Chapter 32

Inductance

Inductance

Self-inductance

- A time-varying current in a circuit produces an induced emf opposing the emf that initially set up the time-varying current.
  - Basis of the electrical circuit element called an *inductor*
- Energy is stored in the magnetic field of an inductor.
- There is an energy density associated with the magnetic field.

Mutual induction

- An emf is induced in a coil as a result of a changing magnetic flux produced by a second coil.

Circuits may contain inductors as well as resistors and capacitors.

Joseph Henry

1797 – 1878

American physicist

First director of the Smithsonian

First president of the Academy of Natural Science

Improved design of electromagnet

Constructed one of the first motors

Discovered self-inductance

- Didn't publish his results

Unit of inductance is named in his honor

Some Terminology

Use *emf* and *current* when they are caused by batteries or other sources.

Use *induced emf* and *induced current* when they are caused by changing magnetic fields.

When dealing with problems in electromagnetism, it is important to distinguish between the two situations.

Self-Inductance

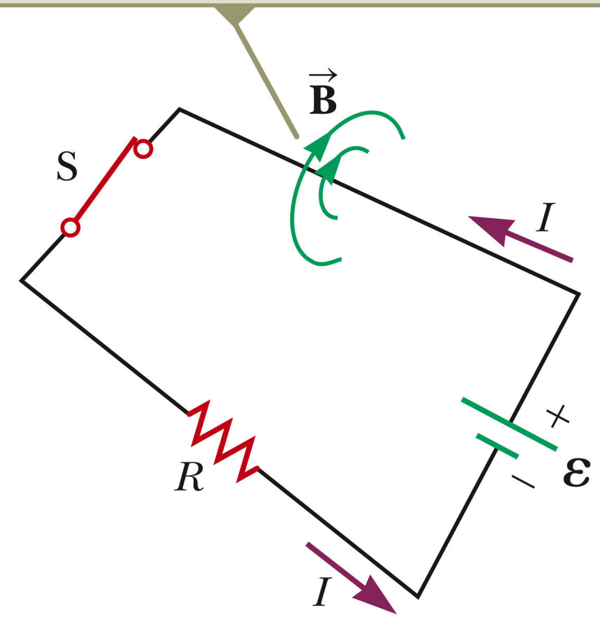
When the switch is closed, the current does not immediately reach its maximum value.

Faraday's law of electromagnetic induction can be used to describe the effect.

As the current increases with time, the magnetic flux through the circuit loop due to this current also increases with time.

This increasing flux creates an induced emf in the circuit.

After the switch is closed, the current produces a magnetic flux through the area enclosed by the loop. As the current increases toward its equilibrium value, this magnetic flux changes in time and induces an emf in the loop.



Self-Inductance, cont.

The direction of the induced emf is such that it would cause an induced current in the loop which would establish a magnetic field opposing the change in the original magnetic field.

The direction of the induced emf is opposite the direction of the emf of the battery.

This results in a gradual increase in the current to its final equilibrium value.

This effect is called **self-inductance.**

- Because the changing flux through the circuit and the resultant induced emf arise from the circuit itself.

The emf ε_L is called a **self-induced emf.**

Self-Inductance, Equations

An induced emf is always proportional to the time rate of change of the current.

- The emf is proportional to the flux, which is proportional to the field and the field is proportional to the current.

$$\varepsilon_L = -L\frac{dI}{dt}$$

L is a constant of proportionality called the **inductance** of the coil.

- It depends on the geometry of the coil and other physical characteristics.

Inductance of a Coil

A closely spaced coil of N turns carrying current *I* has an inductance of

$$L = \frac{N\Phi_B}{I} = -\frac{\varepsilon_L}{dI/dt}$$

The inductance is a measure of the opposition to a change in current.

Inductance Units

The SI unit of inductance is the **henry** (H)

$$1\,\text{H} = 1\,\frac{\text{V} \cdot \text{s}}{\text{A}}$$

Named for Joseph Henry

Example 32.1 Inductance of a Solenoid

Assume a uniformly wound solenoid having N turns and length ℓ.

- Assume ℓ is much greater than the radius of the solenoid.

The flux through each turn of area A is

$$\Phi_B = BA = \mu_o n I A = \mu_o \frac{N}{\ell} I A$$

The inductance is

$$L = \frac{N\Phi_B}{I} = \frac{\mu_o N^2 A}{\ell} = \mu_o n^2 V$$

This shows that L depends on the geometry of the object.

Example 32.2 Calculating Inductance and emf

(A) Calculate the inductance of an air-core solenoid containing 300 turns if the length of the solenoid is 25.0 cm and its cross-sectional area is 4.00 cm².

$$L = \frac{\mu_0 N^2 A}{\ell}$$

$$= \frac{(4\pi \times 10^{-7}\ \mathrm{T \cdot m/A})(300)^2(4.00 \times 10^{-4}\ \mathrm{m}^2)}{25.0 \times 10^{-2}\ \mathrm{m}}$$

$$= 1.81 \times 10^{-4}\ \mathrm{T \cdot m^2/A} = \boxed{0.181\ \mathrm{mH}}$$

(B) Calculate the self-induced emf in the solenoid if the current it carries is decreasing at the rate of 50.0 A/s.

$$\mathcal{E}_L = -L\frac{dI}{dt} = -(1.81 \times 10^{-4}\ \mathrm{H})(-50.0\ \mathrm{A/s})$$

$$= \boxed{9.05\ \mathrm{mV}}$$

Problem 32.6

An emf of 24.0 mV is induced in a 500-turn coil at an instant when the current is 4.00 A and is changing at the rate of 10.0 A/s. What is the magnetic flux through each turn of the coil?

From $|\varepsilon| = L\left(\frac{\Delta I}{\Delta t}\right)$, we have $L = \frac{\varepsilon}{(\Delta I/\Delta t)} = \frac{24.0 \times 10^{-3}\ \text{V}}{10.0\ \text{A/s}} = 2.40 \times 10^{-3}\ \text{H}$.

From $L = \frac{N\Phi_B}{I}$, we have $\Phi_B = \frac{LI}{N} = \frac{(2.40 \times 10^{-3}\ \text{H})(4.00\ \text{A})}{500} = \boxed{19.2\ \mu\text{T}\cdot\text{m}^2}$.

Problem 32.7

An inductor in the form of a solenoid contains 420 turns, is 16.0 cm in length, and has a cross-sectional area of 3.00 cm². What uniform rate of decrease of current through the inductor induces an emf of 175 μV?

$$L = \frac{\mu_0 N^2 A}{\ell} = \frac{\mu_0 (420)^2 \left(3.00 \times 10^{-4}\right)}{0.160} = 4.16 \times 10^{-4}\ \text{H}$$

$$\varepsilon = -L\frac{dI}{dt} \rightarrow \frac{dI}{dt} = \frac{-\varepsilon}{L} = \frac{-175 \times 10^{-6}\ \text{V}}{4.16 \times 10^{-4}\ \text{H}} = \boxed{-0.421\ \text{A/s}}$$

Problem 32.9

A 40.0-mA current is carried by a uniformly wound air-core solenoid with 450 turns, a 15.0-mm diameter, and 12.0-cm length. Compute (a) the magnetic field inside the solenoid, (b) the magnetic flux through each turn, and (c) the inductance of the solenoid. (d) What If? If the current were different, which of these quantities would change?

(a) $$B = \mu_0 n I = \mu_0 \left(\frac{450}{0.120} \right) (0.040\,0 \text{ A}) = \boxed{188\ \mu\text{T}}$$

(b) $$\Phi_B = BA = \boxed{3.33 \times 10^{-8}\ \text{T} \cdot \text{m}^2}$$

(c) $$L = \frac{N\Phi_B}{I} = \boxed{0.375\ \text{mH}}$$

(d) $$\boxed{B \text{ and } \Phi_B \text{ are proportional to current; } L \text{ is independent of current}}$$

Energy in a Magnetic Field

In a circuit with an inductor, the battery must supply more energy than in a circuit without an inductor.

Part of the energy supplied by the battery appears as internal energy in the resistor.

The remaining energy is stored in the magnetic field of the inductor.

Energy in a Magnetic Field, cont.

Looking at this energy (in terms of rate)

$$I\varepsilon = I^2 R + L I \frac{dI}{dt}$$

- $I\mathcal{E}$ is the rate at which energy is being supplied by the battery.
- I^2R is the rate at which the energy is being delivered to the resistor.
- Therefore, $LI\,(dI/dt)$ must be the rate at which the energy is being stored in the magnetic field.

Energy in a Magnetic Field, final

Let U denote the energy stored in the inductor at any time.

The rate at which the energy is stored is

$$\frac{dU}{dt} = L I \frac{dI}{dt}$$

To find the total energy, integrate and

$$U = L \int_0^I I \; dI = \frac{1}{2} L I^2$$

Energy Density of a Magnetic Field

Given $U = \frac{1}{2} L I^2$ and assume (for simplicity) a solenoid with $L = \mu_o n^2 V$

$$U = \frac{1}{2}\mu_o n^2 V\left(\frac{B}{\mu_o n}\right)^2 = \frac{B^2}{2\mu_o}V$$

Since V is the volume of the solenoid, the magnetic energy density, u_B is

$$u_B = \frac{U}{V} = \frac{B^2}{2\mu_o}$$

This applies to any region in which a magnetic field exists (not just the solenoid).

Energy Storage Summary

A resistor, inductor and capacitor all store energy through different mechanisms.

- Charged capacitor
  - Stores energy as electric potential energy
- Inductor
  - When it carries a current, stores energy as magnetic potential energy
- Resistor
  - Energy delivered is transformed into internal energy

Problem 32.29

Calculate the energy associated with the magnetic field of a 200-turn solenoid in which a current of 1.75 A produces a flux of 3.70 x 10^{-4} Wb in each turn.

$$L = \frac{N\Phi_B}{I} = \frac{200\left(3.70\times 10^{-4}\right)}{1.75} = 42.3\text{ mH} \text{ so } U = \frac{1}{2}LI^2 = \frac{1}{2}(0.423\text{ H})(1.75\text{ A})^2 = \boxed{0.064\,8\text{ J}}.$$

Problem 32.30

The magnetic field inside a superconducting solenoid is 4.50 T. The solenoid has an inner diameter of 6.20 cm and a length of 26.0 cm. Determine (a) the magnetic energy density in the field and (b) the energy stored in the magnetic field within the solenoid.

(a) The magnetic energy density is given by

$$\mu = \frac{B^2}{2\mu_0} = \frac{(4.50\ \mathrm{T})^2}{2\left(1.26\times10^{-6}\ \mathrm{T\cdot m/A}\right)} = \boxed{8.06\times10^{6}\ \mathrm{J/m^3}}.$$

(b) The magnetic energy stored in the field equals u times the volume of the solenoid (the volume in which B is non-zero).

$$U = uV = \left(8.06\times10^{6}\ \mathrm{J/m^3}\right)\left[(0.260\ \mathrm{m})\pi(0.031\,0\ \mathrm{m})^2\right] = \boxed{6.32\ \mathrm{kJ}}$$

Problem 32.31

An air-core solenoid with 68 turns is 8.00 cm long and has a diameter of 1.20 cm. How much energy is stored in its magnetic field when it carries a current of 0.770 A?

$$L = \mu_0 \frac{N^2 A}{\ell} = \mu_0 \frac{(68.0)^2 \left[\pi \left(0.600 \times 10^{-2} \right)^2 \right]}{0.080\,0} = 8.21\ \mu\text{H}$$

$$U = \frac{1}{2} L I^2 = \frac{1}{2} \left(8.21 \times 10^{-6}\ \text{H} \right) (0.770\ \text{A})^2 = \boxed{2.44\ \mu\text{J}}$$

Problem 32.37

A uniform electric field of magnitude 680 kV/m through- out a cylindrical volume results in a total energy of 3.40 μJ. What magnetic field over this same region stores the same total energy?

We have $u = \epsilon_0 \dfrac{E^2}{2}$ and $u = \dfrac{B^2}{2\mu_0}$.

Therefore $\epsilon_0 \dfrac{E^2}{2} = \dfrac{B^2}{2\mu_0}$ so $B^2 = \epsilon_0 \mu_0 E^2$

$$B = E\sqrt{\epsilon_0 \mu_0} = \frac{6.80 \times 10^5 \text{ V/m}}{3.00 \times 10^8 \text{ m/s}} = \boxed{2.27 \times 10^{-3} \text{ T}}.$$