

Coefficient of Variation: CV

$$CV = \frac{\sigma(R_i)}{E(R_i)}$$

The Coefficient of Variation provides an idea about the amount of risk per each unit of Expected Return.

Ex :

$$\begin{cases} \sigma(R_i) = 2.5\% \\ E(R_i) = 10\% \end{cases} \Rightarrow CV(R_i) = \frac{2.5}{10} = \underline{\underline{25\%}}$$

Risk

In Finance, we assume that σ^2 or σ are the best measures of risk.

Computation

Historical Data

$$E(R_i) = \frac{\sum_{i=1}^n R_i}{n}$$

n is the number of historical observations

$$\sigma^2(R_i) = \frac{\sum_{i=1}^n (R_i - E(R_i))^2}{n}$$

in Statistics

$$\sigma^2(R_i) = \frac{\sum_{i=1}^n (R_i - E(R_i))^2}{n-1}$$

Convergent Estimator

Probability distribution

Return Prob

R_1 P_1

R_2 P_2

\vdots \vdots

R_n P_n

$$\sum_{i=1}^n p_i = 1$$

$$E(R) = \sum_{i=1}^n p_i R_i$$

Variance:

$$\sigma^2(R_i) = \sum_{i=1}^n p_i (R_i - E(R_i))^2$$

$$\sigma = \sqrt{\sigma^2}$$

Principle of Dominance.

In Finance, we assume that:

(H₁) - Investors are risk-averse

(H₂) - Investors are RATIONAL.

(H₁): Risk Aversion:

If we have 2 Assets A and B.

$$\begin{cases} E(R_A) = E(R_B) \\ \sigma(R_A) < \sigma(R_B) \end{cases} \Rightarrow \text{Select A having} \\ \text{the lowest level of} \\ \text{risk}$$

(H₂) Investors are Rational

Investors use the principle of Dominance when they are taking their investment decisions.

Let's Consider 2 Assets A and B.
With $E(R_A)$, $E(R_B)$, $\sigma(R_A)$, $\sigma(R_B)$ -

Case 1:

$$\begin{cases} E(R_A) = E(R_B) \\ \sigma_A < \sigma_B \end{cases} \Rightarrow \begin{array}{l} \text{A is better than B} \\ \text{A dominates B} \\ \text{B is dominated by A} \end{array}$$

$A \succ B$

Case 2:

$$\begin{cases} \sigma_A = \sigma_B \\ E(R_A) < E(R_B) \end{cases} \Rightarrow \begin{array}{l} \text{B is better than A} \\ \text{B dominates A} \\ B \succ A \end{array}$$

Case 3:

$$\begin{cases} \sigma_A = \sigma_B \\ E_A = E_B \end{cases} \Rightarrow \begin{array}{l} A \sim B \\ \text{Same level of risk} \\ \text{Same level of } E(R_i) \end{array}$$

Case 4:

$$\begin{bmatrix} E(R_A) \neq E(R_B) \\ \sigma_A \neq \sigma_B \end{bmatrix} \Rightarrow \text{We use the CV}$$

We Select Asset having the lowest
Value of the 4 Coefficient of Variation

(H.D)

$$E(R_i) = \frac{\sum R_i}{n}$$

H.D is a probability distribution.
when we assume that events have the same prob-

$$p_1 = p_2 = \dots = p_n = \frac{1}{n}$$

Prob

$$E(R_i) = \sum_{i=1}^n p_i R_i$$

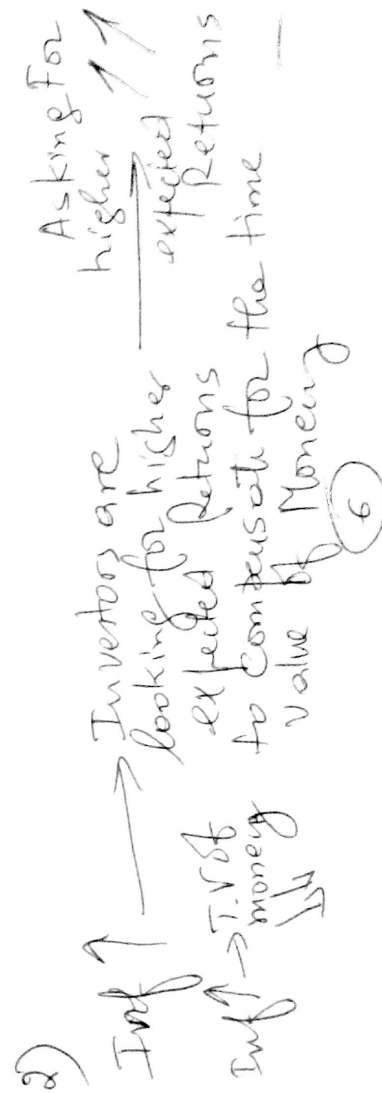
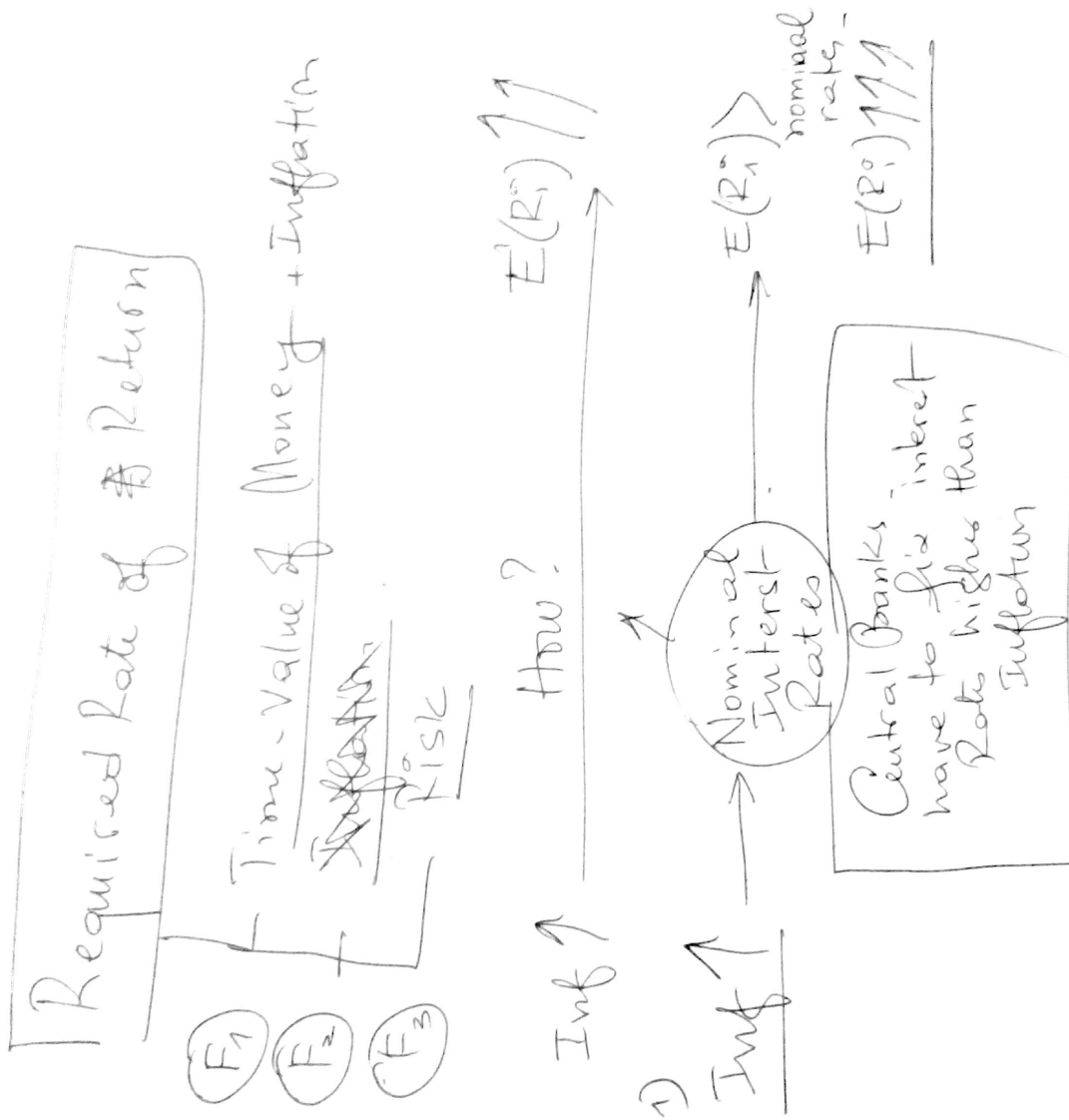
$$\sum_{i=1}^n p_i = 1$$

if

$$p_1 = p_2 = \dots = p_n = \frac{1}{n}$$

$$E(R_i) = \sum_{i=1}^n \frac{1}{n} R_i$$

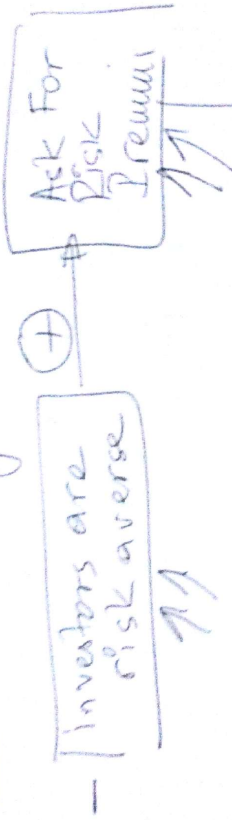
$$E(R_i) = \frac{1}{n} \sum_{i=1}^n R_i$$



- Risk and Required Rate of Return

Risk $\rightarrow E(R_i)$

Different Arguments:



One Component of $E(R_i)$

using the CAPM

$$E(R_i) = RFR + [E(R_M) - RFR] \beta_i$$

β_i Risk

Risk and $E(R_i)$

Risk $\nearrow \nearrow$ Why \rightarrow $\nearrow \nearrow$ \rightarrow $\nearrow \nearrow$

Cost of Equity $\nearrow \nearrow$

$$WACC = \underbrace{\left(K_S \times \frac{S}{D+S} + K_D \frac{D}{D+S} (1-T) \right)}_{\nearrow \nearrow \nearrow}$$

$\left\{ \begin{array}{l} \text{Risk} \uparrow \rightarrow K_S \uparrow \text{ Risk Premium} \\ \text{Risk} \uparrow \rightarrow \text{Debt} \uparrow \rightarrow K_D \uparrow \end{array} \right\}$

WACC $\nearrow \nearrow$

Shareholders will ask
Higher expected
For Returns

\nearrow Required Returns

Nominal and Real Interest Rates:

$$\text{Nominal RFR} = \left[(1 + \text{Real}) (1 + \text{Inf}) \right] - 1$$

$$\text{Real RFR} = \left[\frac{1 + \text{Nominal RFR}}{1 + \text{Inf}} \right] - 1$$

Example:

Let's consider a PF:

$$\left\{ \begin{array}{l} \text{LT Bonds: } 7.5\% , w_{\text{LTB}} = 0.5 \\ \text{US T.Bills: } 5.5\% , w_{\text{T.B.}} = 0.25 \\ \text{Stocks: } 12.5\% , w_S = 0.25 \end{array} \right.$$

$$\sum w_i = 1$$

$$\text{CPI}_{2014} = 174$$

$$\text{CPI}_{2015} = 178$$

1^o Calculate Inflation Rate
2^o the real Return for each asset.

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Q₃) Calculate the Real Return for the whole portfolio -

Answer:

1st Inflation:

$$I_{(2014, 2015)} = \frac{CPI_{2015} - CPI_{2014}}{CPI_{2014}}$$

$$N.A.: \left[\frac{178 - 174}{174} \right] = \frac{4}{174} = 2.29\% \approx \underline{\underline{2.3\%}}$$

2nd Real Return For each asset:

$$\text{Real Return} = \left[\frac{1 + \text{Nominal Return}}{1 + \text{Inf.}} \right] - 1$$

For L.T. Bonds:

$$\text{Real} = \left[\frac{1 + 7.5\%}{1 + 2.3\%} \right] - 1 = 0.05 = \underline{\underline{5\%}}$$

For T. Bills:

$$\text{Real} = \left[\frac{1 + 5.5\%}{1 + 2.3\%} \right] - 1 = 3.13\%$$

For Stocks:

Real =

$$\left[\frac{1 + 12.5\%}{1 + 2.3\%} \right] - 1 = 10.18\%$$

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Comment:

For all the Selected Assets, the real Return ~~is~~ higher is positive because nominal Returns are higher than inflation.

3rd / Total Real Return For the

Portfolios:

$$\left(\begin{array}{c} \text{Real} \\ \text{Return} \\ \text{PF} \end{array} \right) = \frac{\sum_{i=1}^3 W_i \text{ Real Return}_i}{= (0.5 \times 5\%) + (0.25 \times 3.13\%) + (0.25 \times 10\%) = \underline{\hspace{2cm}}}$$

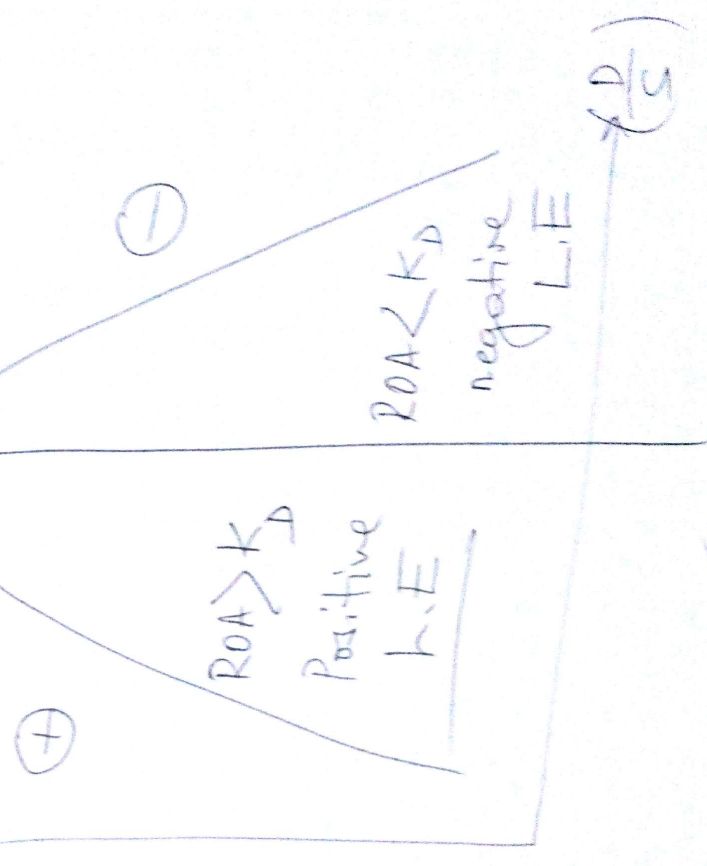
Financial Risk

Debt $\nearrow \nearrow$ (Financial Risk)

leverage effect

$$\frac{(D/S) \ominus \oplus}{ROE}$$

$$\frac{ROE}{NE/E}$$



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