Chapter 24

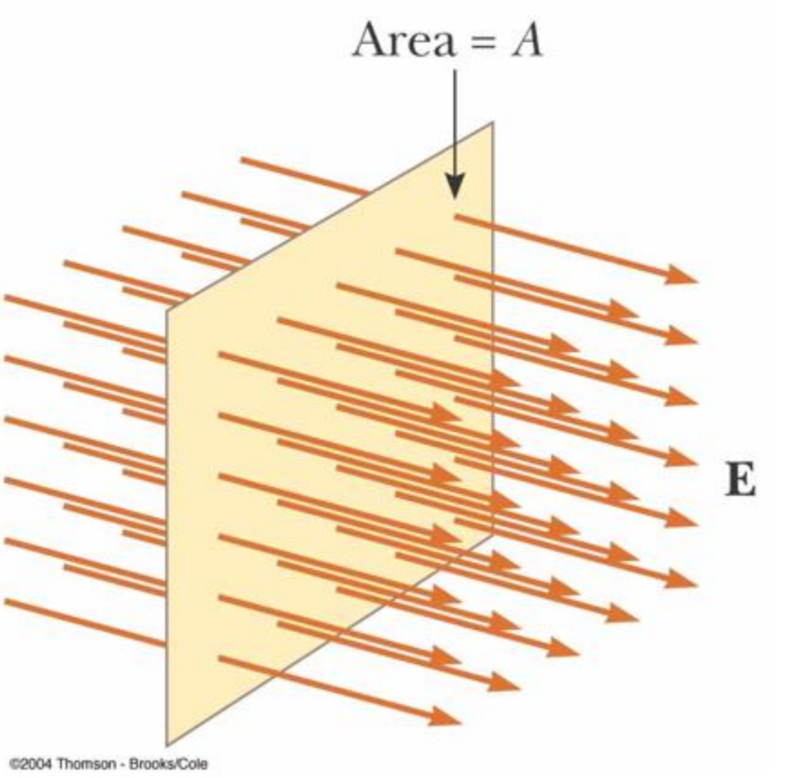
<u>Gauss's Law</u>

Outline

- 24.1 Electric Flux
- 24.2 Gauss's Law
- 24.3 Application of Gauss's Law to Various Charge Distributions
- 24.4 Conductors in Electrostatic Equilibrium

24.1 Electric Flux

- ***Electric flux*** is the product of the magnitude of the electric field and the surface area, *A*, perpendicular to the field
- $\Phi_E = EA$



24.1 Electric Flux

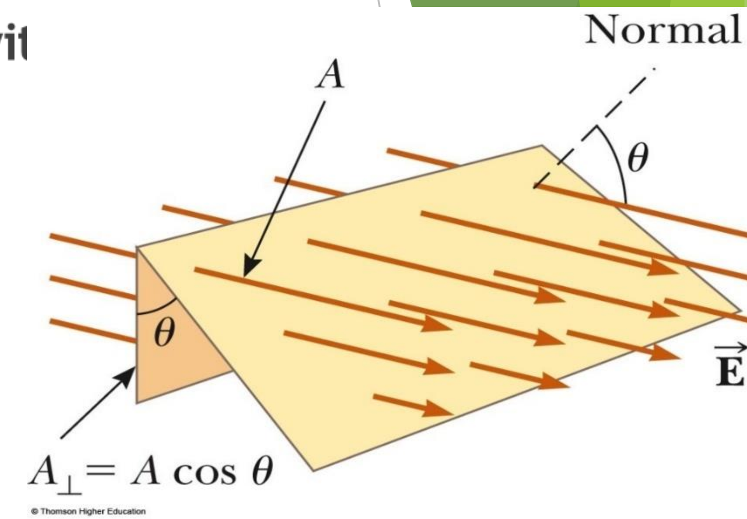
- **Unit of Electric Flux is: <u>Nm^2/C</u>**

<u>Example 24.4 Page 740:</u>

What is the electric flux through a sphere that has a radius of 1.00 m and carries a charge of 1.00μC at its center?

24.1 Electric Flux

- The electric flux is proportional to the number of electric field lines penetrating some surface
- The field lines may make some angle θ wit perpendicular to the surface



$$\Phi_E = EA \cos \theta$$

- Flux has a maximum value **EA** when the surface is perpendicular to the field ($\theta = 0^\circ$)
- Flux is **zero** when the surface is parallel to the field ($\theta = 90^\circ$)

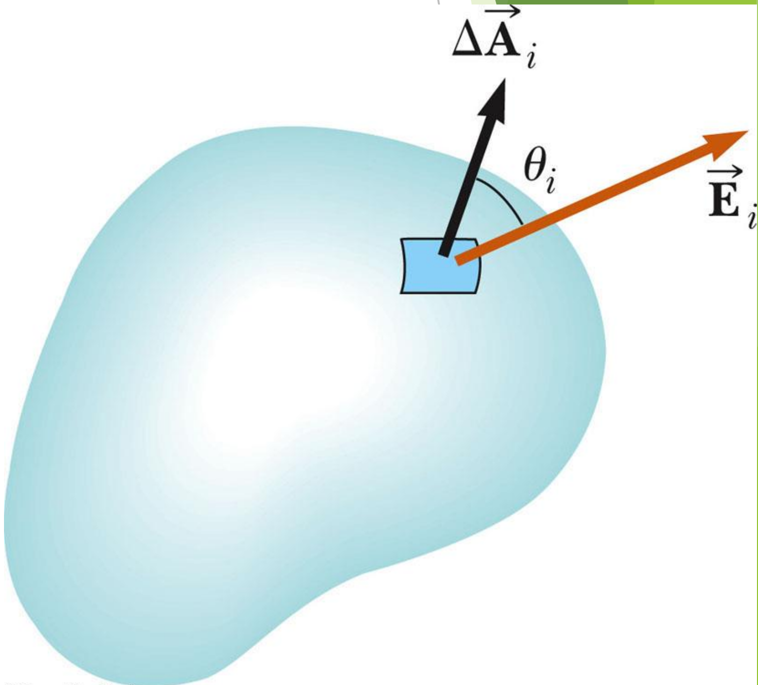
24.1 Electric Flux

- **If the field varies over the surface, Φ = EA cos θ is valid for only over a small element of the area**

$$\Delta\Phi_E = E_i \Delta A_i \cos\theta_i = \vec{\mathbf{E}}_i \cdot \Delta\vec{\mathbf{A}}_i$$

$$\Phi_E = \lim_{\Delta A_i \to 0} \sum E_i \cdot \Delta A_i$$

$$\Phi_E = \int_{\text{surface}} \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}$$



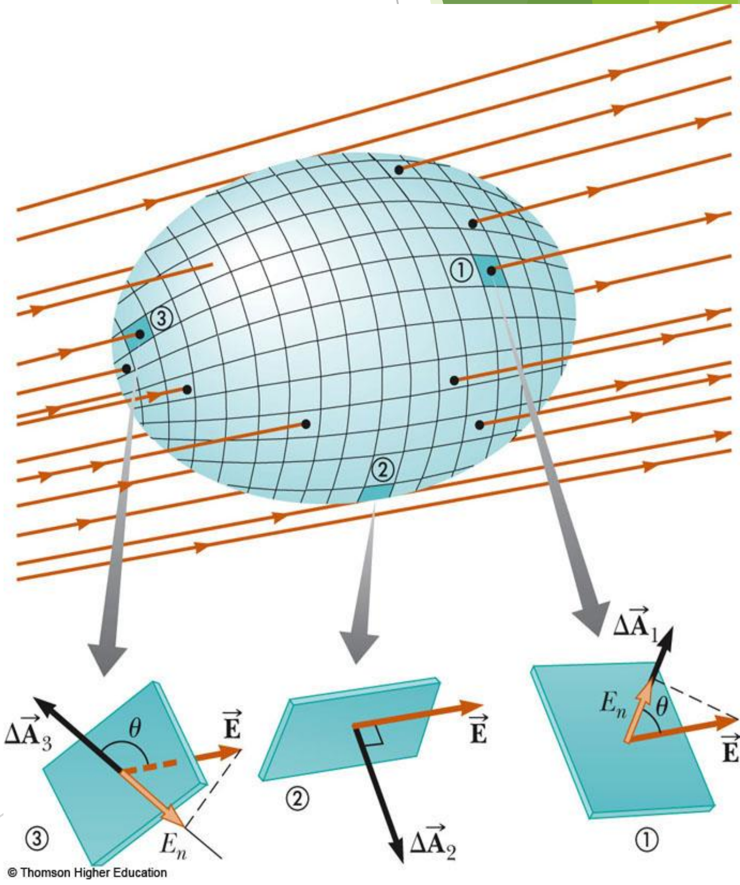
24.1 Electric Flux (Closed Surface)

- **For a closed surface, the flux lines passing into the interior of the volume are negative and those passing out of the interior of the volume are positive**

(1) $\theta < 90^{o}$, $\Phi > 0$
(2) $\theta = 90^{o}$, $\Phi = 0$
(3) $180^{o} > \theta > 90^{o}$, $\Phi < 0$

- The net flux through the surface is proportional to the number of lines leaving the surface minus the number entering the surface

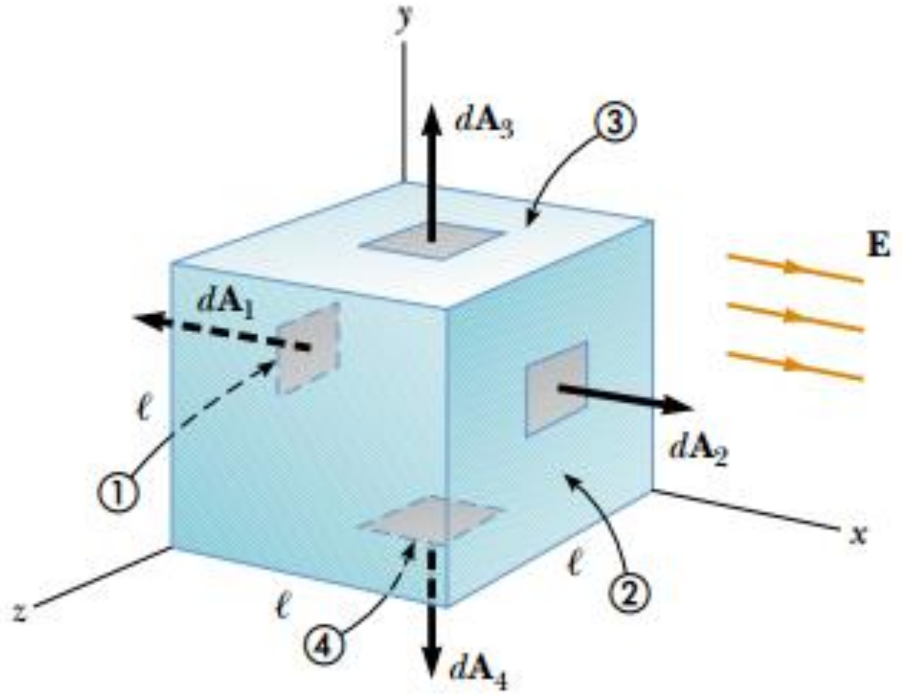
$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \oint E \, dA$$



24.1 Electric Flux

<u>Example 24.2: Flux through a cube</u>

Consider a uniform electric field **E** oriented in the x direction. Find the net electric flux through the surface of a cube of edge length ℓ, oriented as shown in Figure 24.5.



24.1 Electric Flux

Problem 24.4:

Consider a closed triangular box resting within a horizontal electric field of magnitude $E = 7.80 \times 10^4$ N/C as shown in Figure P24.4. Calculate the electric flux through (a) the vertical rectangular surface, (b) the slanted surface, and (c) the entire surface of the box.

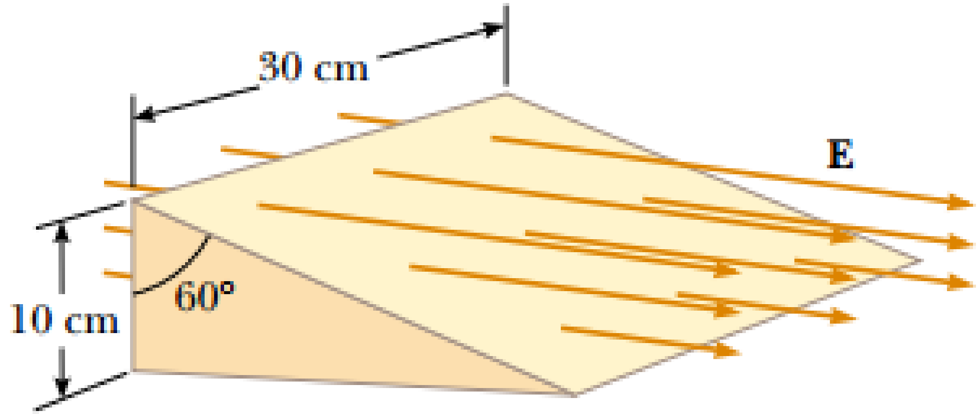
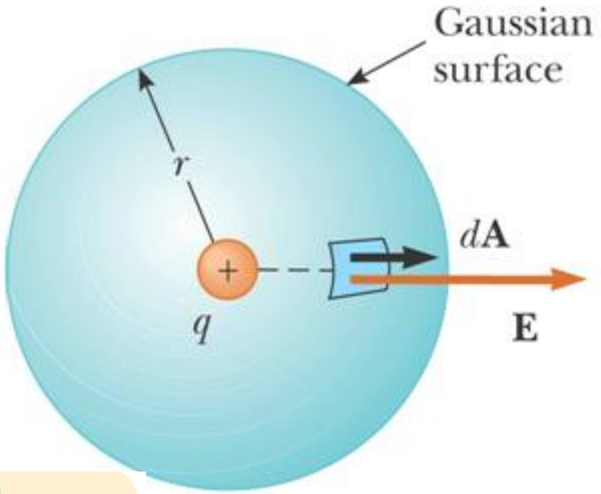


Figure P24.4

24.2 Gauss's Law

- Find a relationship between the net electric flux through a closed surface (often called a *gaussian surface*) and the charge enclosed by the surface.
- Consider a positive point charge q located at the center of a sphere of radius r

Net Flux = $\Phi = \oint \vec{E} \bullet d\vec{A} = \oint E\cos\theta dA = \oint E dA$

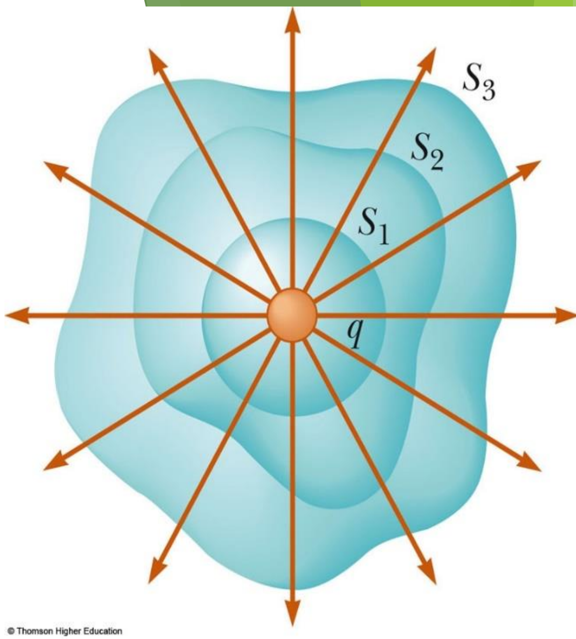


The electric flux Φ_E through any closed surface is equal to the net charge inside the surface, Q_{inside}, divided by ϵ_0:

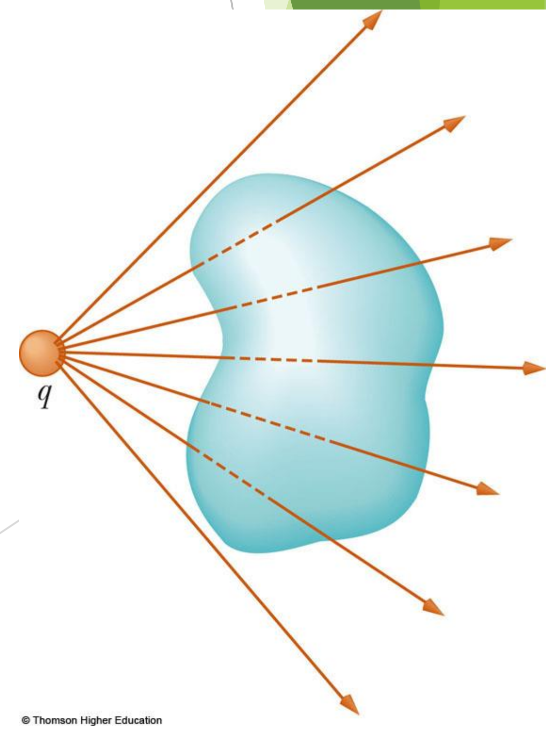
$$\Phi_E = \frac{Q_{\text{inside}}}{\epsilon_0} \qquad [15.11]$$

24.2 Gauss's Law

- The net flux through any closed surface surrounding a point charge q is given by *q/ε_o* and is independent of the shape of the surface

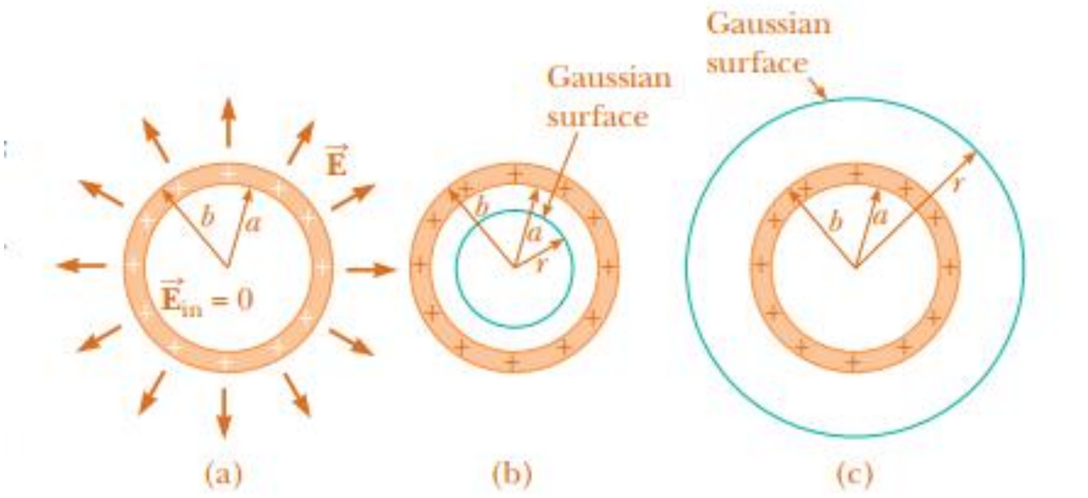


- The If the charge is outside the closed surface of an arbitrary shape, then any field line entering the surface leaves at another point
- net electric flux through a closed surface that surrounds no charge is zero.



24.2 Gauss’s Law

Problem A spherical conducting shell of inner radius a and outer radius b carries a total charge $+Q$ distributed on the surface of a conducting shell (Fig. 15.29a). The quantity Q is taken to be positive. **(a)** Find the electric field in the interior of the conducting shell, for $r < a$, and **(b)** the electric field outside the shell, for $r > b$. **(c)** If an additional



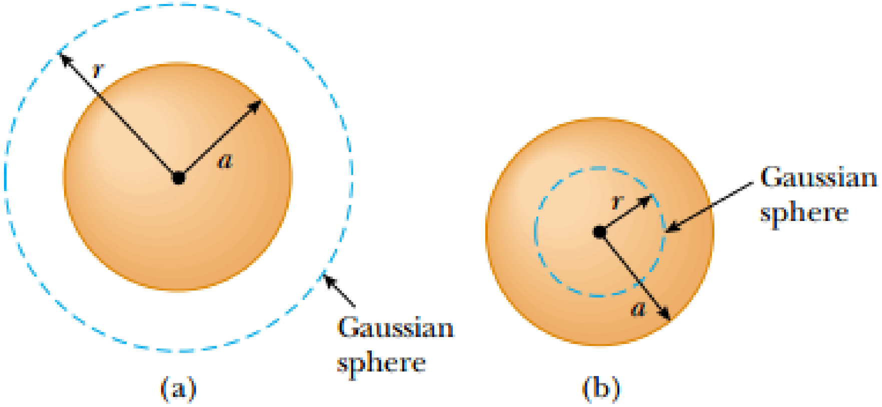
24.3 Application of Gauss's Law to Various Charge Distributions

- ***Example 24.5 Page 747***

An insulating solid sphere of radius a has a uniform volume charge density ρ and carries a total positive charge Q:

A) Calculate the magnitude of the electric field at a point outside the sphere

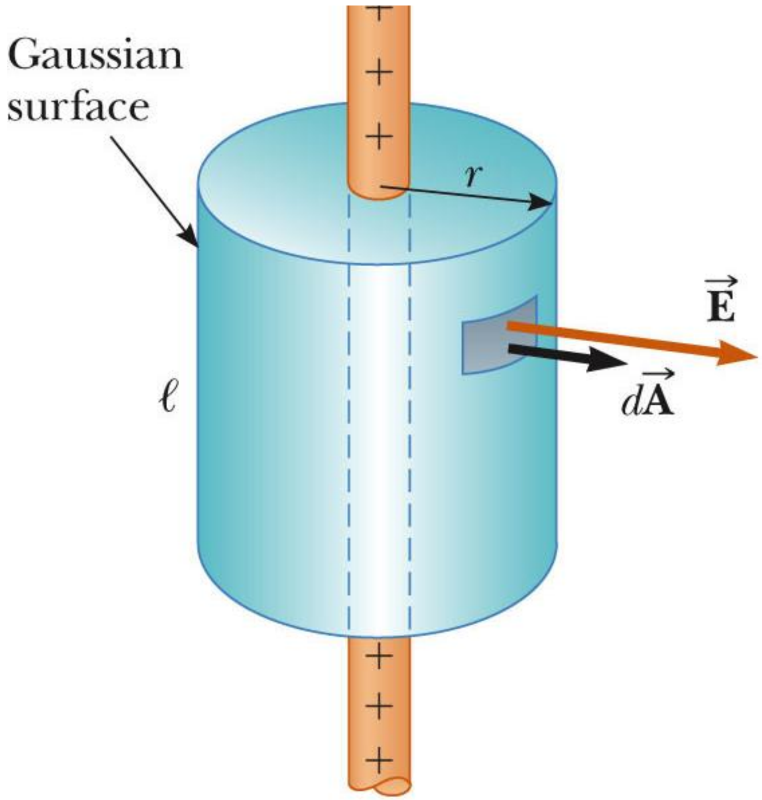
B) Find the magnitude of the electric field at a point inside the sphere



24.3 Application of Gauss's Law to Various Charge Distributions

Example 24.7 Page 748

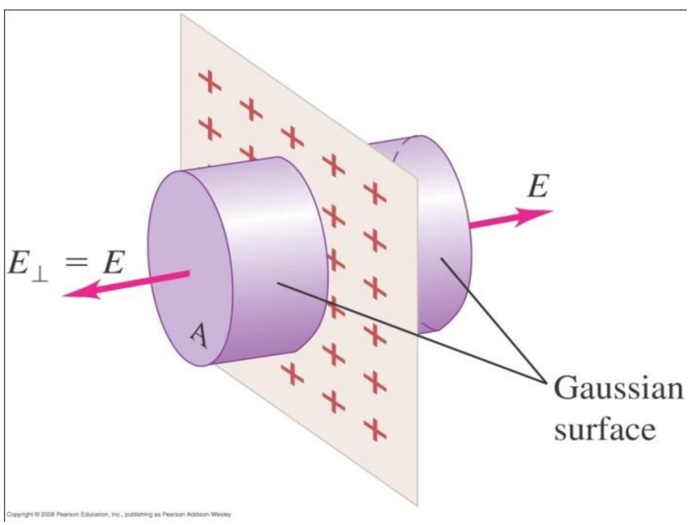
Find the E-field a distance r from a line of positive charge of infinite length and constant charge per unit length λ.

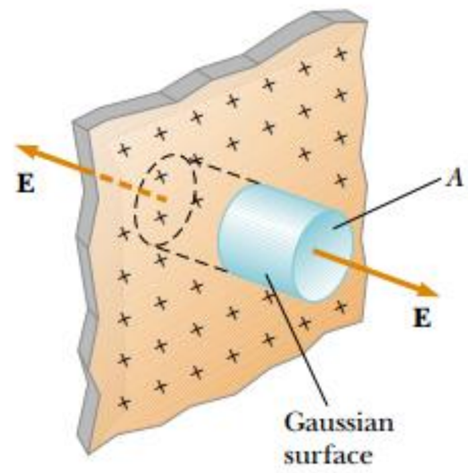


24.3 Application of Gauss's Law to Various Charge Distributions

<u>Example 24.8 Page 749</u>

Find the electric field due to an infinite plane of positive charge with uniform surface charge density σ.





24.3 Application of Gauss's Law to Various Charge Distributions

<u>Problem 24 Page 757</u>

A solid sphere of radius 40.0 cm has a total positive charge of 26.0 μc, uniformly distributed throughout its volume. Calculate the magnitude of the electric field (a) 0 cm, (b) 10.0 cm, (c) 40.0 cm, and (d) 60.0 cm from the center of the sphere.

<u>Problem 37 Page 757</u>

A large, flat, horizontal sheet of charge has a charge per unit area of 7.10 μC/m2. Find the electric field just above the middle of the sheet.

23.4 Conductors in Electrostatic Equilibrium

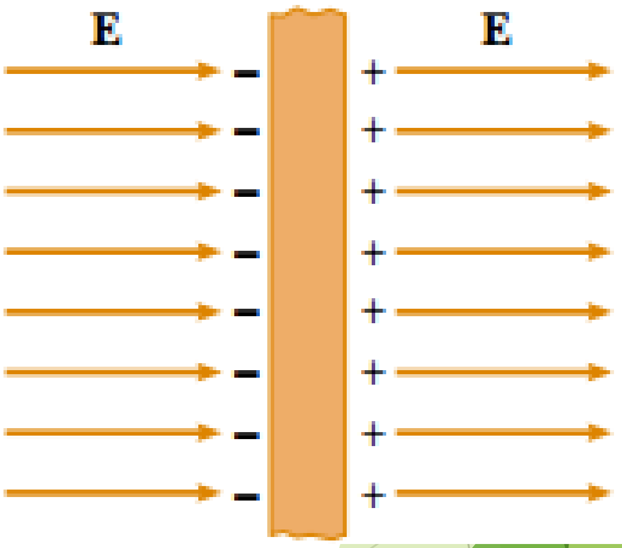
- ***When there is no net motion of charge within a conductor, the conductor is in electrostatic equilibrium***

A conductor in electrostatic equilibrium has the following properties:

- **The electric field is zero everywhere inside the conductor**
- **If an isolated conductor carries a charge, the charge resides on its surface**
- **The electric field just outside a charged conductor is perpendicular to the conductor's surface and has a magnitude** $\frac{\sigma}{\varepsilon_0}$
- **On an irregularly shaped conductor, the surface charge density is greatest at locations where the radius of curvature of the surface is smallest.**

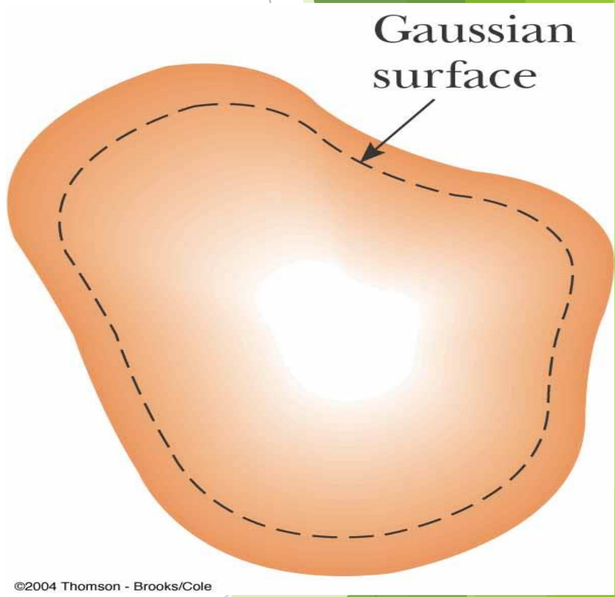
The electric field is zero everywhere inside the conductor

- The electric field inside the conductor must be zero under the assumption that we have electrostatic equilibrium
- If the field were not zero, free electrons in the conductor would experience an electric force(F=qE) and would accelerate due to this force (this motion means conductor is not in electrostatic equilibrium)
- When the external field is applied, the free electrons accelerate to the left, causing a plane of negative charge to be present on the left surface. The movement of electrons to the left results in a plane of positive charge on the right surface. These planes of charge create an additional electric field inside the conductor that opposes the external field. As the electrons move, the surface charge densities on the left and right surfaces increase until the magnitude of the internal field equals that of the external field, resulting in a net field of zero inside the conductor



If an isolated conductor carries a charge, the charge resides on its surface

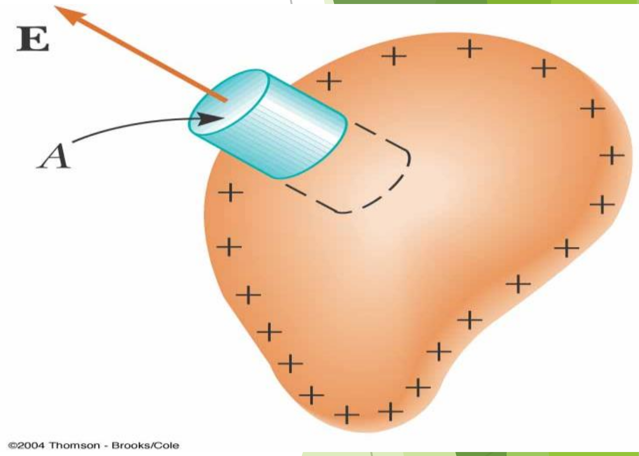
- **From Gauss's law, we conclude that the net charge inside the gaussian surface is zero.**



- **Because there can be no net charge inside the gaussian surface (which is arbitrarily close to the conductor's surface), *any net charge on the conductor must reside on its surface*.**

The electric field just outside a charged conductor is perpendicular to the conductor's surface

- To determine the magnitude of the electric field, we draw a gaussian surface in the shape of a small cylinder whose end faces are parallel to the surface of the conductor



$$\Phi_E = EA = \frac{\sigma A}{\varepsilon_0} \qquad E = \frac{\sigma}{\varepsilon_0}$$

Summary

Typical Electric Field Calculations Using Gauss's Law		
Charge Distribution	**Electric Field**	**Location**
Insulating sphere of radius R, uniform charge density, and total charge Q	$\begin{cases} k_e \dfrac{Q}{r^2} \\ k_e \dfrac{Q}{R^3} r \end{cases}$	$r > R$ $r < R$
Thin spherical shell of radius R and total charge Q	$\begin{cases} k_e \dfrac{Q}{r^2} \\ 0 \end{cases}$	$r > R$ $r < R$
Line charge of infinite length and charge per unit length λ	$2k_e \dfrac{\lambda}{r}$	Outside the line
Infinite charged plane having surface charge density σ	$\dfrac{\sigma}{2\epsilon_0}$	Everywhere outside the plane
Conductor having surface charge density σ	$\begin{cases} \dfrac{\sigma}{\epsilon_0} \\ 0 \end{cases}$	Just outside the conductor Inside the conductor

Chapter Problems

س10) الفيض الكهربي Φ الكلي حول بروتون يساوي:

Q10) The total electric flux Φ around proton equals:

A. 1.6×10^{-19} **B.** 1.4×10^{-30} **C.** 18×10^{-9} **D.** 55×10^{6}

س11) كرة عازلة مصمته نصف قطرها a ويتوزع بانتظام على حجمها شحنة مقدارها Q. مقدار المجال الكهربي عند نقطة تبعد مسافه r من مركز الكرة بحيث $(r < a)$ يعطى بالعلاقه:

Q11) An insulator solid sphere of radius a has a total positive charge Q uniformly distributed throughout its volume. The magnitude of the electric field at a point at distance r from the center of the sphere $(r < a)$ is given by the relation:

A. $\dfrac{k\,Q\,a}{r^3}$ **B.** $\dfrac{k\,Q\,r}{a^3}$ **C.** $\dfrac{k\,Q}{r^2}$ **D.** $\dfrac{k\,Q\,r^2}{a^3}$

Chapter Problems

س12) كرة عازلة مصمته نصف قطرها 12 cm تحوي شحنة مقدارها 40 μC موزعة بانتظام خلال حجمها. مقدار المجال الكهربي عند سطح الكرة يساوي:

Q12) An insulator solid sphere of radius 12 cm has a charge of 40 μC uniformly distributed throughout its volume. The magnitude of the electric field at the sphere surface equals:

A. 3 MN/C **B.** 25 MN/C **C.** 9.4 MN/C **D.** Zero

س13) في السؤال السابق (12) إذا كانت الكرة موصلة فان مقدار المجال الكهربي عند نقطة تبعد 10 cm من مركز الكرة يساوي:

Q13) In the previous question (12), if the sphere is conductor, the magnitude of the electric field at a point 10 cm from the center of the sphere equals:

A. Zero **B.** 20.8 MN/C **C.** 36 MN/C **D.** 0.5 MN/C

س14) فتيل طويل جدا شحنته لوحدة الأطوال 50 nC/m فاذا كان مقدار المجال الكهربي له عند نقطة حول منتصفه هو 60 N/C فان بعد هذة النقطة من منتصف الفتيل يساوي:

Q14) A very long filament has charge per unit length 50 nC/m. If the electric field at a point around its middle is 60 N/C , the distance of the point from the filament equals:

A. 30 cm **B.** 25 cm **C.** 15 m **D.** 12 m

س15) إذا كانت كثافة الشحنه السطحيه (σ) لشريحة لانهائية عازلة هي 35.4×10^{-12} C/m^2 فان المجال الكهربي مباشرة فوق الشريحة يساوي:

Q15) If the surface charge density (σ) of an infinite insulator sheet is 35.4×10^{-12} C/m^2, the electric field just above the sheet equals:

A. 2 N/C **B.** 4 N/C **C.** 8.85 MN/C **D.** Zero

Chapter Problems

(8 إذا مُلئَ مكعب طول ضلعه 8 cm بشحنة كثافتها الحجمية 40 nC/m3 فان الفيض الكهربي خلال أسطح المكعب يساوي:

Q8) If a cube of 8 cm edges is filled with a charge of uniform volume density of 40 nC/m^3, the total electric flux through the surfaces of the cube equals:

A. 2.9 B. 1.8 C. 2 D. 2.3

س9) تحمل قشرة كرويه رقيقه نصف قطرها 16 cm شحنة μC 32 موزعه بانتظام على سطحها. مقدار المجال الكهربي عند نقطه تبعد 10 cm من مركز الشريحه يساوي:

Q9) A thin spherical shell of radius 16 cm carry a total charge of 32 μC distributed uniformly on its surface. The electric field at a point 10 cm from the center of the shell equals:

A. 7×10^6 B. 28.8×10^6 C. 46×10^6 D. Zero

س10) اذا كان المجال الكهربي عند نقطة تبعد 18 mm من منتصف فتيل مستقيم طويل يساوي 9×10^6 N/C فان شحنة الفتيل لوحدة الأطوال λ تساوي:

Q10) If the electric field at a point of 18 mm from the center of a long straight filament is 9×10^6 N/C, the filament charge per unit length λ equals:

A. 9 C/m B. 2 μC/m C. 9 μC/m D. 162 mC/m