Chapter 26

Capacitance and Dielectric

Outline

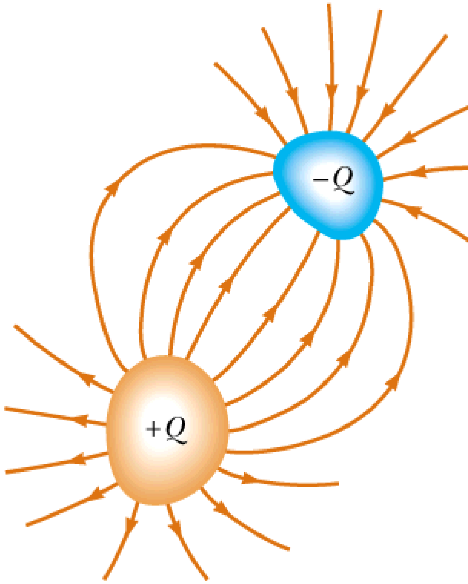
- 26.1 Definition of Capacitance
- 26.2 Calculating Capacitance
- 26.3 Combinations of Capacitors
- 26.4 Energy Stored in a Charged Capacitor
- 26.5 Capacitors with Dielectrics

26.1 Definition of Capacitance

- A capacitor consists of *two conductors* (known as plates) carrying charges of *equal magnitude* and *opposite sign*.

- A potential difference ΔV exists between the conductors due to the presence of the charges

- Capacitors are used in many applications such as to tune the frequency of radio receivers and store short-term energy for rapid release in electronic flash units



26.1 Definition of Capacitance

- Capacitance is a measure of a capacitor's ability to store charge

The **capacitance** C of a capacitor is defined as the ratio of the magnitude of the charge on either conductor to the magnitude of the potential difference between the conductors:

$$C \equiv \frac{Q}{\Delta V} \tag{26.1}$$

- *Important Note: the total charge on the capacitor is zero (because there is as much excess positive charge on one conductor as there is excess negative charge on the other)*

26.1 Definition of Capacitance

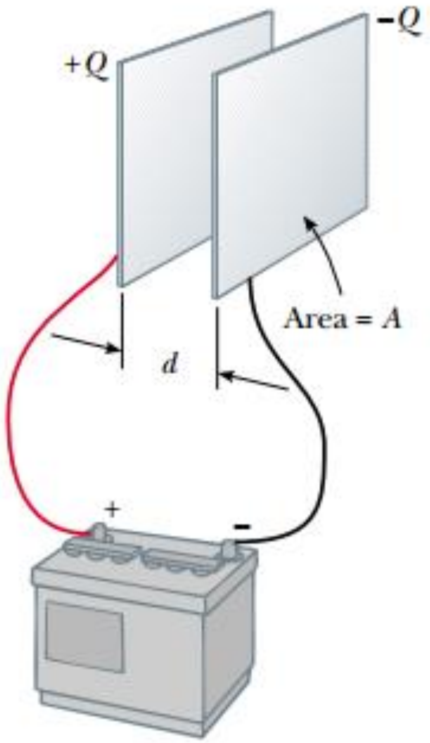
- Capacitance is always a positive quantity (because we take the magnitude of the charge on either conductor)

- SI Unit of Capacitance: farad (F), 1F = 1 C/V

- In practice, the units used ranges from microfarads (mF=10^{-6}F), nanofarads (nF=10^{-9} F), and picofarads (pF=10^{-12} F)

26.1 Definition of Capacitance

- Let us consider a typical design for a capacitor as the figure (two parallel metal plates)



- The plates are connected to the positive and negative terminals of a battery. When this connection is made, electrons are pulled off one of the plates, leaving it with a charge of ***+Q***, and are transferred through the battery to the other plate, leaving it with a charge of ***–Q***. The transfer of charge **stops** when

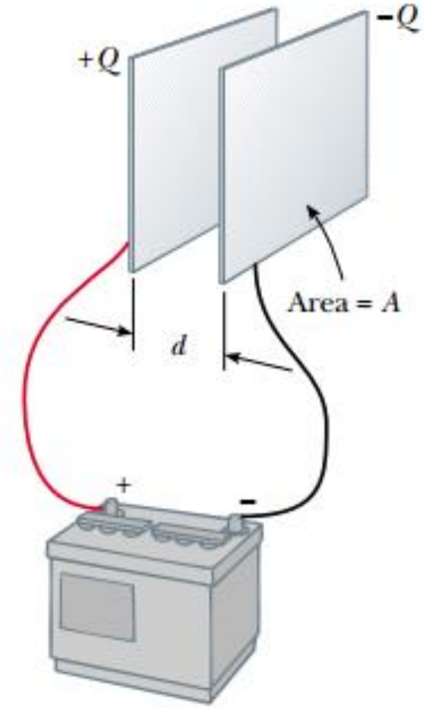
ΔV (between the plates) = ΔV (between the terminals)

- A charged capacitor is a device that stores energy that can be reclaimed when needed for a specific application

26.1 Definition of Capacitance

- A capacitor rated at 4pF. This rating means that the capacitor can store 4pC of charge for each volt of potential difference between the two conductors.

- If a 9-V battery is connected across this capacitor, one of the conductor sends up with a net charge of ***-36pC*** and the other ends up with a net charge of ***+36pC***



26.1 Definition of Capacitance

- ***Quick Quiz 26.1***

A capacitor stores charge Q at a potential difference ΔV. If the voltage applied by a battery to the capacitor is doubled to 2 ΔV

1- The capacitance falls to half its initial value and the charge remains the same

2- The capacitance and the charge both fall to half their initial values

3- the capacitance and the charge both double

4- The capacitance remains the same and the charge doubles

26.2 Calculating Capacitance

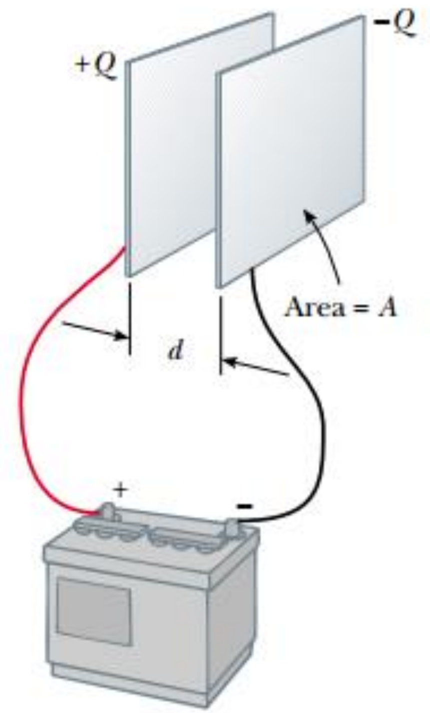
- **Capacitance of an Isolated Sphere:**
- A single conductor also has a capacitance

$$C = \frac{Q}{\Delta V} = \frac{Q}{k_e Q / R} = \frac{R}{k_e} = 4\pi\epsilon_0 R$$

- The capacitance of an isolated charged sphere is proportional to its radius and is ***independent*** of both the charge on the sphere and the potential difference

26.2 Calculating Capacitance

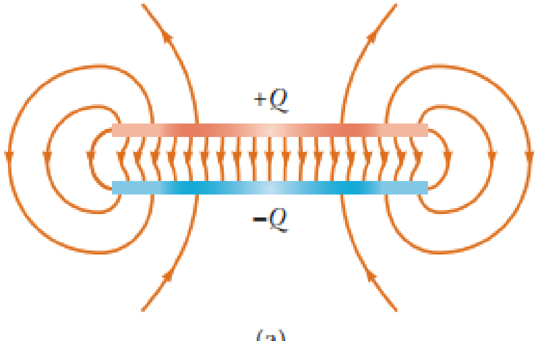
- <u>Parallel-Plate Capacitors:</u>



- The capacitance of a parallel-plate capacitor is given by:

$$C = \frac{\epsilon_0 A}{d}$$

- The capacitance of a parallel-plate capacitor is proportional to the area of its plates and inversely proportional to the plate separation
- This means plates with larger area can store more charge

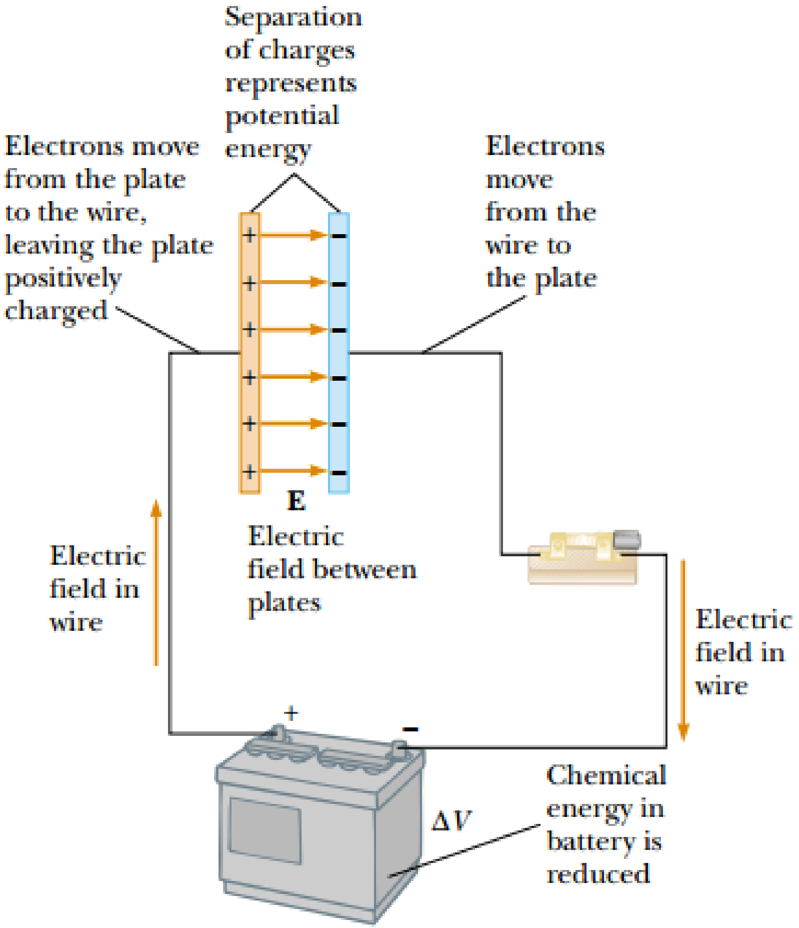


- Electric field is uniform in the central region between the plates, but is nonuniform at the edges of the plate

26.2 Calculating Capacitance

➢ Capacitors Storage of Enegry :

When the switch is closed, some of the *chemical energy* in the battery is converted to *electric potential energy* related to the separation of positive and negative charges on the plates



26.2 Calculating Capacitance

- ***Example 26.1***

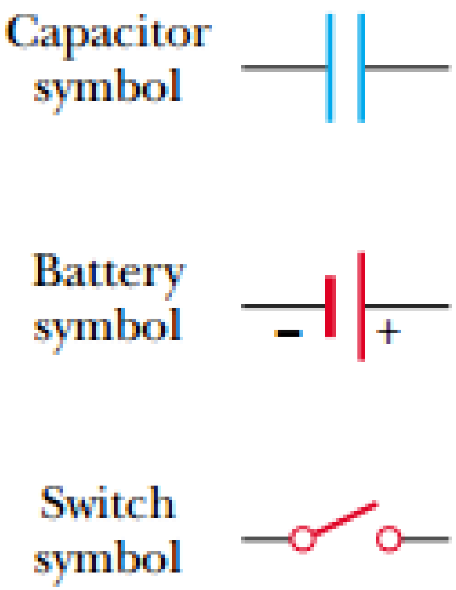
A parallel-plate capacitor with air between the plates has an area

$A=2.00x10^4$ m2 and a plate separation d=1.00mm.

1) Find its capacitance.
2) How much charge is on the positive plate if the capacitor is connected to a 3.00-V battery?
3) Calculate the charge density on the positive plate, assuming the density is uniform
4) Calculate the magnitude of the electric field between the plates.

26.3 Combinations of Capacitors

➢ Circuit symbols for capacitors



26.3 Combinations of Capacitors

- <u>Parallel Combination</u>

- The total charge Q stored by the two capacitors is

$$Q = Q_1 + Q_2$$

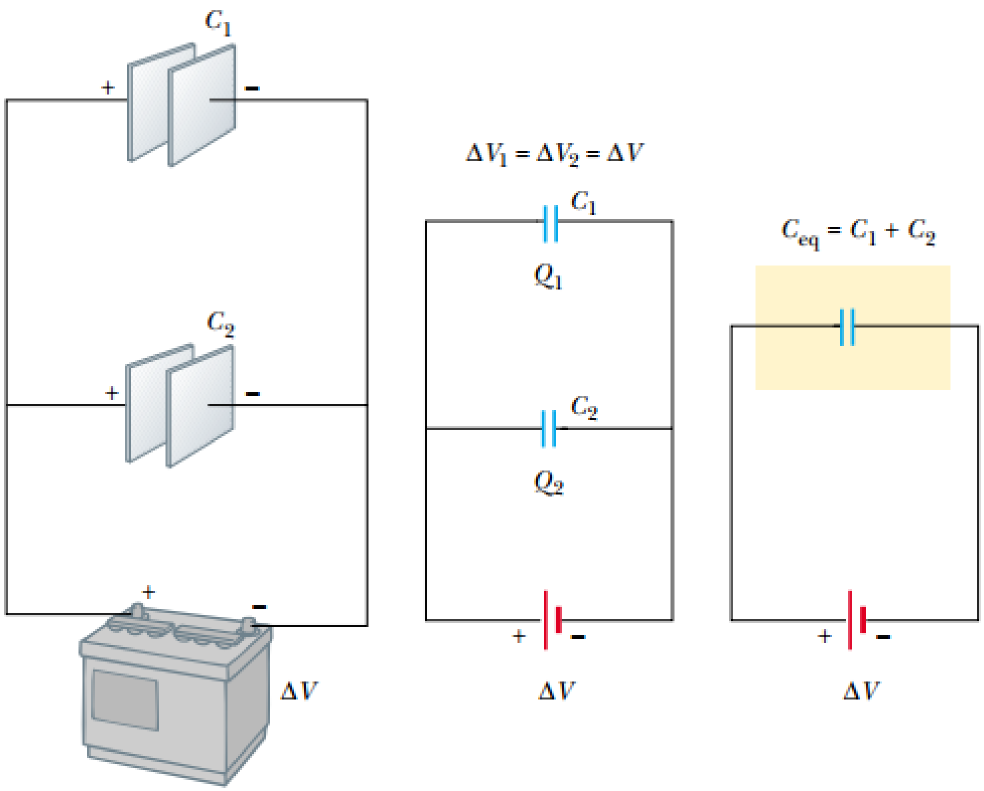
$$Q_1 = C_1 \, \Delta V$$

$$Q_2 = C_2 \, \Delta V$$

$$Q_{ea} = C_{ea} \, \Delta V$$

<u>Equivalent capacitance:</u>

$$C_{eq} = C_1 + C_2 + C_3 + C_4 + \ldots..$$



26.3 Combinations of Capacitors

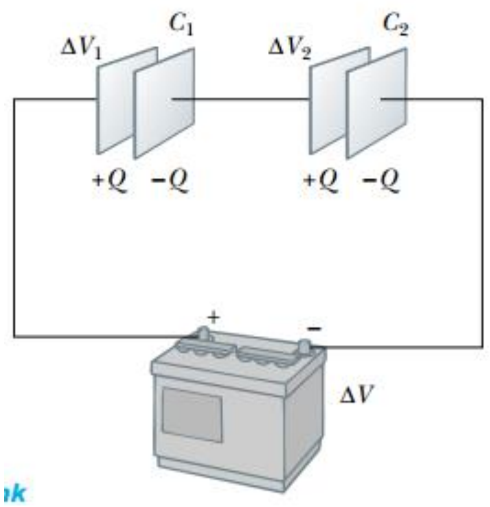
➢ Series Combination

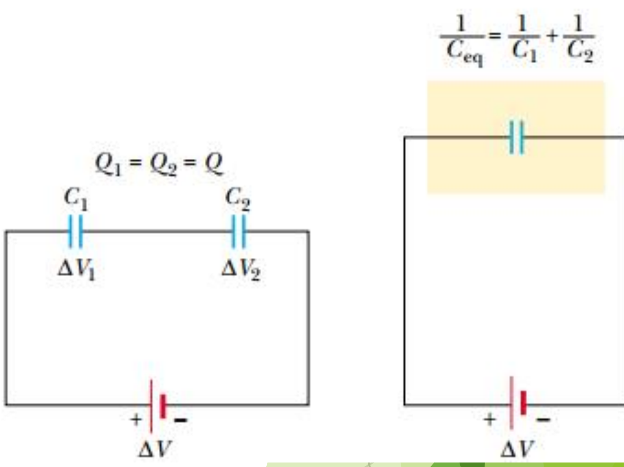
$$\Delta V = \Delta V_1 + \Delta V_2$$

$$\Delta V = \frac{Q}{C_{eq}}$$

$$\Delta V_1 = \frac{Q}{C_1}$$

$$\Delta V_2 = \frac{Q}{C_2}$$





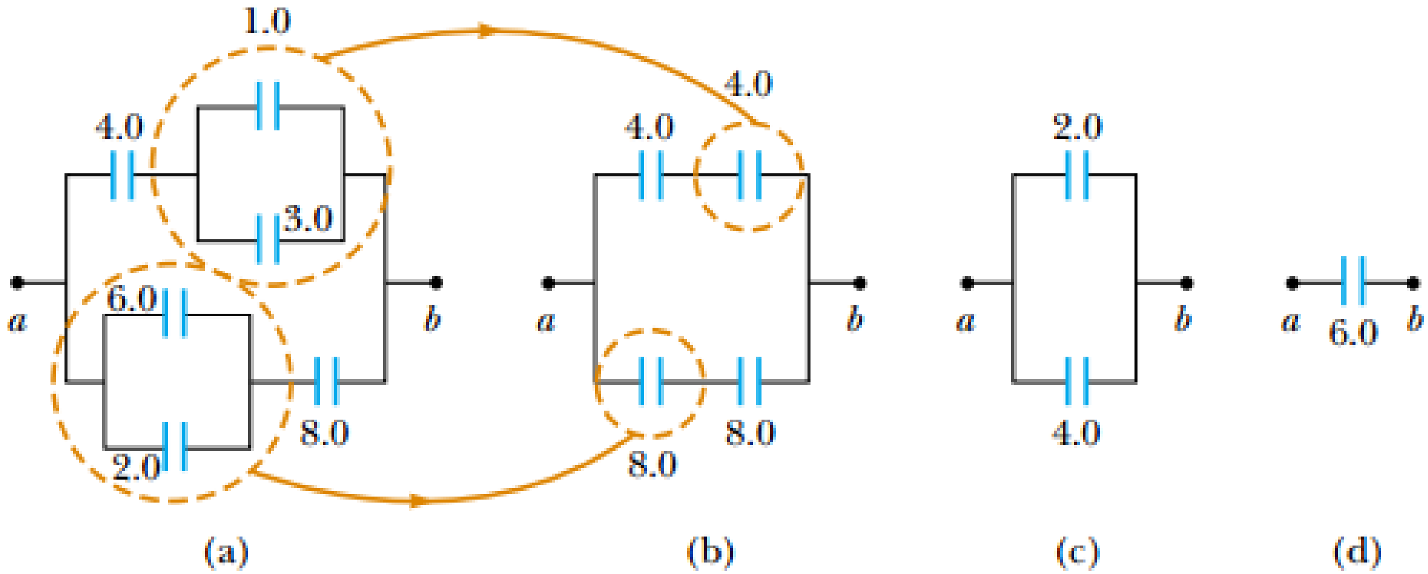
Equivalent capacitance:

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots\ldots$$

26.3 Combinations of Capacitors

- ***Example 26.4***

Find the equivalent capacitance between a and b for the combination of capacitors shown in the Figure below . All capacitances are in microfarads.



26.3 Combinations of Capacitors

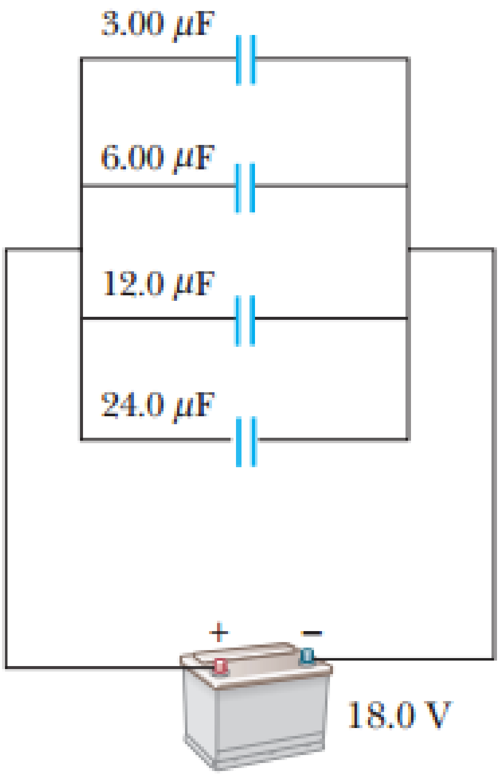
- ***Example***

(a) Determine the capacitance of the single capacitor that is equivalent

to the parallel combination of capacitors shown in the Figure below

(b) Find the charge on the 12 μF capacitor

(C) Find the total charge contained in the configuration



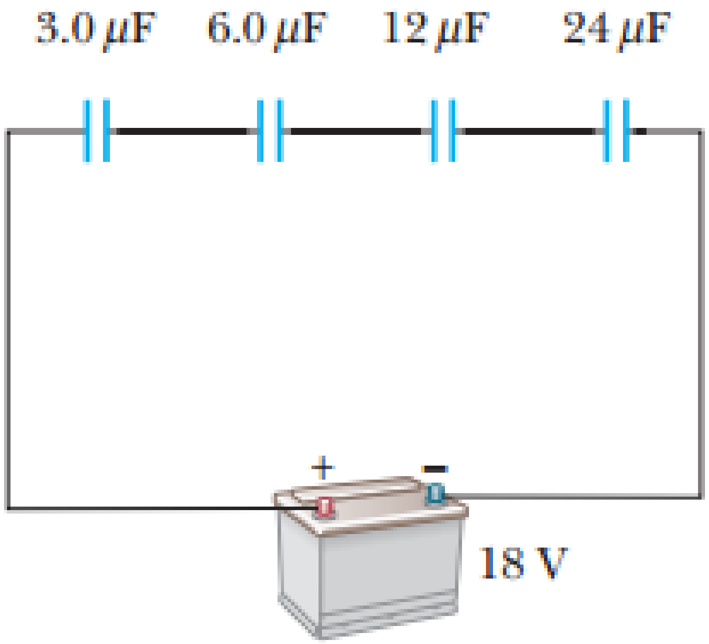
26.3 Combinations of Capacitors

➢ ***Example***

(a) Calculate the capacitance of the equivalent capacitor

(b) Compute the charge on the 12 μF capacitor

(C) Find the voltage drop across the 12 μF capacitor



26.4 Energy Stored in a Charged Capacitor

- <u>Calculating the energy stored in the capacitor</u>
  - If a capacitor is initially uncharged (both plates are neutral) so that the plates are at the same potential, very little work is required to transfer a small amount of charge from one plate to the other.
  - Once ΔV appears between the plates, work must be done to transfer additional charge against this potential difference.
  - The work ΔW required to move more charge ΔQ through this potential difference is given by

$$dW = \Delta V\, dq = \frac{q}{C}\, dq$$

26.4 Energy Stored in a Charged Capacitor

- **The total work required to charge the capacitor from q=0 to some final charge q=Q is**

$$W_B = \int_0^Q dW_B = \int_0^Q \frac{q}{C}\, dq = \frac{Q^2}{2C} = \Delta U$$

- **The potential energy stored in a charged capacitor**

$$U = \frac{Q^2}{2C} = \tfrac{1}{2} Q\,\Delta V = \tfrac{1}{2} C (\Delta V)^2$$

Notes:
This result applies to any capacitor, regardless of its geometry

The energy stored increases as the charge increases and as the potential difference (voltage) increases

26.4 Energy Stored in a Charged Capacitor

- **Energy density in an electric field**

$$u_E = \frac{1}{2}\epsilon_0 E^2$$

26.4 Energy Stored in a Charged Capacitor

- ***Quick Quiz 26.5***

You have three capacitors and a battery. In which of the following combinations of the three capacitors will the maximum possible energy be stored when the combination is attached to the battery?

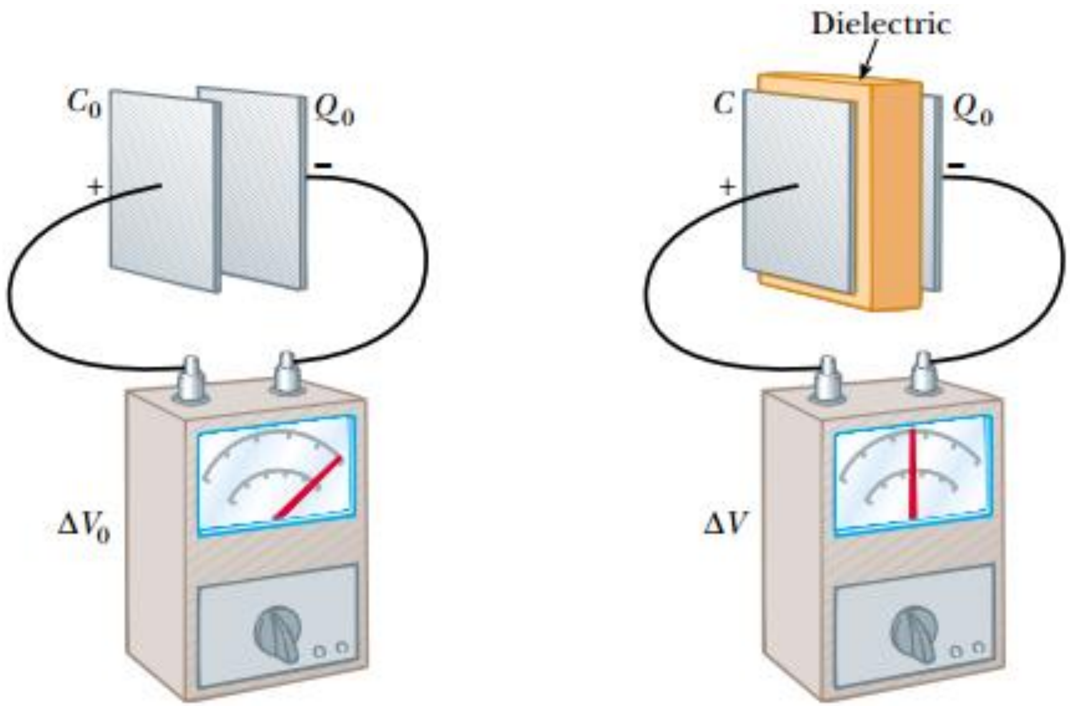
1/ series

2/ parallel

3/ Both combinations will store the same amount of energy.

26.5 Capacitors with Dielectrics

- A dielectric is a *nonconducting* material, such as rubber, glass, or waxed paper.
- When a dielectric is inserted between the plates of a capacitor, the capacitance ***increases***.



26.5 Capacitors with Dielectrics

➢ ***Capacitance of a capacitor filled with a material of dielectric constant k***

$$C = \frac{Q_0}{\Delta V} = \frac{Q_0}{\Delta V_0/\kappa} = \kappa\frac{Q_0}{\Delta V_0}$$

$$\boxed{C = \kappa C_0}$$

$$C = \kappa\frac{\epsilon_0 A}{d}$$

A dielectric provides the following advantages:

- **Increase in capacitance**
- **Increase in maximum operating voltage**
- **Possible mechanical support between the plates, which allows the plates to be close together without touching, thereby decreasing d and increasing C**

26.5 Capacitors with Dielectrics

Approximate Dielectric Constants and Dielectric Strengths of Various Materials at Room Temperature

Material	Dielectric Constant κ	Dielectric Strength[a] (10^6 V/m)
Air (dry)	1.000 59	3
Bakelite	4.9	24
Fused quartz	3.78	8
Mylar	3.2	7
Neoprene rubber	6.7	12
Nylon	3.4	14
Paper	3.7	16
Paraffin-impregnated paper	3.5	11
Polystyrene	2.56	24
Polyvinyl chloride	3.4	40
Porcelain	6	12
Pyrex glass	5.6	14
Silicone oil	2.5	15
Strontium titanate	233	8
Teflon	2.1	60
Vacuum	1.000 00	—
Water	80	—

26.5 Capacitors with Dielectrics

- ***Example 26.6***

A parallel-plate capacitor has plates of dimensions 2.0cm by 3.0cm separated by a 1.0-mm thickness of paper

(A)Find its capacitance.

(B) What is the maximum charge that can be placed on the capacitor

Chapter Problems

1) The plates of a capacitor are connected to a battery. What happens to the charge on the plates if the connecting wires are removed from the battery? What happens to the charge if the wires are removed from the battery and connected to each other?

2) If the potential difference across a capacitor is doubled, by what factor does the energy stored change?

Problem 1 page 822

3) How much charge is on each plate of a 4.00-%F capacitor when it is connected to a 12.0-V battery? (b) If this same capacitor is connected to a 1.50-V battery, what charge is stored?

Chapter Problems

Problem 7 page 823

An air-filled capacitor consists of two parallel plates, each with an area of 7.60cm, separated by a distance of 1.80mm. A 20.0-V potential difference is applied to these plates.

Calculate

(a) the electric field between the plates,

(b) the surface charge density,

(c) the capacitance,

(d) the charge on each plate.

Chapter Problems

A parallel-plate capacitor in air has a plate separation of 1.50cm and a plate area of 25.0cm2 The plates are charged to a potential difference of 250V and disconnected from the source. The capacitor is then immersed in distilled water.

Determine

(a) the charge on the plates before and after immersion,

(b) the capacitance after immersion,

Chapter Problems

Problem 27 page 824

Find the equivalent capacitance between points *a* and *b* for the group of capacitors connected as shown in Figure P26.27. Take $C_1 = 5.00\ \mu F$, $C_2 = 10.0\ \mu F$, and $C_3 = 2.00\ \mu F$.