 \text{ kPa}$$

$$\sigma_{\min} = -10 - 41.2 = -51.2 \text{ kPa}$$

From the circle, counterclockwise angle 2θ, measured from the CA to the $-\sigma$ axis is,

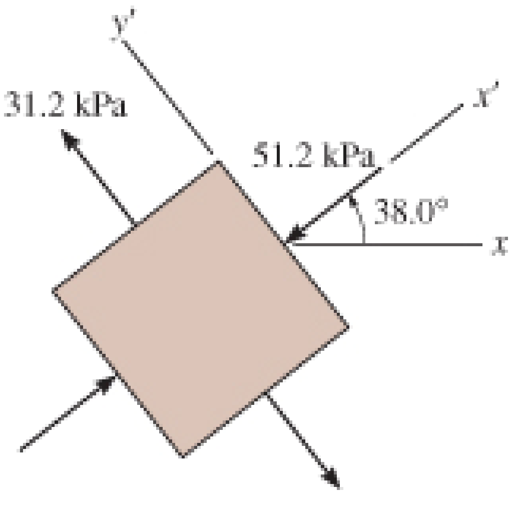
$$2\theta = \tan^{-1}\left(\frac{40}{20-10}\right) = 76.0°$$

Thus, $\theta = 38.0°$

EXAMPLE 9.14 (SOLN)

Principal stresses

This counterclockwise rotation defines the direction of the x' axis or min and its associated principal plane. Since there is no principal stress on the element in the z direction, we have



(c)

$$\sigma_{\max} = 31.2 \text{ kPa}$$

$$\sigma_{\text{int}} = 0$$

$$\sigma_{\min} = -51.2 \text{ kPa}$$

EXAMPLE 9.14 (SOLN)

Absolute maximum shear stress

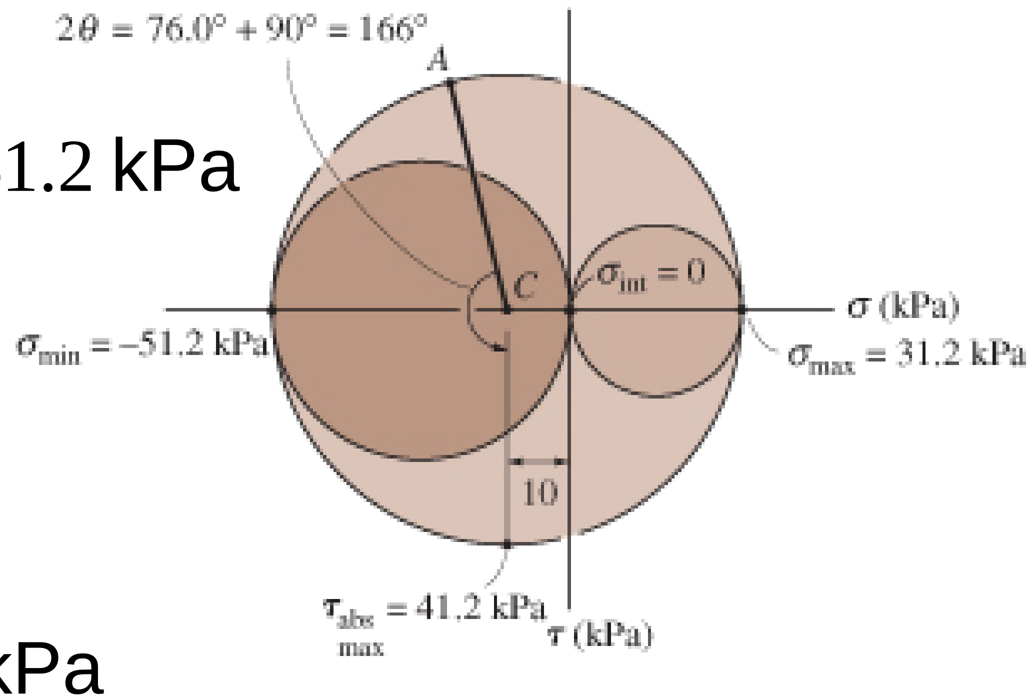
Applying Eqns. 9-13 and 9-14,

$$\tau_{\substack{\text{abs}\\ \text{max}}} = \frac{\sigma_{\text{max}} - \sigma_{\text{min}}}{2}$$

$$= \frac{31.2 - (-51.2))}{2} = 41.2 \text{ kPa}$$

$$\sigma_{\text{avg}} = \frac{\sigma_{\text{max}} + \sigma_{\text{min}}}{2}$$

$$= \frac{31.2 - 51.2}{2} = -10 \text{ kPa}$$

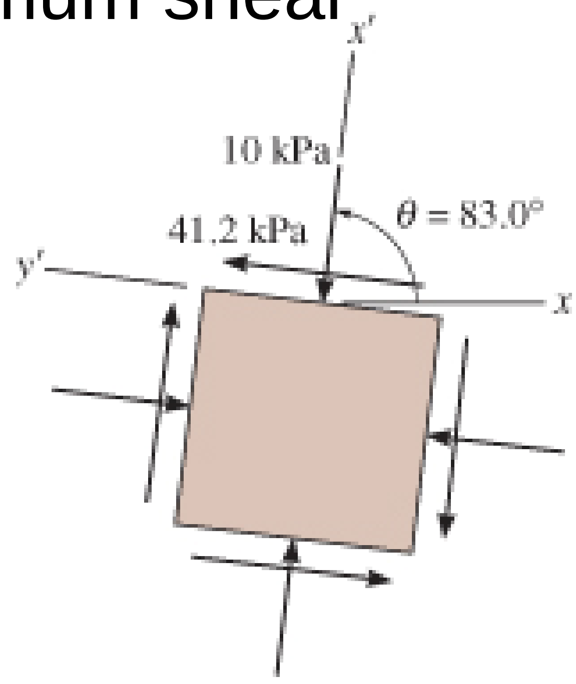


(d)

EXAMPLE 9.14 (SOLN)

Absolute maximum shear stress

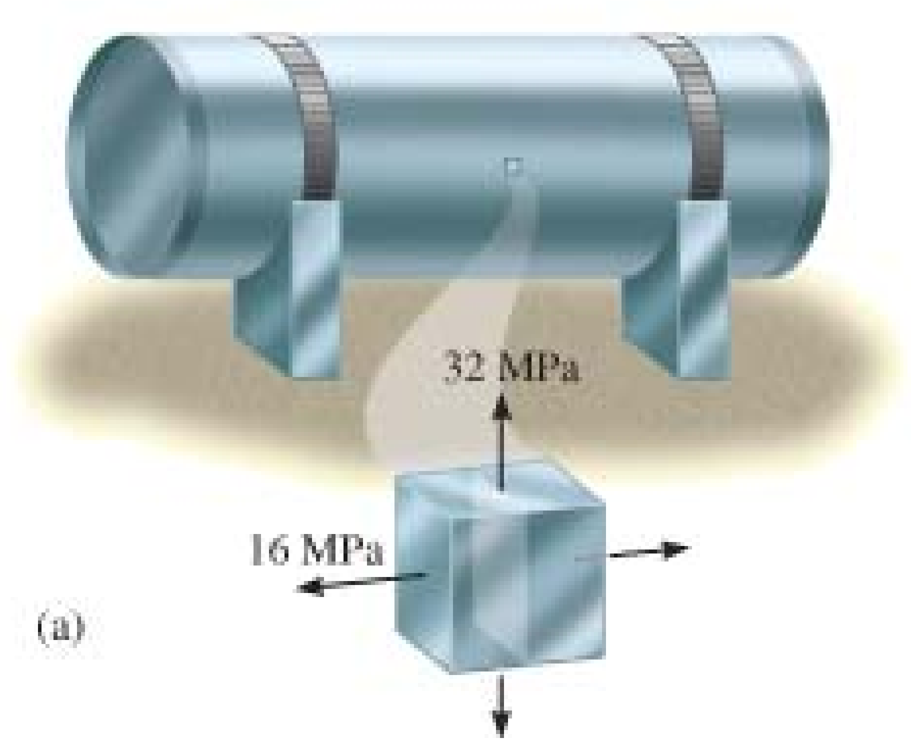
These same results can be obtained by drawing Mohr's circle for each orientation of an element about the x', y', and z' axes. Since σ_{max} and σ_{min} are of opposite signs, then the absolute maximum shear stress equals the maximum in-plane shear stress. This results from a 45° rotation of the element about the z' axis, so that the properly oriented element is shown.



(e)

EXAMPLE 9.15

The pt on the surface of the cylindrical pressure vessel is subjected to the state of plane stress. Determine the absolute maximum shear stress at this pt.



(a)

EXAMPLE 9.15 (SOLN)

Principal stresses are σ_{max} = 32 MPa, σ_{int} = 16 MPa, and σ_{min} = 0. If these stresses are plotted along the axis, the 3 Mohr's circles can be constructed that describe the stress state viewed in each of the three perpendicular planes.

The largest circle has a radius of 16 MPa and describes the state of stress in the plane containing σ_{max} = 32 MPa and σ_{min} = 0.

An orientation of an element 45° within this plane yields the state of absolute maximum shear stress and the associated average normal stress, namely,

EXAMPLE 9.15 (SOLN)

An orientation of an element 45° within this plane yields the state of absolute maximum shear stress and the associated average normal stress, namely,

$$\tau_{\substack{\text{abs}\\ \text{max}}} = 16\text{ MPa} \qquad \sigma_{\text{avg}} = 16\text{ MPa}$$

Or we can apply Eqns 9-13 and 9-14:

$$\tau_{\substack{\text{abs}\\ \text{max}}} = \frac{\sigma_{\max} - \sigma_{\min}}{2} = \frac{32-0}{2} = 16\text{ MPa}$$

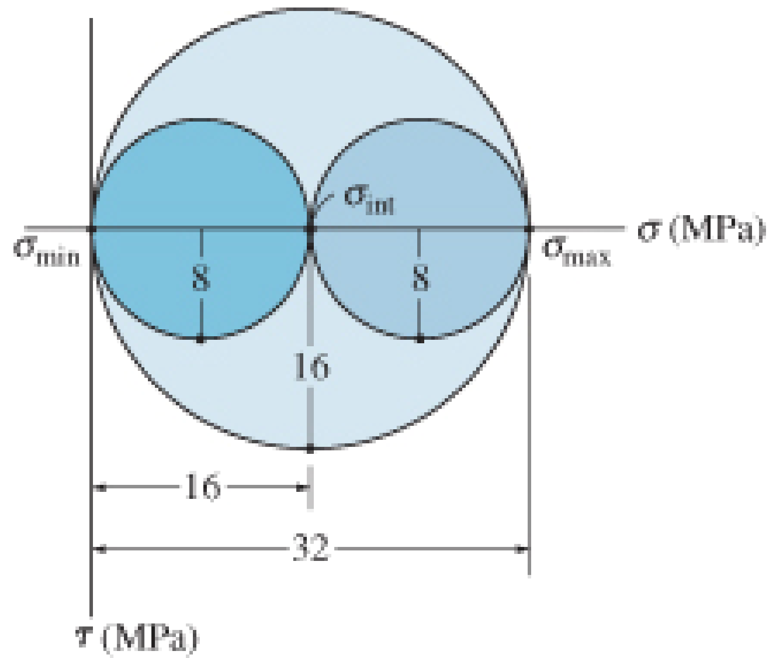
$$\sigma_{\text{avg}} = \frac{\sigma_{\max} + \sigma_{\min}}{2} = \frac{32+0}{2} = 16\text{ MPa}$$

EXAMPLE 9.15 (SOLN)

By comparison, maximum in-plane shear stress can be determined from the Mohr's circle drawn between σ_{max} = 32 MPa and σ_{int} = 16 MPa, which gives a value of

$$\tau_{\substack{abs\\max}} = \frac{32-16}{2} = 8\text{ MPa}$$

$$\sigma_{avg} = 16 + \frac{32-16}{2} = 24\text{ MPa}$$



(b)

CHAPTER REVIEW

- Plane stress occurs when the material at a pt is subjected to two normal stress components σ_x and σ_y and a shear stress τ_{xy}.
- Provided these components are known, then the stress components acting on an element having a different orientation can be determined using the two force equations of equilibrium or the equations of stress transformation.

CHAPTER REVIEW

- For design, it is important to determine the orientations of the element that produces the maximum principal normal stresses and the maximum in-plane shear stress.
- Using the stress transformation equations, we find that no shear stress acts on the planes of principal stress.
- The planes of maximum in-plane shear stress are oriented 45° from this orientation, and on these shear planes there is an associated average normal stress $(\sigma_x + \sigma y)/2$.

CHAPTER REVIEW

- Mohr's circle provides a semi-graphical aid for finding the stress on any plane, the principal normal stresses, and the maximum in-plane shear stress.
- To draw the circle, the σ and τ axes are established, the center of the circle $[(\sigma_x + \sigma_y)/2, 0]$, and the controlling pt (σ_x, τ_{xy}) are plotted.
- The radius of the circle extends between these two points and is determined from trigonometry.

CHAPTER REVIEW

- The absolute maximum shear stress will be equal to the maximum in-plane shear stress, provided the in-plane principal stresses have the opposite sign.
- If they are of the same sign, then the absolute maximum shear stress will lie out of plane. Its value is $\tau_{\max}^{abs} = (\sigma_{\max} - 0)/2$.