Table 26.2

Capacitance and Geometry

Geometry	Capacitance	Equation
Isolated sphere of radius R (second spherical conductor assumed to have infinite radius)	$C = 4\pi\epsilon_0 R$	26.2
Parallel-plate capacitor of plate area A and plate separation d	$C = \epsilon_0 \dfrac{A}{d}$	26.3
Cylindrical capacitor of length ℓ and inner and outer radii a and b, respectively	$C = \dfrac{\ell}{2k_e \ln(b/a)}$	26.4
Spherical capacitor with inner and outer radii a and b, respectively	$C = \dfrac{ab}{k_e(b-a)}$	26.6

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots \quad (26.10)$$

These two equations enable you to simplify many electric circuits by replacing multiple capacitors with a single equivalent capacitance.

Energy is stored in a capacitor because the charging process is equivalent to the transfer of charges from one conductor at a lower electric potential to another conductor at a higher potential. The energy stored in a capacitor with charge Q is

$$U = \frac{Q^2}{2C} = \tfrac{1}{2}Q\Delta V = \tfrac{1}{2}C(\Delta V)^2 \quad (26.11)$$

When a dielectric material is inserted between the plates of a capacitor, the capacitance increases by a dimensionless factor κ, called the **dielectric constant:**

$$C = \kappa C_0 \quad (26.14)$$

where C_0 is the capacitance in the absence of the dielectric. The increase in capacitance is due to a decrease in the magnitude of the electric field in the presence of the dielectric. The decrease in the magnitude of $\mathbf{E}$ arises from an internal electric field produced by aligned dipoles in the dielectric.

The **electric dipole moment p** of an electric dipole has a magnitude

$$p \equiv 2aq \quad (26.16)$$

The direction of the electric dipole moment vector is from the negative charge toward the positive charge.

The torque acting on an electric dipole in a uniform electric field $\mathbf{E}$ is

$$\boldsymbol{\tau} = \mathbf{p} \times \mathbf{E} \quad (26.18)$$

The potential energy of the system of an electric dipole in a uniform external electric field $\mathbf{E}$ is

$$U = -\mathbf{p} \cdot \mathbf{E} \quad (26.20)$$

QUESTIONS

1. The plates of a capacitor are connected to a battery. What happens to the charge on the plates if the connecting wires are removed from the battery? What happens to the charge if the wires are removed from the battery and connected to each other?

2. A farad is a very large unit of capacitance. Calculate the length of one side of a square, air-filled capacitor that has a capacitance of 1 F and a plate separation of 1 m.

3. A pair of capacitors are connected in parallel while an identical pair are connected in series. Which pair would be more dangerous to handle after being connected to the same battery? Explain.

4. If you are given three different capacitors C_1, C_2, C_3, how many different combinations of capacitance can you produce?

5. What advantage might there be in using two identical capacitors in parallel connected in series with another identical parallel pair, rather than using a single capacitor?

6. Is it always possible to reduce a combination of capacitors to one equivalent capacitor with the rules we have developed? Explain.

7. The sum of the charges on both plates of a capacitor is zero. What does a capacitor store?

8. Because the charges on the plates of a parallel-plate capacitor are opposite in sign, they attract each other. Hence, it would take positive work to increase the plate separation. What type of energy in the system changes due to the external work done in this process?

9. Why is it dangerous to touch the terminals of a high-voltage capacitor even after the applied potential difference has been turned off? What can be done to make the capacitor safe to handle after the voltage source has been removed?

10. Explain why the work needed to move a charge Q through a potential difference ΔV is $W = Q\Delta V$ whereas the energy stored in a charged capacitor is $U = \frac{1}{2}Q\Delta V$. Where does the $\frac{1}{2}$ factor come from?

11. If the potential difference across a capacitor is doubled, by what factor does the energy stored change?

12. It is possible to obtain large potential differences by first charging a group of capacitors connected in parallel and then activating a switch arrangement that in effect disconnects the capacitors from the charging source and from each other and reconnects them in a series arrangement. The group of charged capacitors is then discharged in series. What is the maximum potential difference that can be obtained in this manner by using ten capacitors each of 500 μF and a charging source of 800 V?

13. Assume you want to increase the maximum operating voltage of a parallel-plate capacitor. Describe how you can do this for a fixed plate separation.

14. An air-filled capacitor is charged, then disconnected from the power supply, and finally connected to a voltmeter. Explain how and why the potential difference changes when a dielectric is inserted between the plates of the capacitor.

15. Using the polar molecule description of a dielectric, explain how a dielectric affects the electric field inside a capacitor.

16. Explain why a dielectric increases the maximum operating voltage of a capacitor although the physical size of the capacitor does not change.

17. What is the difference between dielectric strength and the dielectric constant?

18. Explain why a water molecule is permanently polarized. What type of molecule has no permanent polarization?

19. If a dielectric-filled capacitor is heated, how will its capacitance change? (Ignore thermal expansion and assume that the dipole orientations are temperature-dependent.)

20. If you were asked to design a capacitor where small size and large capacitance were required, what factors would be important in your design?

PROBLEMS

1, 2, 3 = straightforward, intermediate, challenging ☐ = full solution available in the *Student Solutions Manual and Study Guide*

= coached solution with hints available at http://www.pse6.com = computer useful in solving problem

= paired numerical and symbolic problems

Section 26.1 Definition of Capacitance

1. (a) How much charge is on each plate of a 4.00-μF capacitor when it is connected to a 12.0-V battery? (b) If this same capacitor is connected to a 1.50-V battery, what charge is stored?

2. Two conductors having net charges of $+10.0\ \mu$C and $-10.0\ \mu$C have a potential difference of 10.0 V between them. (a) Determine the capacitance of the system. (b) What is the potential difference between the two conductors if the charges on each are increased to $+100\ \mu$C and $-100\ \mu$C?

Section 26.2 Calculating Capacitance

3. An isolated charged conducting sphere of radius 12.0 cm creates an electric field of 4.90×10^4 N/C at a distance 21.0 cm from its center. (a) What is its surface charge density? (b) What is its capacitance?

4. (a) If a drop of liquid has capacitance 1.00 pF, what is its radius? (b) If another drop has radius 2.00 mm, what is its capacitance? (c) What is the charge on the smaller drop if its potential is 100 V?

5. Two conducting spheres with diameters of 0.400 m and 1.00 m are separated by a distance that is large compared with the diameters. The spheres are connected by a thin wire and are charged to 7.00 μC. (a) How is this total charge shared between the spheres? (Ignore any charge on the wire.) (b) What is the potential of the system of spheres when the reference potential is taken to be $V = 0$ at $r = \infty$?

6. Regarding the Earth and a cloud layer 800 m above the Earth as the "plates" of a capacitor, calculate the capacitance. Assume the cloud layer has an area of 1.00 km^2 and that the air between the cloud and the ground is pure and dry. Assume charge builds up on the cloud and on the ground until a uniform electric field of 3.00×10^6 N/C throughout the space between them makes the air break down and conduct electricity as a lightning bolt. What is the maximum charge the cloud can hold?

7. An air-filled capacitor consists of two parallel plates, each with an area of 7.60 cm^2, separated by a distance of 1.80 mm. A 20.0-V potential difference is applied to these plates. Calculate (a) the electric field between the plates, (b) the surface charge density, (c) the capacitance, and (d) the charge on each plate.

8. A 1-megabit computer memory chip contains many 60.0-fF capacitors. Each capacitor has a plate area of 21.0×10^{-12} m^2. Determine the plate separation of such a capacitor (assume a parallel-plate configuration). The order of magnitude of the diameter of an atom is 10^{-10} m = 0.1 nm. Express the plate separation in nanometers.

9. When a potential difference of 150 V is applied to the plates of a parallel-plate capacitor, the plates carry a surface charge density of 30.0 nC/cm^2. What is the spacing between the plates?

10. A variable air capacitor used in a radio tuning circuit is made of N semicircular plates each of radius R and positioned a distance d from its neighbors, to which it is electrically connected. As shown in Figure P26.10, a second identical set of plates is enmeshed with its plates halfway between those of the first set. The second set can rotate as a unit. Determine the capacitance as a function of the angle of rotation θ, where $\theta = 0$ corresponds to the maximum capacitance.

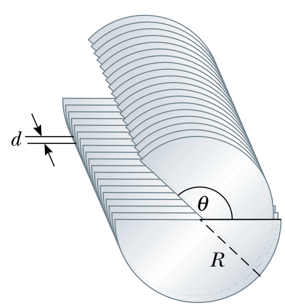

Figure P26.10

11. A 50.0-m length of coaxial cable has an inner conductor that has a diameter of 2.58 mm and carries a charge of 8.10 μC. The surrounding conductor has an inner diameter of 7.27 mm and a charge of -8.10 μC. (a) What is the capacitance of this cable? (b) What is the potential difference between the two conductors? Assume the region between the conductors is air.

12. A 20.0-μF spherical capacitor is composed of two concentric metal spheres, one having a radius twice as large as the other. The region between the spheres is a vacuum. Determine the volume of this region.

13. An air-filled spherical capacitor is constructed with inner and outer shell radii of 7.00 and 14.0 cm, respectively. (a) Calculate the capacitance of the device. (b) What potential difference between the spheres results in a charge of 4.00 μC on the capacitor?

14. A small object of mass m carries a charge q and is suspended by a thread between the vertical plates of a parallel-plate capacitor. The plate separation is d. If the thread makes an angle θ with the vertical, what is the potential difference between the plates?

15. Find the capacitance of the Earth. (*Suggestion:* The outer conductor of the "spherical capacitor" may be considered as a conducting sphere at infinity where V approaches zero.)

Section 26.3 Combinations of Capacitors

16. Two capacitors, $C_1 = 5.00$ μF and $C_2 = 12.0$ μF, are connected in parallel, and the resulting combination is connected to a 9.00-V battery. (a) What is the equivalent capacitance of the combination? What are (b) the potential difference across each capacitor and (c) the charge stored on each capacitor?

17. **What If?** The two capacitors of Problem 16 are now connected in series and to a 9.00-V battery. Find (a) the equivalent capacitance of the combination, (b) the potential difference across each capacitor, and (c) the charge on each capacitor.

18. Evaluate the equivalent capacitance of the configuration shown in Figure P26.18. All the capacitors are identical, and each has capacitance C.

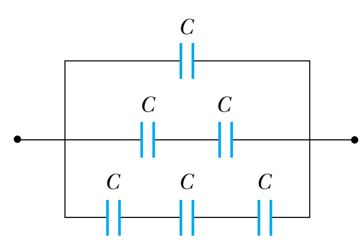

Figure P26.18

19. Two capacitors when connected in parallel give an equivalent capacitance of 9.00 pF and give an equivalent capacitance of 2.00 pF when connected in series. What is the capacitance of each capacitor?

20. Two capacitors when connected in parallel give an equivalent capacitance of C_p and an equivalent capacitance of C_s when connected in series. What is the capacitance of each capacitor?

21. Four capacitors are connected as shown in Figure P26.21. (a) Find the equivalent capacitance between points a and b. (b) Calculate the charge on each capacitor if $\Delta V_{ab} = 15.0$ V.

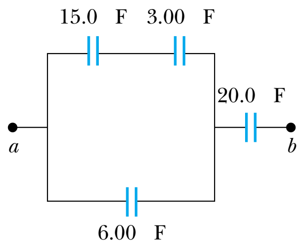

Figure P26.21

22. Three capacitors are connected to a battery as shown in Figure P26.22. Their capacitances are $C_1 = 3C$, $C_2 = C$, and $C_3 = 5C$. (a) What is the equivalent capacitance of this set of capacitors? (b) State the ranking of the capacitors according to the charge they store, from largest to smallest. (c) Rank the capacitors according to the potential differences across them, from largest to smallest. (d) **What If?** If C_3 is increased, what happens to the charge stored by each of the capacitors?

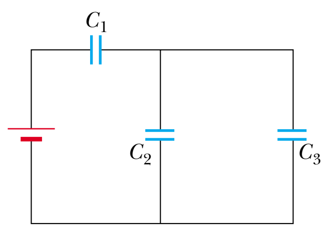

Figure P26.22

23. Consider the circuit shown in Figure P26.23, where $C_1 = 6.00\ \mu$F, $C_2 = 3.00\ \mu$F, and $\Delta V = 20.0$ V. Capacitor C_1 is first charged by the closing of switch S_1. Switch S_1 is then opened, and the charged capacitor is connected to the uncharged capacitor by the closing of S_2. Calculate the initial charge acquired by C_1 and the final charge on each capacitor.

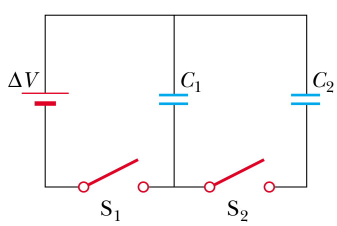

Figure P26.23

24. According to its design specification, the timer circuit delaying the closing of an elevator door is to have a capacitance of 32.0 μF between two points A and B. (a) When one circuit is being constructed, the inexpensive but durable capacitor installed between these two points is found to have capacitance 34.8 μF. To meet the specification, one additional capacitor can be placed between the two points. Should it be in series or in parallel with the 34.8-μF capacitor? What should be its capacitance? (b) **What If?** The next circuit comes down the assembly line with capacitance 29.8 μF between A and B. What additional capacitor should be installed in series or in parallel in that circuit, to meet the specification?

25. A group of identical capacitors is connected first in series and then in parallel. The combined capacitance in parallel is 100 times larger than for the series connection. How many capacitors are in the group?

26. Consider three capacitors C_1, C_2, C_3, and a battery. If C_1 is connected to the battery, the charge on C_1 is 30.8 μC. Now C_1 is disconnected, discharged, and connected in series with C_2. When the series combination of C_2 and C_1 is connected across the battery, the charge on C_1 is 23.1 μC. The circuit is disconnected and the capacitors discharged. Capacitor C_3, capacitor C_1, and the battery are connected in series, resulting in a charge on C_1 of 25.2 μC. If, after being disconnected and discharged, C_1, C_2, and C_3 are connected in series with one another and with the battery, what is the charge on C_1?

27. Find the equivalent capacitance between points a and b for the group of capacitors connected as shown in Figure P26.27. Take $C_1 = 5.00\ \mu$F, $C_2 = 10.0\ \mu$F, and $C_3 = 2.00\ \mu$F.

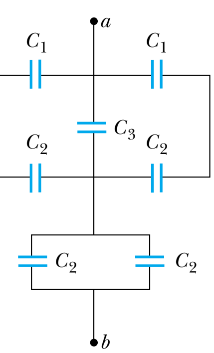

Figure P26.27 Problems 27 and 28.

28. For the network described in the previous problem, if the potential difference between points a and b is 60.0 V, what charge is stored on C_3?

29. Find the equivalent capacitance between points a and b in the combination of capacitors shown in Figure P26.29.

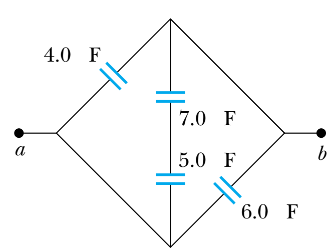

Figure P26.29

30. Some physical systems possessing capacitance continuously distributed over space can be modeled as an infinite array of discrete circuit elements. Examples are a microwave waveguide and the axon of a nerve cell. To practice analysis of an infinite array, determine the equivalent capacitance C between terminals X and Y of the infinite set of capacitors represented in Figure P26.30. Each capacitor has capacitance C_0. *Suggestion:* Imagine that the ladder is cut at the line AB, and note that the equivalent capacitance of the infinite section to the right of AB is also C.

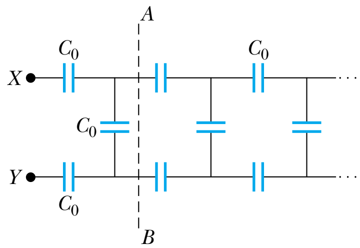

Figure P26.30

Section 26.4 Energy Stored in a Charged Capacitor

31. (a) A 3.00-μF capacitor is connected to a 12.0-V battery. How much energy is stored in the capacitor? (b) If the capacitor had been connected to a 6.00-V battery, how much energy would have been stored?

32. The immediate cause of many deaths is ventricular fibrillation, uncoordinated quivering of the heart as opposed to proper beating. An electric shock to the chest can cause momentary paralysis of the heart muscle, after which the heart will sometimes start organized beating again. A *defibrillator* (Fig. 26.14) is a device that applies a strong electric shock to the chest over a time interval of a few milliseconds. The device contains a capacitor of several microfarads, charged to several thousand volts. Electrodes called paddles, about 8 cm across and coated with conducting paste, are held against the chest on both sides of the heart. Their handles are insulated to prevent injury to the operator, who calls, "Clear!" and pushes a button on one paddle to discharge the capacitor through the patient's chest. Assume that an energy of 300 J is to be delivered from a 30.0-μF capacitor. To what potential difference must it be charged?

33. Two capacitors, $C_1 = 25.0\ \mu$F and $C_2 = 5.00\ \mu$F, are connected in parallel and charged with a 100-V power supply. (a) Draw a circuit diagram and calculate the total energy stored in the two capacitors. (b) **What If?** What potential difference would be required across the same two capacitors connected in series in order that the combination stores the same amount of energy as in (a)? Draw a circuit diagram of this circuit.

34. A parallel-plate capacitor is charged and then disconnected from a battery. By what fraction does the stored energy change (increase or decrease) when the plate separation is doubled?

35. As a person moves about in a dry environment, electric charge accumulates on his body. Once it is at high voltage, either positive or negative, the body can discharge via sometimes noticeable sparks and shocks. Consider a human body well separated from ground, with the typical capacitance 150 pF. (a) What charge on the body will produce a potential of 10.0 kV? (b) Sensitive electronic devices can be destroyed by electrostatic discharge from a person. A particular device can be destroyed by a discharge releasing an energy of 250 μJ. To what voltage on the body does this correspond?

36. A uniform electric field $E = 3\,000$ V/m exists within a certain region. What volume of space contains an energy equal to 1.00×10^{-7} J? Express your answer in cubic meters and in liters.

37. A parallel-plate capacitor has a charge Q and plates of area A. What force acts on one plate to attract it toward the other plate? Because the electric field between the plates is $E = Q/A\epsilon_0$, you might think that the force is $F = QE = Q^2/A\epsilon_0$. This is wrong, because the field E includes contributions from both plates, and the field created by the positive plate cannot exert any force on the positive plate. Show that the force exerted on each plate is actually $F = Q^2/2\epsilon_0 A$. (*Suggestion:* Let $C = \epsilon_0 A/x$ for an arbitrary plate separation x; then require that the work done in separating the two charged plates be $W = \int F\,dx$.) The force exerted by one charged plate on another is sometimes used in a machine shop to hold a workpiece stationary.

38. The circuit in Figure P26.38 consists of two identical parallel metal plates connected by identical metal springs to a 100-V battery. With the switch open, the plates are uncharged, are separated by a distance $d = 8.00$ mm, and have a capacitance $C = 2.00\ \mu$F. When the switch is closed, the distance between the plates decreases by a factor of 0.500. (a) How much charge collects on each plate and (b) what is the spring constant for each spring? (*Suggestion:* Use the result of Problem 37.)

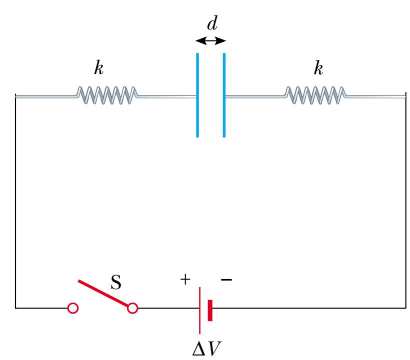

Figure P26.38

39. **Review problem.** A certain storm cloud has a potential of 1.00×10^8 V relative to a tree. If, during a lightning storm, 50.0 C of charge is transferred through this potential difference and 1.00% of the energy is absorbed by the tree, how much sap in the tree can be boiled away? Model the sap as water initially at 30.0°C. Water has a specific heat of 4 186 J/kg°C, a boiling point of 100°C, and a latent heat of vaporization of 2.26×10^6 J/kg.

40. Two identical parallel-plate capacitors, each with capacitance C, are charged to potential difference ΔV and connected in parallel. Then the plate separation in one of the capacitors is doubled. (a) Find the total energy of the system of two capacitors *before* the plate separation is doubled. (b) Find the potential difference across each capacitor *after* the plate separation is doubled. (c) Find the total energy of the system *after* the plate separation is doubled. (d) Reconcile the difference in the answers to parts (a) and (c) with the law of conservation of energy.

41. Show that the energy associated with a conducting sphere of radius R and charge Q surrounded by a vacuum is $U = k_e Q^2/2R$.

42. Consider two conducting spheres with radii R_1 and R_2. They are separated by a distance much greater than either radius. A total charge Q is shared between the spheres, subject to the condition that the electric potential energy of the system has the smallest possible value. The total charge Q is equal to $q_1 + q_2$, where q_1 represents the charge on the first sphere and q_2 the charge on the second. Because the spheres are very far apart, you can assume that the charge of each is uniformly distributed over its surface. You may use the result of Problem 41. (a) Determine the values of q_1 and q_2 in terms of Q, R_1, and R_2. (b) Show that the potential difference between the spheres is zero. (We saw in Chapter 25 that two conductors joined by a conducting wire will be at the same potential in a static situation. This problem illustrates the general principle that static charge on a conductor will distribute itself so that the electric potential energy of the system is a minimum.)

Section 26.5 Capacitors with Dielectrics

43. Determine (a) the capacitance and (b) the maximum potential difference that can be applied to a Teflon-filled parallel-plate capacitor having a plate area of 1.75 cm^2 and plate separation of 0.040 0 mm.

44. (a) How much charge can be placed on a capacitor with air between the plates before it breaks down, if the area of each of the plates is 5.00 cm^2? (b) **What If?** Find the maximum charge if polystyrene is used between the plates instead of air.

45. A commercial capacitor is to be constructed as shown in Figure 26.17a. This particular capacitor is made from two strips of aluminum separated by a strip of paraffin-coated paper. Each strip of foil and paper is 7.00 cm wide. The foil is 0.004 00 mm thick, and the paper is 0.025 0 mm thick and has a dielectric constant of 3.70. What length should the strips have, if a capacitance of 9.50×10^{-8} F is desired before the capacitor is rolled up? (Adding a second strip of paper and rolling the capacitor effectively doubles its capacitance, by allowing charge storage on both sides of each strip of foil.)

46. The supermarket sells rolls of aluminum foil, of plastic wrap, and of waxed paper. Describe a capacitor made from supermarket materials. Compute order-of-magnitude estimates for its capacitance and its breakdown voltage.

47. A parallel-plate capacitor in air has a plate separation of 1.50 cm and a plate area of 25.0 cm^2. The plates are charged to a potential difference of 250 V and disconnected from the source. The capacitor is then immersed in distilled water. Determine (a) the charge on the plates before and after immersion, (b) the capacitance and potential difference after immersion, and (c) the change in energy of the capacitor. Assume the liquid is an insulator.

48. A wafer of titanium dioxide ($\kappa = 173$) of area 1.00 cm^2 has a thickness of 0.100 mm. Aluminum is evaporated on the parallel faces to form a parallel-plate capacitor. (a) Calculate the capacitance. (b) When the capacitor is charged with a 12.0-V battery, what is the magnitude of charge delivered to each plate? (c) For the situation in part (b), what are the free and induced surface charge densities? (d) What is the magnitude of the electric field?

49. Each capacitor in the combination shown in Figure P26.49 has a breakdown voltage of 15.0 V. What is the breakdown voltage of the combination?

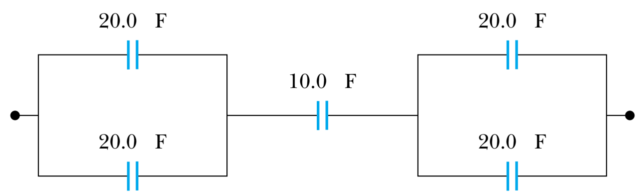

Figure P26.49

Section 26.6 Electric Dipole in an Electric Field

50. A small rigid object carries positive and negative 3.50-nC charges. It is oriented so that the positive charge has coordinates (−1.20 mm, 1.10 mm) and the negative charge is at the point (1.40 mm, −1.30 mm). (a) Find the electric dipole moment of the object. The object is placed in an electric field $\mathbf{E} = (7\,800\hat{\mathbf{i}} - 4\,900\hat{\mathbf{j}})$ N/C. (b) Find the torque acting on the object. (c) Find the potential energy of the object–field system when the object is in this orientation. (d) If the orientation of the object can change, find the difference between the maximum and minimum potential energies of the system.

51. A small object with electric dipole moment $\mathbf{p}$ is placed in a nonuniform electric field $\mathbf{E} = E(x)\hat{\mathbf{i}}$. That is, the field is in the x direction and its magnitude depends on the coordinate x. Let θ represent the angle between the dipole moment and the x direction. (a) Prove that the dipole feels a net force

$$F = p\left(\frac{dE}{dx}\right)\cos\theta$$

in the direction toward which the field increases. (b) Consider a spherical balloon centered at the origin, with radius 15.0 cm and carrying charge 2.00 μC. Evaluate dE/dx at the point (16 cm, 0, 0). Assume a water droplet at this point has an induced dipole moment of $6.30\hat{\mathbf{i}}$ nC·m. Find the force on it.

Section 26.7 An Atomic Description of Dielectrics

52. A detector of radiation called a Geiger tube consists of a closed, hollow, conducting cylinder with a fine wire along its axis. Suppose that the internal diameter of the cylinder is 2.50 cm and that the wire along the axis has a diameter of 0.200 mm. The dielectric strength of the gas between the central wire and the cylinder is 1.20×10^6 V/m. Calculate the maximum potential difference that can be applied between the wire and the cylinder before breakdown occurs in the gas.

53. The general form of Gauss's law describes how a charge creates an electric field in a material, as well as in vacuum. It is

$$\oint \mathbf{E} \cdot d\mathbf{A} = \frac{q}{\epsilon}$$

where $\epsilon = \kappa\epsilon_0$ is the permittivity of the material. (a) A sheet with charge Q uniformly distributed over its area A is surrounded by a dielectric. Show that the sheet creates a uniform electric field at nearby points, with magnitude $E = Q/2A\epsilon$. (b) Two large sheets of area A, carrying opposite charges of equal magnitude Q, are a small distance d apart. Show that they create uniform electric field in the space between them, with magnitude $E = Q/A\epsilon$. (c) Assume that the negative plate is at zero potential. Show that the positive plate is at potential $Qd/A\epsilon$. (d) Show that the capacitance of the pair of plates is $A\epsilon/d = \kappa A\epsilon_0/d$.

Additional Problems

54. For the system of capacitors shown in Figure P26.54, find (a) the equivalent capacitance of the system, (b) the potential across each capacitor, (c) the charge on each capacitor, and (d) the total energy stored by the group.

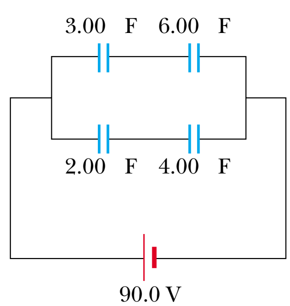

Figure P26.54

55. Four parallel metal plates P_1, P_2, P_3, and P_4, each of area 7.50 cm^2, are separated successively by a distance $d = 1.19$ mm, as shown in Figure P26.55. P_1 is connected to the negative terminal of a battery, and P_2 to the positive terminal. The battery maintains a potential difference of 12.0 V. (a) If P_3 is connected to the negative terminal, what is the capacitance of the three-plate system $P_1P_2P_3$? (b) What is the charge on P_2? (c) If P_4 is now connected to the positive terminal of the battery, what is the capacitance of the four-plate system $P_1P_2P_3P_4$? (d) What is the charge on P_4?

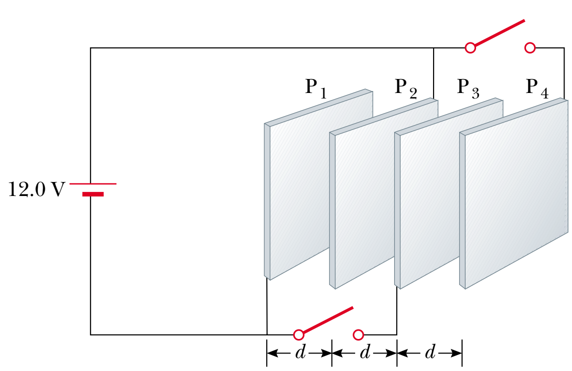

Figure P26.55

56. One conductor of an overhead electric transmission line is a long aluminum wire 2.40 cm in radius. Suppose that at a particular moment it carries charge per length 1.40 μC/m and is at potential 345 kV. Find the potential 12.0 m below the wire. Ignore the other conductors of the transmission line and assume the electric field is everywhere purely radial.

57. Two large parallel metal plates are oriented horizontally and separated by a distance $3d$. A grounded conducting wire joins them, and initially each plate carries no charge. Now a third identical plate carrying charge Q is inserted between the two plates, parallel to them and located a distance d from the upper plate, as in Figure P26.57. (a) What induced charge appears on each of the two original plates? (b) What potential difference appears between the middle plate and each of the other plates? Each plate has area A.

Figure P26.57

58. A 2.00-nF parallel-plate capacitor is charged to an initial potential difference $\Delta V_i = 100$ V and then isolated. The dielectric material between the plates is mica, with a dielectric constant of 5.00. (a) How much work is required to withdraw the mica sheet? (b) What is the potential difference of the capacitor after the mica is withdrawn?

59. A parallel-plate capacitor is constructed using a dielectric material whose dielectric constant is 3.00 and whose dielectric strength is 2.00×10^8 V/m. The desired capacitance is 0.250 μF, and the capacitor must withstand a maximum potential difference of 4 000 V. Find the minimum area of the capacitor plates.

60. A 10.0-μF capacitor has plates with vacuum between them. Each plate carries a charge of magnitude 1 000 μC. A particle with charge -3.00 μC and mass 2.00×10^{-16} kg is fired from the positive plate toward the negative plate with an initial speed of 2.00×10^6 m/s. Does it reach the negative plate? If so, find its impact speed. If not, what fraction of the way across the capacitor does it travel?

61. A parallel-plate capacitor is constructed by filling the space between two square plates with blocks of three dielectric materials, as in Figure P26.61. You may assume that $\ell >> d$. (a) Find an expression for the capacitance of the device in terms of the plate area A and d, κ_1, κ_2, and κ_3. (b) Calculate the capacitance using the values $A = 1.00$ cm^2, $d = 2.00$ mm, $\kappa_1 = 4.90$, $\kappa_2 = 5.60$, and $\kappa_3 = 2.10$.

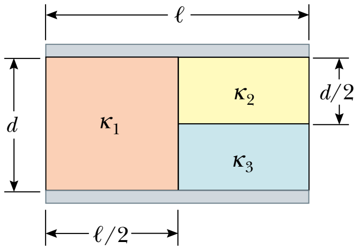

Figure P26.61

62. A 10.0-μF capacitor is charged to 15.0 V. It is next connected in series with an uncharged 5.00-μF capacitor. The series combination is finally connected across a 50.0-V battery, as diagrammed in Figure P26.62. Find the new potential differences across the 5-μF and 10-μF capacitors.

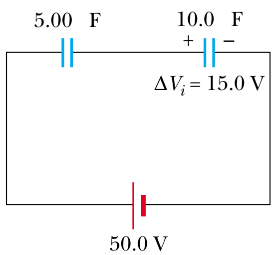

Figure P26.62

63. (a) Two spheres have radii a and b and their centers are a distance d apart. Show that the capacitance of this system is

$$C = \frac{4\pi\epsilon_0}{\dfrac{1}{a} + \dfrac{1}{b} - \dfrac{2}{d}}$$

provided that d is large compared with a and b. (*Suggestion:* Because the spheres are far apart, assume that the potential of each equals the sum of the potentials due to each sphere, and when calculating those potentials assume that $V = k_eQ/r$ applies.) (b) Show that as d approaches infinity the above result reduces to that of two spherical capacitors in series.

64. A capacitor is constructed from two square plates of sides ℓ and separation d. A material of dielectric constant κ is inserted a distance x into the capacitor, as shown in Figure P26.64. Assume that d is much smaller than x. (a) Find the equivalent capacitance of the device. (b) Calculate the energy stored in the capacitor, letting ΔV represent the potential difference. (c) Find the direction and magnitude of the force exerted on the dielectric, assuming a constant potential difference ΔV. Ignore friction. (d) Obtain a numerical value for the force assuming that $\ell = 5.00$ cm, $\Delta V = 2\,000$ V, $d = 2.00$ mm, and the dielectric is glass ($\kappa = 4.50$). (*Suggestion:* The system can be considered as two capacitors connected in parallel.)

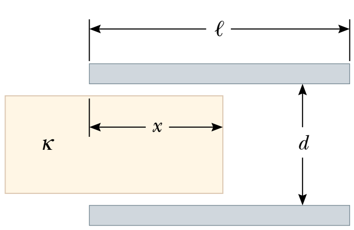

Figure P26.64 Problems 64 and 65.

65. A capacitor is constructed from two square plates of sides ℓ and separation d, as suggested in Figure P26.64. You may assume that d is much less than ℓ. The plates carry charges $+Q_0$ and $-Q_0$. A block of metal has a width ℓ, a length ℓ, and a thickness slightly less than d. It is inserted a distance x into the capacitor. The charges on the plates are not disturbed as the block slides in. In a static situation, a metal prevents an electric field from penetrating inside it. The metal can be thought of as a perfect dielectric, with $\kappa \to \infty$. (a) Calculate the stored energy as a function of x. (b) Find the direction and magnitude of the force that acts on the metallic block. (c) The area of the advancing front face of the block is essentially equal to ℓd. Considering the force on the block as acting on this face, find the stress (force per area) on it. (d) For comparison, express the energy density in the electric field between the capacitor plates in terms of Q_0, ℓ, d, and ϵ_0.

66. When considering the energy supply for an automobile, the energy per unit mass of the energy source is an important parameter. Using the following data, compare the energy per unit mass (J/kg) for gasoline, lead–acid batteries, and capacitors. (The ampere A will be introduced in the next chapter as the SI unit of electric current. 1 A = 1 C/s.)
Gasoline: 126 000 Btu/gal; density = 670 kg/m^3.
Lead–acid battery: 12.0 V; 100 A · h; mass = 16.0 kg.
Capacitor: potential difference at full charge = 12.0 V; capacitance = 0.100 F; mass = 0.100 kg.

67. An isolated capacitor of unknown capacitance has been charged to a potential difference of 100 V. When the charged capacitor is then connected in parallel to an uncharged 10.0-μF capacitor, the potential difference across the combination is 30.0 V. Calculate the unknown capacitance.

68. To repair a power supply for a stereo amplifier, an electronics technician needs a 100-μF capacitor capable of withstanding a potential difference of 90 V between the plates. The only available supply is a box of five 100-μF capacitors, each having a maximum voltage capability of 50 V. Can the technician substitute a combination of these capacitors that has the proper electrical characteristics? If so, what will be the maximum voltage across any of the capacitors used? (*Suggestion:* The technician may not have to use all the capacitors in the box.)

69. A parallel-plate capacitor of plate separation d is charged to a potential difference ΔV_0. A dielectric slab of thickness d and dielectric constant κ is introduced between the plates while the battery remains connected to the plates. (a) Show that the ratio of energy stored after the dielectric is introduced to the energy stored in the empty capacitor is $U/U_0 = \kappa$. Give a physical explanation for this increase in stored energy. (b) What happens to the charge on the capacitor? (Note that this situation is not the same as in

Example 26.7, in which the battery was removed from the circuit before the dielectric was introduced.)

70. A vertical parallel-plate capacitor is half filled with a dielectric for which the dielectric constant is 2.00 (Fig. P26.70a). When this capacitor is positioned horizontally, what fraction of it should be filled with the same dielectric (Fig. P26.70b) in order for the two capacitors to have equal capacitance?

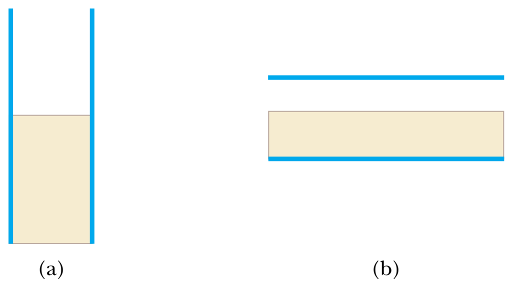

Figure P26.70

71. Capacitors $C_1 = 6.00\ \mu F$ and $C_2 = 2.00\ \mu F$ are charged as a parallel combination across a 250-V battery. The capacitors are disconnected from the battery and from each other. They are then connected positive plate to negative plate and negative plate to positive plate. Calculate the resulting charge on each capacitor.

72. Calculate the equivalent capacitance between the points a and b in Figure P26.72. Note that this is not a simple series or parallel combination. (*Suggestion:* Assume a potential difference ΔV between points a and b. Write expressions for ΔV_{ab} in terms of the charges and capacitances for the various possible pathways from a to b, and require conservation of charge for those capacitor plates that are connected to each other.)

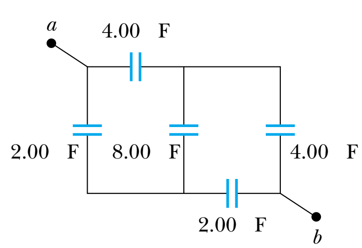

Figure P26.72

73. The inner conductor of a coaxial cable has a radius of 0.800 mm, and the outer conductor's inside radius is 3.00 mm. The space between the conductors is filled with polyethylene, which has a dielectric constant of 2.30 and a dielectric strength of 18.0×10^6 V/m. What is the maximum potential difference that this cable can withstand?

74. You are optimizing coaxial cable design for a major manufacturer. Show that for a given outer conductor radius b, maximum potential difference capability is attained when the radius of the inner conductor is $a = b/e$ where e is the base of natural logarithms.

75. Determine the equivalent capacitance of the combination shown in Figure P26.75. (*Suggestion:* Consider the symmetry involved.)

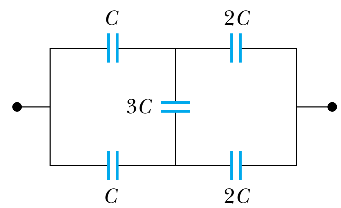

Figure P26.75

76. Consider two long, parallel, and oppositely charged wires of radius d with their centers separated by a distance D. Assuming the charge is distributed uniformly on the surface of each wire, show that the capacitance per unit length of this pair of wires is

$$\frac{C}{\ell} = \frac{\pi \epsilon_0}{\ln[(D - d)/d]}$$

77. Example 26.2 explored a cylindrical capacitor of length ℓ and radii a and b of the two conductors. In the **What If?** section, it was claimed that increasing ℓ by 10% is more effective in terms of increasing the capacitance than increasing a by 10% if $b > 2.85a$. Verify this claim mathematically.

Answers to Quick Quizzes

26.1 (d). The capacitance is a property of the physical system and does not vary with applied voltage. According to Equation 26.1, if the voltage is doubled, the charge is doubled.

26.2 (a). When the key is pressed, the plate separation is decreased and the capacitance increases. Capacitance depends only on how a capacitor is constructed and not on the external circuit.

26.3 (a). When connecting capacitors in series, the inverses of the capacitances add, resulting in a smaller overall equivalent capacitance.

26.4 (a). When capacitors are connected in series, the voltages add, for a total of 20 V in this case. If they are combined in parallel, the voltage across the combination is still 10 V.

26.5 (b). For a given voltage, the energy stored in a capacitor is proportional to C: $U = C(\Delta V)^2/2$. Thus, you want to maximize the equivalent capacitance. You do this by connecting the three capacitors in parallel, so that the capacitances add.

26.6 (a) C decreases (Eq. 26.3). (b) Q stays the same because there is no place for the charge to flow. (c) E remains constant (see Eq. 24.8 and the paragraph following it). (d) ΔV increases because $\Delta V = Q/C$, Q is constant (part b), and C decreases (part a). (e) The energy stored in the capacitor is proportional to both Q and ΔV (Eq.

26.11) and thus increases. The additional energy comes from the work you do in pulling the two plates apart.

26.7 (a) C decreases (Eq. 26.3). (b) Q decreases. The battery supplies a constant potential difference ΔV; thus, charge must flow out of the capacitor if $C = Q/\Delta V$ is to decrease. (c) E decreases because the charge density on the plates decreases. (d) ΔV remains constant because of the presence of the battery. (e) The energy stored in the capacitor decreases (Eq. 26.11).

26.8 Increase. The dielectric constant of wood (and of all other insulating materials, for that matter) is greater than 1; therefore, the capacitance increases (Eq. 26.14). This increase is sensed by the stud-finder's special circuitry, which causes an indicator on the device to light up.

26.9 (a) C increases (Eq. 26.14). (b) Q increases. Because the battery maintains a constant ΔV, Q must increase if C increases. (c) E between the plates remains constant because $\Delta V = Ed$ and neither ΔV nor d changes. The electric field due to the charges on the plates increases because more charge has flowed onto the plates. The induced surface charges on the dielectric create a field that opposes the increase in the field caused by the greater number of charges on the plates (see Section 26.7). (d) The battery maintains a constant ΔV.