Coefficient of Variation: CV

$$CV = \frac{\sigma(R_i)}{E(R_i)}$$

The Coefficient of Variation provides an idea about the amount of risk per each unit of Expected Return.

Ex: $\begin{cases} \sigma(R_i) = 2.5\% \\ E(R_i) = 10\% \end{cases}$, $\Rightarrow CV(R_i) = \frac{2.5}{10} = 25\%$

Risk

In Finance, we assume that (σ^2) or (σ) are the best measures of risk.

Computation

Historical Data

$$\boxed{E(R_i) = \frac{\sum_{i=1}^{n} R_i}{n}}$$

n is the number of historical observations

$$\sigma^2(R_i) = \frac{\sum_{i=1}^{n} \left(R_i - E(R_i)\right)^2}{n}$$

in Statistics

$$\sigma^2(R_i) = \frac{\sum \left(R_i - E(R_i)\right)^2}{n-1}$$

→ Convergent Estimator

Probability distribution

Return	Prob
R_1	p_1
R_2	p_2
⋮	⋮
R_n	p_n

$$\sum_{i=1}^{n} p_i = 1$$

$$\boxed{E(R) = \sum_{i=1}^{n} p_i R_i}$$

Variance:

$$\boxed{\sigma^2(R_i) = \sum_{i=1}^{n} p_i \left(R_i - E(R_i)\right)^2}$$

2 $\sigma = \sqrt{\sigma^2}$

Principle of Dominance.

In Finance, we assume that

H_1 – Investors are risk-averse

H_2 – Investors are RATIONAL

H_1 : Risk Aversion :

If we have 2 Assets A and B.

$$\begin{cases} E(R_A) = E(R_B) \\ \sigma(R_A) < \sigma(R_B) \end{cases} \Rightarrow \text{Select A having the lowest level of risk}$$

H_2 Investors are Rational

Investors use the principle of Dominance when they are taking their investment decisions.

Let's Consider 2 Assets A and B With $E(R_A)$, $E(R_B)$, $\sigma(R_A)$, $\sigma(R_B)$.

Case 1:

$$\begin{cases} E(R_A) = E(R_B) \\ \sigma_A < \sigma_B \end{cases} \Rightarrow$$

A is better than B

A dominates B

B is dominated by A

$\boxed{A \succ B}$

Case 2:

$$\begin{cases} \sigma_A = \sigma_B \\ E(R_A) < E(R_B) \end{cases} \Rightarrow$$

B is better than A

B dominates A.

$B \succ A$

Case 3:

$$\begin{cases} \sigma_A = \sigma_B \\ E_A = E_B \end{cases} \Rightarrow A \simeq B$$

Same level of risk

Same level of $E(R_i)$.

Case 4:

$$\begin{bmatrix} E(R_A) \neq E(R_B) \\ \sigma_A \neq \sigma_B \end{bmatrix} \Rightarrow$$ We use the CV

We Select Asset having the lowest Value of the Coefficient of Variation

H.D

$$E(R_i) = \frac{\sum R_i}{n}.$$

H.D is a probability distribution. when we assume that events have the same prob.

$$p_1 = p_2 \text{ — } = p_n = \frac{1}{n}.$$

Prob

$$E(R_i) = \sum_{i=1}^{n} p_i R_i$$

$$\sum_{i=1}^{n} p_i = 1$$

if

$$p_1 = p_2 = \cdots p_n = \frac{1}{n}.$$

$$E(R_i) = \sum_{i=1}^{n} \frac{1}{n} R_i$$

$$\boxed{E(R_i) = \frac{1}{n} \sum_{i=1}^{n} R_i}$$

Required Rate of ~~R~~ Return

- F1 — Time-Value of Money + Inflation
- F2 — ~~Inflation~~
- F3 — Risk

Inf ↑ $\xrightarrow{\text{How?}}$ $E(R_i)$ ↑↑

1)

Inf ↑ → Nominal Interest Rates ↗ → $E(R_i)$ > nominal rate.

Central Banks have to fix interest Rates higher than Inflation

$E(R_i)$ ↑↑↑

2)

Inf ↑ → Investors are looking for higher expected Returns to compensate for the time value of Money → Asking for higher expected Returns ↑↑

Inf ↑ → T.V of money ↓↓

– Risk and Required Rate of Return

$\boxed{\text{Risk}} \uparrow\uparrow \longrightarrow E(R_i) \uparrow\uparrow\uparrow$

Different Arguments:

– $\boxed{\text{Investors are risk averse}}\ \uparrow\uparrow \xrightarrow{+} \boxed{\text{Ask For Risk Premium}}\ \uparrow\uparrow$

$\downarrow$

$\boxed{\text{One Component of } E(R_i)}$

$\downarrow$

using the CAPM

$$\uparrow\uparrow\uparrow\ E(R_i) = RFR + \underbrace{[E(R_M) - RFR]}_{\uparrow\uparrow\uparrow}\ \beta_i \rightarrow \text{Risk}$$

Risk and $E(R_i)$

Risk $\uparrow\uparrow$ $\xrightarrow{\text{Why}}$ Fund $\Downarrow$

Cost of Equity $\uparrow\uparrow$

$$\boxed{WACC = K_S \times \frac{S}{D+S} + K_D \frac{D}{D+S}(1-T)}$$

$\uparrow\uparrow\uparrow\uparrow$

Risk $\uparrow \longrightarrow K_S \uparrow\uparrow$, Risk Premium

Risk $\uparrow \longrightarrow$ Debt $\uparrow\uparrow \longrightarrow K_D \uparrow\uparrow$

WACC $\uparrow\uparrow\uparrow$

Shareholders will ask for Higher expected Returns

$\uparrow$ Required Returns

Nominal and Real Interest Rates:

$$\text{Nominal RFR} = \left[\left(1 + \text{Real}_{RFR}\right)\left(1 + \text{Inf}\right)\right] - 1 .$$

$$\text{Real RFR} = \left[\frac{1 + \text{Nominal RFR}}{1 + \text{Inf}}\right] - 1$$

Example:

Let's Consider a PF:

- L.T. Bonds: 7,5%, $W_{LTB} = 0{,}5$
- U.S. T.Bills: 5,5%, $W_{T.B} = 0{,}25$
- Stocks: 12,5%, $W_S = 0{,}25$

$$\sum w_i = 1$$

$CPI_{2014} = 174$

$CPI_{2015} = 178$

1°/ Calcule Inflation Rate.

2°/ ——— the real Return for each asset.

Q$_3$) Calculate the Real Return for the whole portfolio.

Answer:

1°/ **Inflation:**

$$I_{(2014,\,2015)} = \frac{CPI_{2015} - CPI_{2014}}{CPI_{2014}}$$

N.A:

$$\left[\frac{178 - 174}{174}\right] = \cancel{5{,}88}\ /\ 2{,}29\% \simeq \underline{2{,}3\%}$$

2°/ **Real Return For each asset:**

$$\text{Real Return} = \left[\frac{1 + \text{Nominal Return}}{1 + \text{Inf}}\right] - 1$$

For L.T. Bonds:

$$\text{Real} = \left[\frac{1 + 7{,}5\%}{1 + 2{,}3\%}\right] - 1 = 0{,}05 = \underline{5\%}$$

For T. Bills:

$$\text{Real} = \left[\frac{1 + 5{,}5\%}{1 + 2{,}3\%}\right] - 1 = \underline{3{,}13\%}$$

$\boxed{10\%}$

For Stocks:

$$\text{Real} = \left[\frac{1 + 12{,}5\%}{1 + 2{,}3\%}\right] - 1 = \cancel{9{,}79}$$

Comment :

For all the Selected Assets, the real Return ~~is~~ ~~higher~~ is positive because nominal Returns are higher than inflation.

3°/ Total Real Return For the Portfolio :

$$\boxed{\left(\text{Real Return PF}\right) = \sum_{i=1}^{3} W_i \, \text{Real Return}_i}$$

$$= (0{,}5 \times 5\%) + (0{,}25 \times 3.13\%) + (0{,}25 \times 10\%) = \_\_\_\_$$

Financial Risk

Debt ↑↑ → (Financial Risk) ↑↑

leverage effect

$\left(\frac{D}{S}\right)$ (−) (+) ⇄ ROE