Table 24.1

Typical Electric Field Calculations Using Gauss's Law

Charge Distribution	Electric Field	Location
Insulating sphere of radius R, uniform charge density, and total charge Q	$k_e \frac{Q}{r^2}$	$r > R$
	$k_e \frac{Q}{R^2} r$	$r < R$
Thin spherical shell of radius R and total charge Q	$k_e \frac{Q}{r^2}$	$r > R$
	0	$r < R$
Line charge of infinite length and charge per unit length λ	$2k_e \frac{\lambda}{r}$	Outside the line
Infinite charged plane having surface charge density σ	$\frac{\sigma}{2\epsilon_0}$	Everywhere outside the plane
Conductor having surface charge density σ	$\frac{\sigma}{\epsilon_0}$	Just outside the conductor
	0	Inside the conductor

You should be able to apply Equations 24.2 and 24.3 in a variety of situations, particularly those in which symmetry simplifies the calculation.

Gauss's law says that the net electric flux Φ_E through any closed gaussian surface is equal to the *net* charge q_{in} inside the surface divided by ϵ_0:

$$\Phi_E = \oint \mathbf{E} \cdot d\mathbf{A} = \frac{q_{\text{in}}}{\epsilon_0} \tag{24.6}$$

Using Gauss's law, you can calculate the electric field due to various symmetric charge distributions. Table 24.1 lists some typical results.

A conductor in **electrostatic equilibrium** has the following properties:

1. The electric field is zero everywhere inside the conductor.
2. Any net charge on the conductor resides entirely on its surface.
3. The electric field just outside the conductor is perpendicular to its surface and has a magnitude σ/ϵ_0, where σ is the surface charge density at that point.
4. On an irregularly shaped conductor, the surface charge density is greatest where the radius of curvature of the surface is the smallest.

QUESTIONS

1. The Sun is lower in the sky during the winter months than it is in the summer. How does this change the flux of sunlight hitting a given area on the surface of the Earth? How does this affect the weather?

2. If the electric field in a region of space is zero, can you conclude that no electric charges are in that region? Explain.

3. If more electric field lines leave a gaussian surface than enter it, what can you conclude about the net charge enclosed by that surface?

4. A uniform electric field exists in a region of space in which there are no charges. What can you conclude about the net electric flux through a gaussian surface placed in this region of space?

5. If the total charge inside a closed surface is known but the distribution of the charge is unspecified, can you use Gauss's law to find the electric field? Explain.

6. Explain why the electric flux through a closed surface with a given enclosed charge is independent of the size or shape of the surface.

7. Consider the electric field due to a nonconducting infinite plane having a uniform charge density. Explain why the electric field does not depend on the distance from the plane, in terms of the spacing of the electric field lines.

8. Use Gauss's law to explain why electric field lines must begin or end on electric charges. (*Suggestion:* Change the size of the gaussian surface.)

9. On the basis of the repulsive nature of the force between like charges and the freedom of motion of charge within a conductor, explain why excess charge on an isolated conductor must reside on its surface.

10. A person is placed in a large hollow metallic sphere that is insulated from ground. If a large charge is placed on the sphere, will the person be harmed upon touching the inside of the sphere? Explain what will happen if the person also has an initial charge whose sign is opposite that of the charge on the sphere.

11. Two solid spheres, both of radius R, carry identical total charges, Q. One sphere is a good conductor while the other is an insulator. If the charge on the insulating sphere is uniformly distributed throughout its interior volume, how do the electric fields outside these two spheres compare? Are the fields identical inside the two spheres?

12. A common demonstration involves charging a rubber balloon, which is an insulator, by rubbing it on your hair, and touching the balloon to a ceiling or wall, which is also an insulator. The electrical attraction between the charged balloon and the neutral wall results in the balloon sticking to the wall. Imagine now that we have two infinitely large flat sheets of insulating material. One is charged and the other is neutral. If these are brought into contact, will an attractive force exist between them, as there was for the balloon and the wall?

13. You may have heard that one of the safer places to be during a lightning storm is inside a car. Why would this be the case?

PROBLEMS

1, 2, 3 = straightforward, intermediate, challenging; □ = full solution available in the *Student Solutions Manual and Study Guide*

= coached solution with hints available at http://www.pse6.com; = computer useful in solving problem

= paired numerical and symbolic problems

Section 24.1 Electric Flux

1. An electric field with a magnitude of 3.50 kN/C is applied along the x axis. Calculate the electric flux through a rectangular plane 0.350 m wide and 0.700 m long assuming that (a) the plane is parallel to the yz plane; (b) the plane is parallel to the xy plane; (c) the plane contains the y axis, and its normal makes an angle of 40.0° with the x axis.

2. A vertical electric field of magnitude 2.00×10^4 N/C exists above the Earth's surface on a day when a thunderstorm is brewing. A car with a rectangular size of 6.00 m by 3.00 m is traveling along a roadway sloping downward at 10.0°. Determine the electric flux through the bottom of the car.

3. A 40.0-cm-diameter loop is rotated in a uniform electric field until the position of maximum electric flux is found. The flux in this position is measured to be $5.20 \times 10^5\ \text{N}\cdot\text{m}^2/\text{C}$. What is the magnitude of the electric field?

4. Consider a closed triangular box resting within a horizontal electric field of magnitude $E = 7.80 \times 10^4$ N/C as shown in Figure P24.4. Calculate the electric flux through (a) the vertical rectangular surface, (b) the slanted surface, and (c) the entire surface of the box.

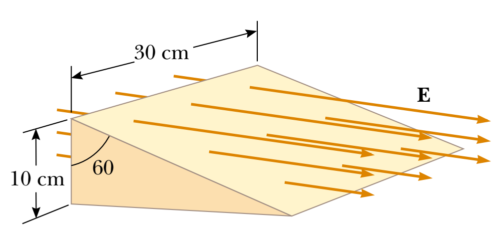

Figure P24.4

5. A uniform electric field $a\hat{\mathbf{i}} + b\hat{\mathbf{j}}$ intersects a surface of area A. What is the flux through this area if the surface lies (a) in the yz plane? (b) in the xz plane? (c) in the xy plane?

6. A point charge q is located at the center of a uniform ring having linear charge density λ and radius a, as shown in Figure P24.6. Determine the total electric flux through a sphere centered at the point charge and having radius R, where $R < a$.

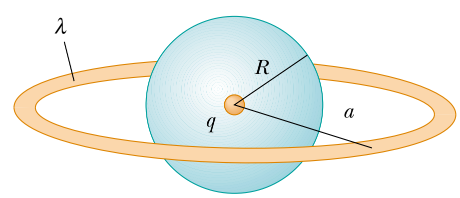

Figure P24.6

7. A pyramid with horizontal square base, 6.00 m on each side, and a height of 4.00 m is placed in a vertical electric field of 52.0 N/C. Calculate the total electric flux through the pyramid's four slanted surfaces.

8. A cone with base radius R and height h is located on a horizontal table. A horizontal uniform field E penetrates the cone, as shown in Figure P24.8. Determine the electric flux that enters the left-hand side of the cone.

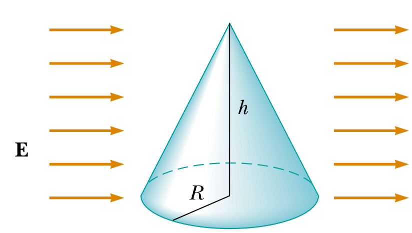

Figure P24.8

Section 24.2 Gauss's Law

9. The following charges are located inside a submarine: 5.00 μC, -9.00 μC, 27.0 μC, and -84.0 μC. (a) Calculate

the net electric flux through the hull of the submarine. (b) Is the number of electric field lines leaving the submarine greater than, equal to, or less than the number entering it?

10. The electric field everywhere on the surface of a thin spherical shell of radius 0.750 m is measured to be 890 N/C and points radially toward the center of the sphere. (a) What is the net charge within the sphere's surface? (b) What can you conclude about the nature and distribution of the charge inside the spherical shell?

11. Four closed surfaces, S_1 through S_4, together with the charges $-2Q$, Q, and $-Q$ are sketched in Figure P24.11. (The colored lines are the intersections of the surfaces with the page.) Find the electric flux through each surface.

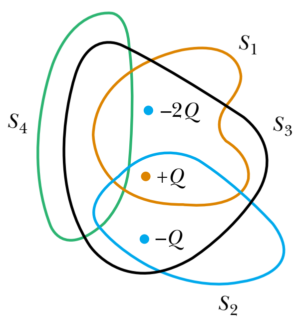

Figure P24.11

12. (a) A point charge q is located a distance d from an infinite plane. Determine the electric flux through the plane due to the point charge. (b) **What If?** A point charge q is located a *very small* distance from the center of a *very large* square on the line perpendicular to the square and going through its center. Determine the approximate electric flux through the square due to the point charge. (c) Explain why the answers to parts (a) and (b) are identical.

13. Calculate the total electric flux through the paraboloidal surface due to a uniform electric field of magnitude E_0 in the direction shown in Figure P24.13.

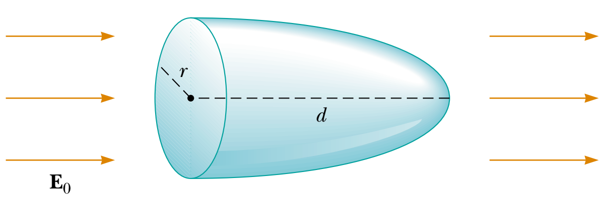

Figure P24.13

14. A point charge of 12.0 μC is placed at the center of a spherical shell of radius 22.0 cm. What is the total electric flux through (a) the surface of the shell and (b) any hemispherical surface of the shell? (c) Do the results depend on the radius? Explain.

15. A point charge Q is located just above the center of the flat face of a hemisphere of radius R as shown in Figure P24.15. What is the electric flux (a) through the curved surface and (b) through the flat face?

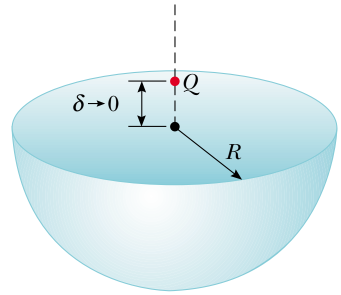

Figure P24.15

16. In the air over a particular region at an altitude of 500 m above the ground the electric field is 120 N/C directed downward. At 600 m above the ground the electric field is 100 N/C downward. What is the average volume charge density in the layer of air between these two elevations? Is it positive or negative?

17. A point charge $Q = 5.00\ \mu$C is located at the center of a cube of edge $L = 0.100$ m. In addition, six other identical point charges having $q = -1.00\ \mu$C are positioned symmetrically around Q as shown in Figure P24.17. Determine the electric flux through one face of the cube.

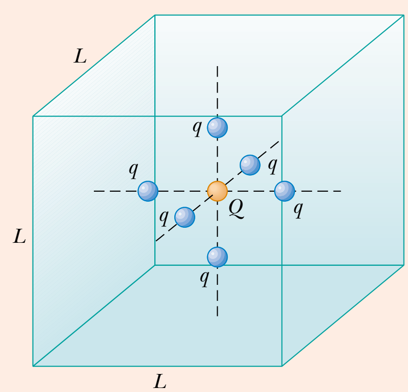

Figure P24.17 Problems 17 and 18.

18. A positive point charge Q is located at the center of a cube of edge L. In addition, six other identical negative point charges q are positioned symmetrically around Q as shown in Figure P24.17. Determine the electric flux through one face of the cube.

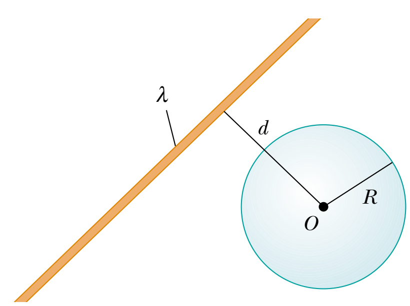

Figure P24.19

19. An infinitely long line charge having a uniform charge per unit length λ lies a distance d from point O as shown in Figure P24.19. Determine the total electric flux through the surface of a sphere of radius R centered at O resulting from this line charge. Consider both cases, where $R < d$ and $R > d$.

20. An uncharged nonconducting hollow sphere of radius 10.0 cm surrounds a 10.0-μC charge located at the origin of a cartesian coordinate system. A drill with a radius of 1.00 mm is aligned along the z axis, and a hole is drilled in the sphere. Calculate the electric flux through the hole.

21. A charge of 170 μC is at the center of a cube of edge 80.0 cm. (a) Find the total flux through each face of the cube. (b) Find the flux through the whole surface of the cube. (c) **What If?** Would your answers to parts (a) or (b) change if the charge were not at the center? Explain.

22. The line ag in Figure P24.22 is a diagonal of a cube. A point charge q is located on the extension of line ag, very close to vertex a of the cube. Determine the electric flux through each of the sides of the cube which meet at the point a.

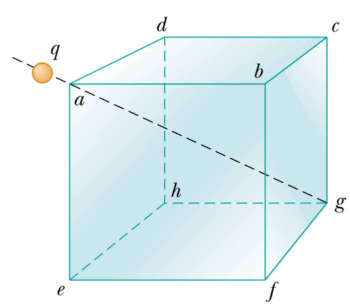

Figure P24.22

Section 24.3 Application of Gauss's Law to Various Charge Distributions

23. Determine the magnitude of the electric field at the surface of a lead-208 nucleus, which contains 82 protons and 126 neutrons. Assume the lead nucleus has a volume 208 times that of one proton, and consider a proton to be a sphere of radius 1.20×10^{-15} m.

24. A solid sphere of radius 40.0 cm has a total positive charge of 26.0 μC uniformly distributed throughout its volume. Calculate the magnitude of the electric field (a) 0 cm, (b) 10.0 cm, (c) 40.0 cm, and (d) 60.0 cm from the center of the sphere.

25. A 10.0-g piece of Styrofoam carries a net charge of -0.700 μC and floats above the center of a large horizontal sheet of plastic that has a uniform charge density on its surface. What is the charge per unit area on the plastic sheet?

26. A cylindrical shell of radius 7.00 cm and length 240 cm has its charge uniformly distributed on its curved surface. The magnitude of the electric field at a point 19.0 cm radially outward from its axis (measured from the midpoint of the shell) is 36.0 kN/C. Find (a) the net charge on the shell and (b) the electric field at a point 4.00 cm from the axis, measured radially outward from the midpoint of the shell.

27. A particle with a charge of -60.0 nC is placed at the center of a nonconducting spherical shell of inner radius 20.0 cm and outer radius 25.0 cm. The spherical shell carries charge with a uniform density of -1.33 μC/m^3. A proton moves in a circular orbit just outside the spherical shell. Calculate the speed of the proton.

28. A nonconducting wall carries a uniform charge density of 8.60 μC/cm^2. What is the electric field 7.00 cm in front of the wall? Does your result change as the distance from the wall is varied?

29. Consider a long cylindrical charge distribution of radius R with a uniform charge density ρ. Find the electric field at distance r from the axis where $r < R$.

30. A solid plastic sphere of radius 10.0 cm has charge with uniform density throughout its volume. The electric field 5.00 cm from the center is 86.0 kN/C radially inward. Find the magnitude of the electric field 15.0 cm from the center.

31. Consider a thin spherical shell of radius 14.0 cm with a total charge of 32.0 μC distributed uniformly on its surface. Find the electric field (a) 10.0 cm and (b) 20.0 cm from the center of the charge distribution.

32. In nuclear fission, a nucleus of uranium-238, which contains 92 protons, can divide into two smaller spheres, each having 46 protons and a radius of 5.90×10^{-15} m. What is the magnitude of the repulsive electric force pushing the two spheres apart?

33. Fill two rubber balloons with air. Suspend both of them from the same point and let them hang down on strings of equal length. Rub each with wool or on your hair, so that they hang apart with a noticeable separation from each other. Make order-of-magnitude estimates of (a) the force on each, (b) the charge on each, (c) the field each creates at the center of the other, and (d) the total flux of electric field created by each balloon. In your solution state the quantities you take as data and the values you measure or estimate for them.

34. An insulating solid sphere of radius a has a uniform volume charge density and carries a total positive charge Q. A spherical gaussian surface of radius r, which shares a common center with the insulating sphere, is inflated starting from $r = 0$. (a) Find an expression for the electric flux passing through the surface of the gaussian sphere as a function of r for $r < a$. (b) Find an expression for the electric flux for $r > a$. (c) Plot the flux versus r.

35. A uniformly charged, straight filament 7.00 m in length has a total positive charge of 2.00 μC. An uncharged cardboard cylinder 2.00 cm in length and 10.0 cm in radius surrounds the filament at its center, with the filament as the axis of the cylinder. Using reasonable approximations, find (a) the electric field at the surface of the cylinder and (b) the total electric flux through the cylinder.

36. An insulating sphere is 8.00 cm in diameter and carries a 5.70-μC charge uniformly distributed throughout its interior volume. Calculate the charge enclosed by a concentric spherical surface with radius (a) $r = 2.00$ cm and (b) $r = 6.00$ cm.

37. A large flat horizontal sheet of charge has a charge per unit area of 9.00 μC/m^2. Find the electric field just above the middle of the sheet.

38. The charge per unit length on a long, straight filament is $-90.0\ \mu\text{C/m}$. Find the electric field (a) 10.0 cm, (b) 20.0 cm, and (c) 100 cm from the filament, where distances are measured perpendicular to the length of the filament.

Section 24.4 Conductors in Electrostatic Equilibrium

39. A long, straight metal rod has a radius of 5.00 cm and a charge per unit length of 30.0 nC/m. Find the electric field (a) 3.00 cm, (b) 10.0 cm, and (c) 100 cm from the axis of the rod, where distances are measured perpendicular to the rod.

40. On a clear, sunny day, a vertical electric field of about 130 N/C points down over flat ground. What is the surface charge density on the ground for these conditions?

41. A very large, thin, flat plate of aluminum of area A has a total charge Q uniformly distributed over its surfaces. Assuming the same charge is spread uniformly over the *upper* surface of an otherwise identical glass plate, compare the electric fields just above the center of the upper surface of each plate.

42. A solid copper sphere of radius 15.0 cm carries a charge of 40.0 nC. Find the electric field (a) 12.0 cm, (b) 17.0 cm, and (c) 75.0 cm from the center of the sphere. (d) **What If?** How would your answers change if the sphere were hollow?

43. A square plate of copper with 50.0-cm sides has no net charge and is placed in a region of uniform electric field of 80.0 kN/C directed perpendicularly to the plate. Find (a) the charge density of each face of the plate and (b) the total charge on each face.

44. A solid conducting sphere of radius 2.00 cm has a charge of $8.00\ \mu\text{C}$. A conducting spherical shell of inner radius 4.00 cm and outer radius 5.00 cm is concentric with the solid sphere and has a total charge of $-4.00\ \mu\text{C}$. Find the electric field at (a) $r = 1.00$ cm, (b) $r = 3.00$ cm, (c) $r = 4.50$ cm, and (d) $r = 7.00$ cm from the center of this charge configuration.

45. Two identical conducting spheres each having a radius of 0.500 cm are connected by a light 2.00-m-long conducting wire. A charge of $60.0\ \mu\text{C}$ is placed on one of the conductors. Assume that the surface distribution of charge on each sphere is uniform. Determine the tension in the wire.

46. The electric field on the surface of an irregularly shaped conductor varies from 56.0 kN/C to 28.0 kN/C. Calculate the local surface charge density at the point on the surface where the radius of curvature of the surface is (a) greatest and (b) smallest.

47. A long, straight wire is surrounded by a hollow metal cylinder whose axis coincides with that of the wire. The wire has a charge per unit length of λ, and the cylinder has a net charge per unit length of 2λ. From this information, use Gauss's law to find (a) the charge per unit length on the inner and outer surfaces of the cylinder and (b) the electric field outside the cylinder, a distance r from the axis.

48. A conducting spherical shell of radius 15.0 cm carries a net charge of $-6.40\ \mu\text{C}$ uniformly distributed on its surface. Find the electric field at points (a) just outside the shell and (b) inside the shell.

49. A thin square conducting plate 50.0 cm on a side lies in the xy plane. A total charge of 4.00×10^{-8} C is placed on the plate. Find (a) the charge density on the plate, (b) the electric field just above the plate, and (c) the electric field just below the plate. You may assume that the charge density is uniform.

50. A conducting spherical shell of inner radius a and outer radius b carries a net charge Q. A point charge q is placed at the center of this shell. Determine the surface charge density on (a) the inner surface of the shell and (b) the outer surface of the shell.

51. A hollow conducting sphere is surrounded by a larger concentric spherical conducting shell. The inner sphere has charge $-Q$, and the outer shell has net charge $+3Q$. The charges are in electrostatic equilibrium. Using Gauss's law, find the charges and the electric fields everywhere.

52. A positive point charge is at a distance $R/2$ from the center of an uncharged thin conducting spherical shell of radius R. Sketch the electric field lines set up by this arrangement both inside and outside the shell.

Section 24.5 Formal Derivation of Gauss's Law

53. A sphere of radius R surrounds a point charge Q, located at its center. (a) Show that the electric flux through a circular cap of half-angle θ (Fig. P24.53) is

$$\Phi_E = \frac{Q}{2\epsilon_0}(1 - \cos\theta)$$

What is the flux for (b) $\theta = 90°$ and (c) $\theta = 180°$?

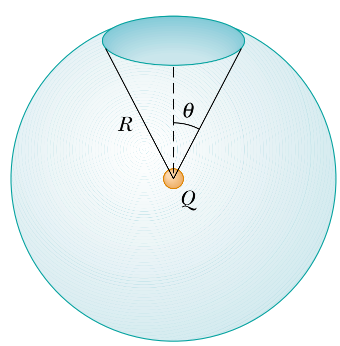

Figure P24.53

Additional Problems

54. A nonuniform electric field is given by the expression $\mathbf{E} = ay\hat{\mathbf{i}} + bz\hat{\mathbf{j}} + cx\hat{\mathbf{k}}$, where a, b, and c are constants. Determine the electric flux through a rectangular surface in the xy plane, extending from $x = 0$ to $x = w$ and from $y = 0$ to $y = h$.

55. A solid insulating sphere of radius a carries a net positive charge $3Q$, uniformly distributed throughout its volume. Concentric with this sphere is a conducting spherical shell with inner radius b and outer radius c, and having a net charge $-Q$, as shown in Figure P24.55. (a) Construct a spherical gaussian surface of radius $r > c$ and find the net charge enclosed by this surface. (b) What is the direction of the electric field at $r > c$? (c) Find the electric field at $r > c$. (d) Find the electric field in the region with radius r where $c > r > b$. (e) Construct a spherical gaussian surface of radius r, where $c > r > b$, and find the net charge enclosed by this surface. (f) Construct a spherical gaussian surface of radius r, where $b > r > a$, and find the net charge enclosed by this surface. (g) Find the electric field in the region $b > r > a$. (h) Construct a spherical gaussian surface of radius $r < a$, and find an expression for the net charge enclosed by this surface, as a function of r. Note that the charge inside this surface is less than $3Q$. (i) Find the electric field in the region $r < a$. (j) Determine the charge on the inner surface of the conducting shell. (k) Determine the charge on the outer surface of the conducting shell. (l) Make a plot of the magnitude of the electric field versus r.

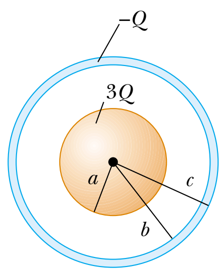

Figure P24.55

56. Consider two identical conducting spheres whose surfaces are separated by a small distance. One sphere is given a large net positive charge while the other is given a small net positive charge. It is found that the force between them is attractive even though both spheres have net charges of the same sign. Explain how this is possible.

57. A solid, insulating sphere of radius a has a uniform charge density ρ and a total charge Q. Concentric with this sphere is an uncharged, conducting hollow sphere whose inner and outer radii are b and c, as shown in Figure P24.57. (a) Find the magnitude of the electric field in the regions $r < a$, $a < r < b$, $b < r < c$, and $r > c$. (b) Determine the induced charge per unit area on the inner and outer surfaces of the hollow sphere.

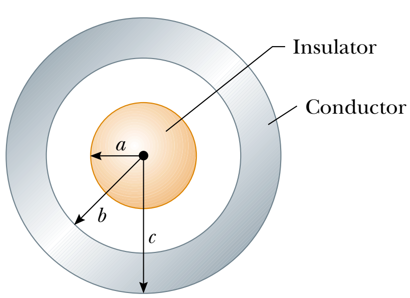

Figure P24.57 Problems 57 and 58.

58. For the configuration shown in Figure P24.57, suppose that $a = 5.00$ cm, $b = 20.0$ cm, and $c = 25.0$ cm. Furthermore, suppose that the electric field at a point 10.0 cm from the center is measured to be 3.60×10^3 N/C radially inward while the electric field at a point 50.0 cm from the center is 2.00×10^2 N/C radially outward. From this information, find (a) the charge on the insulating sphere, (b) the net charge on the hollow conducting sphere, and (c) the charges on the inner and outer surfaces of the hollow conducting sphere.

59. A particle of mass m and charge q moves at high speed along the x axis. It is initially near $x = -\infty$, and it ends up near $x = +\infty$. A second charge Q is fixed at the point $x = 0$, $y = -d$. As the moving charge passes the stationary charge, its x component of velocity does not change appreciably, but it acquires a small velocity in the y direction. Determine the angle through which the moving charge is deflected. *Suggestion:* The integral you encounter in determining v_y can be evaluated by applying Gauss's law to a long cylinder of radius d, centered on the stationary charge.

60. **Review problem.** An early (incorrect) model of the hydrogen atom, suggested by J. J. Thomson, proposed that a positive cloud of charge $+e$ was uniformly distributed throughout the volume of a sphere of radius R, with the electron an equal-magnitude negative point charge $-e$ at the center. (a) Using Gauss's law, show that the electron would be in equilibrium at the center and, if displaced from the center a distance $r < R$, would experience a restoring force of the form $F = -Kr$, where K is a constant. (b) Show that $K = k_e e^2/R^3$. (c) Find an expression for the frequency f of simple harmonic oscillations that an electron of mass m_e would undergo if displaced a small distance ($<R$) from the center and released. (d) Calculate a numerical value for R that would result in a frequency of 2.47×10^{15} Hz, the frequency of the light radiated in the most intense line in the hydrogen spectrum.

61. An infinitely long cylindrical insulating shell of inner radius a and outer radius b has a uniform volume charge density ρ. A line of uniform linear charge density λ is placed along the axis of the shell. Determine the electric field everywhere.

62. Two infinite, nonconducting sheets of charge are parallel to each other, as shown in Figure P24.62. The sheet on the left has a uniform surface charge density σ, and the one

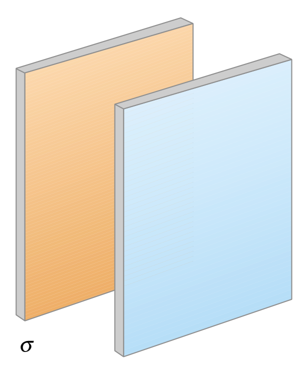

Figure P24.62

on the right has a uniform charge density $-\sigma$. Calculate the electric field at points (a) to the left of, (b) in between, and (c) to the right of the two sheets.

63. **What If?** Repeat the calculations for Problem 62 when both sheets have *positive* uniform surface charge densities of value σ.

64. A sphere of radius $2a$ is made of a nonconducting material that has a uniform volume charge density ρ. (Assume that the material does not affect the electric field.) A spherical cavity of radius a is now removed from the sphere, as shown in Figure P24.64. Show that the electric field within the cavity is uniform and is given by $E_x = 0$ and $E_y = \rho a/3\epsilon_0$. (*Suggestion:* The field within the cavity is the superposition of the field due to the original uncut sphere, plus the field due to a sphere the size of the cavity with a uniform negative charge density $-\rho$.)

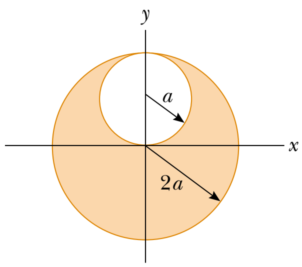

Figure P24.64

65. A uniformly charged spherical shell with surface charge density σ contains a circular hole in its surface. The radius of the hole is small compared with the radius of the sphere. What is the electric field at the center of the hole? (*Suggestion:* This problem, like Problem 64, can be solved by using the idea of superposition.)

66. A closed surface with dimensions $a = b = 0.400$ m and $c = 0.600$ m is located as in Figure P24.66. The left edge of the closed surface is located at position $x = a$. The electric field throughout the region is nonuniform and given by $\mathbf{E} = (3.0 + 2.0x^2)\hat{\mathbf{i}}$ N/C, where x is in meters. Calculate the net electric flux leaving the closed surface. What net charge is enclosed by the surface?

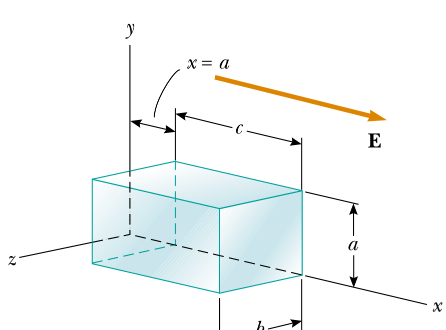

Figure P24.66

67. A solid insulating sphere of radius R has a nonuniform charge density that varies with r according to the expression $\rho = Ar^2$, where A is a constant and $r < R$ is measured from the center of the sphere. (a) Show that the magnitude of the electric field outside $(r > R)$ the sphere is $E = AR^5/5\epsilon_0 r^2$. (b) Show that the magnitude of the electric field inside $(r < R)$ the sphere is $E = Ar^3/5\epsilon_0$. (*Suggestion:* The total charge Q on the sphere is equal to the integral of $\rho\,dV$, where r extends from 0 to R; also, the charge q within a radius $r < R$ is less than Q. To evaluate the integrals, note that the volume element dV for a spherical shell of radius r and thickness dr is equal to $4\pi r^2 dr$.)

68. A point charge Q is located on the axis of a disk of radius R at a distance b from the plane of the disk (Fig. P24.68). Show that if one fourth of the electric flux from the charge passes through the disk, then $R = \sqrt{3}b$.

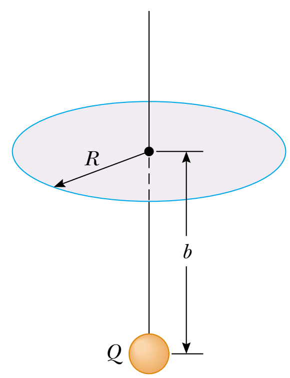

Figure P24.68

69. A spherically symmetric charge distribution has a charge density given by $\rho = a/r$, where a is constant. Find the electric field as a function of r. (*Suggestion:* The charge within a sphere of radius R is equal to the integral of $\rho\,dV$, where r extends from 0 to R. To evaluate the integral, note that the volume element dV for a spherical shell of radius r and thickness dr is equal to $4\pi r^2 dr$.)

70. An infinitely long insulating cylinder of radius R has a volume charge density that varies with the radius as

$$\rho = \rho_0\left(a - \frac{r}{b}\right)$$

where ρ_0, a, and b are positive constants and r is the distance from the axis of the cylinder. Use Gauss's law to determine the magnitude of the electric field at radial distances (a) $r < R$ and (b) $r > R$.

71. **Review problem.** A slab of insulating material (infinite in two of its three dimensions) has a uniform positive charge density ρ. An edge view of the slab is shown in Figure P24.71. (a) Show that the magnitude of the electric field a distance x from its center and inside the slab is $E = \rho x/\epsilon_0$. (b) **What If?** Suppose an electron of charge $-e$ and mass m_e can move freely within the slab. It is released from rest at a distance x from the center. Show that the electron exhibits simple harmonic motion with a frequency

$$f = \frac{1}{2\pi}\sqrt{\frac{\rho e}{m_e \epsilon_0}}$$

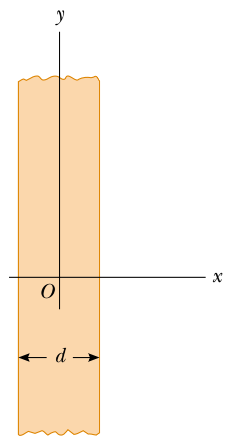

Figure P24.71 Problems 71 and 72.

72. A slab of insulating material has a nonuniform positive charge density $\rho = Cx^2$, where x is measured from the center of the slab as shown in Figure P24.71, and C is a constant. The slab is infinite in the y and z directions. Derive expressions for the electric field in (a) the exterior regions and (b) the interior region of the slab $(-d/2 < x < d/2)$.

73. (a) Using the mathematical similarity between Coulomb's law and Newton's law of universal gravitation, show that Gauss's law for gravitation can be written as

$$\oint \mathbf{g} \cdot d\mathbf{A} = -4\pi G m_{\text{in}}$$

where m_{in} is the net mass inside the gaussian surface and $\mathbf{g} = \mathbf{F}_g/m$ represents the gravitational field at any point on the gaussian surface. (b) Determine the gravitational field at a distance r from the center of the Earth where $r < R_E$, assuming that the Earth's mass density is uniform.

Answers to Quick Quizzes

24.1 (e). The same number of field lines pass through a sphere of any size. Because points on the surface of the sphere are closer to the charge, the field is stronger.

24.2 (d). All field lines that enter the container also leave the container so that the total flux is zero, regardless of the nature of the field or the container.

24.3 (b) and (d). Statement (a) is not necessarily true because an equal number of positive and negative charges could be present inside the surface. Statement (c) is not necessarily true, as can be seen from Figure 24.8: a nonzero electric field exists everywhere on the surface, but the charge is not enclosed within the surface; thus, the net flux is zero.

24.4 (c). The charges q_1 and q_4 are outside the surface and contribute zero net flux through S'.

24.5 (d). We don't need the surfaces to realize that any given point in space will experience an electric field due to all local source charges.

24.6 (a). Charges added to the metal cylinder by your brother will reside on the outer surface of the conducting cylinder. If you are on the inside, these charges cannot transfer to you from the inner surface. For this same reason, you are safe in a metal automobile during a lightning storm.