Table 25.1 lists electric potentials due to several charge distributions.

Table 25.1

Electric Potential Due to Various Charge Distributions

Charge Distribution	Electric Potential	Location
Uniformly charged ring of radius a	$V = k_e \dfrac{Q}{\sqrt{x^2 + a^2}}$	Along perpendicular central axis of ring, distance x from ring center
Uniformly charged disk of radius a	$V = 2\pi k_e \sigma[(x^2 + a^2)^{1/2} - x]$	Along perpendicular central axis of disk, distance x from disk center
Uniformly charged, *insulating* solid sphere of radius R and total charge Q	$V = k_e \dfrac{Q}{r}$	$r \geq R$
	$V = \dfrac{k_e Q}{2R}\left(3 - \dfrac{r^2}{R^2}\right)$	$r < R$
Isolated *conducting* sphere of radius R and total charge Q	$V = k_e \dfrac{Q}{r}$	$r > R$
	$V = k_e \dfrac{Q}{R}$	$r \leq R$

QUESTIONS

1. Distinguish between electric potential and electric potential energy.
2. A negative charge moves in the direction of a uniform electric field. Does the potential energy of the charge–field system increase or decrease? Does the charge move to a position of higher or lower potential?
3. Give a physical explanation of the fact that the potential energy of a pair of charges with the same sign is positive whereas the potential energy of a pair of charges with opposite signs is negative.
4. A uniform electric field is parallel to the x axis. In what direction can a charge be displaced in this field without any external work being done on the charge?
5. Explain why equipotential surfaces are always perpendicular to electric field lines.
6. Describe the equipotential surfaces for (a) an infinite line of charge and (b) a uniformly charged sphere.
7. Explain why, under static conditions, all points in a conductor must be at the same electric potential.
8. The electric field inside a hollow, uniformly charged sphere is zero. Does this imply that the potential is zero inside the sphere? Explain.
9. The potential of a point charge is defined to be zero at an infinite distance. Why can we not define the potential of an infinite line of charge to be zero at $r = \infty$?
10. Two charged conducting spheres of different radii are connected by a conducting wire as shown in Figure 25.25. Which sphere has the greater charge density?
11. What determines the maximum potential to which the dome of a Van de Graaff generator can be raised?
12. Explain the origin of the glow sometimes observed around the cables of a high-voltage power line.
13. Why is it important to avoid sharp edges or points on conductors used in high-voltage equipment?
14. How would you shield an electronic circuit or laboratory from stray electric fields? Why does this work?
15. Two concentric spherical conducting shells of radii $a = 0.400$ m and $b = 0.500$ m are connected by a thin wire as shown in Figure Q25.15. If a total charge $Q = 10.0\ \mu$C is placed on the system, how much charge settles on each sphere?

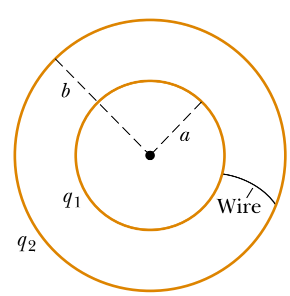


Figure Q25.15

16. Study Figure 23.4 and the accompanying text discussion of charging by induction. You may also compare to Figure

25.24. When the grounding wire is touched to the rightmost point on the sphere in Figure 23.4c, electrons are drained away from the sphere to leave the sphere positively charged. Suppose instead that the grounding wire is touched to the leftmost point on the sphere. Will electrons still drain away, moving closer to the negatively charged rod as they do so? What kind of charge, if any, will remain on the sphere?

PROBLEMS

1, 2, 3 = straightforward, intermediate, challenging   □ = full solution available in the *Student Solutions Manual and Study Guide*

= coached solution with hints available at http://www.pse6.com   = computer useful in solving problem

= paired numerical and symbolic problems

Section 25.1 Potential Difference and Electric Potential

1. How much work is done (by a battery, generator, or some other source of potential difference) in moving Avogadro's number of electrons from an initial point where the electric potential is 9.00 V to a point where the potential is −5.00 V? (The potential in each case is measured relative to a common reference point.)

2. An ion accelerated through a potential difference of 115 V experiences an increase in kinetic energy of 7.37×10^{-17} J. Calculate the charge on the ion.

3. (a) Calculate the speed of a proton that is accelerated from rest through a potential difference of 120 V. (b) Calculate the speed of an electron that is accelerated through the same potential difference.

4. What potential difference is needed to stop an electron having an initial speed of 4.20×10^{5} m/s?

Section 25.2 Potential Differences in a Uniform Electric Field

5. A uniform electric field of magnitude 250 V/m is directed in the positive x direction. A +12.0-μC charge moves from the origin to the point $(x, y) = (20.0 \text{ cm}, 50.0 \text{ cm})$. (a) What is the change in the potential energy of the charge–field system? (b) Through what potential difference does the charge move?

6. The difference in potential between the accelerating plates in the electron gun of a TV picture tube is about 25 000 V. If the distance between these plates is 1.50 cm, what is the magnitude of the uniform electric field in this region?

7. An electron moving parallel to the x axis has an initial speed of 3.70×10^{6} m/s at the origin. Its speed is reduced to 1.40×10^{5} m/s at the point $x = 2.00$ cm. Calculate the potential difference between the origin and that point. Which point is at the higher potential?

8. Suppose an electron is released from rest in a uniform electric field whose magnitude is 5.90×10^{3} V/m. (a) Through what potential difference will it have passed after moving 1.00 cm? (b) How fast will the electron be moving after it has traveled 1.00 cm?

9. A uniform electric field of magnitude 325 V/m is directed in the negative y direction in Figure P25.9. The coordinates of point A are (−0.200, −0.300) m, and those of point B are (0.400, 0.500) m. Calculate the potential difference $V_B - V_A$, using the blue path.

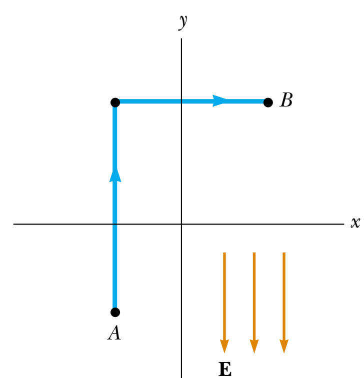


Figure P25.9

10. Starting with the definition of work, prove that at every point on an equipotential surface the surface must be perpendicular to the electric field there.

11. **Review problem.** A block having mass m and charge $+Q$ is connected to a spring having constant k. The block lies on a frictionless horizontal track, and the system is immersed in a uniform electric field of magnitude E, directed as shown in Figure P25.11. If the block is released from rest when the spring is unstretched (at $x = 0$), (a) by what maximum amount does the spring expand? (b) What is the equilibrium position of the block? (c) Show that the block's motion is simple harmonic, and determine its period. (d) **What If?** Repeat part (a) if the coefficient of kinetic friction between block and surface is μ_k.

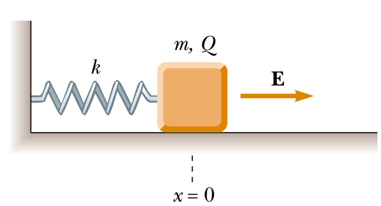


Figure P25.11

12. On planet Tehar, the free-fall acceleration is the same as that on Earth but there is also a strong downward electric field that is uniform close to the planet's surface. A 2.00-kg ball having a charge of 5.00 μC is thrown upward at a speed of 20.1 m/s, and it hits the ground after an interval of 4.10 s. What is the potential difference between the starting point and the top point of the trajectory?

13. An insulating rod having linear charge density λ = 40.0 μC/m and linear mass density μ = 0.100 kg/m is released from rest in a uniform electric field E = 100 V/m directed perpendicular to the rod (Fig. P25.13). (a) Determine the speed of the rod after it has traveled 2.00 m. (b) **What If?** How does your answer to part (a) change if the electric field is not perpendicular to the rod? Explain.

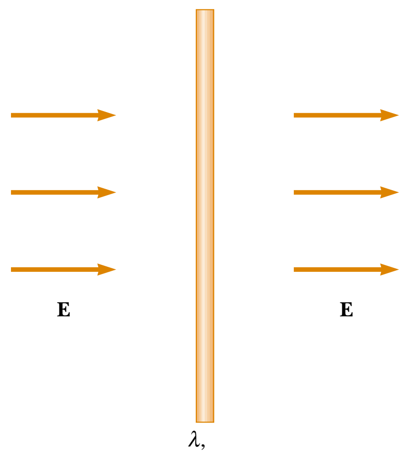


Figure P25.13

14. A particle having charge q = +2.00 μC and mass m = 0.010 0 kg is connected to a string that is L = 1.50 m long and is tied to the pivot point P in Figure P25.14. The particle, string and pivot point all lie on a frictionless horizontal table. The particle is released from rest when the string makes an angle θ = 60.0° with a uniform electric field of magnitude E = 300 V/m. Determine the speed of the particle when the string is parallel to the electric field (point a in Fig. P25.14).

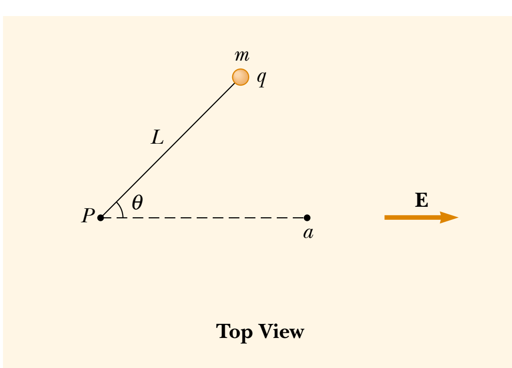


Figure P25.14

Section 25.3 Electric Potential and Potential Energy Due to Point Charges

> *Note:* Unless stated otherwise, assume the reference level of potential is $V = 0$ at $r = \infty$.

15. (a) Find the potential at a distance of 1.00 cm from a proton. (b) What is the potential difference between two points that are 1.00 cm and 2.00 cm from a proton? (c) **What If?** Repeat parts (a) and (b) for an electron.

16. Given two 2.00-μC charges, as shown in Figure P25.16, and a positive test charge $q = 1.28 \times 10^{-18}$ C at the origin, (a) what is the net force exerted by the two 2.00-μC charges on the test charge q? (b) What is the electric field at the origin due to the two 2.00-μC charges? (c) What is the electric potential at the origin due to the two 2.00-μC charges?

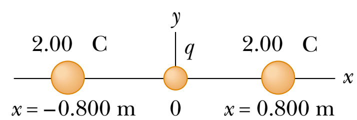


Figure P25.16

17. At a certain distance from a point charge, the magnitude of the electric field is 500 V/m and the electric potential is −3.00 kV. (a) What is the distance to the charge? (b) What is the magnitude of the charge?

18. A charge $+q$ is at the origin. A charge $-2q$ is at x = 2.00 m on the x axis. For what finite value(s) of x is (a) the electric field zero? (b) the electric potential zero?

19. The three charges in Figure P25.19 are at the vertices of an isosceles triangle. Calculate the electric potential at the midpoint of the base, taking q = 7.00 μC.

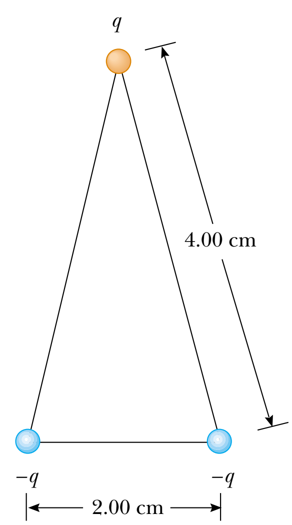


Figure P25.19

20. Two point charges, $Q_1 = +5.00$ nC and $Q_2 = -3.00$ nC, are separated by 35.0 cm. (a) What is the potential energy of the pair? What is the significance of the algebraic sign of your answer? (b) What is the electric potential at a point midway between the charges?

21. *Compare this problem with Problem 57 in Chapter 23.* Four identical point charges ($q = +10.0\ \mu$C) are located on the corners of a rectangle as shown in Figure P23.57. The dimensions of the rectangle are $L = 60.0$ cm and $W = 15.0$ cm. Calculate the change in electric potential energy of the system as the charge at the lower left corner in Figure P23.57 is brought to this position from infinitely far away. Assume that the other three charges in Figure P23.57 remain fixed in position.

22. *Compare this problem with Problem 20 in Chapter 23.* Two point charges each of magnitude 2.00 μC are located on the x axis. One is at $x = 1.00$ m, and the other is at $x = -1.00$ m. (a) Determine the electric potential on the y axis at $y = 0.500$ m. (b) Calculate the change in electric potential energy of the system as a third charge of $-3.00\ \mu$C is brought from infinitely far away to a position on the y axis at $y = 0.500$ m.

23. Show that the amount of work required to assemble four identical point charges of magnitude Q at the corners of a square of side s is $5.41k_eQ^2/s$.

24. *Compare this problem with Problem 23 in Chapter 23.* Five equal negative point charges $-q$ are placed symmetrically around a circle of radius R. Calculate the electric potential at the center of the circle.

25. *Compare this problem with Problem 41 in Chapter 23.* Three equal positive charges q are at the corners of an equilateral triangle of side a as shown in Figure P23.41. (a) At what point, if any, in the plane of the charges is the electric potential zero? (b) What is the electric potential at the point P due to the two charges at the base of the triangle?

26. **Review problem.** Two insulating spheres have radii 0.300 cm and 0.500 cm, masses 0.100 kg and 0.700 kg, and uniformly distributed charges of $-2.00\ \mu$C and $3.00\ \mu$C. They are released from rest when their centers are separated by 1.00 m. (a) How fast will each be moving when they collide? (*Suggestion:* consider conservation of energy and of linear momentum.) (b) **What If?** If the spheres were conductors, would the speeds be greater or less than those calculated in part (a)? Explain.

27. **Review problem.** Two insulating spheres have radii r_1 and r_2, masses m_1 and m_2, and uniformly distributed charges $-q_1$ and q_2. They are released from rest when their centers are separated by a distance d. (a) How fast is each moving when they collide? (*Suggestion:* consider conservation of energy and conservation of linear momentum.) (b) **What If?** If the spheres were conductors, would their speeds be greater or less than those calculated in part (a)? Explain.

28. Two particles, with charges of 20.0 nC and -20.0 nC, are placed at the points with coordinates (0, 4.00 cm) and (0, -4.00 cm), as shown in Figure P25.28. A particle with charge 10.0 nC is located at the origin. (a) Find the electric potential energy of the configuration of the three fixed charges. (b) A fourth particle, with a mass of 2.00×10^{-13} kg and a charge of 40.0 nC, is released from rest at the point (3.00 cm, 0). Find its speed after it has moved freely to a very large distance away.

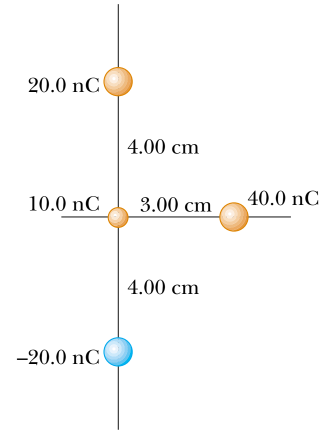


Figure P25.28

29. **Review problem.** A light unstressed spring has length d. Two identical particles, each with charge q, are connected to the opposite ends of the spring. The particles are held stationary a distance d apart and then released at the same time. The system then oscillates on a horizontal frictionless table. The spring has a bit of internal kinetic friction, so the oscillation is damped. The particles eventually stop vibrating when the distance between them is $3d$. Find the increase in internal energy that appears in the spring during the oscillations. Assume that the system of the spring and two charges is isolated.

30. Two point charges of equal magnitude are located along the y axis equal distances above and below the x axis, as shown in Figure P25.30. (a) Plot a graph of the potential at points along the x axis over the interval $-3a < x < 3a$. You should plot the potential in units of k_eQ/a. (b) Let the charge located at $-a$ be negative and plot the potential along the y axis over the interval $-4a < y < 4a$.

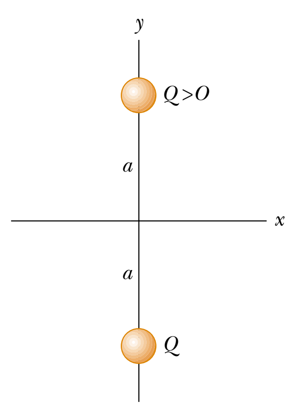


Figure P25.30

31. A small spherical object carries a charge of 8.00 nC. At what distance from the center of the object is the potential equal to 100 V? 50.0 V? 25.0 V? Is the spacing of the equipotentials proportional to the change in potential?

32. In 1911 Ernest Rutherford and his assistants Geiger and Marsden conducted an experiment in which they scattered alpha particles from thin sheets of gold. An alpha particle, having charge $+2e$ and mass 6.64×10^{-27} kg, is a product of certain radioactive decays. The results of the experiment led Rutherford to the idea that most of the mass of an atom is in a very small nucleus, with electrons in orbit around it—his planetary model of the atom. Assume an alpha particle, initially very far from a gold nucleus, is fired with a velocity of 2.00×10^7 m/s directly toward the nucleus (charge $+79e$). How close does the alpha particle get to the nucleus before turning around? Assume the gold nucleus remains stationary.

33. An electron starts from rest 3.00 cm from the center of a uniformly charged insulating sphere of radius 2.00 cm and total charge 1.00 nC. What is the speed of the electron when it reaches the surface of the sphere?

34. Calculate the energy required to assemble the array of charges shown in Figure P25.34, where $a = 0.200$ m, $b = 0.400$ m, and $q = 6.00\ \mu$C.

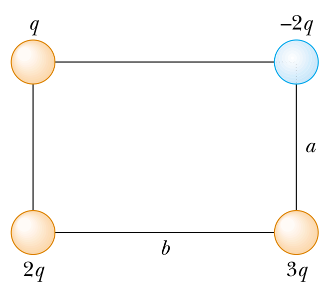


Figure P25.34

35. Four identical particles each have charge q and mass m. They are released from rest at the vertices of a square of side L. How fast is each charge moving when their distance from the center of the square doubles?

36. How much work is required to assemble eight identical point charges, each of magnitude q, at the corners of a cube of side s?

Section 25.4 Obtaining the Value of the Electric Field from the Electric Potential

37. The potential in a region between $x = 0$ and $x = 6.00$ m is $V = a + bx$, where $a = 10.0$ V and $b = -7.00$ V/m. Determine (a) the potential at $x = 0$, 3.00 m, and 6.00 m, and (b) the magnitude and direction of the electric field at $x = 0$, 3.00 m, and 6.00 m.

38. The electric potential inside a charged spherical conductor of radius R is given by $V = k_eQ/R$, and the potential outside is given by $V = k_eQ/r$. Using $E_r = -dV/dr$, derive the electric field (a) inside and (b) outside this charge distribution.

39. Over a certain region of space, the electric potential is $V = 5x - 3x^2y + 2yz^2$. Find the expressions for the x, y, and z components of the electric field over this region. What is the magnitude of the field at the point P that has coordinates $(1, 0, -2)$ m?

40. Figure P25.40 shows several equipotential lines each labeled by its potential in volts. The distance between the lines of the square grid represents 1.00 cm. (a) Is the magnitude of the field larger at A or at B? Why? (b) What is **E** at B? (c) Represent what the field looks like by drawing at least eight field lines.

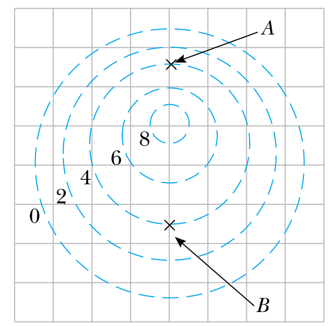


Figure P25.40

41. It is shown in Example 25.7 that the potential at a point P a distance a above one end of a uniformly charged rod of length ℓ lying along the x axis is

$$V = \frac{k_eQ}{\ell} \ln\left(\frac{\ell + \sqrt{\ell^2 + a^2}}{a}\right)$$

Use this result to derive an expression for the y component of the electric field at P. (*Suggestion:* Replace a with y.)

Section 25.5 Electric Potential Due to Continuous Charge Distributions

42. Consider a ring of radius R with the total charge Q spread uniformly over its perimeter. What is the potential difference between the point at the center of the ring and a point on its axis a distance $2R$ from the center?

43. A rod of length L (Fig. P25.43) lies along the x axis with its left end at the origin. It has a nonuniform charge density $\lambda = \alpha x$, where α is a positive constant. (a) What are the units of α? (b) Calculate the electric potential at A.

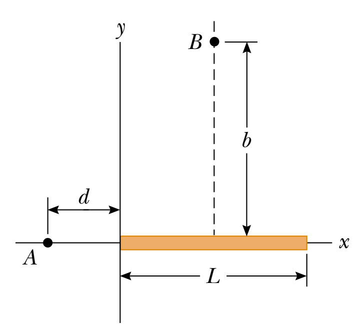


Figure P25.43 Problems 43 and 44.

44. For the arrangement described in the previous problem, calculate the electric potential at point B, which lies on the perpendicular bisector of the rod a distance b above the x axis.

45. *Compare this problem with Problem 33 in Chapter 23.* A uniformly charged insulating rod of length 14.0 cm is bent into the shape of a semicircle as shown in Figure P23.33. The rod has a total charge of $-7.50\ \mu$C. Find the electric potential at O, the center of the semicircle.

46. Calculate the electric potential at point P on the axis of the annulus shown in Figure P25.46, which has a uniform charge density σ.

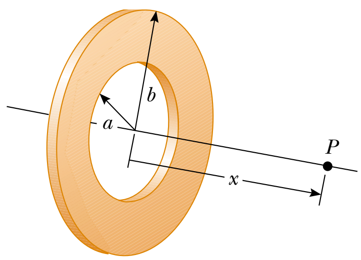


Figure P25.46

47. A wire having a uniform linear charge density λ is bent into the shape shown in Figure P25.47. Find the electric potential at point O.

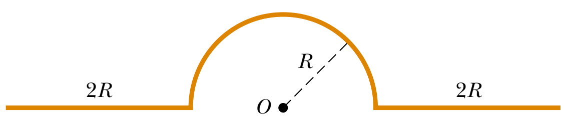


Figure P25.47

Section 25.6 Electric Potential Due to a Charged Conductor

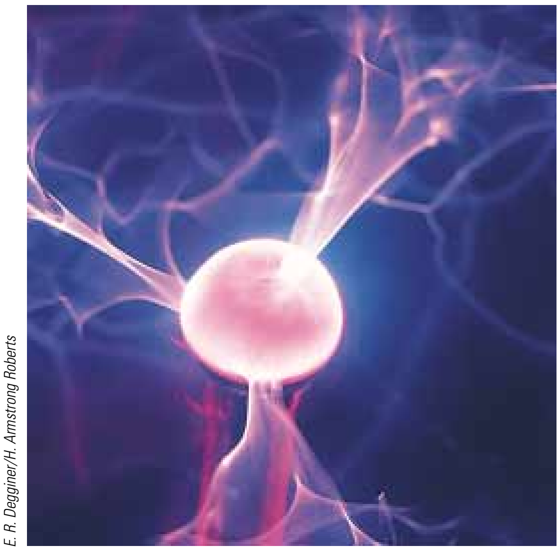
48. How many electrons should be removed from an initially uncharged spherical conductor of radius 0.300 m to produce a potential of 7.50 kV at the surface?

49. A spherical conductor has a radius of 14.0 cm and charge of 26.0 μC. Calculate the electric field and the electric potential (a) $r = 10.0$ cm, (b) $r = 20.0$ cm, and (c) $r = 14.0$ cm from the center.

50. Electric charge can accumulate on an airplane in flight. You may have observed needle-shaped metal extensions on the wing tips and tail of an airplane. Their purpose is to allow charge to leak off before much of it accumulates. The electric field around the needle is much larger than the field around the body of the airplane, and can become large enough to produce dielectric breakdown of the air, discharging the airplane. To model this process, assume that two charged spherical conductors are connected by a long conducting wire, and a charge of 1.20 μC is placed on the combination. One sphere, representing the body of the airplane, has a radius of 6.00 cm, and the other, representing the tip of the needle, has a radius of 2.00 cm. (a) What is the electric potential of each sphere? (b) What is the electric field at the surface of each sphere?

Section 25.8 Applications of Electrostatics

51. Lightning can be studied with a Van de Graaff generator, essentially consisting of a spherical dome on which charge is continuously deposited by a moving belt. Charge can be added until the electric field at the surface of the dome becomes equal to the dielectric strength of air. Any more charge leaks off in sparks, as shown in Figure P25.51. Assume the dome has a diameter of 30.0 cm and is surrounded by dry air with dielectric strength 3.00×10^6 V/m. (a) What is the maximum potential of the dome? (b) What is the maximum charge on the dome?



E. R. Degginer/H. Armstrong Roberts

Figure P25.51 Problems 51 and 52.

52. The spherical dome of a Van de Graaff generator can be raised to a maximum potential of 600 kV; then additional charge leaks off in sparks, by producing dielectric breakdown of the surrounding dry air, as shown in Figure P25.51. Determine (a) the charge on the dome and (b) the radius of the dome.

Additional Problems

53. The liquid-drop model of the atomic nucleus suggests that high-energy oscillations of certain nuclei can split the nucleus into two unequal fragments plus a few neutrons. The fission products acquire kinetic energy from their mutual Coulomb repulsion. Calculate the electric potential energy (in electron volts) of two spherical fragments from a uranium nucleus having the following charges and radii: $38e$ and 5.50×10^{-15} m; $54e$ and 6.20×10^{-15} m. Assume that the charge is distributed uniformly throughout the volume of each spherical fragment and that just before separating they are at rest with their surfaces in contact. The electrons surrounding the nucleus can be ignored.

54. On a dry winter day you scuff your leather-soled shoes across a carpet and get a shock when you extend the tip of one finger toward a metal doorknob. In a dark room you see a spark perhaps 5 mm long. Make order-of-magnitude estimates of (a) your electric potential and (b) the charge on your body before you touch the doorknob. Explain your reasoning.

55. The Bohr model of the hydrogen atom states that the single electron can exist only in certain allowed orbits around the proton. The radius of each Bohr orbit is $r = n^2(0.052\,9\text{ nm})$ where $n = 1, 2, 3, \ldots$. Calculate the electric potential energy of a hydrogen atom when the electron (a) is in the first allowed orbit, with $n = 1$, (b) is in the second allowed orbit, $n = 2$, and (c) has escaped from the atom, with $r = \infty$. Express your answers in electron volts.

56. An electron is released from rest on the axis of a uniform positively charged ring, 0.100 m from the ring's center. If the linear charge density of the ring is $+0.100\ \mu\text{C/m}$ and the radius of the ring is 0.200 m, how fast will the electron be moving when it reaches the center of the ring?

57. As shown in Figure P25.57, two large parallel vertical conducting plates separated by distance d are charged so that their potentials are $+V_0$ and $-V_0$. A small conducting ball of mass m and radius R (where $R \ll d$) is hung midway between the plates. The thread of length L supporting the ball is a conducting wire connected to ground, so the potential of the ball is fixed at $V = 0$. The ball hangs straight down in stable equilibrium when V_0 is sufficiently small. Show that the equilibrium of the ball is unstable if V_0 exceeds the critical value $k_e d^2 mg/(4RL)$. (*Suggestion:* consider the forces on the ball when it is displaced a distance $x \ll L$.)

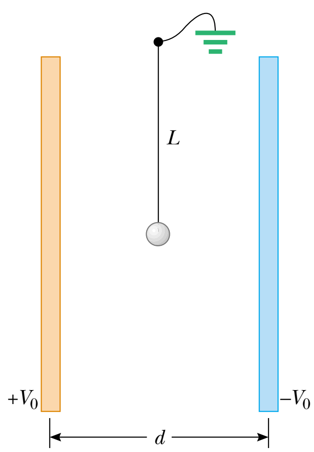


Figure P25.57

58. *Compare this problem with Problem 34 in Chapter 23.* (a) A uniformly charged cylindrical shell has total charge Q, radius R, and height h. Determine the electric potential at a point a distance d from the right end of the cylinder, as shown in Figure P25.58. (*Suggestion:* use the result of Example 25.5 by treating the cylinder as a collection of ring charges.) (b) **What If?** Use the result of Example 25.6 to solve the same problem for a solid cylinder.

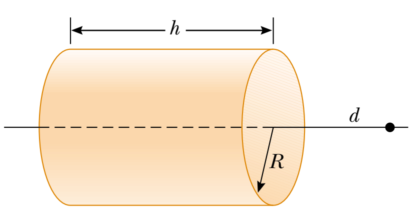


Figure P25.58

59. Calculate the work that must be done to charge a spherical shell of radius R to a total charge Q.

60. Two parallel plates having charges of equal magnitude but opposite sign are separated by 12.0 cm. Each plate has a surface charge density of 36.0 nC/m^2. A proton is released from rest at the positive plate. Determine (a) the potential difference between the plates, (b) the kinetic energy of the proton when it reaches the negative plate, (c) the speed of the proton just before it strikes the negative plate, (d) the acceleration of the proton, and (e) the force on the proton. (f) From the force, find the magnitude of the electric field and show that it is equal to the electric field found from the charge densities on the plates.

61. A Geiger tube is a radiation detector that essentially consists of a closed, hollow metal cylinder (the cathode) of inner radius r_a and a coaxial cylindrical wire (the anode) of radius r_b (Fig. P25.61). The charge per unit length on the anode is λ, while the charge per unit length on the cathode is $-\lambda$. A gas fills the space between the electrodes. When a high-energy elementary particle passes through this space, it can ionize an atom of the gas. The strong electric field makes the resulting ion and electron accelerate in opposite directions. They strike other molecules of the gas to ionize them, producing an avalanche of electrical discharge. The pulse of electric current between the wire and the cylinder is counted by an external circuit. (a) Show that the magnitude of the potential difference between the wire and the cylinder is

$$\Delta V = 2k_e\lambda \ln\left(\frac{r_a}{r_b}\right)$$

(b) Show that the magnitude of the electric field in the space between cathode and anode is given by

$$E = \frac{\Delta V}{\ln(r_a/r_b)}\left(\frac{1}{r}\right)$$

where r is the distance from the axis of the anode to the point where the field is to be calculated.

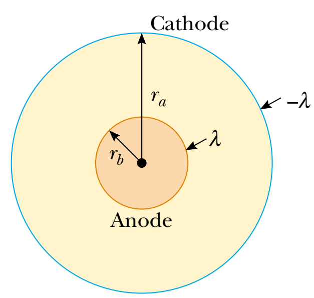


Figure P25.61 Problems 61 and 62.

62. The results of Problem 61 apply also to an electrostatic precipitator (Figures 25.30 and P25.61). An applied voltage $\Delta V = V_a - V_b = 50.0\text{ kV}$ is to produce an electric field of magnitude 5.50 MV/m at the surface of the central wire. Assume the outer cylindrical wall has uniform radius $r_a = 0.850\text{ m}$. (a) What should be the radius r_b of the central wire? You will need to solve a transcendental equation. (b) What is the magnitude of the electric field at the outer wall?

63. From Gauss's law, the electric field set up by a uniform line of charge is

$$\mathbf{E} = \left(\frac{\lambda}{2\pi\epsilon_0 r}\right)\hat{\mathbf{r}}$$

where $\hat{\mathbf{r}}$ is a unit vector pointing radially away from the line and λ is the linear charge density along the line. Derive an expression for the potential difference between $r = r_1$ and $r = r_2$.

64. Four balls, each with mass m, are connected by four nonconducting strings to form a square with side a, as shown in Figure P25.64. The assembly is placed on a horizontal nonconducting frictionless surface. Balls 1 and 2 each have charge q, and balls 3 and 4 are uncharged. Find the maximum speed of balls 1 and 2 after the string connecting them is cut.

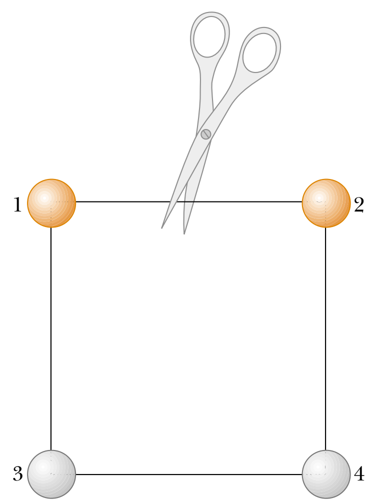


Figure P25.64

65. A point charge q is located at $x = -R$, and a point charge $-2q$ is located at the origin. Prove that the equipotential surface that has zero potential is a sphere centered at $(-4R/3, 0, 0)$ and having a radius $r = 2R/3$.

66. Consider two thin, conducting, spherical shells as shown in Figure P25.66. The inner shell has a radius $r_1 = 15.0$ cm and a charge of 10.0 nC. The outer shell has a radius $r_2 = 30.0$ cm and a charge of -15.0 nC. Find (a) the electric field $\mathbf{E}$ and (b) the electric potential V in regions A, B, and C, with $V = 0$ at $r = \infty$.

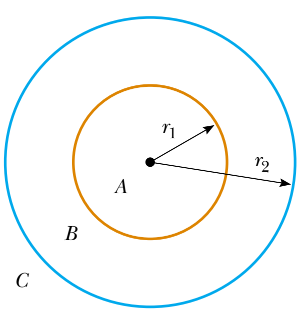


Figure P25.66

67. The x axis is the symmetry axis of a stationary uniformly charged ring of radius R and charge Q (Fig. P25.67). A point charge Q of mass M is located initially at the center of the ring. When it is displaced slightly, the point charge accelerates along the x axis to infinity. Show that the ultimate speed of the point charge is

$$v = \left(\frac{2k_eQ^2}{MR}\right)^{1/2}$$

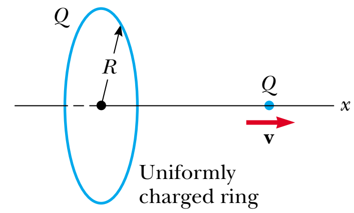


Figure P25.67

68. The thin, uniformly charged rod shown in Figure P25.68 has a linear charge density λ. Find an expression for the electric potential at P.

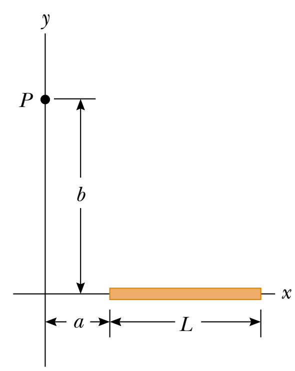


Figure P25.68

69. An electric dipole is located along the y axis as shown in Figure P25.69. The magnitude of its electric dipole moment is defined as $p = 2qa$. (a) At a point P, which is far from the dipole $(r \gg a)$, show that the electric potential is

$$V = \frac{k_e p\cos\theta}{r^2}$$

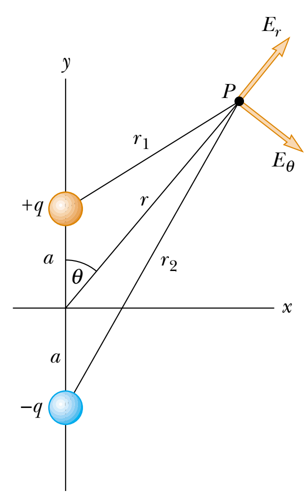


Figure P25.69

(b) Calculate the radial component E_r and the perpendicular component E_θ of the associated electric field. Note that $E_\theta = -(1/r)(\partial V/\partial\theta)$. Do these results seem reasonable for $\theta = 90°$ and $0°$? for $r = 0$? (c) For the dipole arrangement shown, express V in terms of Cartesian coordinates using $r = (x^2 + y^2)^{1/2}$ and

$$\cos\theta = \frac{y}{(x^2 + y^2)^{1/2}}$$

Using these results and again taking $r \gg a$, calculate the field components E_x and E_y.

70. When an uncharged conducting sphere of radius a is placed at the origin of an xyz coordinate system that lies in an initially uniform electric field $\mathbf{E} = E_0\hat{\mathbf{k}}$, the resulting electric potential is $V(x, y, z) = V_0$ for points inside the sphere and

$$V(x, y, z) = V_0 - E_0 z + \frac{E_0 a^3 z}{(x^2 + y^2 + z^2)^{3/2}}$$

for points outside the sphere, where V_0 is the (constant) electric potential on the conductor. Use this equation to determine the x, y, and z components of the resulting electric field.

71. A disk of radius R (Fig. P25.71) has a nonuniform surface charge density $\sigma = Cr$, where C is a constant and r is measured from the center of the disk. Find (by direct integration) the potential at P.

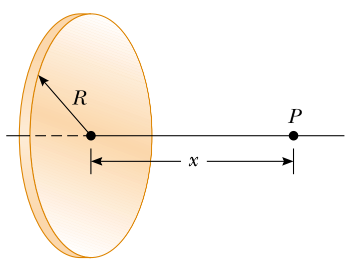


Figure P25.71

72. A solid sphere of radius R has a uniform charge density ρ and total charge Q. Derive an expression for its total electric potential energy. (*Suggestion:* imagine that the sphere is constructed by adding successive layers of concentric shells of charge $dq = (4\pi r^2\, dr)\rho$ and use $dU = V\, dq$.)

73. Charge is uniformly distributed with a density of 100.0 μC/m^3 throughout the volume of a cube 10.00 cm on each edge. (a) Find the electric potential at a distance of 5.000 cm from the center of one face of the cube, measured along a perpendicular to the face. Determine the potential to four significant digits. Use a numerical method that divides the cube into a sufficient number of smaller cubes, treated as point charges. Symmetry considerations will reduce the number of actual calculations. (b) **What If?** If the charge on the cube is redistributed into a uniform sphere of charge with the same center, by how much does the potential change?

Answers to Quick Quizzes

25.1 (b). When moving straight from A to B, $\mathbf{E}$ and $d\mathbf{s}$ both point toward the right. Thus, the dot product $\mathbf{E}\cdot d\mathbf{s}$ in Equation 25.3 is positive and ΔV is negative.

25.2 (a). From Equation 25.3, $\Delta U = q_0\,\Delta V$, so if a negative test charge is moved through a negative potential difference, the potential energy is positive. Work must be done to move the charge in the direction opposite to the electric force on it.

25.3 $B \rightarrow C$, $C \rightarrow D$, $A \rightarrow B$, $D \rightarrow E$. Moving from B to C decreases the electric potential by 2 V, so the electric field performs 2 J of work on each coulomb of positive charge that moves. Moving from C to D decreases the electric potential by 1 V, so 1 J of work is done by the field. It takes no work to move the charge from A to B because the electric potential does not change. Moving from D to E increases the electric potential by 1 V, and thus the field does -1 J of work per unit of positive charge that moves.

25.4 (f). The electric field points in the direction of decreasing electric potential.

25.5 (b) and (f). The electric potential is inversely proportion to the radius (see Eq. 25.11). Because the same number of field lines passes through a closed surface of any shape or size, the electric flux through the surface remains constant.

25.6 (c). The potential is established only by the source charge and is independent of the test charge.

25.7 (a). The potential energy of the two-charge system is initially negative, due to the products of charges of opposite sign in Equation 25.13. When the sign of q_2 is changed, both charges are negative, and the potential energy of the system is positive.

25.8 (a). If the potential is constant (zero in this case), its derivative along this direction is zero.

25.9 (b). If the electric field is zero, there is no change in the electric potential and it must be constant. This constant value *could be* zero but does not *have to be* zero.

25.10 The graph would look like the sketch below. Notice the flat plateaus at each conductor, representing the constant electric potential inside a conductor.