Time Series and Forecasting

Chapter 16

McGraw-Hill/Irwin

Goals

- Define the components of a time series
- Compute moving average
- Determine a linear trend equation
- Compute a trend equation for a nonlinear trend
- Use a trend equation to forecast future time periods and to develop seasonally adjusted forecasts
- Determine and interpret a set of seasonal indexes
- Deseasonalize data using a seasonal index
- Test for autocorrelation

Time Series

What is a time series?

- a collection of data recorded over a period of time (weekly, monthly, quarterly)
- an analysis of history, it can be used by management to make current decisions and plans based on long-term forecasting
- Usually assumes past pattern to continue into the future

Components of a Time Series

- Secular Trend – the smooth long term direction of a time series
- Cyclical Variation – the rise and fall of a time series over periods longer than one year
- Seasonal Variation – Patterns of change in a time series within a year which tends to repeat each year
- Irregular Variation – classified into:
  - Episodic – unpredictable but identifiable
  - Residual – also called chance fluctuation and unidentifiable

Cyclical Variation – Sample Chart

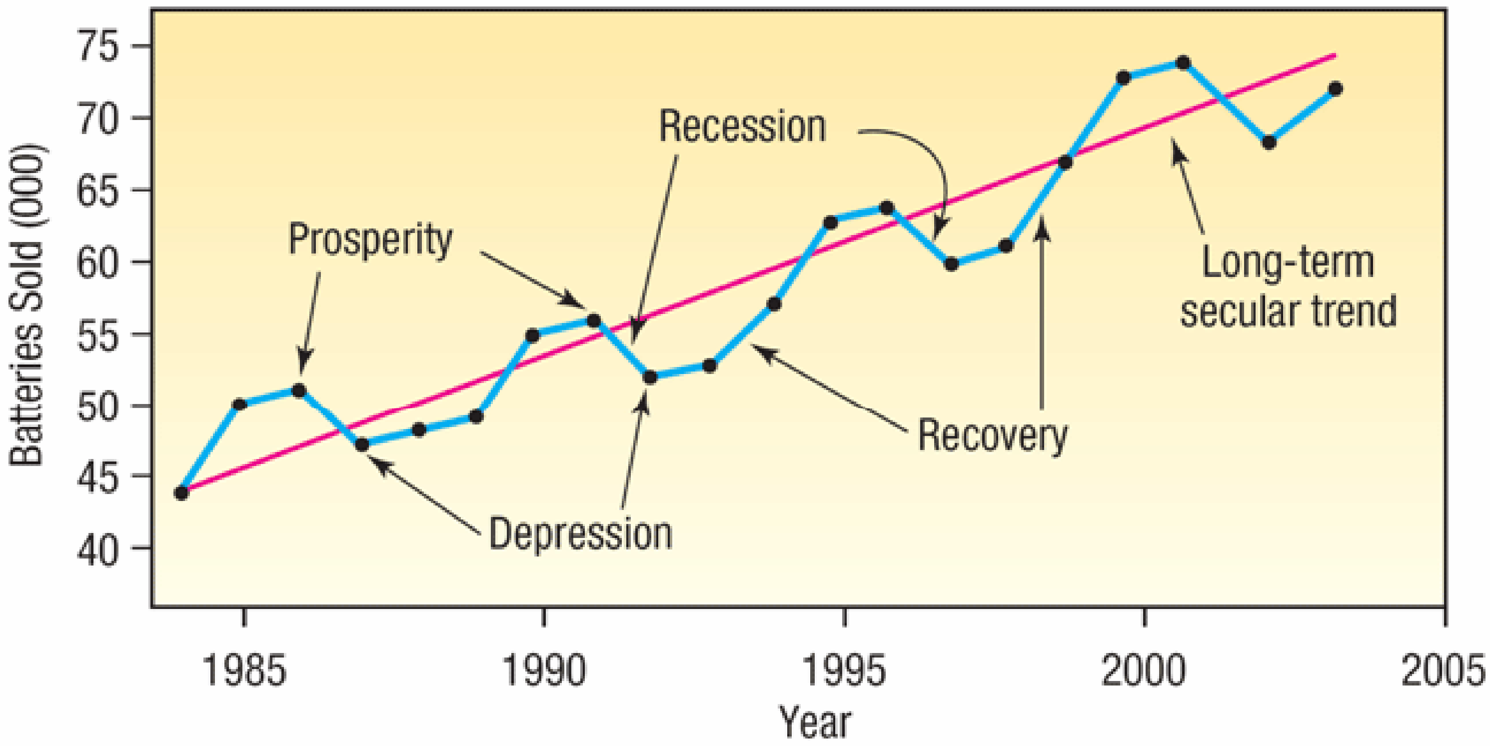


CHART 16–1 Batteries Sold by National Battery Retailers, Inc., from 1984 to 2004

Seasonal Variation – Sample Chart

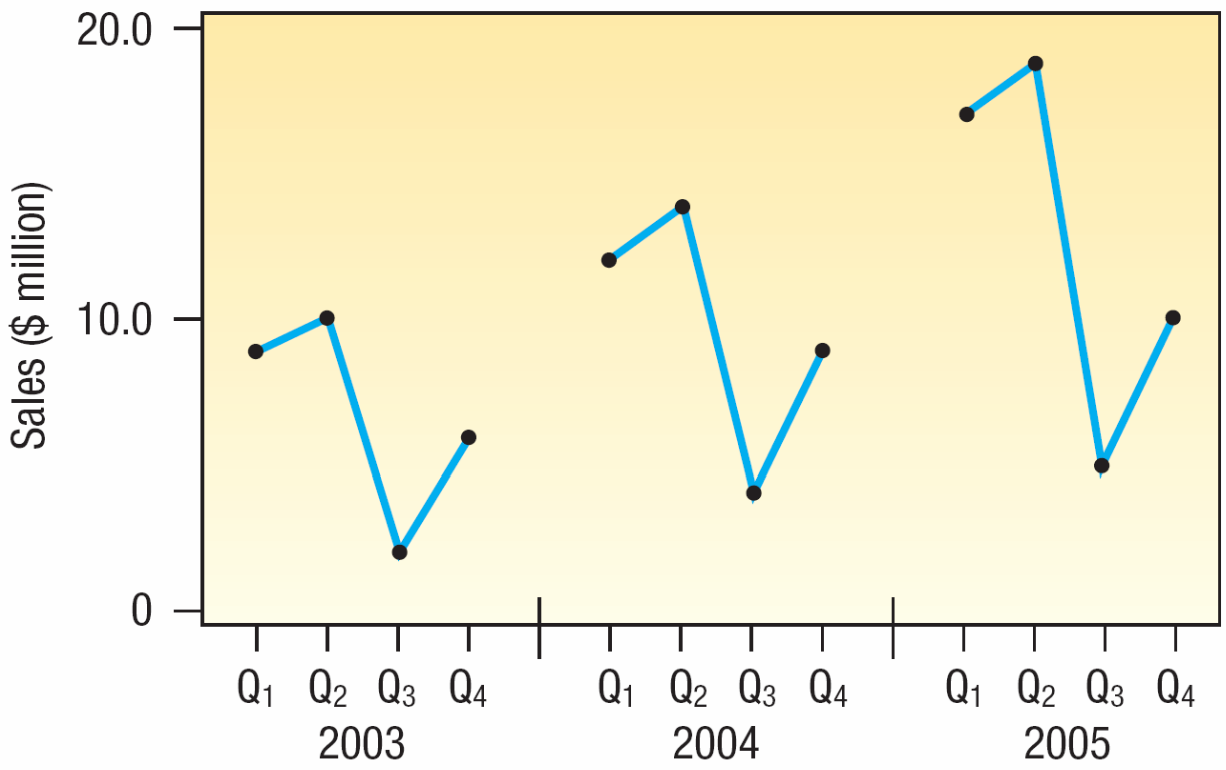
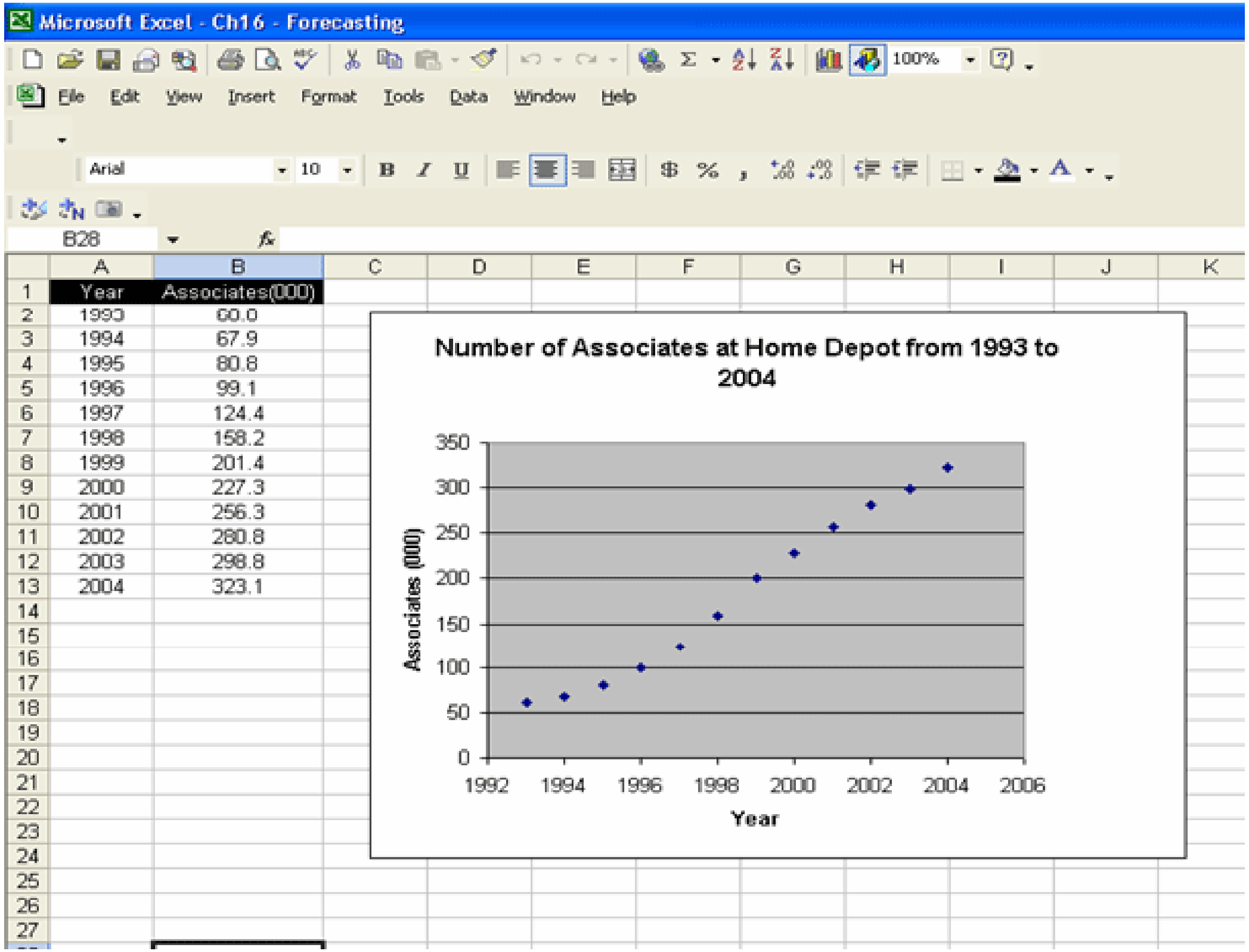


CHART 16–2 Sales of Baseball and Softball Equipment, Hercher Sporting Goods, 2003–2005 by Quarter

Secular Trend – Home Depot Example



Year	Associates(000)
1993	60.0
1994	67.9
1995	80.8
1996	99.1
1997	124.4
1998	158.2
1999	201.4
2000	227.3
2001	256.3
2002	280.8
2003	298.8
2004	323.1

Secular Trend – EMS Calls Example

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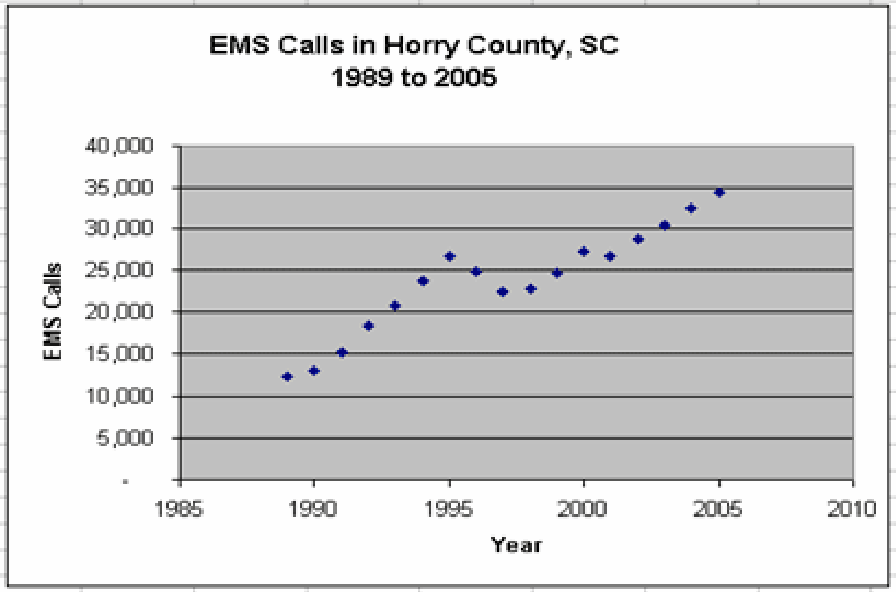
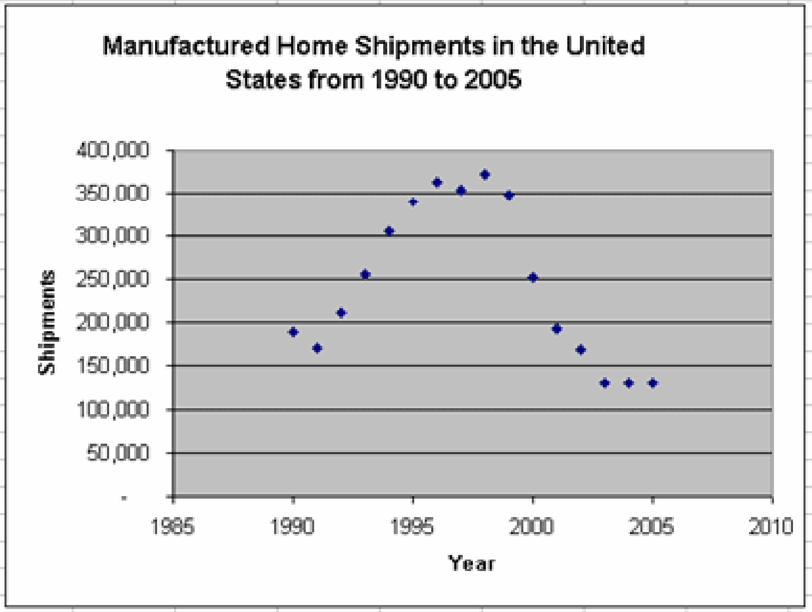
Year	EMS Calls
1989	12,269
1990	13,000
1991	15,219
1992	18,436
1993	20,665
1994	23,702
1995	26,666
1996	24,814
1997	22,461
1998	22,692
1999	24,610
2000	27,149
2001	26,646
2002	28,706
2003	30,341
2004	32,314
2005	34,572



Secular Trend – Manufactured Home Shipments in the U.S.

Year	Shipments
1990	188,172
1991	170,713
1992	210,787
1993	254,276
1994	303,932
1995	339,601
1996	363,411
1997	353,377
1998	372,843
1999	348,671
2000	250,560
2001	193,229
2002	168,491
2003	130,815
2004	130,748
2005	129,368

2005 estimated



The Moving Average Method

- Useful in smoothing time series to see its trend
- Basic method used in measuring seasonal fluctuation
- Applicable when time series follows fairly linear trend that have definite rhythmic pattern

Moving Average Method - Example

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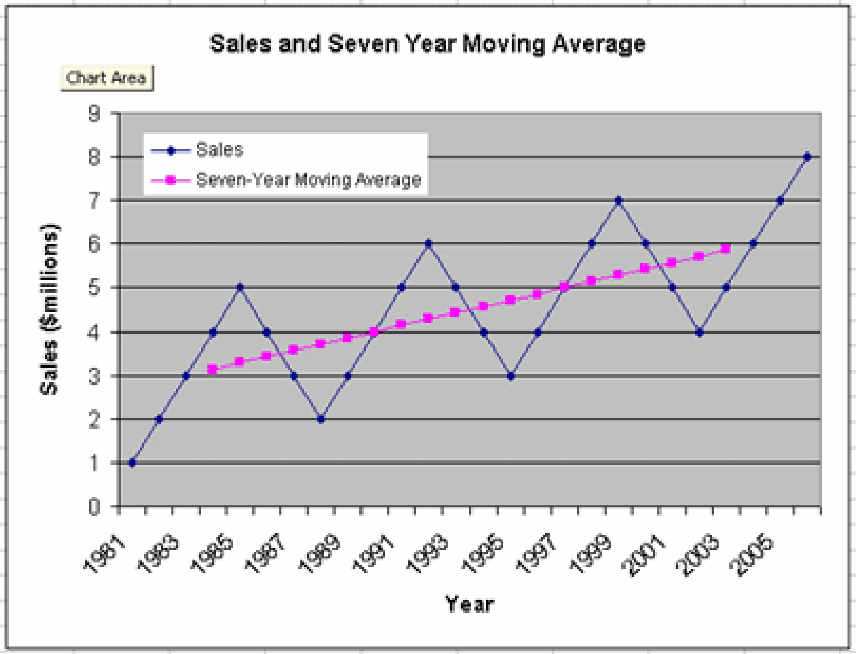
C5 f_x =AVERAGE(B2:B8)

	A	B	C
1	Year	Sales ($M)	MA7
2	1981	1	
3	1982	2	
4	1983	3	
5	1984	4	3.143
6	1985	5	3.286
7	1986	4	3.429
8	1987	3	3.571
9	1988	2	3.714
10	1989	3	3.857
11	1990	4	4.000
12	1991	5	4.143
13	1992	6	4.286
14	1993	5	4.429
15	1994	4	4.571
16	1995	3	4.714
17	1996	4	4.857
18	1997	5	5.000
19	1998	6	5.143
20	1999	7	5.286
21	2000	6	5.429
22	2001	5	5.571
23	2002	4	5.714
24	2003	5	5.857
25	2004	6	
26	2005	7	
27	2006	8	



Three-year and Five-Year Moving Averages

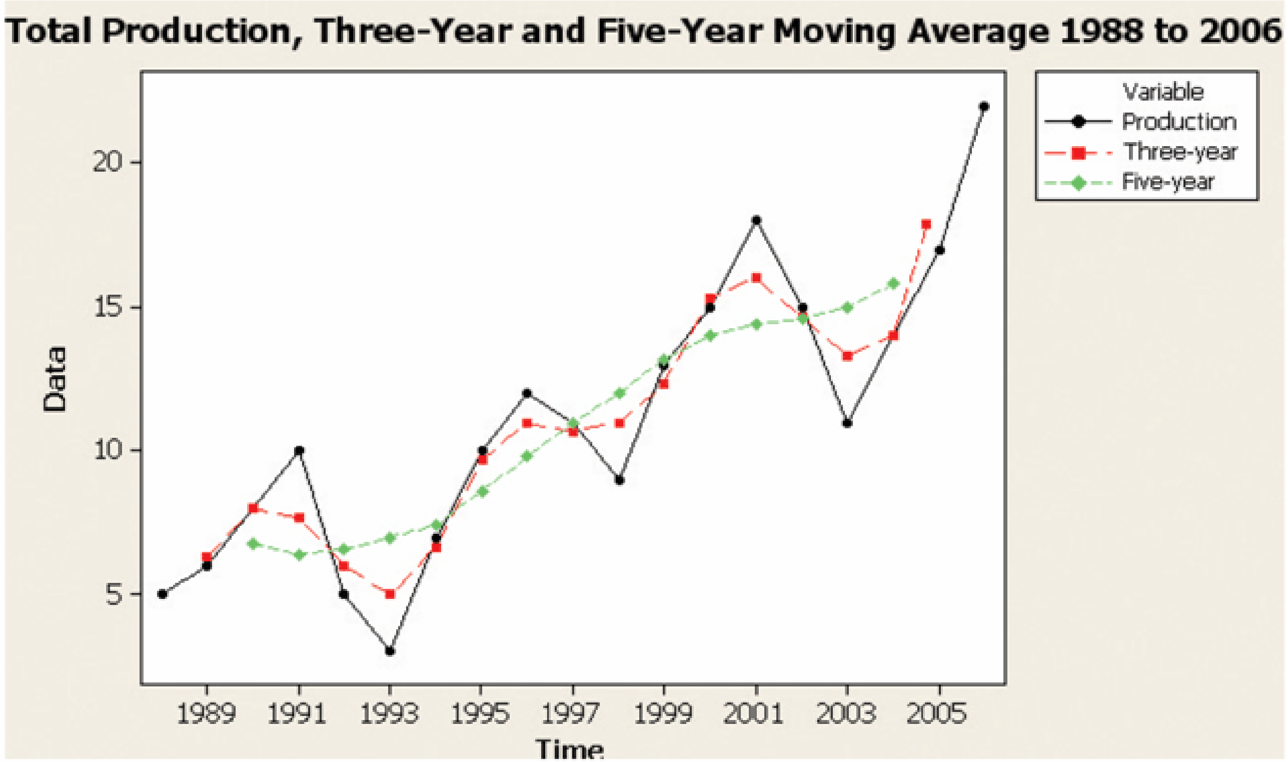


CHART 16–4 A Three-Year and Five-Year Moving Average 1988 to 2006

Weighted Moving Average

- A simple moving average assigns the same weight to each observation in averaging
- Weighted moving average assigns different weights to each observation
- Most recent observation receives the most weight, and the weight decreases for older data values
- In either case, the sum of the weights = 1

Weighted Moving Average - Example

Cedar Fair operates seven amusement parks and five separately gated water parks. Its combined attendance (in thousands) for the last 12 years is given in the following table. A partner asks you to study the trend in attendance. Compute a three-year moving average and a three-year weighted moving average with weights of 0.2, 0.3, and 0.5 for successive years.

Year	Attendance (000)
1993	5,761
1994	6,148
1995	6,783
1996	7,445
1997	7,405
1998	11,450
1999	11,224
2000	11,703
2001	11,890
2002	12,380
2003	12,181
2004	12,557

Weighted Moving Average - Example

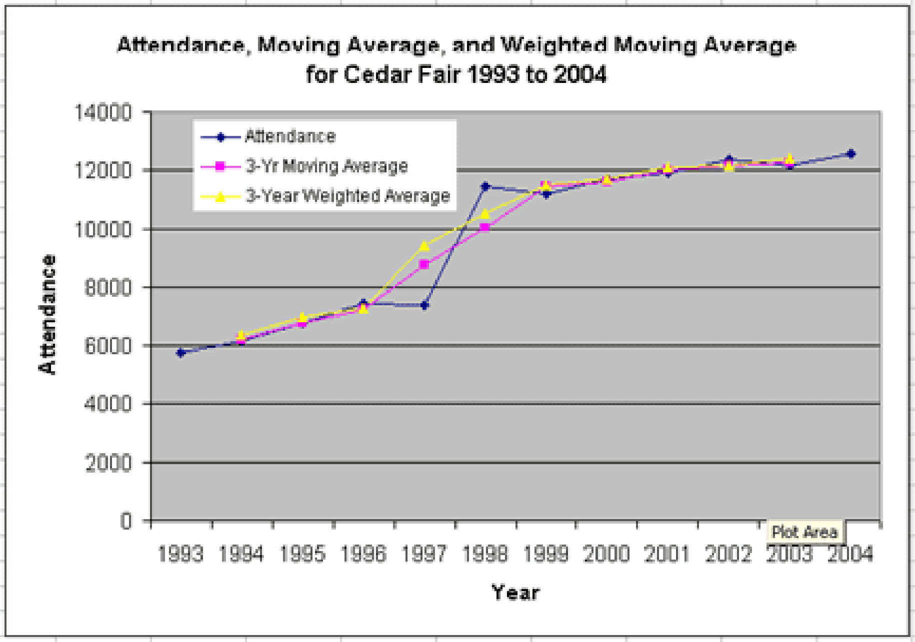
Year	Attendance (000)	Weighted Moving Average	Found by
1993	5,761		
1994	6,148	6,388	.2(5,761) + .3(6,148) + .5(6,783)
1995	6,783	6,987	.2(6,148) + .3(6,783) + .5(7,445)
1996	7,445	7,293	.2(6,783) + .3(7,445) + .5(7,405)
1997	7,405	9,436	.2(7,445) + .3(7,405) + .5(11,450)
1998	11,450	10,528	.2(7,405) + .3(11,450) + .5(11,224)
1999	11,224	11,509	.2(11,450) + .3(11,224) + .5(11,703)
2000	11,703	11,701	.2(11,224) + .3(11,703) + .5(11,890)
2001	11,890	12,098	.2(11,703) + .3(11,890) + .5(12,380)
2002	12,380	12,183	.2(11,890) + .3(12,380) + .5(12,181)
2003	12,181	12,409	.2(12,380) + .3(12,181) + .5(12,557)
2004	12,557		

Weighed Moving Average – An Example

Microsoft Excel - Ch16 - Forecasting

D3 f_x =(0.2)*B2+(0.3)*(B3)+(0.5)*(B4)

	A	B	C	D
1	Year	Attendance	Moving Average	Weighted Average
2	1993	5761		
3	1994	6148	6230.7	6388.1
4	1995	6783	6792.0	6987
5	1996	7445	7211.0	7292.6
6	1997	7405	8766.7	9435.5
7	1998	11450	10026.3	10528
8	1999	11224	11459.0	11508.7
9	2000	11703	11605.7	11700.7
10	2001	11890	11991.0	12097.6
11	2002	12380	12150.3	12182.5
12	2003	12181	12280.5	12408.8
13	2004	12557		



Linear Trend

- The long term trend of many business series often approximates a straight line

Linear Trend Equation: $\hat{Y} = a + bt$

where:

$\hat{Y}$ – read "Y hat", is the projected value of the variable of interest (response variable)

a – the Y-intercept (estimated value of Y when $t = 0$)

b – the slope of the line (average change in Y for each unit change in t)

t – any value of time (coded) that is selected

Linear Trend Plot

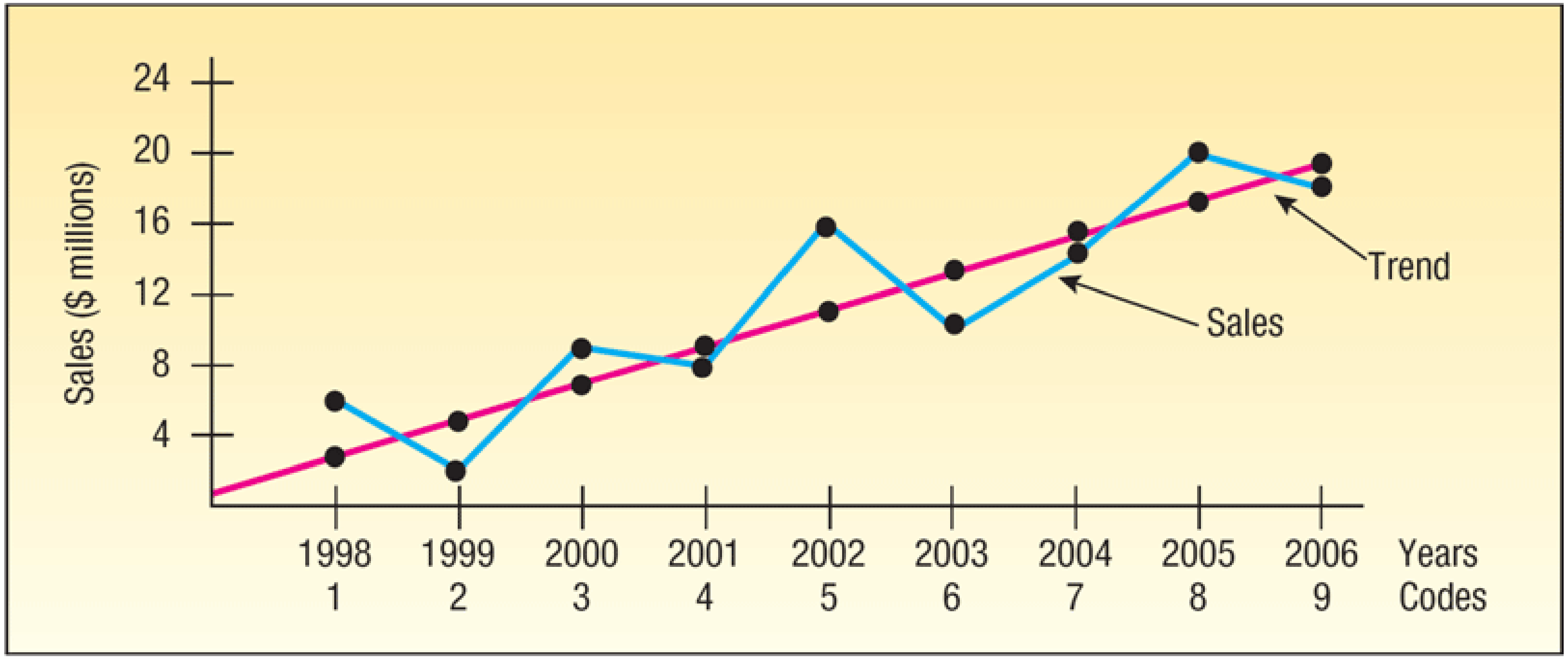


CHART 16–5 A Straight Line Fitted to Sales Data

Linear Trend – Using the Least Squares Method

- Use the least squares method in Simple Linear Regression (Chapter 13) to find the best linear relationship between 2 variables
- Code time (*t*) and use it as the independent variable
- E.g. let *t* be 1 for the first year, 2 for the second, and so on (if data are annual)

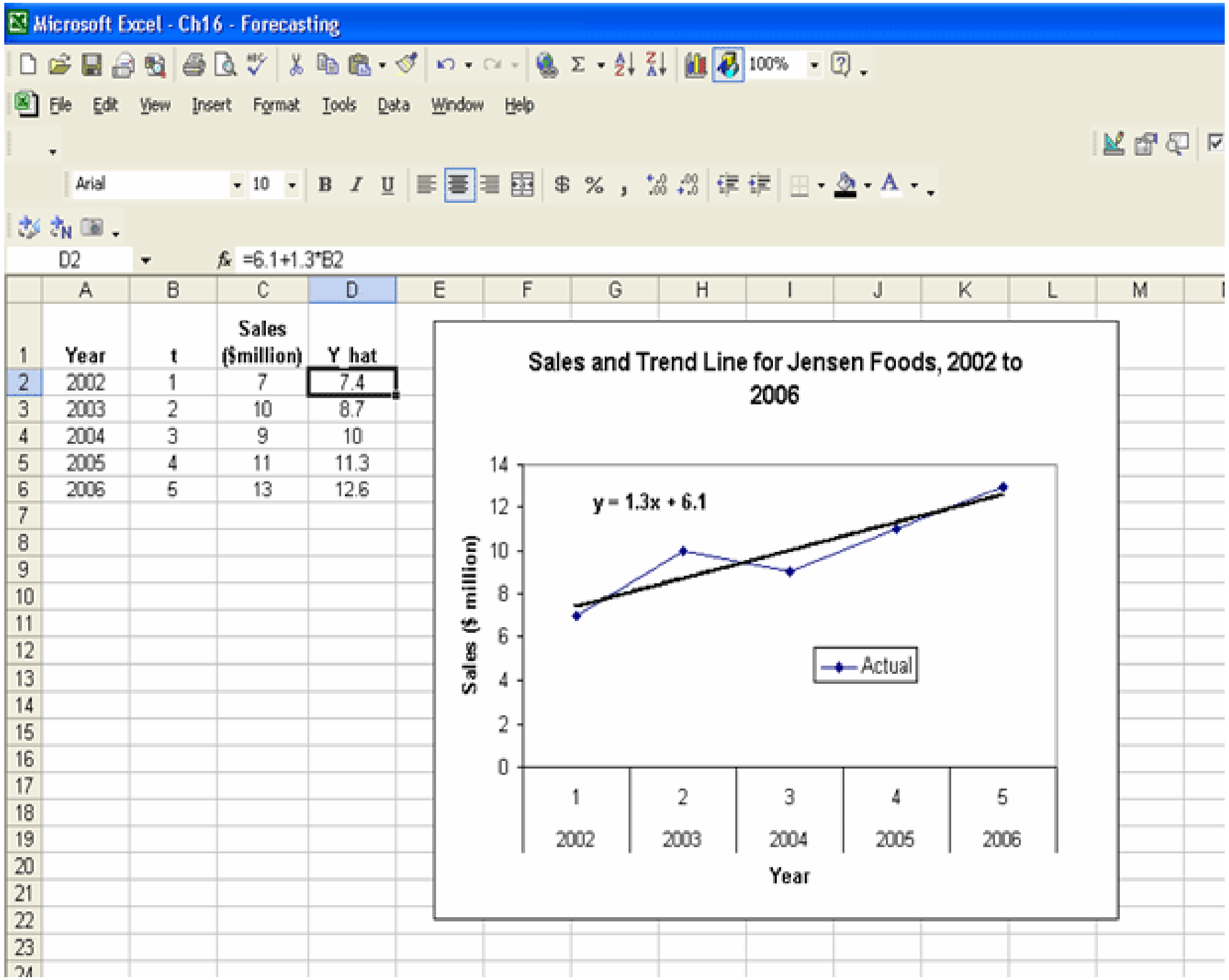
Linear Trend – Using the Least Squares Method: An Example

The sales of Jensen Foods, a small grocery chain located in southwest Texas, since 2002 are:

Year	Sales ($ mil.)
2002	7
2003	10
2004	9
2005	11
2006	13

Year	*t*	Sales ($ mil.)
2002	1	7
2003	2	10
2004	3	9
2005	4	11
2006	5	13

Linear Trend – Using the Least Squares Method: An Example Using Excel



Year	t	Sales ($million)	Y hat
2002	1	7	7.4
2003	2	10	8.7
2004	3	9	10
2005	4	11	11.3
2006	5	13	12.6

Nonlinear Trends

- A linear trend equation is used when the data are increasing (or decreasing) by <u>equal</u> amounts
- A nonlinear trend equation is used when the data are increasing (or decreasing) by <u>increasing</u> amounts over time
- When data increase (or decrease) by equal *percents or proportions* plot will show curvilinear pattern

Log Trend Equation – Gulf Shores Importers Example

- Top graph is plot of the original data
- Bottom graph is the log base 10 of the original data which now is linear

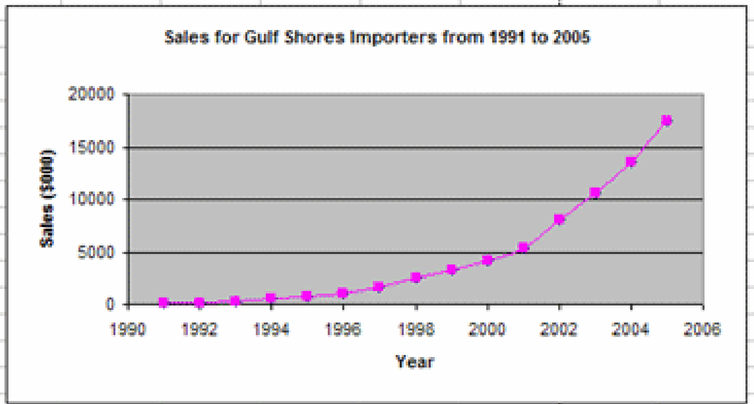
  (Excel function:

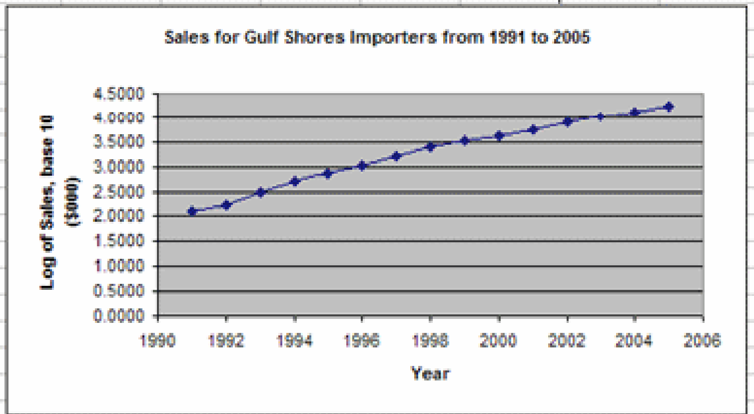
  =log(x) or log(x,10)
- Using Data Analysis in Excel, generate the linear equation
- Regression output shown in next slide

Microsoft Excel - Ch16 - Forecasting.xls [Read-Only]

C2 =LOG(B2,10)

Year	Sales	Log-Sales
1991	124.2	2.0941
1992	175.6	2.2445
1993	306.9	2.4870
1994	524.2	2.7195
1995	714	2.8537
1996	1052	3.0220
1997	1638.3	3.2144
1998	2463.2	3.3915
1999	3358.2	3.5261
2000	4181.3	3.6213
2001	5388.5	3.7315
2002	8027.4	3.9046
2003	10587.2	4.0248
2004	13537.4	4.1315
2005	17515.6	4.2434





Log Trend Equation – Gulf Shores Importers Example

The Linear Equation is :

$$\hat{y} = 2.053805 + 0.153357\,t$$

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Year	Sales	Log-Sales	t
1991	124.2	2.0941	1
1992	175.6	2.2445	2
1993	306.9	2.4870	3
1994	524.2	2.7195	4
1995	714	2.8537	5
1996	1052	3.0220	6
1997	1638.3	3.2144	7
1998	2463.2	3.3915	8
1999	3358.2	3.5261	9
2000	4181.3	3.6213	10
2001	5388.5	3.7315	11
2002	8027.4	3.9046	12
2003	10587.2	4.0248	13
2004	13537.4	4.1315	14
2005	17515.6	4.2434	15

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.993953
R Square	0.987943
Adjusted R Squ	0.987016
Standard Error	0.078625
Observations	15

ANOVA

	df	*SS*	*MS*	*F*	*ignificance F*
Regression	1	6.585166	6.585166	1065.228	7.36E-14
Residual	13	0.080365	0.006182		
Total	14	6.665531			

	Coefficients	*tandard Err*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*ower 95.0%*	*pper 95.0%*
Intercept	2.053805	0.042722	48.07407	4.98E-16	1.961511	2.1461	1.961511	2.1461
t	0.153357	0.004699	32.63783	7.36E-14	0.143206	0.163508	0.143206	0.163508

Log Trend Equation – Gulf Shores Importers Example

Estimate the Import for the year 2009 using the linear trend

$$\hat{y} = 2.053807 + 0.153357t$$

Substitute into the linear equation above the code (19) for 2009

$$\hat{y} = 2.053805 + 0.153357(19)$$

$$\hat{y} = 4.967588$$

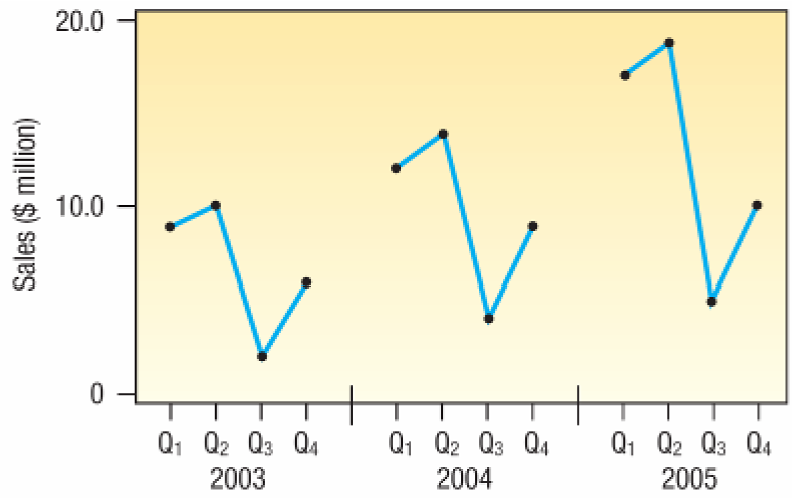
Then find the antilog of $\hat{y} = 10^{\hat{Y}}$

$$= 10^{4.967588}$$

$$= 92{,}808$$

Seasonal Variation

- One of the components of a time series
- Seasonal variations are fluctuations that coincide with certain seasons and are repeated year after year
- Understanding seasonal fluctuations help plan for sufficient goods and materials on hand to meet varying seasonal demand
- Analysis of seasonal fluctuations over a period of years help in evaluating current sales



Seasonal Index

- A number, usually expressed in percent, that expresses the relative value of a season with respect to the average for the year (100%)
- Ratio-to-moving-average method
  - The method most commonly used to compute the typical seasonal pattern
  - It eliminates the trend (*T*), cyclical (*C*), and irregular (*I*) components from the time series

Seasonal Index – An Example

The table below shows the quarterly sales for Toys International for the years 2001 through 2006. The sales are reported in millions of dollars. Determine a quarterly seasonal index using the ratio-to-moving-average method.

Year	Winter	Spring	Summer	Fall
2001	6.7	4.6	10.0	12.7
2002	6.5	4.6	9.8	13.6
2003	6.9	5.0	10.4	14.1
2004	7.0	5.5	10.8	15.0
2005	7.1	5.7	11.1	14.5
2006	8.0	6.2	11.4	14.9

Step (1) – Organize time series data in column form

Step (2) Compute the 4-quarter moving totals

Step (3) Compute the 4-quarter moving averages

Step (4) Compute the centered moving averages by getting the average of two 4-quarter moving averages

Step (5) Compute ratio by dividing actual sales by the centered moving averages

Year	Quarter	(1) Sales ($ millions)	(2) Four-Quarter Total	(3) Four-Quarter Moving Average	(4) Centered Moving Average	(5) Specific Seasonal
2001	Winter	6.7				
	Spring	4.6				
			34.0	8.500		
	Summer	10.0			8.475	1.180
			33.8	8.450		
	Fall	12.7			8.450	1.503
			33.8	8.450		
2002	Winter	6.5			8.425	0.772
			33.6	8.400		
	Spring	4.6			8.513	0.540
			34.5	8.625		
	Summer	9.8			8.675	1.130
			34.9	8.725		
	Fall	13.6			8.775	1.550
			35.3	8.825		
2003	Winter	6.9			8.900	0.775
			35.9	8.975		
	Spring	5.0			9.038	0.553
			36.4	9.100		
	Summer	10.4			9.113	1.141
			36.5	9.125		
	Fall	14.1			9.188	1.535
			37.0	9.250		
2004	Winter	7.0			9.300	0.753
			37.4	9.350		
	Spring	5.5			9.463	0.581
			38.3	9.575		
	Summer	10.8			9.588	1.126
			38.4	9.600		
	Fall	15.0			9.625	1.558
			38.6	9.650		
2005	Winter	7.1			9.688	0.733
			38.9	9.725		
	Spring	5.7			9.663	0.590
			38.4	9.600		
	Summer	11.1			9.713	1.143
			39.3	9.825		
	Fall	14.5			9.888	1.466
			39.8	9.950		
2006	Winter	8.0			9.888	0.801
			40.1	10.025		
	Spring	6.2			10.075	0.615
			40.5	10.125		
	Summer	11.4				
	Fall	14.9				

Seasonal Index – An Example

Year	Winter	Spring	Summer	Fall	
2001			1.180	1.503	
2002	0.772	0.540	1.130	1.550	
2003	0.775	0.553	1.141	1.535	
2004	0.753	0.581	1.126	1.558	
2005	0.733	0.590	1.143	1.466	
2006	0.801	0.615			
Total	3.834	2.879	5.720	7.612	
Mean	0.767	0.576	1.144	1.522	4.009
Adjusted	0.765	0.575	1.141	1.519	4.000
Index	76.5	57.5	114.1	151.9	

CORRECTION FACTOR FOR ADJUSTING QUARTERLY MEANS

$$\text{Correction factor} = \frac{4.00}{\text{Total of four means}}$$

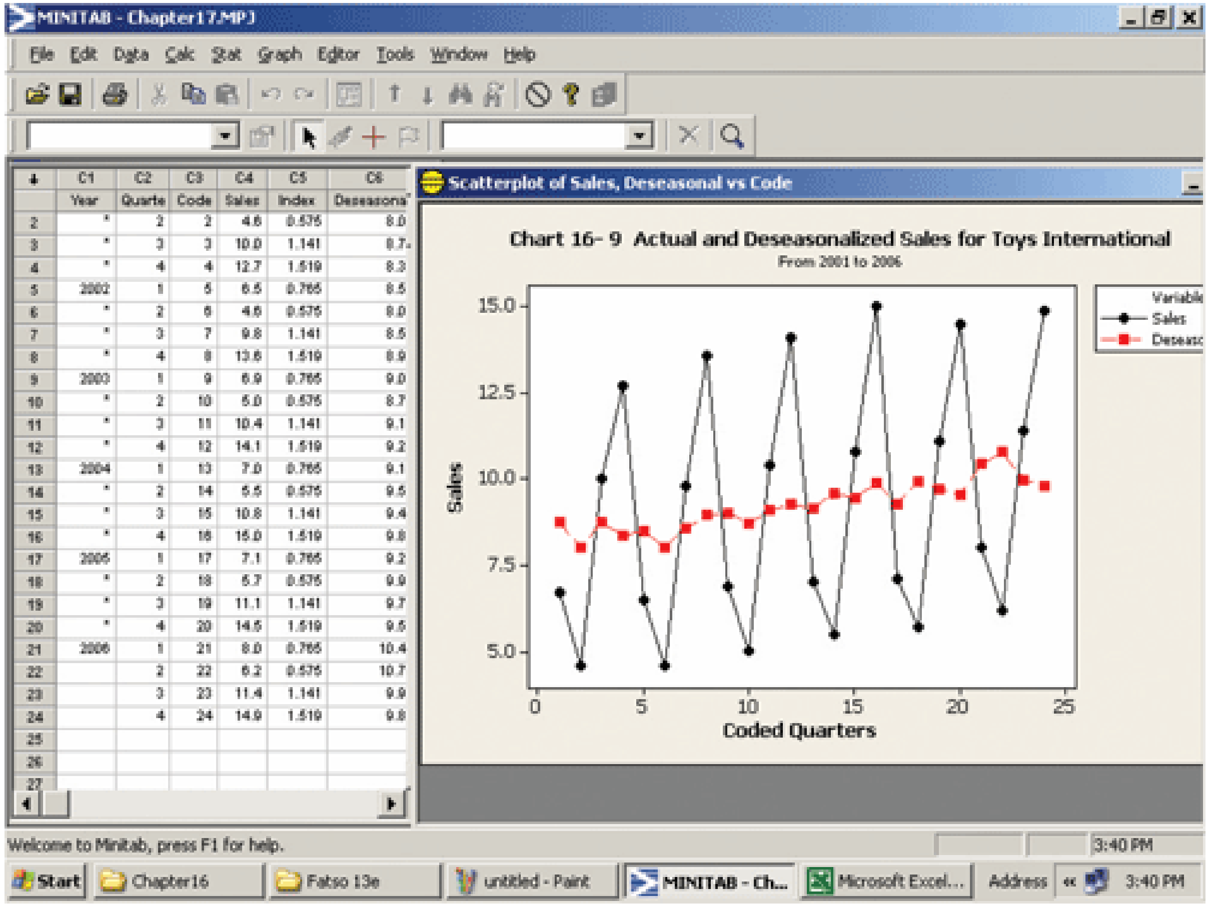
$$\text{Correction factor} = \frac{4.00}{4.009} = 0.997755$$

Actual versus Deseasonalized Sales for Toys International

Deseasonalized Sales = Sales / Seasonal Index

Year	Quarter	(1) Sales	(2) Seasonal Index	(3) Deseasonalized Sales
2001	Winter	6.7	0.765	8.76
	Spring	4.6	0.575	8.00
	Summer	10.0	1.141	8.76
	Fall	12.7	1.519	8.36
2002	Winter	6.5	0.765	8.50
	Spring	4.6	0.575	8.00
	Summer	9.8	1.141	8.59
	Fall	13.6	1.519	8.95
2003	Winter	6.9	0.765	9.02
	Spring	5.0	0.575	8.70
	Summer	10.4	1.141	9.11
	Fall	14.1	1.519	9.28
2004	Winter	7.0	0.765	9.15
	Spring	5.5	0.575	9.57
	Summer	10.8	1.141	9.47
	Fall	15.0	1.519	9.87
2005	Winter	7.1	0.765	9.28
	Spring	5.7	0.575	9.91
	Summer	11.1	1.141	9.73
	Fall	14.5	1.519	9.55
2006	Winter	8.0	0.765	10.46
	Spring	6.2	0.575	10.79
	Summer	11.4	1.141	9.99
	Fall	14.9	1.519	9.81

Actual versus Deseasonalized Sales for Toys International – Time Series Plot using Minitab



Seasonal Index – An Example Using Excel

Microsoft Excel - Ch16 - Forecasting.xls

t	Year	Quarter	(1) Sales ($ millions)	(2) Four-Quarter Moving Average	(3) Centered Moving Average	(4) Ratio to CMA	(5) Seasonal Indexes	Sales Deseasonalized
1	2001	Winter	6.7				0.765	8.759
2		Spring	4.6				0.575	8.004
3		Summer	10	8.5000	8.4750	1.180	1.141	8.761
4		Fall	12.7	8.4500	8.4500	1.503	1.519	8.361
5	2002	Winter	6.5	8.4500	8.4250	0.772	0.765	8.498
6		Spring	4.6	8.4000	8.5125	0.540	0.575	8.004
7		Summer	9.8	8.6250	8.6750	1.130	1.141	8.586
8		Fall	13.6	8.7250	8.7750	1.550	1.519	8.953
9	2003	Winter	6.9	8.8250	8.9000	0.775	0.765	9.021
10		Spring	5	8.9750	9.0375	0.553	0.575	8.700
11		Summer	10.4	9.1000	9.1125	1.141	1.141	9.112
12		Fall	14.1	9.1250	9.1875	1.535	1.519	9.283
13	2004	Winter	7	9.2500	9.3000	0.753	0.765	9.151
14		Spring	5.5	9.3500	9.4625	0.581	0.575	9.570
15		Summer	10.8	9.5750	9.5875	1.126	1.141	9.462
16		Fall	15	9.6000	9.6250	1.558	1.519	9.875
17	2005	Winter	7.1	9.6500	9.6875	0.733	0.765	9.282
18		Spring	5.7	9.7250	9.6625	0.590	0.575	9.918
19		Summer	11.1	9.6000	9.7125	1.143	1.141	9.725
20		Fall	14.5	9.8250	9.8875	1.466	1.519	9.546
21	2006	Winter	8	9.9500	9.9875	0.801	0.765	10.459
22		Spring	6.2	10.0250	10.0750	0.615	0.575	10.788
23		Summer	11.4	10.1250			1.141	9.988
24		Fall	14.9				1.519	9.809

Calculation of Seasonal Indexes

	Winter	Spring	Summer	Fall	
2001			1.180	1.503	
2002	0.772	0.540	1.130	1.550	
2003	0.775	0.553	1.141	1.535	
2004	0.753	0.581	1.126	1.558	
2005	0.733	0.590	1.143	1.466	
2006	0.801	0.615			
Mean	0.767	0.576	1.144	1.522	4.009
Adjusted	0.765	0.575	1.141	1.519	4.000

=AVERAGE(E5:E6)

=AVERAGE(D3:D6)

=D5/F5

=D5/H5

=AVERAGE(C32:C37)

=4/G38*C38

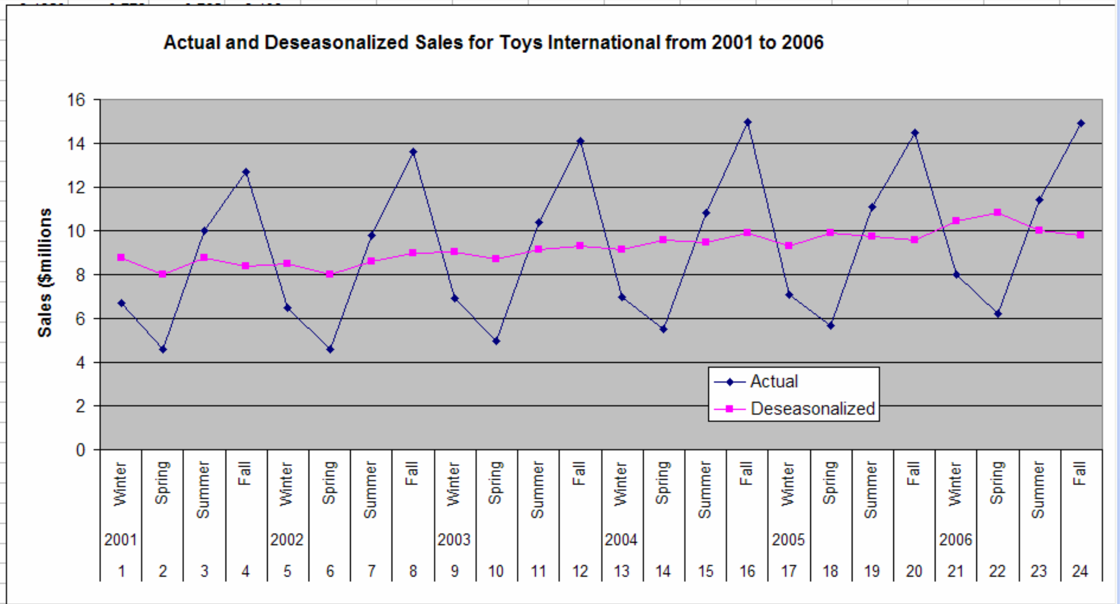
Seasonal Index – An Example Using Excel

Microsoft Excel - Ch16 - Forecasting.xls

Year	Quarter	Sales ($ millions) (1)	Four-Quarter Moving Average (2)	Centered Moving Average (3)	Ratio to CMA (4)	Seasonal Indexes (5)	Sales Deseasonalized
2001	Winter	6.7				0.765	8.759
	Spring	4.6				0.575	8.004
	Summer	10	8.5000	8.4750	1.180	1.141	8.761
	Fall	12.7	8.4500	8.4500	1.503	1.519	8.361
2002	Winter	6.5	8.4500				
	Spring	4.6	8.4000				
	Summer	9.8	8.6250				
	Fall	13.6	8.7250				
2003	Winter	6.9	8.8250				
	Spring	5	8.9750				
	Summer	10.4	9.1000				
	Fall	14.1	9.1250				
2004	Winter	7	9.2500				
	Spring	5.5	9.3500				
	Summer	10.8	9.5750				
	Fall	15	9.6000				
2005	Winter	7.1	9.6500				
	Spring	5.7	9.7250				
	Summer	11.1	9.6000				
	Fall	14.5	9.8250				
2006	Winter	8	9.9500				
	Spring	6.2	10.0250				
	Summer	11.4	10.1250				
	Fall	14.9					

h of Seasonal Indexes

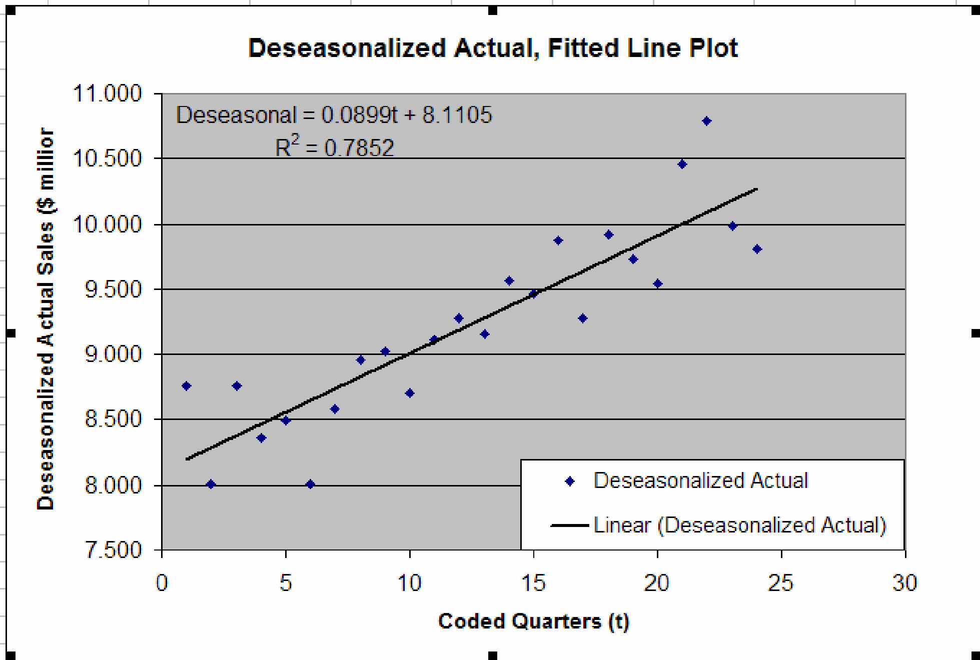
	Quarter				
	Winter	Spring	Summer		
2001			1.180		
2002	0.772	0.540	1.130		
2003	0.775	0.553	1.141		
2004	0.753	0.581	1.126		
2005	0.733	0.590	1.143		
2006	0.801	0.615			
Mean	0.767	0.576	1.144	1.522	4.009
Adjusted:	0.765	0.575	1.141	1.519	4.000



Seasonal Index – An Excel Example using Toys International Sales

Microsoft Excel - Ch16 - Forecasting.xls

Quarter	t	Sales ($ millions)	Seasonal Indexes	Sales Deseasonalized
Winter	1	6.7	0.765	8.759
Spring	2	4.6	0.575	8.004
Summer	3	10	1.141	8.761
Fall	4	12.7	1.519	8.361
Winter	5	6.5	0.765	8.498
Spring	6	4.6	0.575	8.004
Summer	7	9.8	1.141	8.586
Fall	8	13.6	1.519	8.953
Winter	9	6.9	0.765	9.021
Spring	10	5	0.575	8.700
Summer	11	10.4	1.141	9.112
Fall	12	14.1	1.519	9.283
Winter	13	7	0.765	9.151
Spring	14	5.5	0.575	9.570
Summer	15	10.8	1.141	9.462
Fall	16	15	1.519	9.875
Winter	17	7.1	0.765	9.282
Spring	18	5.7	0.575	9.918
Summer	19	11.1	1.141	9.725
Fall	20	14.5	1.519	9.546
Winter	21	8	0.765	10.459
Spring	22	6.2	0.575	10.788
Summer	23	11.4	1.141	9.988
Fall	24	14.9	1.519	9.809



Seasonal Index – An Example Using Excel

Given the deseasonalized linear equation for Toys International sales as $\hat{Y}$=8.109 + 0.0899t, generate the seasonally adjusted forecast for the each of the quarters of 2007

Quarter	t	$\hat{Y}$ (unadjusted forecast)	Seasonal Index	Quarterly Forecast (seasonally adjusted forecast)
Winter	25	10.35675	0.765	7.923
Spring	26	10.44666	0.575	6.007
Summer	27	10.53657	1.141	12.022
Fall	28	10.62648	1.519	16.142

$\hat{Y}$ = 8.109 + 0.0899(28)

$\hat{Y}$ X SI = 10.62648 X 1.519

Durbin-Watson Statistic

- Tests the autocorrelation among the residuals
- The Durbin-Watson statistic, *d*, is computed by first determining the residuals for each observation: $e_t = (Y_t - \hat{Y}_t)$
- Then compute *d* using the following equation:

DURBIN-WATSON STATISTIC

$$d = \frac{\sum_{t=2}^{n} (e_t - e_{t-1})^2}{\sum_{t=1}^{n} (e_t)^2} \qquad [16\text{–}4]$$

Durbin-Watson Test for Autocorrelation – Interpretation of the Statistic

- Range of *d* is 0 to 4

d = 2	No autocorrelation
d close to 0	Positive autocorrelation
d beyond 2	Negative autocorrelation

- Hypothesis Test:

  H_0: No residual correlation ($\rho = 0$)
  H_1: Positive residual correlation ($\rho > 0$)

- Critical values for *d* are found in *Appendix B.10* using
  - α - significance level
  - n – sample size
  - K – the number of predictor variables

Durbin-Watson Critical Values (α=.05)

	$k = 1$		$k = 2$		$k = 3$		$k = 4$	
n	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U
15	1.08	1.36	0.95	1.54	0.82	1.75	0.69	1.97
16	1.10	1.37	0.98	1.54	0.86	1.73	0.74	1.93
17	1.13	1.38	1.02	1.54	0.90	1.71	0.78	1.90
18	1.16	1.39	1.05	1.53	0.93	1.69	0.82	1.87
19	1.18	1.40	1.08	1.53	0.97	1.68	0.86	1.85
20	1.20	1.41	1.10	1.54	1.00	1.68	0.90	1.83
21	1.22	1.42	1.13	1.54	1.03	1.67	0.93	1.81
22	1.24	1.43	1.15	1.54	1.05	1.66	0.96	1.80
23	1.26	1.44	1.17	1.54	1.08	1.66	0.99	1.79
24	1.27	1.45	1.19	1.55	1.10	1.66	1.01	1.78
25	1.29	1.45	1.21	1.55	1.12	1.66	1.04	1.77
26	1.30	1.46	1.22	1.55	1.14	1.65	1.06	1.76
27	1.32	1.47	1.24	1.56	1.16	1.65	1.08	1.76
28	1.33	1.48	1.26	1.56	1.18	1.65	1.10	1.75
29	1.34	1.48	1.27	1.56	1.20	1.65	1.12	1.74
30	1.35	1.49	1.28	1.57	1.21	1.65	1.14	1.74
31	1.36	1.50	1.30	1.57	1.23	1.65	1.16	1.74
32	1.37	1.50	1.31	1.57	1.24	1.65	1.18	1.73
33	1.38	1.51	1.32	1.58	1.26	1.65	1.19	1.73
34	1.39	1.51	1.33	1.58	1.27	1.65	1.21	1.73
35	1.40	1.52	1.34	1.58	1.28	1.65	1.22	1.73
36	1.41	1.52	1.35	1.59	1.29	1.65	1.24	1.73
37	1.42	1.53	1.36	1.59	1.31	1.66	1.25	1.72
38	1.43	1.54	1.37	1.59	1.32	1.66	1.26	1.72
39	1.43	1.54	1.38	1.60	1.33	1.66	1.27	1.72
40	1.44	1.54	1.39	1.60	1.34	1.66	1.29	1.72
45	1.48	1.57	1.43	1.62	1.38	1.67	1.34	1.72
50	1.50	1.59	1.46	1.63	1.42	1.67	1.38	1.72
55	1.53	1.60	1.49	1.64	1.45	1.68	1.41	1.72
60	1.55	1.62	1.51	1.65	1.48	1.69	1.44	1.73
65	1.57	1.63	1.54	1.66	1.50	1.70	1.47	1.73
70	1.58	1.64	1.55	1.67	1.52	1.70	1.49	1.74
75	1.60	1.65	1.57	1.68	1.54	1.71	1.51	1.74
80	1.61	1.66	1.59	1.69	1.56	1.72	1.53	1.74
85	1.62	1.67	1.60	1.70	1.57	1.72	1.55	1.75
90	1.63	1.68	1.61	1.70	1.59	1.73	1.57	1.75
95	1.64	1.69	1.62	1.71	1.60	1.73	1.58	1.75
100	1.65	1.69	1.63	1.72	1.61	1.74	1.59	1.76

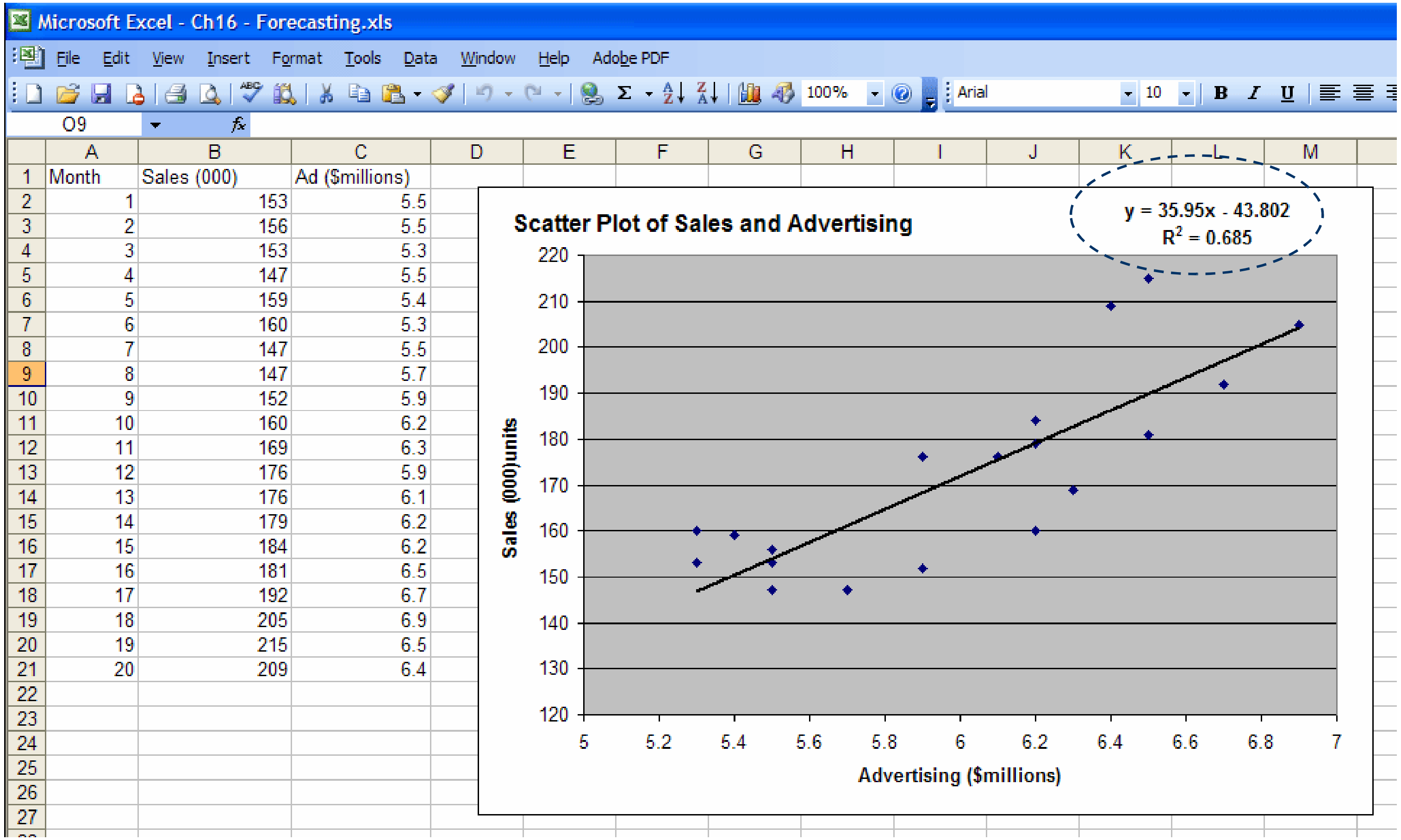
Durbin-Watson Test for Autocorrelation: An Example

The Banner Rock Company manufactures and markets its own rocking chair. The company developed special rocker for senior citizens which it advertises extensively on TV. Banner's market for the special chair is the Carolinas, Florida and Arizona, areas where there are many senior citizens and retired people The president of Banner Rocker is studying the association between his advertising expense (*X*) and the number of rockers sold over the last 20 months (*Y*). He collected the following data. He would like to use the model to forecast sales, based on the amount spent on advertising, but is concerned that because he gathered these data over consecutive months that there might be problems of autocorrelation.

Month	Sales (000)	Ad ($millions)
1	153	5.5
2	156	5.5
3	153	5.3
4	147	5.5
5	159	5.4
6	160	5.3
7	147	5.5
8	147	5.7
9	152	5.9
10	160	6.2
11	169	6.3
12	176	5.9
13	176	6.1
14	179	6.2
15	184	6.2
16	181	6.5
17	192	6.7
18	205	6.9
19	215	6.5
20	209	6.4

Durbin-Watson Test for Autocorrelation: An Example

- Step 1: Generate the regression equation



	A	B	C
1	Month	Sales (000)	Ad ($millions)
2	1	153	5.5
3	2	156	5.5
4	3	153	5.3
5	4	147	5.5
6	5	159	5.4
7	6	160	5.3
8	7	147	5.5
9	8	147	5.7
10	9	152	5.9
11	10	160	6.2
12	11	169	6.3
13	12	176	5.9
14	13	176	6.1
15	14	179	6.2
16	15	184	6.2
17	16	181	6.5
18	17	192	6.7
19	18	205	6.9
20	19	215	6.5
21	20	209	6.4

Durbin-Watson Test for Autocorrelation: An Example

- The resulting equation is: $\hat{Y} = -43.802 + 35.95X$
- The coefficient (r) is 0.828
- The coefficient of determination (r^2) is 68.5%

  (note: Excel reports r^2 as a ratio. Multiply by 100 to convert into percent)
- There is a strong, positive association between sales and advertising
- Is there potential problem with autocorrelation?

Durbin-Watson Test for Autocorrelation: An Example

Microsoft Excel - Ch16 - Forecasting

E3 =B3-D3

	A	B	C	D	E	F	G	H
1	Month	Sales (000)	Ad ($millions)	Predicted Sales (000)	Residuals	Lagged		
2		Y	X	$\hat{Y}$	$e_i = Y - \hat{Y}$	e_{i-1}	$(e_i - e_{i-1})^2$	$(e_i)^2$
3	1	153	5.5	153.923	-0.923			0.8519
4	2	156	5.5	153.923	2.077	-0.923	9.0000	4.3139
5	3	153	5.3	146.733	6.267	2.077	17.5561	39.2753
6	4	147	5.5	153.923	-6.923	6.267	173.9761	47.9279
7	5	159	5.4	150.328	8.672	-6.923	243.2040	75.2036
8	6	160	5.3	146.733	13.267	8.672	21.1140	176.0133
9	7	147	5.5	153.923	-6.923	13.267	407.6361	47.9279
10	8	147	5.7	161.113	-14.113	-6.923	51.6961	199.1768
11	9	152	5.9	168.303	-16.303	-14.113	4.7961	265.7878
12	10	160	6.2	179.088	-19.088	-16.303	7.7562	364.3517
13	11	169	6.3	182.683	-13.683	-19.088	29.2140	187.2245
14	12	176	5.9	168.303	7.697	-13.683	457.1044	59.2438
15	13	176	6.1	175.493	0.507	7.697	51.6961	0.2570
16	14	179	6.2	179.088	-0.088	0.507	0.3540	0.0077
17	15	184	6.2	179.088	4.912	-0.088	25.0000	24.1277
18	16	181	6.5	189.873	-8.873	4.912	190.0262	78.7301
19	17	192	6.7	197.063	-5.063	-8.873	14.5161	25.6340
20	18	205	6.9	204.253	0.747	-5.063	33.7561	0.5580
21	19	215	6.5	189.873	25.127	0.747	594.3844	631.3661
22	20	209	6.4	186.278	22.722	25.127	5.7840	516.2893
23								
24							2338.5702	2744.2686

=-43.802+35.95*C3

=(E4-F4)^2

=E4^2

=B3-D3

=E3

$\sum(e_i - e_{i-1})^2$

$\sum(e_i)^2$

Durbin-Watson Test for Autocorrelation: An Example

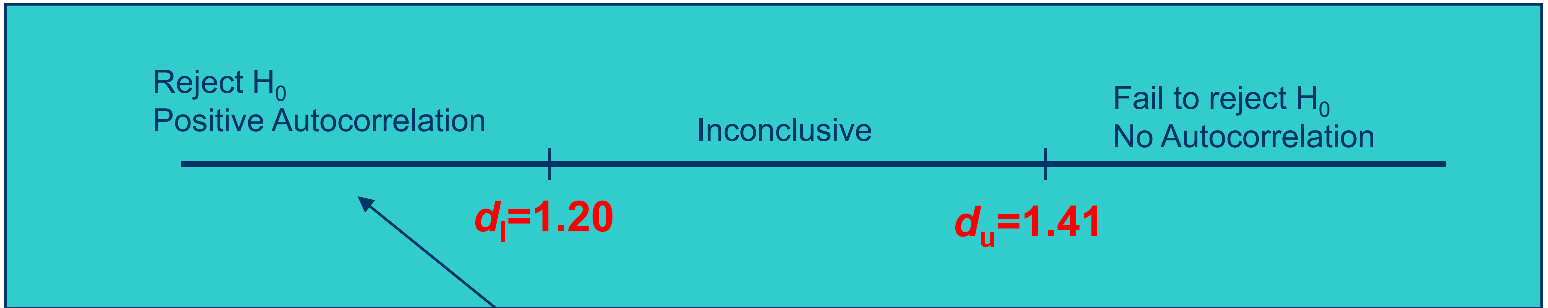
- Hypothesis Test:

  H_0: No residual correlation (ρ = 0)

  H_1: Positive residual correlation (ρ > 0)

- Critical values for d given α=0.5, n=20, k=1 found in Appendix B.10

  d_l=1.20 d_u=1.41



$$d = \frac{\sum_{t=2}^{n}(e_t - e_{t-1})^2}{\sum_{t=1}^{n}(e_t)^2} = \frac{2338.5829}{2744.2685} = 0.8522$$

END OF CHAPTER 16