14.5 Geometry Factors *I* and *J* (Z_I and Y_J)

- The determination of *I* and *J* depends upon the ***face-contact ratio*** $\boldsymbol{m_F}$.
- This is defined as:

$$m_F = \frac{F}{p_x} \qquad (14\text{-}19)$$

- Where P_x is the axial pitch and *F* is the face width.
- For spur gears, $m_F = 0$
- Low-contact-ratio (LCR) helical gears having a small helix angle or a thin face width, or both, have face-contact ratios less than unity ($m_F \leq 1$), and will not be considered here.

- Such gears have a noise level not too different from that for spur gears.

- ***Consequently we shall consider here only:***

  Spur gears with $m_F = 0$

  Conventional helical gears with $m_F > 1$.

Bending-Strength Geometry Factor J (Y_J)

- The AGMA factor J formula is:

$$J = \frac{Y}{K_f m_N} \qquad (14\text{-}20)$$

- Where
- Y = ***a modified value of the Lewis form factor***
- K_f = ***fatigue stress-concentration factor***
- m_N = ***tooth load-sharing ratio***

- **It is important to note that** the form factor Y in equation (14-20) is not the Lewis factor at all. The value of Y here is obtained from a generated layout of the tooth profile in the normal plane and is based on the highest point of single-tooth contact.
- K_f can be calculated from equation (14-9)
- The load –sharing ratio $\boldsymbol{m_N}$ is equal to **the face width divided by the minimum total length of the lines of contact.**

- Factor m_N depends on:
  - The transverse contact ratio m_p
  - The face-contact ratio m_F
  - The effects of any profile modifications
  - The tooth deflection

- For spur gears, $m_N = 1.0$
- For helical gears having a face-contact ratio $m_F > 2.0$, a conservative approximation is given by the equation

$$m_N = \frac{P_N}{0.95Z} \qquad (14\text{-}21)$$

- where P_N is the normal base pitch and Z is the length of the line of action in the transverse plane (distance L_{ab} in figure 13-15 page 684)
- In our study we can obtain the geometry factor J for:
- For Spur gears having a 20^o pressure angle and full-depth teeth: from figure 14-6.

Figure 14-6: Spur-gear geometry factors J having a 20^{o} normal pressure angle and and full depth teeth.

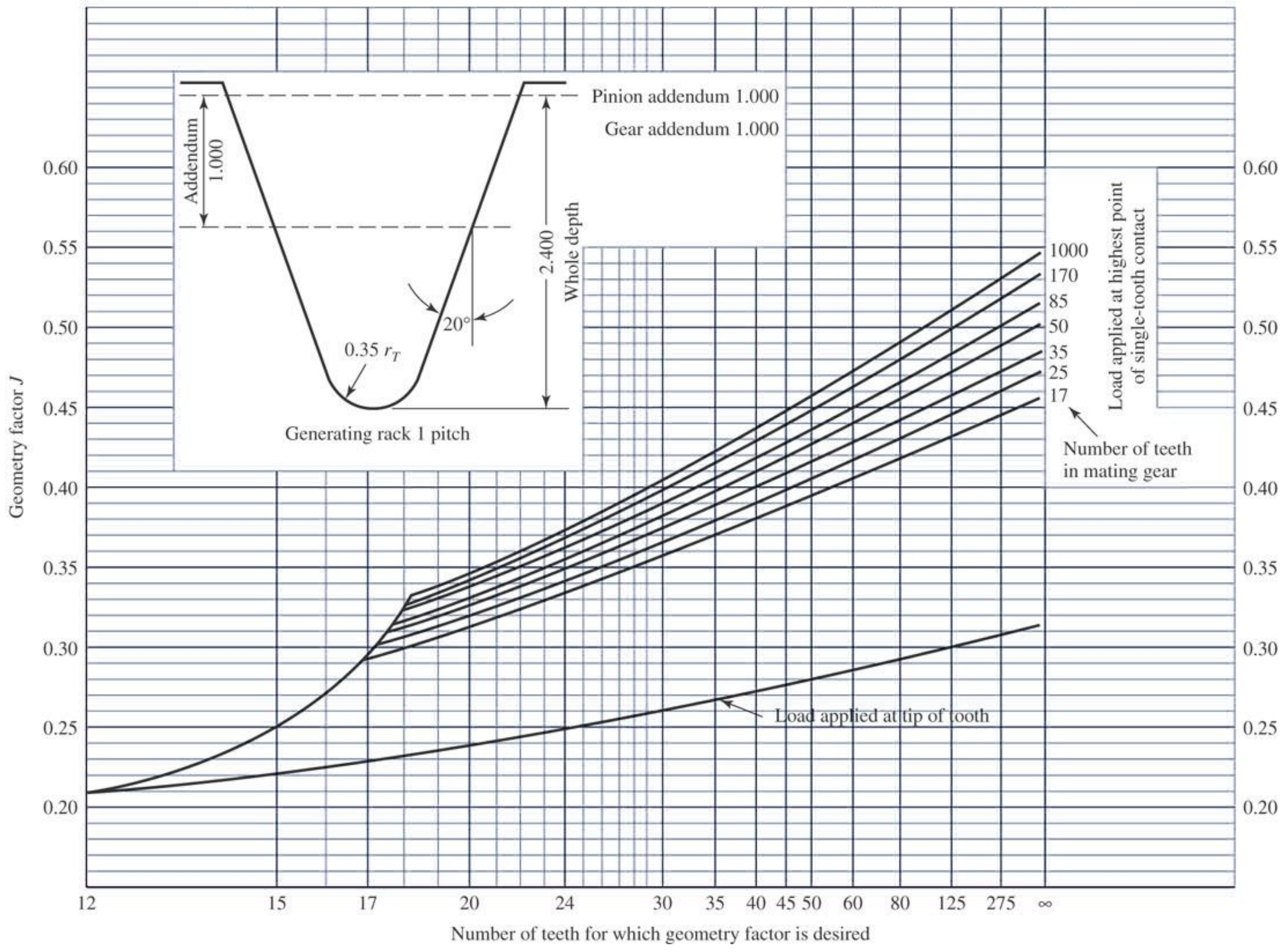


Figure 14-7: Helical-gear geometry factors J' having a 20° normal pressure angle and face contact ratio m_f=2.0 or greater

- For Helical gears having a 20° normal pressure angle and face-contact ratios of $m_F \geq 2$ from figure 14-7 and 14-8.

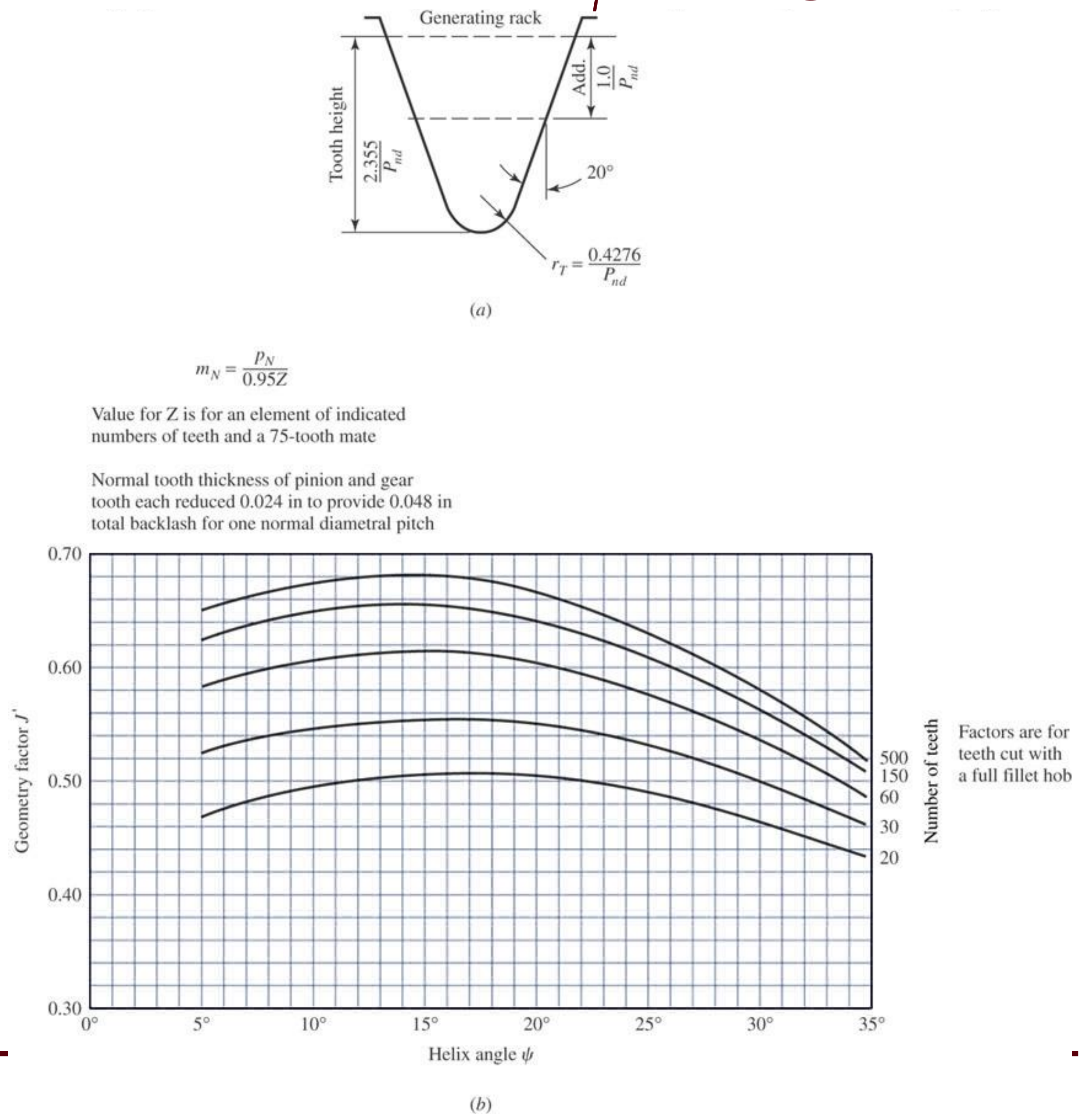
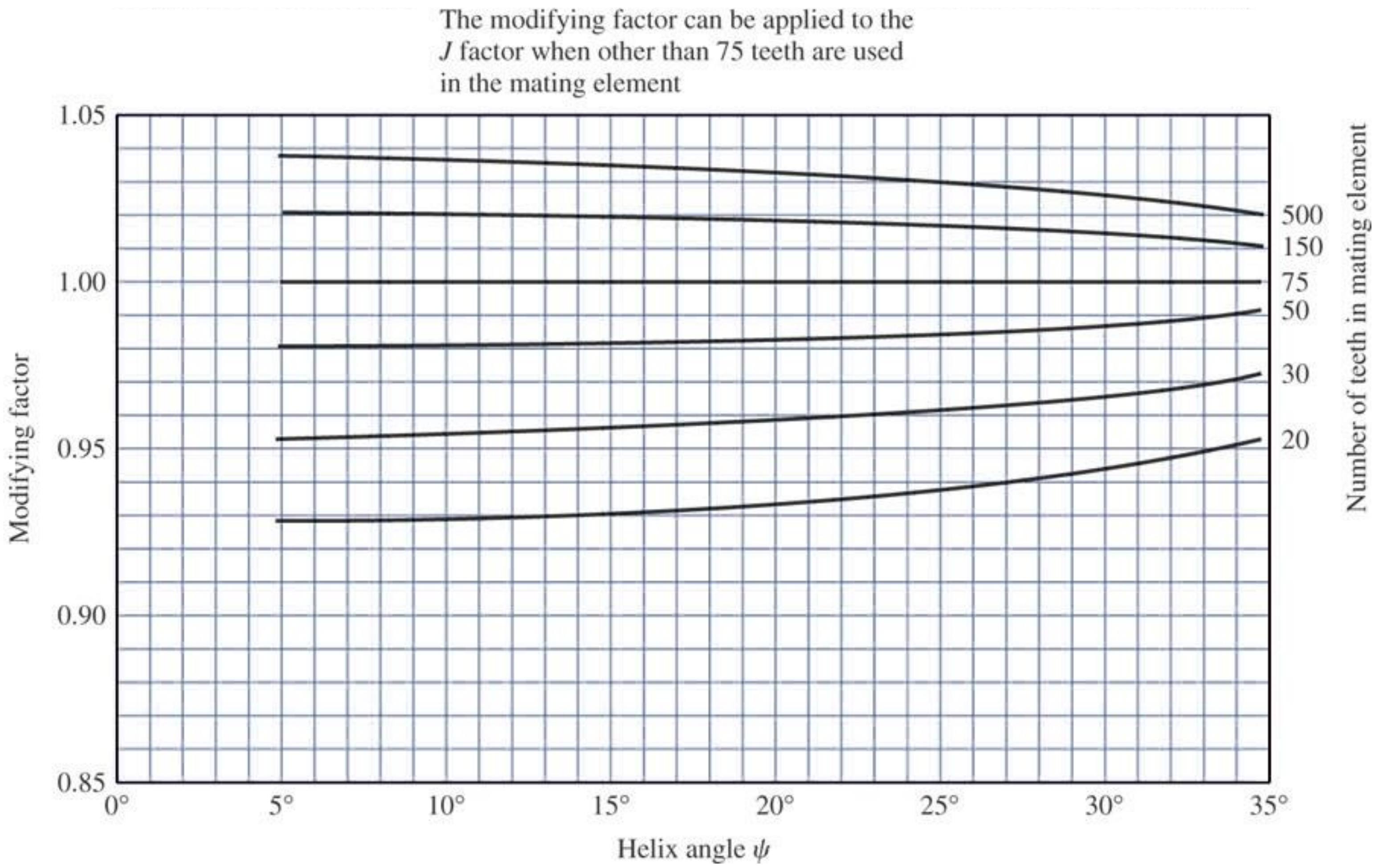


Figure 14-8: *J'* factor multipliers for use with fig. 14-7 to find *J*



Surface Strength Geometry Factor I (Z_I)

- The factor I is called the pitting-resistance geometry factor
- The sum of the reciprocals of equation (14-14) is:

$$\frac{1}{r_1} + \frac{1}{r_2}$$

Knowing that

$$r_1 = \frac{d_P \sin \phi}{2} \qquad r_2 = \frac{d_G \sin \phi}{2}$$

Thus

$$\frac{1}{r_1} + \frac{1}{r_2} = \frac{2}{\sin \phi_t}\left(\frac{1}{d_P} + \frac{1}{d_G}\right) \qquad \text{(a)}$$

- Here we have used the transverse pressure angle so that we can apply this relation the helical gear too.
- Defined the speed ratio m_G as:

$$m_G = \frac{N_G}{N_P} = \frac{d_G}{d_P} \qquad (14\text{-}22)$$

- Thus equation (a) can be written as:

$$\frac{1}{r_1} + \frac{1}{r_2} = \frac{2}{d_P \sin\phi_t} \frac{m_G + 1}{m_G} \qquad \text{(b)}$$

- Now substitute equation (b) in equation (14-14). The result is found to be:

$$\sigma_c = -\sigma_C = C_P \left[\frac{K_\upsilon W^t}{d_P F} \frac{1}{\dfrac{\cos\phi_t \sin\phi_t}{2} \dfrac{m_G}{m_G + 1}} \right]^{1/2} \qquad \text{(c)}$$

- The geometry factor *I* for external spur and helical gears is the denominator of the second term in the brackets in equation (c).

- By adding the load-shearing ratio m_N, we obtain a factor valid for both spur and helical gears. The equation is then written as:

$$I = \begin{cases} \dfrac{\cos\phi_t \sin\phi_t}{2m_N} \dfrac{m_G}{m_G + 1} & \text{external gears} \\ \dfrac{\cos\phi_t \sin\phi_t}{2m_N} \dfrac{m_G}{m_G - 1} & \text{internal gears} \end{cases} \qquad (14\text{-}23)$$

where $m_N = 1$ for spur gears

- In solving equation (14-21) for m_N, note that

$$p_N = p_n \cos\phi_n \qquad (14\text{-}24)$$

where p_n is the normal circular pitch.

- If the quantity Z. for use in equation (14-21), can be obtained from the equation:

$$Z = \left[(r_P + a)^2 - r_{bP}^2\right]^{1/2} + \left[(r_G + a)^2 - r_{bG}^2\right] - (r_P + r_G)\sin\phi_t \qquad (14\text{-}25)$$

Where r_P and r_G are the pitch radii and r_{bP} and r_{bG} the base-circle radii of the pinion and gear, respectively and the radius of the base circle is

$$r_b = r\cos\phi_t \qquad (14\text{-}26)$$

14.6 The Elastic Coefficient C_p (Z_E)

- To compute C_P use either:

  Equation 14-13 or Table 14-8

- Table 14-8: Elastic Coefficient C_P (Z_E),

Pinion Material	Pinion Modulus of Elasticity E_p psi (MPa)*	Gear Material and Modulus of Elasticity E_G, lbf/in² (MPa)*: Steel 30×10^6 (2×10^5)	Malleable Iron 25×10^6 (1.7×10^5)	Nodular Iron 24×10^6 (1.7×10^5)	Cast Iron 22×10^6 (1.5×10^5)	Aluminum Bronze 17.5×10^6 (1.2×10^5)	Tin Bronze 16×10^6 (1.1×10^5)
Steel	30×10^6 (2×10^5)	2300 (191)	2180 (181)	2160 (179)	2100 (174)	1950 (162)	1900 (158)
Malleable iron	25×10^6 (1.7×10^5)	2180 (181)	2090 (174)	2070 (172)	2020 (168)	1900 (158)	1850 (154)
Nodular iron	24×10^6 (1.7×10^5)	2160 (179)	2070 (172)	2050 (170)	2000 (166)	1880 (156)	1830 (152)
Cast iron	22×10^6 (1.5×10^5)	2100 (174)	2020 (168)	2000 (166)	1960 (163)	1850 (154)	1800 (149)
Aluminum bronze	17.5×10^6 (1.2×10^5)	1950 (162)	1900 (158)	1880 (156)	1850 (154)	1750 (145)	1700 (141)
Tin bronze	16×10^6 (1.1×10^5)	1900 (158)	1850 (154)	1830 (152)	1800 (149)	1700 (141)	1650 (137)

Poisson's ratio=0.3

14.7 Dynamic Factor K_v

- Dynamic factors are used to account for inaccuracies in the manufacture and meshing of gear teeth in action.

- *Transmission error* is defined as the departure from uniform angular velocity of the gear pair.

- Some of the effects that produce transmission error are:
  - Inaccuracies produced in the generation of the tooth profile; these include errors in tooth spacing, profile lead, and runout
  - Vibration of the tooth during meshing due to the tooth stiffness
  - Magnitude of the pitch-line velocity
  - Dynamic unbalance of the rotating members
  - Wear and permanent deformation of contacting portions of the teeth
  - Gearshaft misalignment and the linear and angular deflection of the shaft
  - Tooth friction

- AGMA has defined a set of *quality-control numbers*. These numbers define the tolerances for gears of various sizes manufactured to a specified quality class.
-
- Classes 3 to 7 will include most commercial-quality gears.
-
- Classes 8 to 12 are of precision quality.
-
- The AGMA *transmissiot accuracy-level number* Q_v can be taken as the same as the quality number.

- The following equations for the dynamic factor are based on these Q_v numbers:

$$K_v = \begin{cases} \left(\dfrac{A+\sqrt{V}}{A}\right)^B & V \text{ in ft/min} \\ \left(\dfrac{A+\sqrt{200V}}{A}\right)^B & V \text{ in m/s} \end{cases} \qquad (14\text{-}27)$$

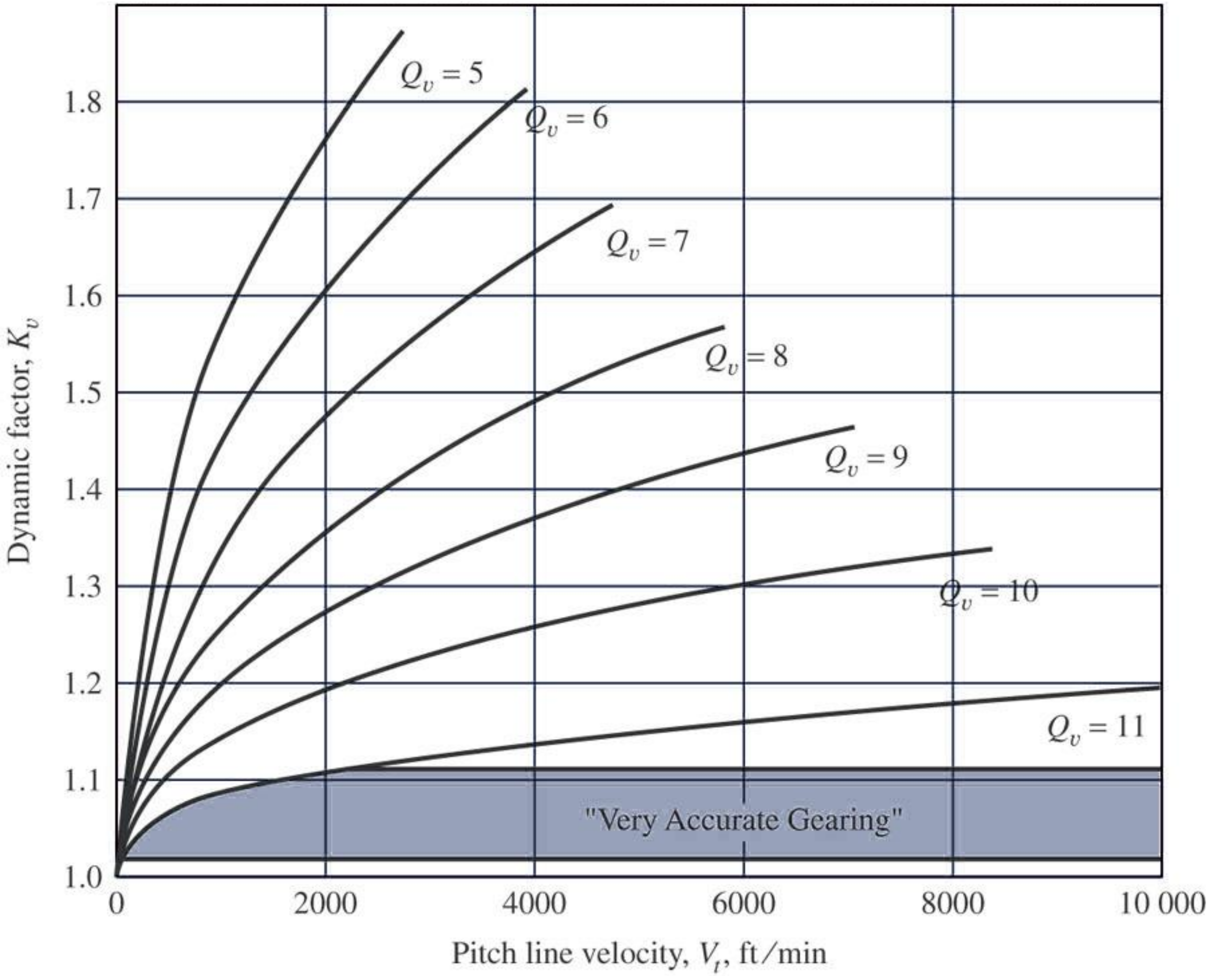
Where

$$A = 50 + 56(1 - B)$$
$$B = 0.25(12 - Q_v)^{2/3} \qquad (14\text{-}28)$$

- The maximum velocity, representing the end point of the Q_v curve, is given by

$$(V_t)_{\max} = \begin{cases} [A + (Q_v - 3)]^2 & \text{ft/min} \\ \dfrac{[A + (Q_v - 3)]^2}{200} & \text{m/s} \end{cases} \qquad (14\text{-}29)$$

Figure 14-9 is a graph of K_v, the dynamic factor, as a function of pitch-line speed for graphical estimates of K_v.



14.8 Overload Factor K_o

- K_o is intended to make allowance for all externally applied loads in excess of the nominal tangential load W^t in a particular application.
- Examples include variations in torque from the mean value due to firing of cylinders in an internal combustion engine or reaction to torque variations in a piston pump drive.
- Others call a similar factor an application factor or a service factor. These are established after considerable field experience in a particular application.

Table of Overload Factors, K_o

	Driven Machine		
Power source	Uniform	Moderate shock	Heavy shock
Uniform	1.00	1.25	1.75
Light shock	1.25	1.50	2.00
Medium shock	1.50	1.75	2.25

14.9 Surface Condition Factor C_f (Z_R)

- The surface condition factor C_f or Z_R is used only in the pitting resistance equation, Eq. (14-16).

- It depends on
  - Surface finish as affected by, but not limited to, cutting, shaving, lapping, grinding, shotpeening
  - Residual stress
  - Plastic effects (work hardening)

- Standard surface conditions for gear teeth have not yet been established. When a detrimental surface finish effect is known to exist, AGMA suggests a value of C_f greater than unity.

14.10 Size Factor K_s

- The size factor reflects nonuniformity of material properties due to size. It depends upon
  - Tooth size
  - Diameter of part
  - Ratio of tooth size to diameter of part
  - Face width
  - Area of stress pattern
  - Ratio of case depth to tooth size
  - Hardenability and heat treatment

- AGMA recommends a size factor $K_s > 1$ for gear teeth in which there is detrimental size effects.
- If there is no detrimental size effect, use $K_s = 1$.
- AGMA suggests $K_s = 1$, which makes K_s a placeholder in Eqs. (14-15) and (14-16) until more information is gathered.
- From Table 13-1, $l = a + b = 2.25/P$.
- The tooth thickness t in Fig. 14-6 is given in Sec. 14-1, Eq. (*b*), as $t = (4lx)^{1/2}$ where $x = 3Y/(2P)$ from Eq. (14-3).

- From Eq. (7-24) the equivalent diameter d_e of a rectangular section in bending is

$$d_e = 0.808(Ft)^{1/2}$$

- From Eq. (7-19):

$$k_b = (d_e/0.3)^{-0.107}$$

Noting that AGMA K_s is the reciprocal of k_b, we find the result of all the algebraic substitution is

$$K_s = \frac{1}{k_b} = 1.192\left(\frac{F\sqrt{Y}}{P}\right)^{0.0535} \quad \text{(a)}$$

$$= 0.904\ (\mathrm{b\ m\ sqrt\ (Y)\ })^{0.0535} \quad \text{(SI units)}$$

- AGMA K_s can be viewed as Lewis's geometry incorporated into the Marin size factor in fatigue.

- You may set AGMA K_s = 1, or you may elect to use the preceding Eq. (a).

- We will use Eq. (a) to remind you that you have a choice. If K_s in Eq. (a) is less than 1, use K_s =1.

14.11 Load-Distribution Factor K_m

- It modified the stress equations to reflect nonuniform distribution of load across the line of contact.
- The ideal is to locate the gear "midspan" between two bearings at the zero slope place when the load is applied. However, this is not always possible.
- The following procedure is applicable to:
  - Net face width to pinion pitch diameter ratio $F/d \leq 2$
  - Gear elements mounted between the bearings
  - Face widths up to 40 in
  - Contact, when loaded, across the full width of the narrowest member

- The load-distribution factor under these conditions is given by

$$K_m = 1 + C_{mc}(C_{pf}C_{pm} + C_{ma}C_e) \tag{14-30}$$

where

$$C_{mc} = \begin{cases} 1 & \text{for uncrowned teeth} \\ 0.8 & \text{for crowned teeth} \end{cases} \tag{14-31}$$

$$C_{pf} = \begin{cases} \dfrac{F}{10d} - 0.025 & F \le 1 \text{ in} \\ \dfrac{F}{10d} - 0.0375 + 0.0125F & 1 < F \le 17 \text{ in} \\ \dfrac{F}{10d} - 0.1109 + 0.0207F - 0.000\,228F^2 & 17 < F \le 40 \text{ in} \end{cases} \tag{14-32}$$

Note that for values of $F/(10d) < 0.05$, $F/(10d) = 0.05$ is used.

$$C_{pm} = \begin{cases} 1 & \text{for straddle-mounted pinion with } S_1/S < 0.175 \\ 1.1 & \text{for straddle-mounted pinion with } S_1/S \geq 0.175 \end{cases} \qquad \text{14-33}$$

$$C_{ma} = A + BF + CF^2 \qquad \text{(see Table 14–9 for values of } A, B, \text{ and } C) \qquad \text{14-34}$$

$$C_e = \begin{cases} 0.8 & \text{for gearing adjusted at assembly, or compatibility} \\ & \text{is improved by lapping, or both} \\ 1 & \text{for all other conditions} \end{cases} \qquad \text{14-35}$$

Definitions of S and S_1 for use with Eq. (14-33) can be shown in Figure 14-10. And see Figure14-11 for graph of C_{ma}

Figure 14-10: Definition of distances S and S_1 used in evaluating C_{pm}, Eq. (14-33)

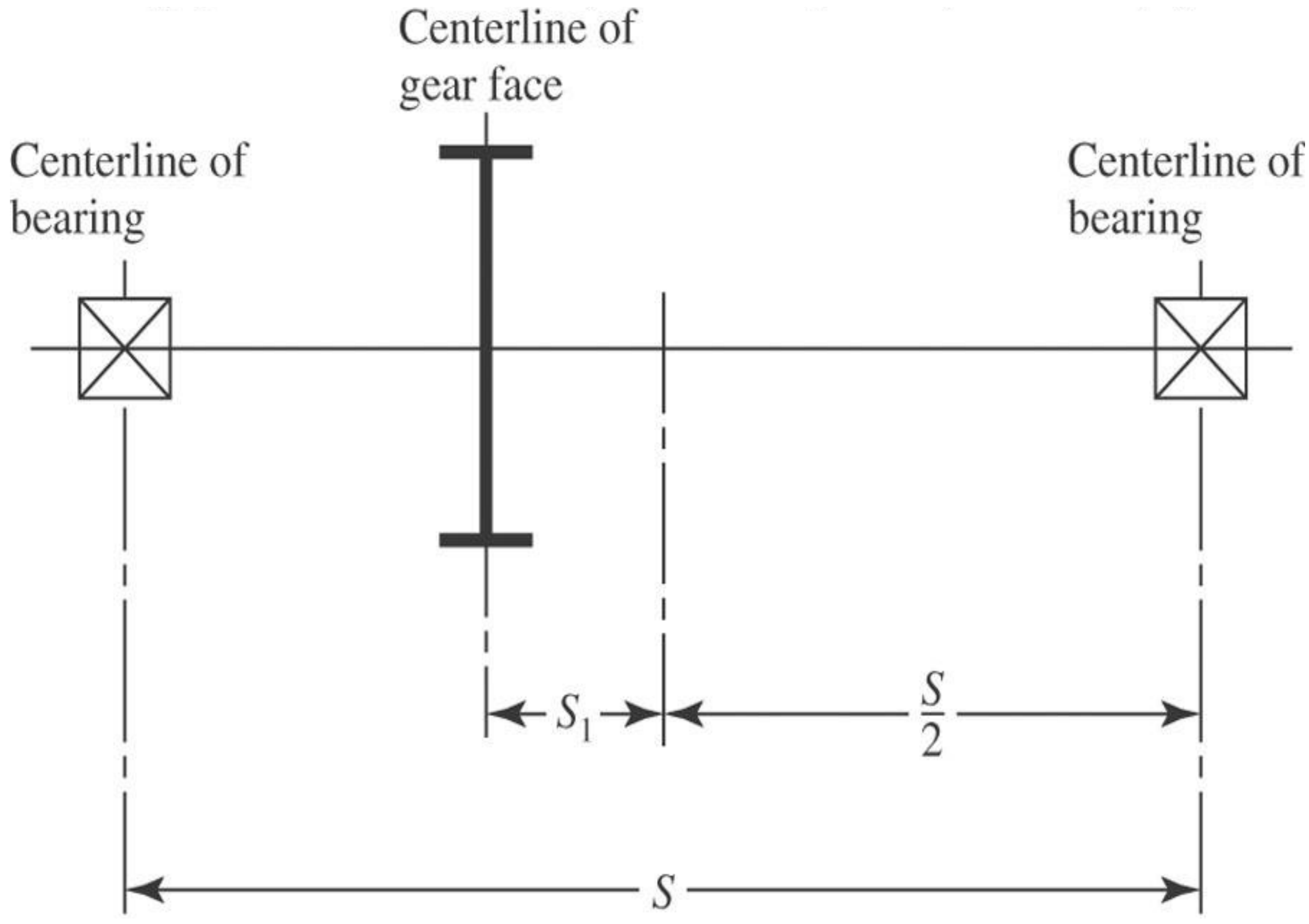


Figure 14-11: mesh alignment factor C_{ma}. Curve-fit equations in Table 14-9

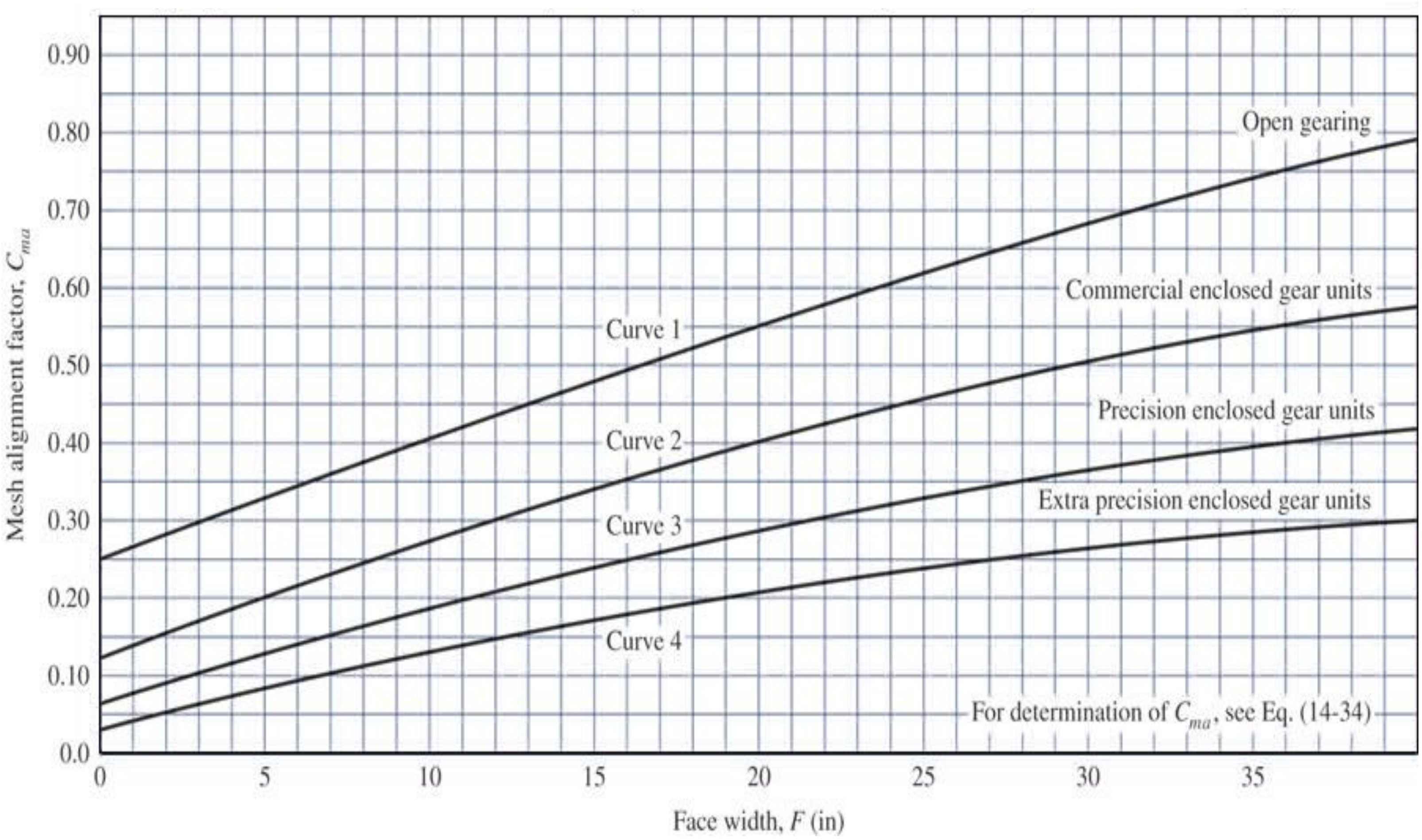


Table 14-9 Empirical Constants *A*, *B*, *C* for Eq. (14-34), Face Width *F* in inches

Condition	A	B	C
Open gearing	0.247	0.0167	$-0.765(10^{-4})$
Commercial, enclosed units	0.127	0.0158	$-0.093(10^{-4})$
Precision, enclosed units	0.0675	0.0128	$-0.926(10^{-4})$
Extraprecision enclosed gear units	0.00360	0.0102	$-0.822(10^{-4})$

14.12 Hardness-Ratio Factor C_H

- The pinion generally has a smaller number of teeth than the gear and consequently is subjected to more cycles of contact stress.
- If both the pinion and the gear are through-hardened, then a uniform surface strength can be obtained by making the pinion harder than the gear.
- A similar effect can be obtained when a surface-hardened pinion is mated with a through-hardened gear.
- The hardness-ratio factor C_H is used *only for the gear*.
- Its purpose is to adjust the surface strengths for this effect.

- The values of C_H are obtained from the equation

$$C_H = 1.0 + A'(m_G - 1.0) \qquad (14\text{-}36)$$

where

$$A' = 8.98(10^{-3})\left(\frac{H_{BP}}{H_{BG}}\right) - 8.29(10^{-3}) \quad 1.2 \le \frac{H_{BP}}{H_{BG}} \le 1.7$$

- The terms H_{BP} and H_{BG} are the Brinell hardness (10-mm ball at 3000-kg load) of the pinion and gear, respectively. The term m_G is the speed ratio and is given by Eq. (14-22). See Fig. 14-12 for a graph of Eq. (14-36). For

$$\frac{H_{BP}}{H_{BG}} < 1.2, \qquad A' = 0$$

$$\frac{H_{BP}}{H_{BG}} > 1.7, \qquad A' = 0.006\,98$$

- When surface-hardened pinions with hardnesses of 48 Rockwell C scale (Rockwell C48) or harder are run with through-hardened gears (180-400 Brinell), a work hardening occurs. The C_H factor is a function of pinion surface finish f_P and the mating gear hardness. Figure 14-13 displays the relationships:

$$C_H = 1 + B'(450 - H_{BG}) \qquad (14\text{-}37)$$

- Where B' =0.00075 exp[-0.0112f_P] and f_P is the surface finish of the pinion expressed as root-mean-square roughness R_a in μ in.

Figure 14-12: Hardness ratio factor *CH* (through-hardened steel)

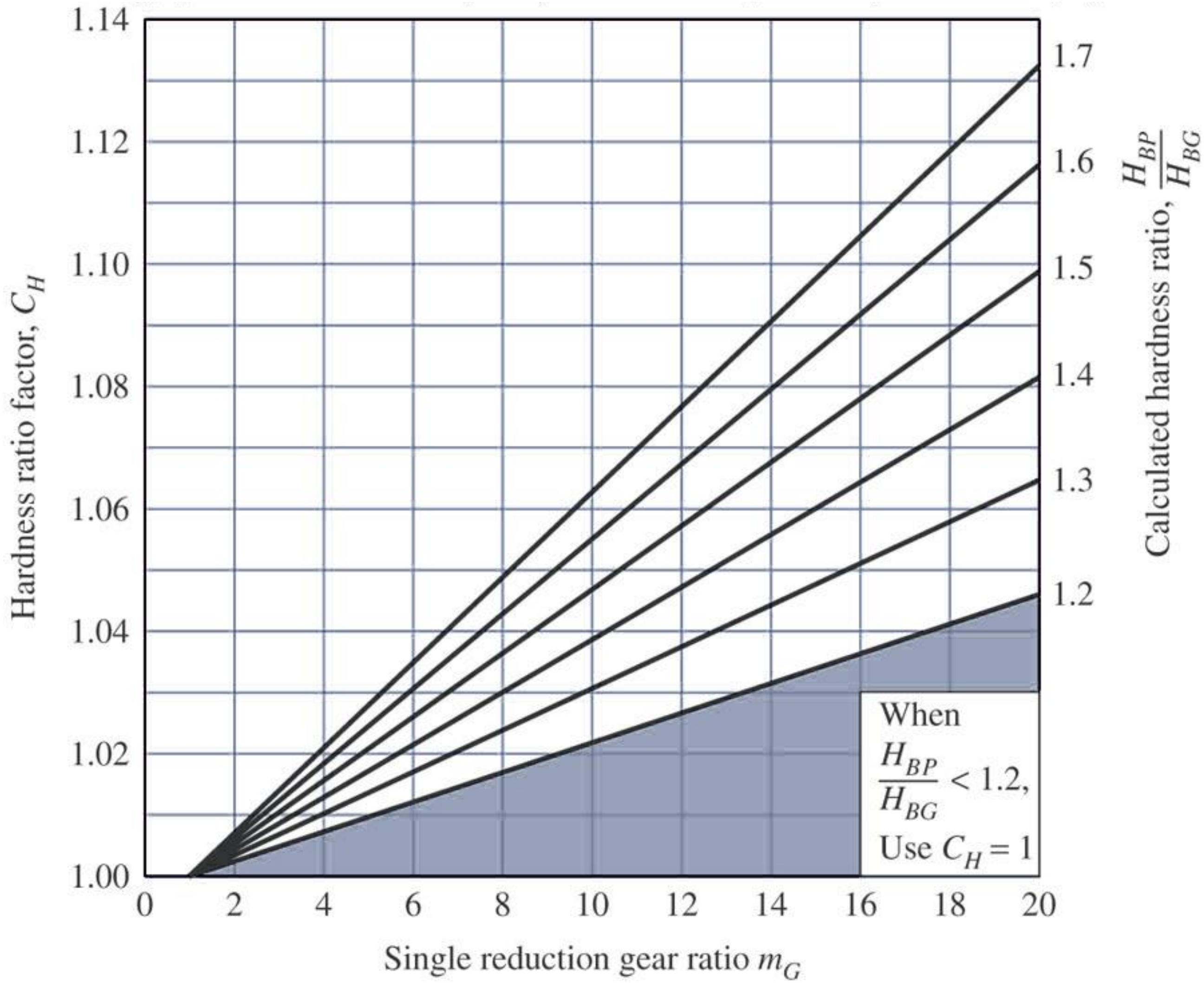
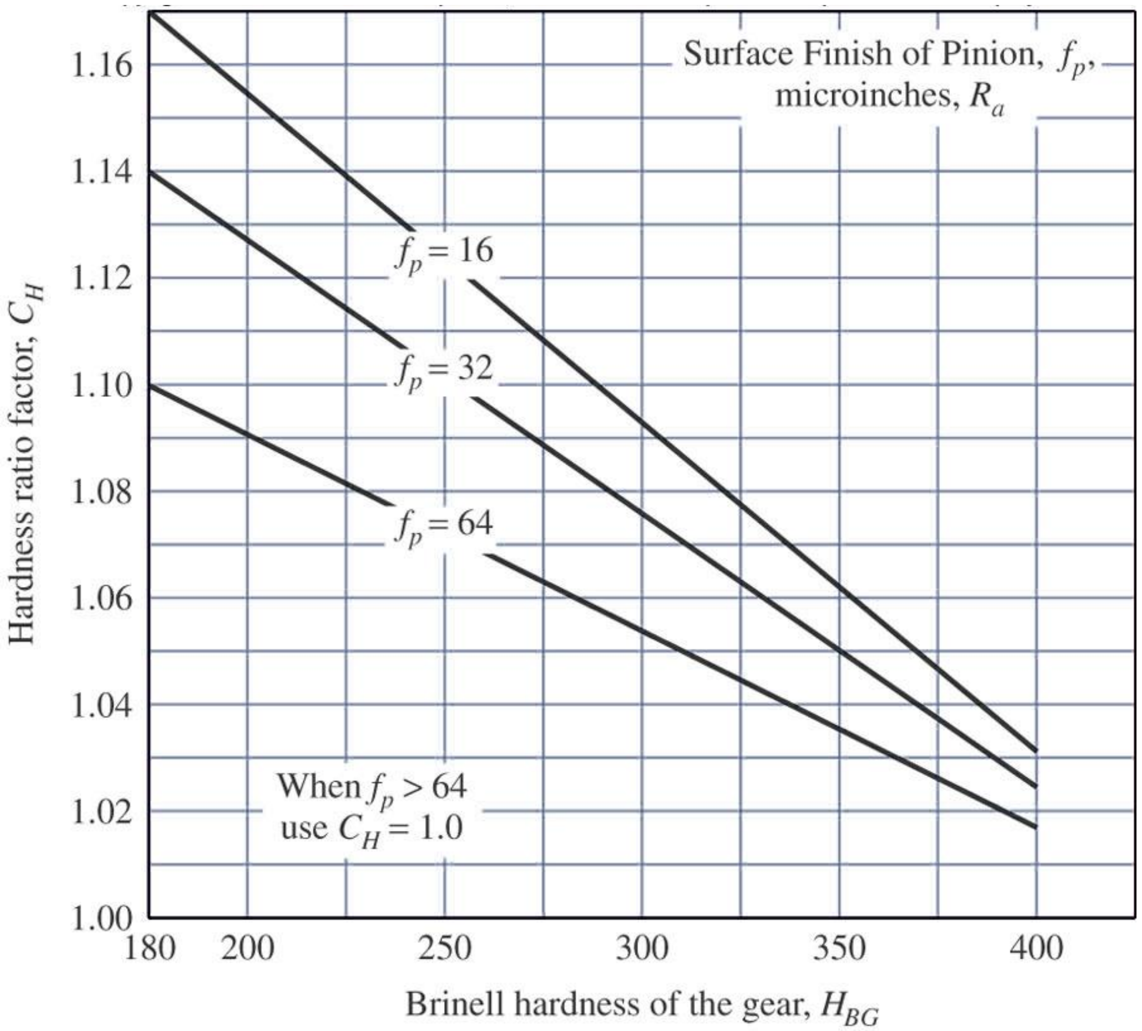


Figure 14-13: Hardness ratio factor *CH* (through-hardened steel pinion)



14.13 Stress Cycle Life Factors Y_N and Z_N

- The AGMA strengths as given in Figs. 14-2 through 14-4, in Tables 14-3 and 14-4 for bending fatigue, and in Fig. 14-5 and Tables 14-5 and 14-6 for contact-stress fatigue are based on 10^7 load cycles repeatedly applied.
- The purpose of the load cycle factors Y_N and Z_N is to modify the AGMA strength for lives other than 10^7 cycles.

- Values for these factors are given in Figs. 14-14 and 14-15.

  Note that for 10^7 cycles $Y_N = Z_N$=1 on each graph. Note also that the equations for Y_N and Z_N change on either side of 10^7 cycles.

- For life goals slightly higher than 10^7 cycles, the mating gear may be experiencing fewer than 10^7 cycles and the equations for $(Y_N)_P$ and $(Y_N)_G$ can be different.
- The same comment applies to $(Z_N)_p$ and $(Z_N)_G$.

Figure 14-14: Repeatedly applied bending strength stress-cycle factor Y_N

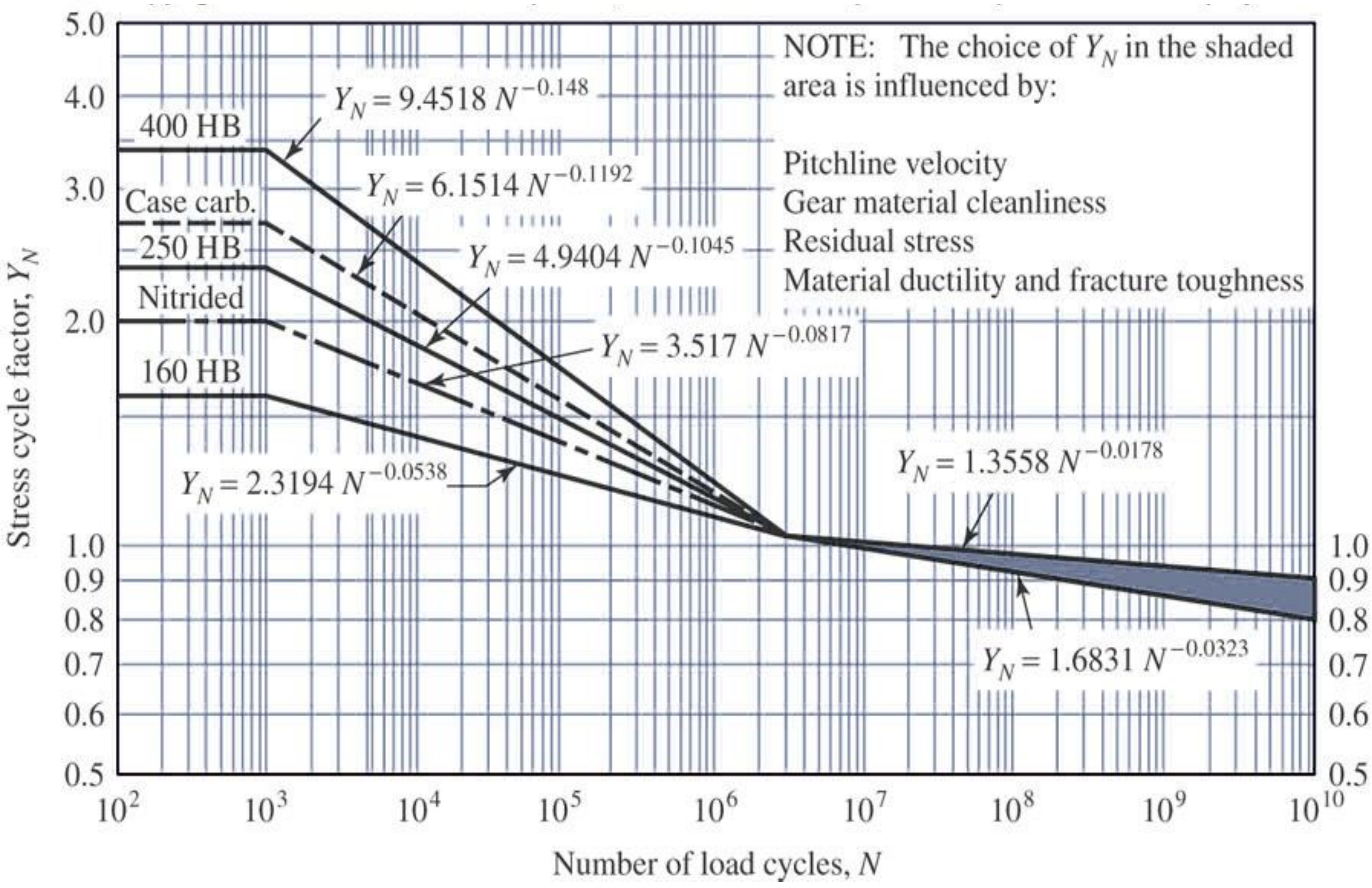
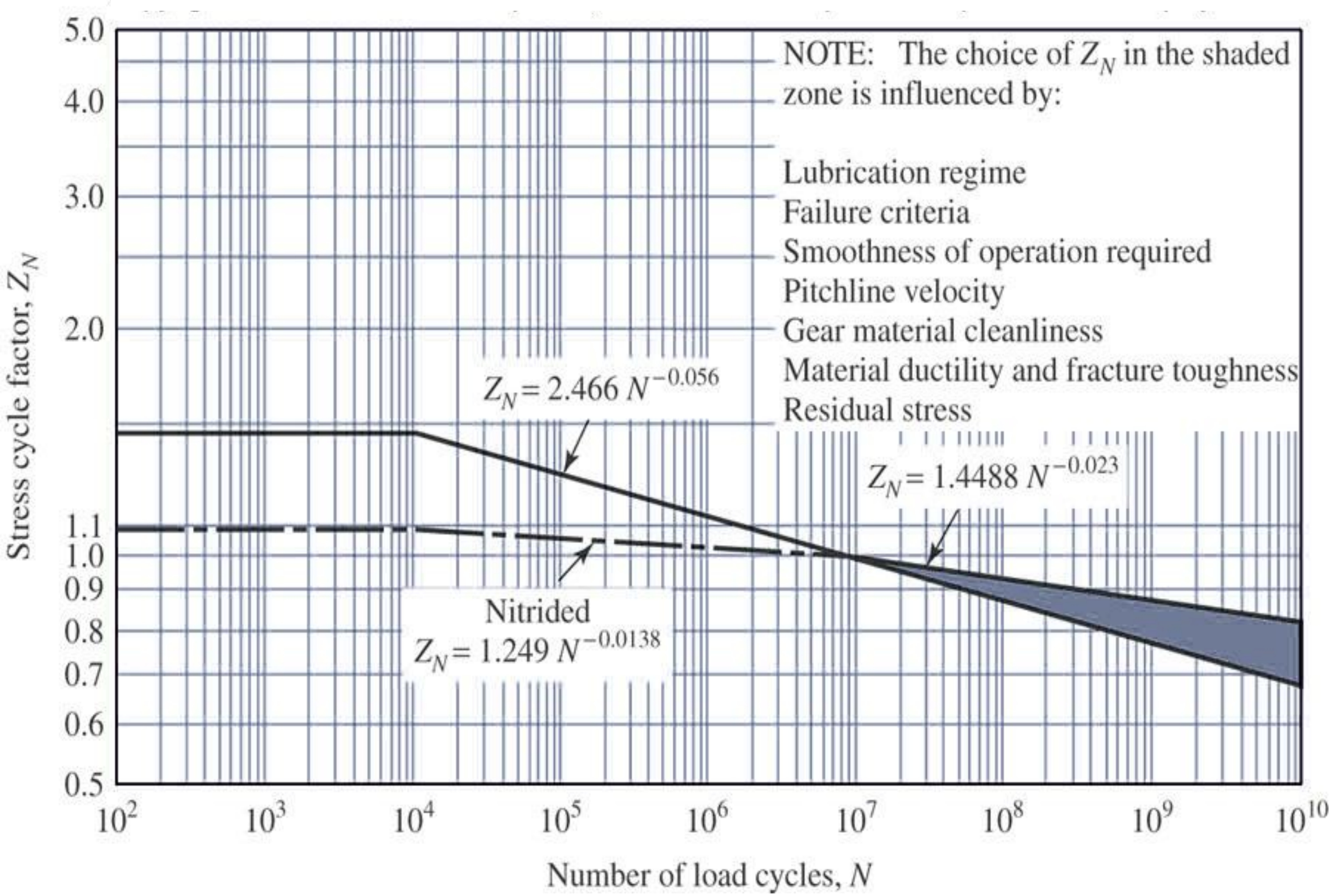


Figure 14-15: Pitting resistance stress-cycle factor Z_N



14.14 Reliability Factor K_R (Y_Z)

- The reliability factor accounts for the effect of the statistical distributions of material fatigue failures. (Load variation is not addressed here).
- The AGMA strengths S_t, and S_c are based on a reliability of 99 percent.
- Table 14-10 is based on data developed by the U.S. Navy for bending and contact-stress fatigue failures.
- The functional relationship between K_R and reliability is highly nonlinear. When interpolation is required, linear interpolation is too crude.

- A log transformation to each quantity produces a linear string. A least-squares regression fit is:

$$K_R = \begin{cases} 0.658 - 0.0759\ln(1-R) & 0.5 < R < 0.99 \\ 0.50 - 0.109\ln(1-R) & 0.99 \le R \le 0.9999 \end{cases} \qquad (14\text{-}38)$$

- For cardinal values of *R*, take *KR* from the table. Otherwise use the logarithmic interpolation afforded by Eqs. (14-38).

Table 14-10: Reliability Factors K_R (Y_Z)

Reliability	K_R (Y_Z)
0.9999	1.50
0.999	1.25
0.99	1.00
0.90	0.85
0.50	0.70

14.15 Temperature Factor K_T

- For oil or gear-blank temperatures up to 250°F (120°C), use $K_T = Y_\theta = 1.0$.
- For higher temperatures, the factor should be greater than unity. Heat exchangers may be used to ensure that operating temperatures are considerably below this value, as is desirable for the lubricant.

14.16 Rim-Thickness Factor K_B

- When the rim thickness is not sufficient to provide full support for the tooth root, the location of bending fatigue failure may be through the gear rim rather than at the tooth fillet.
- In such cases, the use of a stress-modifying factor K_B or (t_R) is recommended.
- The *rim-thickness factor* K_B, adjusts the estimated bending stress for the thin-rimmed gear. It is a function of the backup ratio m_B,

$$m_B = \frac{t_R}{h_t} \tag{14-39}$$

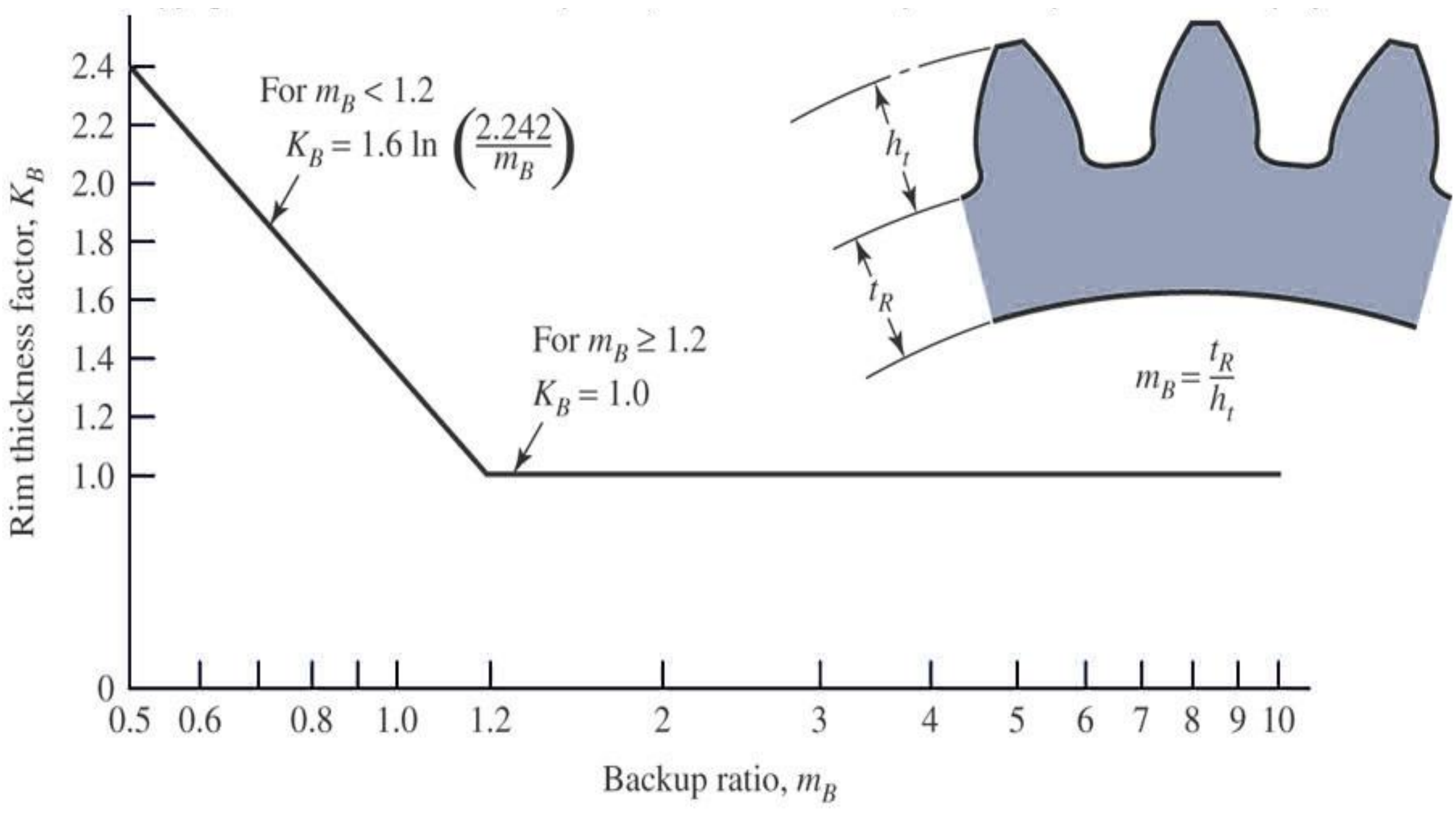
Where t_R =rim thickness below the tooth, in, and h_t = the tooth height. The geometry is illustrated in Fig. 14-16.

- The rim-thickness factor K_B is given by

$$K_B = \begin{cases} 1.6 \ln \dfrac{2.242}{m_B} & m_B < 1.2 \\ 1 & m_B \geq 1.2 \end{cases} \qquad (14\text{-}40)$$

- Figure 14-16 also gives the value of K_B graphically. The rim-thickness factor K_B is applied in addition to the 0.70 reverse-loading factor when applicable.

Figure 14-16: Rim thickness factor K_B



14.17 Safety Factors S_F and S_H

- The ANSI /AGMA standards 200I-C95 and 2101-C95 have reintroduced safety factor S_F guarding against bending fatigue failure and safety factor S_H guarding against pitting failure.
- The definition of S_F, from Eq. (14-17), is:

$$S_F = \frac{S_t Y_N/(K_T K_R)}{\sigma} = \frac{\text{fully corrected bending strength}}{\text{bending stress}} \qquad (14\text{-}41)$$

Where σ is estimated from Eq. (14-15). It is a strength-over-stress definition in a case where the stress is linear with the transmitted load.

- The definition of S_H, from Eq. (.14-18), is

$$S_H = \frac{S_c Z_N C_H/(K_T K_R)}{\sigma_c} = \frac{\text{fully corrected contact strength}}{\text{contact stress}} \qquad (14\text{-}42)$$

Where σ_c is estimated from Eq. (14-16).

- This, too, is a strength-over-stress definition but in a case where the stress, is *not* linear with the transmitted load W^t.

- A caution is required when comparing S_F with S_H in an analysis in order to ascertain the nature and severity of the threat to loss of function.
- To make S_H linear with the transmitted load, W^t it could have been defined as:

$$S_H = \left(\frac{\text{fully corrected contact strength}}{\text{contact stress imposed}} \right)^2 \qquad (14\text{-}43)$$

- With the exponent 2 for linear or helical contact, or an exponent of 3 for crowned teeth (spherical contact).

- With the AGMA definition, Eq. (14-42), compare S_F with S_H^2 (or S_H^3 for crowned teeth) when trying to identify the threat to loss of function with confidence.

- The role of the overload factor K_o is to include predictable excursions of load beyond W^t based on experience.

- A safety factor is intended to account for unquantifiable elements in addition to K_o. When designing a gear mesh, the quantity S_F becomes a design factor $(S_F)_d$ within the meanings used in this book.

- The quantity S_F evaluated as part of a design assessment is a factor of safety. This applies equally well to the quantity S_H.

14.18 Analysis

- Description of the AGMA procedure is highly detailed. The best review is a **road map** for bending fatigue and contact-stress fatigue.
- Figure 14-17 identifies the AGMA bending stress equation, the endurance strength in bending equation, and the factor of safety S_F. (See textbook page 754)
- Figure 14-18 displays the contact-stress equation, the contact fatigue endurance strength equation, and the factor of safety S_H. (See textbook page 755)
- When analyzing a gear problem, this figure is a useful reference.

SPUR GEAR BENDING
Based on ANSI/AGMA 2001-D04

$$d_P = \frac{N_P}{P_d}$$

$$V = \frac{\pi d n}{12}$$

$$W^t = \frac{33\,000\ H}{V}$$

1 [or Eq. (a), Sec. 14–10]; p. 759

Eq. (14–30); p. 759

Eq. (14–40); p. 764

Gear bending stress equation Eq. (14–15)

$$\sigma = W^t K_o K_v K_s \frac{P_d}{F} \frac{K_m K_B}{J}$$

Fig. 14–6; p. 753

Eq. (14–27); p. 756

Table below

$_{0.99}(S_t)_{10^7}$ Tables 14–3, 14–4; pp. 748, 749

Fig. 14–14; p. 763

Gear bending endurance strength equation Eq. (14–17)

$$\sigma_{\text{all}} = \frac{S_t}{S_F} \frac{Y_N}{K_T K_R}$$

Table 14–10, Eq. (14–38); pp. 764, 763

1 if $T < 250°\text{F}$

Bending factor of safety Eq. (14–41)

$$S_F = \frac{S_t Y_N / (K_T K_R)}{\sigma}$$

Remember to compare S_F with S_H^2 when deciding whether bending or wear is the threat to function. For crowned gears compare S_F with S_H^3.

Table of Overload Factors, K_o

	Driven Machine		
Power source	Uniform	Moderate shock	Heavy shock
Uniform	1.00	1.25	1.75
Light shock	1.25	1.50	2.00
Medium shock	1.50	1.75	2.25

Figure 14–17

Roadmap of gear bending equations based on AGMA standards in U.S. customary units. *(ANSI/AGMA 2001-D04.)* The SI version of this figure can be found in Appendix B.

SPUR GEAR WEAR
Based on ANSI/AGMA 2001-D04

$$d_P = \frac{N_P}{P_d}$$

$$V = \frac{\pi d n}{12}$$

$$W^t = \frac{33\,000\ H}{V}$$

1 [or Eq. (a), Sec. 14–10]; p. 759

Eq. (14–30); p. 759

1

Gear contact stress equation Eq. (14–16)

$$\sigma_c = C_p \left(W^t K_o K_v K_s \frac{K_m}{d_P F} \frac{C_f}{I} \right)^{1/2}$$

Eq. (14–23); p. 755

Eq. (14–27); p. 756

Eq. (14–13), Table 14–8; pp. 744, 757

Table below

${}_{0.99}(S_c)_{10^7}$ Tables 14–6, 14–7; pp. 751, 752

Fig. 14–15; p. 763

Section 14–12, gear only; pp. 761, 762

Gear contact endurance strength Eq. (14–18)

$$\sigma_{c,\text{all}} = \frac{S_c Z_N C_H}{S_H K_T K_R}$$

Table 14–10, Eq. (14–38); pp. 763, 764

1 if $T < 250°$F

Gear only

Wear factor of safety Eq. (14–42)

$$S_H = \frac{S_c Z_N C_H/(K_T K_R)}{\sigma_c}$$

Remember to compare S_F with S_H^2 when deciding whether bending or wear is the threat to function. For crowned gears compare S_F with S_H^3.

Table of Overload Factors, K_o

	Driven Machine		
Power source	Uniform	Moderate shock	Heavy shock
Uniform	1.00	1.25	1.75
Light shock	1.25	1.50	2.00
Medium shock	1.50	1.75	2.25

Figure 14–18

Roadmap of gear wear equations based on AGMA standards in U.S. customary units. *(ANSI/AGMA 2001-D04.)* The SI version of this figure can be found in Appendix B.

Example 14-4 (Spur –Gear analysis)

- A 17-tooth 20° pressure angle spur pinion rotates at 1800 rev/min and transmits 4 hp to a 52-tooth disk gear. The diametral pitch is 10 teeth/in, the face width 1.5 in, and the quality standard is No. 6. The gears are straddle-mounted with bearings immediately adjacent. The pinion is a grade 1 steel with a hardness of 240 Brinell tooth surface and through-hardened core. The gear is steel, through-hardened also, grade 1 material, with a Brinell hardness of 200, tooth surface and core. Poisson's ratio is 0.30, J_P = 0.30, J_G = 0.40, and Young's modulus is $30(10^6)$psi. The loading is smooth because of motor and load. Assume a pinion life of 108 cycles and a reliability of 0.90, and use $Y_N = 1.3558N^{-0.0178}$, $Z_N = 1.4488N^{-0.023}$. The tooth profile is uncrowned. This is a commercial enclosed gear unit.

1. Find the factor of safety of the gears in bending.
2. Find the factor of safety of the gears in wear.
3. By examining the factors of safety, identify the threat to each gear and to the mesh.

Solution

- Pinion N_P = 17 teeth, Gear N_G = 52 teeth,
- Pressure angle $\phi = 20^o$
- Power H = 4 hp
- Diametral pitch P_d =10 teeth/in
- Face width F =1.5 in
- Quality standard Q_v = 6
- The pinion is a grade 1 steel, H_{BP} =240
- The gear is grade 1 steel, H_{BG} = 200,
- Poisson's ratio is 0.30, J_P = 0.30, J_G = 0.40,
- Young's modulus E = 30(106) psi.
- The loading is smooth because of motor and load → K_o = 1
- A pinion life $N_P = 10^8$ cycles and a reliability R = 0.90
- Use $Y_N = 1.3558N^{-0.0178}$, $Z_N = 1.4488N^{-0.023}$.
- The tooth profile is uncrowned

Use Figs. 14-17 and 14-18 as guides to what is needed.

$$d_P = N_P/P_d = 17/10 = 1.7 \text{ in} \qquad d_G = 52/10 = 5.2 \text{ in}$$

$$V = \frac{\pi d_P n_P}{12} = \frac{\pi(1.7)1800}{12} = 801.1 \text{ ft/min}$$

$$W^t = \frac{33\,000\, H}{V} = \frac{33\ 000(4)}{801.1} = 164.8 \text{ lbf}$$

- To evaluate K_v, from Eq. (14-28) with a quality number Q_v =6,

$$B = 0.25(12 - 6)^{2/3} = 0.8255$$

$$A = 50 + 56(1 - 0.8255) = 59.77$$

- Then from Eq. (14-27) the dynamic factor is

$$K_v = \left(\frac{59.77 + \sqrt{801.1}}{59.77}\right)^{0.8255} = 1.377$$

- To determine the size factor, K_S, the Lewis form factor is needed. From Table 14-2, with N_P= 17 teeth, Y_P = 0.303. Interpolation for the gear with N_G = 52 teeth yields Y_G =0.412. Thus from Eq. (a) of Sec. 14-10, with F =1.5 in,

$$(K_s)_P = 1.192\left(\frac{1.5\sqrt{0.303}}{10}\right)^{0.0535} = 1.043$$

$$(K_s)_G = 1.192\left(\frac{1.5\sqrt{0.412}}{10}\right)^{0.0535} = 1.052$$

- The load distribution factor K_m is determined from Eq. (14-30), where five terms are needed. They are, where F = 1 .5 in when needed:
- Uncrowned, Eq. (14-30): $C_{mc} = 1$,
- Eq. (14-32):

$$C_{Pf} = 1.5/[10(1.7)] - 0.0375 + 0.0125(1.5) = 0.0695$$

- Bearings immediately adjacent, Eq. (14-33): $C_{pm} = 1$
- Commercial enclosed gear units (Fig. 14-11): $C_{ma} = 0.15$
- Eq. (14-35): $C_e = 1$,
- Thus,

$$K_m = 1 + C_{mc}(C_{pf}C_{pm} + C_{ma}C_e) = 1 + (1)[0.0695(1) + 0.15(1)] = 1.22$$

- Assuming constant thickness gears,

  The rim-thickness factor $K_B = 1$.

  The speed ratio is $m_G = N_G/N_P = 52/17 = 3.059$.

- The load cycle factors given in the problem statement, with $N(_{\text{pinion}}) = 10^8$ cycles and $N(_{\text{gear}}) = 10^8/m_G = 10^8/3.059$ cycles, are:

$$(Y_N)_P = 1.3558(10^8)^{-0.0178} = 0.977$$

$$(Y_N)_G = 1.3558(10^8/3.059)^{-0.0178} = 0.996$$

- From Table 14.10, with a reliability of 0.9, K_R = 0.85. From Fig. 14-18, the temperature and surface condition factors are K_T =1 and C_f = 1. From Eq. (14-23), with m_N = 1 for spur gears,

$$I = \frac{\cos 20^\circ \sin 20^\circ}{2} \frac{3.059}{3.059 + 1} = 0.121$$

- From Table 14-8, C_p = 2300 (psi)$^{1/2}$

- Next, we need the terms for the AGMA endurance strength equations. From Table 14-3, for grade 1 steel with H_{BP} = 240 and H_{BG} = 200, we use Fig. 14-2, which gives

$$(S_t)_P = 77.3(240) + 12800 = 31350 \text{ psi}$$

$$(S_t)_G = 77.3(200) + 12800 = 28260 \text{ psi}$$

- Similarly, from Table 14-6, we use Fig. 14-5, which gives

$$(S_C)_P = 322(240) + 29100 = 106400 \text{ psi}$$

$$(S_C)_G = 322(200) + 29100 = 93500 \text{ psi}$$

- From Fig. 14-15,

$$(Z_N)_P = 1.4488(10^8)^{-0.023} = 0.948$$

$$(Z_N)_G = 1.4488(10^8/3.059)^{-0.023} = 0.973$$

- For the hardness ratio factor C_H, the hardness ratio is

$$H_{BP}/H_{BG} = 240/200 = 1.2.$$

- Then, from Sec. 14-12,

$$A' = 8.98(10^{-3})(H_{BP}/H_{BG}) - 8.29(10^{-3})$$

$$= 8.98(10^{-3})(1.2) - 8.29(10^{-3}) = 0.00249$$

- Thus, from Eq. (14-36),

$$C_H = 1 + 0.00249(3.059 - 1) = 1.005$$

(*a*) **Pinion tooth bending.** Substituting the appropriate terms for the pinion Eq. (14–15) gives

$$(\sigma)_P = \left(W^t K_o K_v K_s \frac{P_d}{F} \frac{K_m K_B}{J} \right)_P = 164.8(1)1.377(1.043) \frac{10}{1.5} \frac{1.22\,(1)}{0.30}$$

$$= 6417 \text{ psi}$$

Substituting the appropriate terms for the pinion into Eq. (14–41) gives

$$(S_F)_P = \left(\frac{S_t Y_N/(K_T K_R)}{\sigma} \right)_P = \frac{31\,350(0.977)/[1(0.85)]}{6417} = 5.62$$

Gear tooth bending. Substituting the appropriate terms for the gear into Eq. (14-15) gives

$$(\sigma)_G = 164.8(1)1.377(1.052) \frac{10}{1.5} \frac{1.22(1)}{0.40} = 4854 \text{ psi}$$

Substituting the appropriate terms for the gear into Eq. (14–41) gives

$$(S_F)_G = \frac{28\,260(0.996)/[1(0.85)]}{4854} = 6.82$$

(*b*) **Pinion tooth wear.** Substituting the appropriate terms for the pinion into Eq. (14–16) gives

$$(\sigma_c)_P = C_p \left(W^t K_o K_v K_s \frac{K_m}{d_P F} \frac{C_f}{I} \right)_P^{1/2}$$

$$= 2300 \left[164.8(1)1.377(1.043) \frac{1.22}{1.7(1.5)} \frac{1}{0.121} \right]^{1/2} = 70\,360 \text{ psi}$$

Substituting the appropriate terms for the pinion into Eq. (14–42) gives

$$(S_H)_P = \left[\frac{S_c Z_N/(K_T K_R)}{\sigma_c} \right]_P = \frac{106\,400(0.948)/[1(0.85)]}{70\,360} = 1.69$$

Gear tooth wear. The only term in Eq. (14–16) that changes for the gear is K_s. Thus

$$(\sigma_c)_G = \left[\frac{(K_s)_G}{(K_s)_P} \right]^{1/2} (\sigma_c)_P = \left(\frac{1.052}{1.043} \right)^{1/2} 70\,360 = 70\,660 \text{ psi}$$

Substituting the appropriate terms for the gear into Eq. (14–42) with $C_H = 1.005$ give

$$(S_H)_G = \frac{93\,500(0.973)1.005/[1(0.85)]}{70\,660} = 1.52$$

- (c) For the pinion, we compare $(S_F)_P$ with $(S_H)_P^2$, or 5.62 with $1.69^2 = 2.86$, so threat in the threat in the pinion is from wear.
- For the gear, we compare $(S_F)_G$ with $(S_H)_G^2$, or 6.82 with $1.52^2 = 2.31$, so the threat in the gear is also from wear.

Example 14-4 (helical gearset under similar circumstances of the pervious example)

- A 17-tooth 20^o normal pitch-angle helical pinion with a right-hand helix angle of 30^o rotates at 1800 rev/min when transmitting 4 hp to a 52-tooth helical gear. The normal diametral pitch is 10 teeth/in, the face width is 1.5 in, and the set has a quality number of 6. The gears are straddle-mounted with bearings immediately adjacent. The pinion and gear are made from a through-hardened steel with surface and core hardnesses of 240 Brinell on ihe pinion an surface and core hardnesses of 200 Brine!! on the gear. The transmission is smooth, connecting an electric motor and a centrifugal pump. Assume a pinion life of 10^8 cycles and a reliability of 0.9 and use upper curves in Figs. 14-14 and 14-15.

(a) Find the factors of safety of the gears in bending.

(b) Find the factors of safety of the gears in wear.

(c) By examining the factors of safety identify the threat to each gear and to the mesh.

Solution:

- All of the parameters in this example are the same as in Ex. 14-4 with the exception that we are using helical gears.
- Thus, several terms will be the same as Ex. 14-4.
- You should verify that the following terms remain unchanged:

K_o = 1, Y_P = 0.303, Y_G = 0.412, m_G = 3.059,
$(K_s)_P$ = 1.043, $(K_S)_G$ = 1.052, $(Y_N)_P$ = 0.977, $(Y_N)_G$ = 0.996,
K_R = 0.85, K_T =1,C_f =1, C_P = 2300(psi)1/2,
$(S_t)_P$ = 31350 psi, $(S_t)_G$ =28260psi,
$(S_c)_P$, = 106380 psi, $(S_c)_G$ = 93500 psi,
$(Z_N)_P$ = 0.948, $(Z_N)_C$ = 0.973, and C_H =1.005

For helical gears, the transverse diametral pitch, given by Eq. (13–18), is

$$P_t = P_n \cos\psi = 10 \cos 30^\circ = 8.660 \text{ teeth/in}$$

Thus, the pitch diameters are $d_P = N_P/P_t = 17/8.660 = 1.963$ in and $d_G = 52/8.660 = 6.005$ in. The pitch-line velocity and transmitted force are

$$V = \frac{\pi d_P n_P}{12} = \frac{\pi(1.963)1800}{12} = 925 \text{ ft/min}$$

$$W^t = \frac{33\,000H}{V} = \frac{33\,000(4)}{925} = 142.7 \text{ lbf}$$

As in Ex. 14–4, for the dynamic factor, $B = 0.8255$ and $A = 59.77$. Thus, Eq. (14–27) gives

$$K_v = \left(\frac{59.77 + \sqrt{925}}{59.77}\right)^{0.8255} = 1.404$$

The geometry factor I for helical gears requires a little work. First, the transverse pressure

angle is given by Eq. (13–19)

$$\phi_t = \tan^{-1}\left(\frac{\tan\phi_n}{\cos\psi}\right) = \tan^{-1}\left(\frac{\tan 20^{\text{o}}}{\cos 30^{\text{o}}}\right) = 22.80^{\text{o}}$$

The radii of the pinion and gear are $r_P = 1.963/2 = 0.9815$ in and $r_G = 6.004/2 = 3.002$ in, respectively. The addendum is $a = 1/P_n = 1/10 = 0.1$, and the base-circle radii of the pinion and gear are given by Eq. (13–6) with $\phi = \phi_t$:

$$(r_b)_P = r_P \cos\phi_t = 0.9815 \cos 22.80^\circ = 0.9048 \text{ in}$$

$$(r_b)_G = 3.002 \cos 22.80^\circ = 2.767 \text{ in}$$

From Eq. (14–25), the surface strength geometry factor

$$Z = \sqrt{(0.9815 + 0.1)^2 - 0.9048^2} + \sqrt{(3.004 + 0.1)^2 - 2.769^2}$$

$$- (0.9815 + 3.004) \sin 22.80^\circ$$

$$= 0.5924 + 1.4027 - 1.5444 = 0.4507 \text{ in}$$

Since the first two terms are less than 1.544 4, the equation for Z stands. From Eq. (14–24) the normal circular pitch p_N is

$$p_N = p_n \cos \phi_n = \frac{\pi}{P_n} \cos 20^\circ = \frac{\pi}{10} \cos 20^\circ = 0.2952 \text{ in}$$

From Eq. (14–21), the load sharing ratio

$$m_N = \frac{p_N}{0.95Z} = \frac{0.2952}{0.95(0.4507)} = 0.6895$$

Substituting in Eq. (14–23), the geometry factor I is

$$I = \frac{\sin 22.80^\circ \cos 22.80^\circ}{2(0.6895)} \frac{3.06}{3.06 + 1} = 0.195$$

From Fig. 14–7, geometry factors $J'_P = 0.45$ and $J'_G = 0.54$. Also from Fig. 14–8 the J-factor multipliers are 0.94 and 0.98, correcting J'_P and J'_G to

$$J_P = 0.45(0.94) = 0.423$$

$$J_G = 0.54(0.98) = 0.529$$

The load-distribution factor K_m is estimated from Eq. (14–32):

$$C_{pf} = \frac{1.5}{10(1.963)} - 0.0375 + 0.0125(1.5) = 0.0577$$

with $C_{mc} = 1$, $C_{pm} = 1$, $C_{ma} = 0.15$ from Fig. 14–11, and $C_e = 1$. Therefore, from Eq. (14–30),

$$K_m = 1 + (1)[0.0577(1) + 0.15(1)] = 1.208$$

(*a*) **Pinion tooth bending.** Substituting the appropriate terms into Eq. (14–15) using P_t gives

$$(\sigma)_P = \left(W^t K_o K_v K_s \frac{P_t}{F} \frac{K_m K_B}{J} \right)_P = 142.7(1)1.404(1.043)\frac{8.66}{1.5}\frac{1.208(1)}{0.423}$$

$$= 3445 \text{ psi}$$

Substituting the appropriate terms for the pinion into Eq. (14–41) gives

$$(S_F)_P = \left(\frac{S_t Y_N/(K_T K_R)}{\sigma} \right)_P = \frac{31\,350(0.977)/[1(0.85)]}{3445} = 10.5$$

Gear tooth bending. Substituting the appropriate terms for the gear into Eq. (14–15) gives

$$(\sigma)_G = 142.7(1)1.404(1.052)\frac{8.66}{1.5}\frac{1.208(1)}{0.529} = 2779 \text{ psi}$$

Substituting the appropriate terms for the gear into Eq. (14–41) gives

$$(S_F)_G = \frac{28\,260(0.996)/[1(0.85)]}{2779} = 11.9$$

(*b*) **Pinion tooth wear.** Substituting the appropriate terms for the pinion into Eq. (14–16) gives

$$(\sigma_c)_P = C_p \left(W^t K_o K_v K_s \frac{K_m}{d_P F} \frac{C_f}{I} \right)_P^{1/2}$$

$$= 2300 \left[142.7(1)1.404(1.043) \frac{1.208}{1.963(1.5)} \frac{1}{0.195} \right]^{1/2} = 48\,230 \text{ psi}$$

Substituting the appropriate terms for the pinion into Eq. (14–42) gives

$$(S_H)_P = \left(\frac{S_c Z_N/(K_T K_R)}{\sigma_c} \right)_P = \frac{106\,400(0.948)/[1(0.85)]}{48\,230} = 2.46$$

Gear tooth wear. The only term in Eq. (14–16) that changes for the gear is K_s. Thus,

$$(\sigma_c)_G = \left[\frac{(K_s)_G}{(K_s)_P}\right]^{1/2} (\sigma_c)_P = \left(\frac{1.052}{1.043}\right)^{1/2} 48\,230 = 48\,440 \text{ psi}$$

Substituting the appropriate terms for the gear into Eq. (14–42) with $C_H = 1.005$ gives

$$(S_H)_G = \frac{93\,500(0.973)1.005/[1(0.85)]}{48\,440} = 2.22$$

(*c*) For the pinion we compare S_F with S_H^2, or 10.5 with $2.46^2 = 6.05$, so the threat in the pinion is from wear. For the gear we compare S_F with S_H^2, or 11.9 with $2.22^2 = 4.93$, so the threat is also from wear in the gear. For the meshing gearset wear controls.

Some necessary relationships between material properties of spur gears in mesh.

- In bending, the AGMA equations are displayed side by side:

$$\sigma_P = \left(W^t K_o K_v K_s \frac{P_d}{F} \frac{K_m K_B}{J} \right)_P \qquad \sigma_G = \left(W^t K_o K_v K_s \frac{P_d}{F} \frac{K_m K_B}{J} \right)_G$$

$$(S_F)_P = \left(\frac{S_t Y_N / (K_T K_R)}{\sigma} \right)_P \qquad (S_F)_G = \left(\frac{S_t Y_N / (K_T K_R)}{\sigma} \right)_G$$

- Equating the factors of safety, substituting for stress and strength, canceling identical terms *(K_s* virtually equal or exactly equal), and solving for $(S_t)_G$ gives

$$(S_t)_G = (S_t)_P \frac{(Y_N)_P}{(Y_N)_G} \frac{J_P}{J_G} \tag{a}$$

- The stress-cycle factor Y_N comes from Fig. 14-14, where for a particular hardness, $Y_N = \alpha N^{\beta}$. For the pinion, $(Y_N)^P = \alpha N_p^{\beta}$, and for the gear, $(Y_N)_G = \alpha (N_P/m_G)^{\beta}$. Substituting these into Eq. (a) and simplifying gives

$$(S_t)_G = (S_t)_P m_G^{\beta} \frac{J_P}{J_G} \tag{14-44}$$

- Normally, $m_G > 1$ and $J_G > J_p$, so equation (14-44) shows that the gear can be less strong (lower Brinell hardness) than the pinion for the same safety factor.

Example 14-6

- In a set of spur gears, a 300-Brinell 18-tooth 16-pitch 20° full-depth pinion meshes with a 64-tooth gear. Both gear and pinion are of grade 1 through-hardened steel. Using β = -0.023, what hardness can the gear have for the same factor of safety?

Solution

- For through-hardened grade 1 steel the pinion strength $(S_t)_p$ is given in Fig. 14-2:
- $(S_t)_P = 77.3(300) + 12\,800 = 35\,990$ psi
- From Fig. 14-6 the form factors are $J_p = 0.32$ and $J_G = 0.41$.
- Equation (14- 44) gives

$$(S_t)_G = 35990\left(\frac{64}{18}\right)^{-0.023}\frac{0.32}{0.41} = 27280\text{psi}$$

- Use the equation in Fig. 14-2 again.

$$(H_B)_G = \frac{27280 - 12800}{77.3} = 187\text{Brinell}$$

- The AGMA contact-stress equations also are displayed side by side:

$$(\sigma_c)_P = C_p \left(W^t K_o K_v K_s \frac{K_m}{d_P F} \frac{C_f}{I} \right)_P^{1/2} \qquad (\sigma_c)_G = C_p \left(W^t K_o K_v K_s \frac{K_m}{d_P F} \frac{C_f}{I} \right)_G^{1/2}$$

$$(S_H)_P = \left(\frac{S_c Z_N/(K_T K_R)}{\sigma_c} \right)_P \qquad (S_H)_G = \left(\frac{S_c Z_N C_H/(K_T K_R)}{\sigma_c} \right)_G$$

- Equating the factors of safety, substituting the stress relations, and canceling identical terms including K_S gives, after solving for $(S_C)_G$,

$$(S_c)_G = (S_c)_P \frac{(Z_N)_P}{(Z_N)_G} \left(\frac{1}{C_H}\right)_G = (S_C)_P m_G^\beta \left(\frac{1}{C_H}\right)_G$$

- Where, as in the development of Eq. (14-44), $(Z_N)_P/(Z_N)_C = m_G^\beta$ and the value of β for wear comes from Fig. 14-15. Since C_H is so close to unity, it is usually neglected; therefore

$$(S_c)_G = (S_c)_P m_G^\beta$$

Example:

- For β = -0.056 for a through-hardened steel, grade 1, continue Ex. 14-6 for wear.
- From Fig. 14-5,

$$(S_C)_P = 322(300) + 29100 = 125700 \text{ psi}$$

- From equation (14-45)

$$(S_c)_G = (S_c)_P \left(\frac{64}{18}\right)^{-0.056} = 125\,700 \left(\frac{64}{18}\right)^{-0.056} = 117\,100 \text{ psi}$$

$$(H_B)_G = \frac{117\,100 - 29\,200}{322} = 273 \text{ Brinell}$$

- Which is slightly less than the pinion hardness of 300 Brinell.

Note: equations (14-44) and (14-45) apply as well to helical gears

14.19 Design of a Gear Mesh

- A useful decision set for spur and helical gears includes:
  - Function: load, speed, reliability, life, K_o
  - Unquantifiable risk: design factor n_d
  - Tooth system: , addendum, dedendum, root fillet radius
  - Gear ratio m_G, N_p, N_G
  - Quality number Q_v
  - Diametral pitch P_d
  - Face width F
  - Pinion material, core hardness, case hardness
  - Gear material, core hardness, case hardness

A priori Decisions

Design Decisions

- The first item to notice is the dimensionality of the decision set.
- There are four design decision categories, eight different decisions if you count them separately.
- It is important to use a design strategy that is convenient in either longhand execution or computer implementation.
- The design decisions have been placed in order of importance

- The steps are, after the a priori decisions have been made,
  - Choose a diametral pitch.
  - Examine implications on face width, pitch diameters, and material properties.
    - If not satisfactory, return to pitch decision for change.
  - Choose a pinion material and examine core and case hardness requirements.
    - If not satisfactory, return to pitch decision and iterate until no decisions are changed.
  - Choose a gear material and examine core and case hardness requirements.
    - If not satisfactory, return to pitch decision and iterate until no decisions are changed.

- With these plan steps in mind, we can consider them in more detail.
- First select a trial diametral pitch.
- *Pinion bending*:
  - Select a median face width for this pitch, $4\pi/P$
  - Find the range of necessary ultimate strengths
  - Choose a material and a core hardness
  - Find face width to meet factor of safety in bending
  - Choose face width
  - Check factor of safety in bending

- *Gear bending*:
  - Find necessary companion core hardness
  - Choose a material and core hardness
  - Check factor of safety in bending

- *Pinion wear*:
  - Find necessary S_c and attendant case hardness
  - Choose a case hardness
  - Check factor of safety in wear
- *Gear wear:*
  - Find companion case hardness
  - Choose a case hardness
  - Check factor of safety in wear

- Completing this set of steps will yield a satisfactory design.
- Additional designs with diametral pitches adjacent to the first satisfactory design will produce several among which to choose.
- A figure of merit is necessary in order to choose the best.
  - **Unfortunately,** a figure of merit in gear design is complex in an academic environment because material and processing cost vary.

- After examining Ex. 14-4 and Ex. 14-5 and seeing the wide range of factors of safety, one might entertain the notion of setting all factors of safety equal.
- In steel gears, wear is usually controlling and $(S_H)_P$ and $(_{SH})_G$ can be brought close to equality.
- The use of softer cores can bring down $(S_F)_P$ and $(S_F)_G$ but there is value in keeping them higher,
  - A tooth broken by bending fatigue not only can destroy the gear set, but can bend shafts, damage bearings, and produce inertial stresses up- and downstream in the power train, causing damage elsewhere if the gear box locks.

Example 14-8

- Design a 4:1 spur-gear reduction for a 100-hp, three-phase squirrel-cage induction motor running at 1120 rev/min. The load is smooth, providing a reliability of 0.95 at 10^9 revolutions of the pinion. Gearing space is meager. Use Nitralloy 135M, grade 1 material to keep the gear size small. The gears are heat-treated first then nitrided.

Make the a priori decisions:

- Function: 100 hp, 1120 rev/min, $R = 0.95$, $N = 10^9$ cycles, $K_o = 1$
- Design factor for unquantifiable exingencies: $n_d = 2$
- Tooth system: $\phi_n = 20^\circ$
- Tooth count: $N_P = 18$ teeth, $N_G = 72$ teeth (no interference)
- Quality number: $Q_v = 6$, use grade 1 material
- Assume $m_B \geq 1.2$ in Eq. (14–40), $K_B = 1$

Pitch: Select a trial diametral pitch of $P_d = 4$ teeth/in. Thus, $d_P = 18/4 = 4.5$ in and $d_G = 72/4 = 18$ in. From Table 14–2, $Y_P = 0.309$, $Y_G = 0.4324$ (interpolated). From Fig. 14–6, $J_P = 0.32$, $J_G = 0.415$.

$$V = \frac{\pi d_P n_P}{12} = \frac{\pi(4.5)1120}{12} = 1319 \text{ ft/min}$$

$$W^t = \frac{33\,000H}{V} = \frac{33\,000(100)}{1319} = 2502 \text{ lbf}$$

From Eqs. (14–28) and (14–27),

$$B = 0.25(12 - Q_v)^{2/3} = 0.25(12 - 6)^{2/3} = 0.8255$$

$$A = 50 + 56(1 - 0.8255) = 59.77$$

$$K_v = \left(\frac{59.77 + \sqrt{1319}}{59.77}\right)^{0.8255} = 1.480$$

From Eq. (14–38), $K_R = 0.658 - 0.0759 \ln(1 - 0.95) = 0.885$. From Fig. 14–14,

$$(Y_N)_P = 1.3558(10^9)^{-0.0178} = 0.938$$

$$(Y_N)_G = 1.3558(10^9/4)^{-0.0178} = 0.961$$

From Fig. 14–15,

$$(Z_N)_P = 1.4488(10^9)^{-0.023} = 0.900$$

$$(Z_N)_G = 1.4488(10^9/4)^{-0.023} = 0.929$$

From the recommendation after Eq. (14–8), $3p \le F \le 5p$. Try $F = 4p = 4\pi/P = 4\pi/4 = 3.14$ in. From Eq. (a), Sec. 14–10,

$$K_s = 1.192\left(\frac{F\sqrt{Y}}{P}\right)^{0.0535} = 1.192\left(\frac{3.14\sqrt{0.309}}{4}\right)^{0.0535} = 1.140$$

From Eqs. (14–31), (14–33), (14–35), $C_{mc} = C_{pm} = C_e = 1$. From Fig. 14–11, $C_{ma} = 0.175$ for commercial enclosed gear units. From Eq. (14–32), $F/(10d_P) = 3.14/[10(4.5)] = 0.0698$. Thus,

$$C_{pf} = 0.0698 - 0.0375 + 0.0125(3.14) = 0.0715$$

From Eq. (14–30),

$$K_m = 1 + (1)[0.0715(1) + 0.175(1)] = 1.247$$

From Table 14–8, for steel gears, $C_p = 2300\sqrt{\text{psi}}$. From Eq. (14–23), with $m_G = 4$ and $m_N = 1$,

$$I = \frac{\cos 20^\circ \sin 20^\circ}{2}\,\frac{4}{4+1} = 0.1286$$

Pinion tooth bending. With the above estimates of K_s and K_m from the trial dimetral pitch, we check to see if the mesh width F is controlled by bending or wear considerations. Equating Eqs. (14–15) and (14–17), substituting $n_d W^t$ for W^t, and solving for the face width $(F)_{\text{bend}}$ necessary to resist bending fatigue, we obtain

$$(F)_{\text{bend}} = n_d W^t K_o K_v K_s P_d \frac{K_m K_B}{J_P} \frac{K_T K_R}{S_t Y_N} \tag{1}$$

Equating Eqs. (14–16) and (14–18), substituting $n_d W^t$ for W^t, and solving for the face width $(F)_{\text{wear}}$ necessary to resist wear fatigue, we obtain

$$(F)_{\text{wear}} = \left(\frac{C_p Z_N}{S_c K_T K_R} \right)^2 n_d W^t K_o K_v K_s \frac{K_m C_f}{d_P I} \tag{2}$$

From Table 14–5 the hardness range of Nitralloy 135M is Rockwell C32–36 (302–335 Brinell). Choosing a midrange hardness as attainable, using 320 Brinell. From Fig. 14–4,

$$S_t = 86.2(320) + 12\,730 = 40\,310 \text{ psi}$$

Inserting the numerical value of S_t in Eq. (1) to estimate the face width gives

$$(F)_{\text{bend}} = 2(2502)(1)1.48(1.14)4\frac{1.247(1)(1)0.885}{0.32(40\,310)0.938} = 3.08 \text{ in}$$

From Table 14–6 for Nitralloy 135M, $S_c = 170\,000$ psi. Inserting this in Eq. (2), we find

$$(F)_{\text{wear}} = \left(\frac{2300(0.900)}{170\,000(1)0.885}\right)^2 2(2502)1(1.48)1.14\frac{1.247(1)}{4.5(0.1286)} = 3.44 \text{ in}$$

Decision Make face width 3.50 in. Correct K_s and K_m:

$$K_s = 1.192\left(\frac{3.50\sqrt{0.309}}{4}\right)^{0.0535} = 1.147$$

$$\frac{F}{10d_P} = \frac{3.50}{10(4.5)} = 0.0778$$

$$C_{pf} = 0.0778 - 0.0375 + 0.0125(3.50) = 0.0841$$

$$K_m = 1 + (1)[0.0841(1) + 0.175(1)] = 1.259$$

The bending stress induced by W^t in bending, from Eq. (14–15), is

$$(\sigma)_P = 2502(1)1.48(1.147)\frac{4}{3.50}\frac{1.259(1)}{0.32} = 19\,100 \text{ psi}$$

The factor of safety in bending of the pinion, from Eq. (14–41), is

$$(S_F)_P = \frac{40\,310(0.938)/[1(0.885)]}{19\,100} = 2.24$$

Decision

Gear tooth bending. Use cast gear blank because of the 18-in pitch diameter. Use the same material, heat treatment, and nitriding. The load-induced bending stress is in the ratio of J_P/J_G. Then

$$(\sigma)_G = 19\,100\frac{0.32}{0.415} = 14\,730 \text{ psi}$$

The factor of safety of the gear in bending is

$$(S_F)_G = \frac{40\,310(0.961)/[1(0.885)]}{14\,730} = 2.97$$

Pinion tooth wear. The contact stress, given by Eq. (14–16), is

$$(\sigma_c)_P = 2300\left[2502(1)1.48(1.147)\frac{1.259}{4.5(3.5)}\frac{1}{0.129}\right]^{1/2} = 118\,000 \text{ psi}$$

The factor of safety from Eq. (14–42), is

$$(S_H)_P = \frac{170\,000(0.900)/[1(0.885)]}{118\,000} = 1.465$$

By our definition of factor of safety, pinion bending is $(S_F)_P = 2.24$, and wear is $(S_H)_P^2 = (1.465)^2 = 2.15$.

Gear tooth wear. The hardness of the gear and pinion are the same. Thus, from Fig. 14–12, $C_H = 1$, the contact stress on the gear is the same as the pinion, $(\sigma_c)_G =$ 118 000 psi. The wear strength is also the same, $S_c = 170\,000$ psi. The factor of safety of the gear in wear is

$$(S_H)_G = \frac{170\,000(0.929)/[1(0.885)]}{118\,000} = 1.51$$

So, for the gear in bending, $(S_F)_G = 2.97$, and wear $(S_H)_G^2 = (1.51)^2 = 2.29$.

Rim. Keep $m_B \geq 1.2$. The whole depth is $h_t =$ addendum + dedendum $= 1/P_d + 1.25/P_d = 2.25/P_d = 2.25/4 = 0.5625$ in. The rim thickness t_R is

$$t_R \geq m_B h_t = 1.2(0.5625) = 0.675 \text{ in}$$

In the design of the gear blank, be sure the rim thickness exceeds 0.675 in; if it does not, review and modify this mesh design.

- This design example showed a satisfactory design for a four-pitch spur-gear mesh.
- Material could be changed, as could pitch.
- There are a number of other satisfactory designs, thus a figure of merit is needed to identify the best.

- One can appreciate that gear design was one of the early applications of the digital computer to mechanical engineering.
- A design program should be interactive, presenting results of calculations, pausing for a decision by the designer, and showing the consequences of the decision, with a loop back to change a decision for the better.
- Standard gears may not be the most economical design that meets the functional requirements, because no application is standard in all respects.
- Methods of designing custom gears are well-understood and frequently used in mobile equipment to provide good weight-to-performance index.
- The required calculations including optimizations are within the capability of a personal computer

EXAMPLE 14–8 A steel spur pinion has a pitch of 6 teeth/in, 17 full-depth milled teeth, and a pressure angle of 20°. The pinion has an ultimate tensile strength at the involute surface of 116 kpsi, a Brinell hardness of 232, and a yield strength of 90 kpsi. Its shaft speed is 1120 rev/min, its face width is 2 in, and its mating gear has 51 teeth. Rate the pinion for power transmission if the design factor is 2.

(*a*) Pinion bending fatigue imposes what power limitation?

(*b*) Pinion surface fatigue imposes what power limitation? The gear has identical strengths to the pinion with regard to material properties.

(*c*) Consider power limitations due to gear bending and wear.

(*d*) Rate the gearset.

Solution Preliminaries:

$$N_P = 17, \quad N_G = 51$$

$$d_P = \frac{N}{P_d} = \frac{17}{6} = 2.833 \text{ in}$$

$$d_G = \frac{51}{6} = 8.500 \text{ in}$$

$$V = \pi d_P n/12 = \pi(2.833)(1120)/12 = 830.7 \text{ ft/min}$$

Eq. (14–4*b*):

$$K_v = (1200 + 830.7)/1200 = 1.692$$

$$\sigma_{\text{all}} = \frac{S_y}{n_d} = \frac{90\,000}{2} = 45\,000 \text{ psi}$$

Table 14–2:

$$Y_P = 0.303, \quad Y_G = 0.410$$

Eq. (14–7):

$$W^t = \frac{FY_P\sigma_{\text{all}}}{K_v P_d} = \frac{2(0.303)(45\,000)}{1.692(6)} = 2686 \text{ lbf}$$

$$H = \frac{W^t V}{33\,000} = \frac{2686(830.7)}{33\,000} = 67.6 \text{ hp}$$

(*a*) **Pinion fatigue.**

Bending

Eq. (6–8) $$S_e' = 0.5S_{ut} = 0.5(116) = 58 \text{ kpsi}$$

Eq. (6–19): $$a = 2.70, \quad b = -0.265, \quad k_a = 2.70(116)^{-0.265} = 0.766$$

Table 13–1: $$l = \frac{1}{P_d} + \frac{1.25}{P_d} = \frac{2.25}{P_d} = \frac{2.25}{6} = 0.375 \text{ in}$$

Eq. (14–3): $$x = \frac{3Y_P}{2P_d} = \frac{3(0.303)}{2(6)} = 0.0758$$

Eq. (*b*), p. 737: $$t = \sqrt{4lx} = \sqrt{4(0.375)(0.0758)} = 0.337 \text{ in}$$

Eq. (6–25): $$d_e = 0.808\sqrt{Ft} = 0.808\sqrt{2(0.337)} = 0.663 \text{ in}$$

Eq. (6–20): $$k_b = \left(\frac{0.663}{0.30}\right)^{-0.107} = 0.919$$

$k_c = k_d = k_e = 1$. Assess two components contributing to k_f. First, based upon one-way bending and the Gerber failure criterion, $k_{f1} = 1.66$ (see Ex. 14–2). Second, due to stress-concentration,

$$r_f = \frac{0.300}{P_d} = \frac{0.300}{6} = 0.050 \text{ in} \quad \text{(see Ex. 14–2)}$$

Fig. A–15–6: $$\frac{r}{d} = \frac{r_f}{t} = \frac{0.05}{0.338} = 0.148$$

Estimate $D/d = \infty$ by setting $D/d = 3$, $K_t = 1.68$. From Fig. 6–20, $q = 0.86$, and Eq. (6–32)

$$K_f = 1 + 0.86(1.68 - 1) = 1.58$$

$$k_{f2} = \frac{1}{K_f} = \frac{1}{1.58} = 0.633$$

$$k_f = k_{f1}k_{f2} = 1.66(0.633) = 1.051$$

$$S_e = 0.766(0.919)(1)(1)(1)(1.051)(58) = 42.9 \text{ kpsi}$$

$$\sigma_{\text{all}} = \frac{S_e}{n_d} = \frac{42.9}{2} = 21.5 \text{ kpsi}$$

$$W^t = \frac{FY_p\sigma_{\text{all}}}{K_v P_d} = \frac{2(0.303)(21\,500)}{1.692(6)} = 1283 \text{ lbf}$$

$$H = \frac{W^t V}{33\,000} = \frac{1283(830.7)}{33\,000} = 32.3 \text{ hp}$$

***(b)* Pinion fatigue.**

Wear

From Table A–5 for steel: $\nu = 0.292, \quad E = 30(10^6)$ psi

Eq. (14–13) or Table 14–8:

$$C_p = \left\{ \frac{1}{2\pi\,[(1 - 0.292^2)/30(10^6)]} \right\}^{1/2} = 2285\sqrt{\text{psi}}$$

In preparation for Eq. (14–14):

Eq. (14–12):

$$r_1 = \frac{d_P}{2}\sin\phi = \frac{2.833}{2}\sin 20^\circ = 0.485 \text{ in}$$

$$r_2 = \frac{d_G}{2}\sin\phi = \frac{8.500}{2}\sin 20^\circ = 1.454 \text{ in}$$

$$\left(\frac{1}{r_1} + \frac{1}{r_2}\right) = \frac{1}{0.485} + \frac{1}{1.454} = 2.750 \text{ in}$$

Eq. (6–68). $(S_C)_{10^8} = 0.4H_B - 10$ kpsi

In terms of gear notation

$$\sigma_C = [0.4(232) - 10]10^3 = 82\,800 \text{ psi}$$

We will introduce the design factor of $nd = 2$ and because it is a contact stress apply it to the load W^t by dividing by $\sqrt{2}$.

$$\sigma_{C,\text{all}} = -\frac{\sigma_c}{\sqrt{2}} = -\frac{82\,800}{\sqrt{2}} = -58\,548 \text{ psi}$$

Solve Eq. (14–14) for W^t:

$$W^t = \left(\frac{-58\,548}{2285}\right)^2 \left[\frac{2\cos 20^\circ}{1.692(2.750)}\right] = 265 \text{ lbf}$$

$$H_{\text{all}} = \frac{265(830.7)}{33\,000} = 6.67 \text{ hp}$$

For 10^8 cycles (turns of pinion), the allowable power is 6.67 hp.

(c) **Gear fatigue due to bending and wear.**

Bending

Eq. (14–3): $$x = \frac{3Y_G}{2P_d} = \frac{3(0.4103)}{2(6)} = 0.1026 \text{ in}$$

Eq. (*b*), p. 737: $$t = \sqrt{4(0.375)(0.1026)} = 0.392 \text{ in}$$

Eq. (6–25): $$d_e = 0.808\sqrt{2(0.392)} = 0.715 \text{ in}$$

Eq. (6–20): $$k_b = \left(\frac{0.715}{0.30}\right)^{-0.107} = 0.911$$

$$k_c = k_d = k_e = 1$$

$$\frac{r}{d} = \frac{r_f}{t} = \frac{0.050}{0.392} = 0.128$$

Approximate $D/d = \infty$ by setting $D/d = 3$, for Fig. A–15–6; $K_t = 1.80$. Fig. 6–20: $q = 0.82$.

$$\text{Eq. (6.32): } k_f = 1 + (0.82)(1.80 - 1) = 1.66$$

$$S_e = 0.766(0.911)(1)(1)(1)(1.66)(58) = 67.2 \text{ kpsi}$$

$$\sigma_{\text{all}} = \frac{S_e}{K_f n_d} = \frac{67.2}{1.66(2)} = 20.2 \text{ kpsi}$$

$$W^t = \frac{F Y_G \sigma_{\text{all}}}{K_v P_d} = \frac{2(0.4103)(20\,200)}{1.692(6)} = 1633 \text{ lbf}$$

$$H_{\text{all}} = \frac{1633(830.7)}{33\,000} = 41.1 \text{ hp}$$

The gear is thus stronger than the pinion in bending.

Wear

Since the material of the pinion and the gear are the same, and the contact stresses are the same, the allowable power transmission of both is the same. Thus, $H_{all} = 6.67$ hp for 10^8 revolutions of each. As yet, we have no way to establish S_C for $10^8/3$ revolutions.

(*d*) Pinion bending: $H_1 = 32.3$ hp

Pinion wear: $H_2 = 6.67$ hp

Gear bending: $H_3 = 41.1$ hp

Gear wear: $H_4 = 6.67$ hp

Power rating of the gear set is thus

Answer $$H_{rated} = \min(32.3, 6.67, 41.1, 6.67) = 6.67 \text{ hp.}$$