Chapter 28
Direct Current Circuits

28.1 Electromotive Force

28.2 Resistors in Series and Parallel

28.3 Kirchhoff's Rules

28.1 Electromotive Force

A constant current can be maintained in a closed circuit through the use of a source of emf, (such as a battery or generator) that produces an electric field and thus may cause charges to move around a circuit.

One can think of a source of emf as a "charge pump" When an electric potential difference exists between two points, the source moves charges from the lower potential to the higher.

The emf describes the work done per unit charge, and hence the SI unit of emf is the volt.

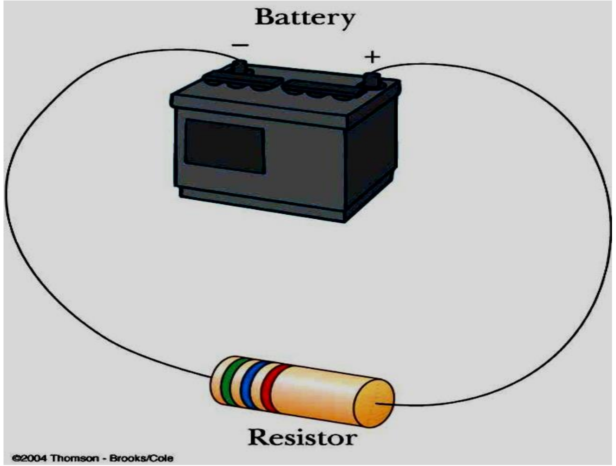
The emf of a battery $\mathcal{E}$ is the maximum possible voltage that the battery can provide between its terminals.

28.1 Electromotive Force

Assume that the connecting wires have no resistance.

The positive terminal of the battery is at a higher potential than the negative terminal. If we neglect the internal resistance of the battery, the potential difference across it (called the terminal voltage) equals its emf.

However, because a real battery always has some internal resistance r, the terminal voltage is not equal to the emf for a battery in a circuit in which there is a current.



28.1 Electromotive Force

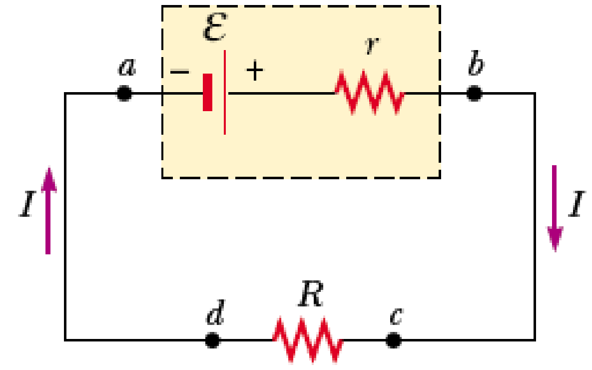
As we pass from the negative terminal to the positive terminal, the potential increases by an amount ε.

As we move through the resistance r, the potential decreases by an amount Ir, where I is the current in the circuit.

$$\Delta V = \varepsilon - Ir$$

ε is equivalent to the open-circuit voltage—that is, the terminal voltage when the current is zero. The emf is the voltage labeled on a battery,….

the terminal voltage V must equal the potential difference across the external resistance R, often called the load resistance.



(a)

28.1 Electromotive Force

The resistor represents a load on the battery because the battery must supply energy to operate the device. The potential difference across the load resistance is

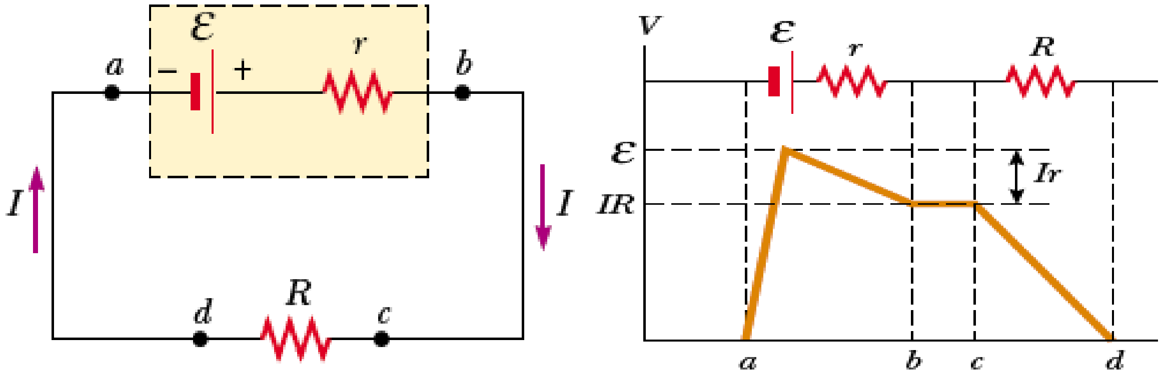
$$\Delta V = IR$$

the total power output $I\varepsilon$ of the battery is delivered to the external load resistance in the amount $I^2 R$ and to the internal resistance in the amount $I^2 r$.

$$\varepsilon = IR + Ir$$

$$I = \frac{\varepsilon}{R + r}$$

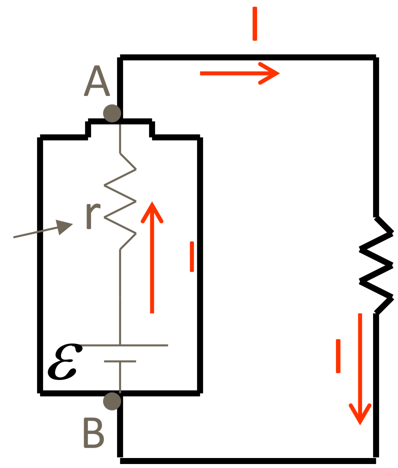
$$I\varepsilon = I^2 R + I^2 r$$



(a) (b)

28.1 Electromotive Force

r = "internal resistance" of the battery



R_L (external resistance, "load")

$$\Rightarrow \quad \varepsilon - Ir = (V_A - V_B)$$

"Terminal voltage"

$$\varepsilon - Ir = V$$

Example 28.1 Terminal Voltage of a Battery

A battery has an emf of 12V and an internal resistance of 0.05Ω. Its terminals are connected to a load resistance of 3Ω. Find:

a) The current in the circuit and the terminal voltage

$$I = \frac{\mathcal{E}}{R + r} = \frac{12.0\ \text{V}}{3.05\ \Omega} = \boxed{3.93\ \text{A}}$$

$$\Delta V = \mathcal{E} - Ir = 12.0\ \text{V} - (3.93\ \text{A})(0.05\ \Omega) = \boxed{11.8\ \text{V}}$$

To check this result, we can calculate the voltage across the load resistance R:

$$\Delta V = IR = (3.93\ \text{A})(3.00\ \Omega) = 11.8\ \text{V}$$

Example 28.1 Terminal Voltage of a Battery

b) The power dissipated in the load, the internal resistance, and the total power delivered by the battery

$$\mathcal{P}_R = I^2R = (3.93\ \text{A})^2\ (3.00\ \Omega) = \boxed{46.3\ \text{W}}$$

The power delivered to the internal resistance is

$$\mathcal{P}_r = I^2r = (3.93\ \text{A})^2\ (0.05\ \Omega) = \boxed{0.772\ \text{W}}$$

Hence, the power delivered by the battery is the sum of these quantities, or 47.1 W. You should check this result, using the expression $\mathcal{P} = I\mathcal{E}$.

28.2 Resistors in Series

When two or more resistors are connected together as are the light- bulbs, they are said to be in series.

In a series connection, all the charges moving through one resistor must also pass through the second resistor.

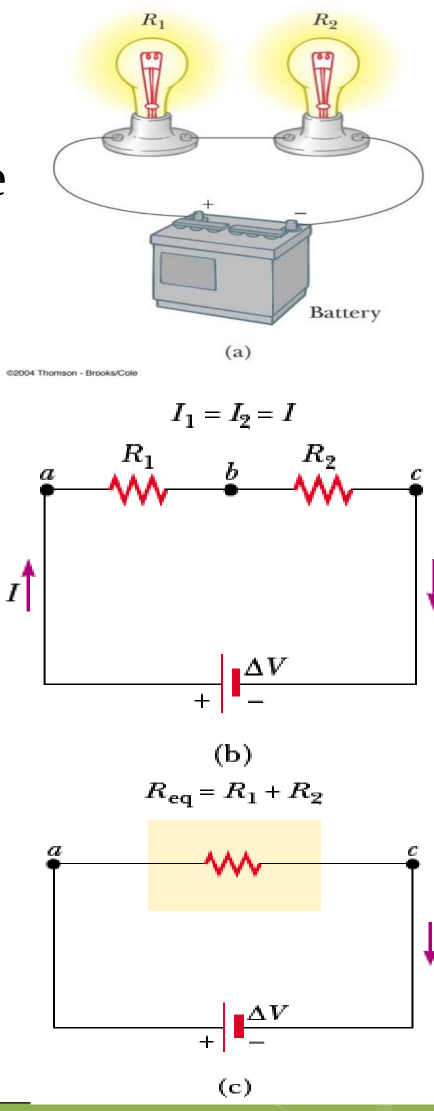
for a series combination of two resistors, the currents are the same in both resistors because the amount of charge that passes through R_1 *must also pass through* R_2

$$\Delta V = IR_1 + IR_2 = I(R_1 + R_2)$$

$$R_{eq} = R_1 + R_2$$

$$R_{eq} = R_1 + R_2 + R_3 + \cdots$$

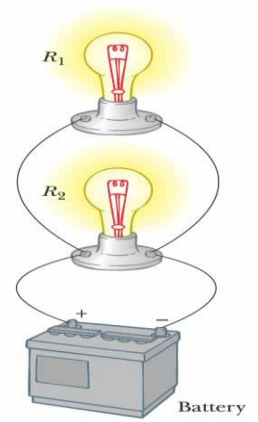
This relationship indicates that the equivalent resistance of a series connection of resistors is always greater than any individual resistance.



28.2 Resistors in Parallel

When the current I reaches point called a junction, it splits into two parts, with I_1 going through R_1 and I_2 going through R_2 .
A junction is any point in a circuit where a current can split

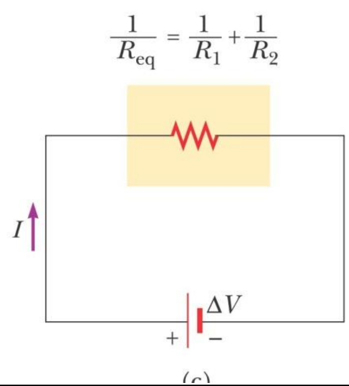
$$I = I_1 + I_2$$

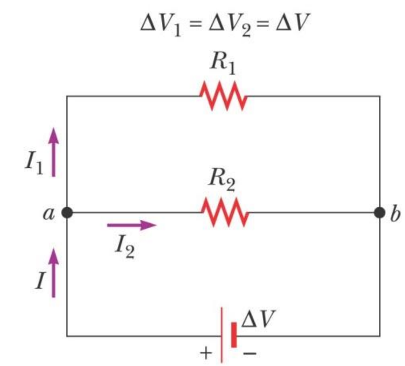


when resistors are connected in parallel, the potential differences across the resistors is the same.

$$I = I_1 + I_2 = \frac{\Delta V}{R_1} + \frac{\Delta V}{R_2} = \Delta V\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{\Delta V}{R_{eq}}$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \qquad R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}}$$



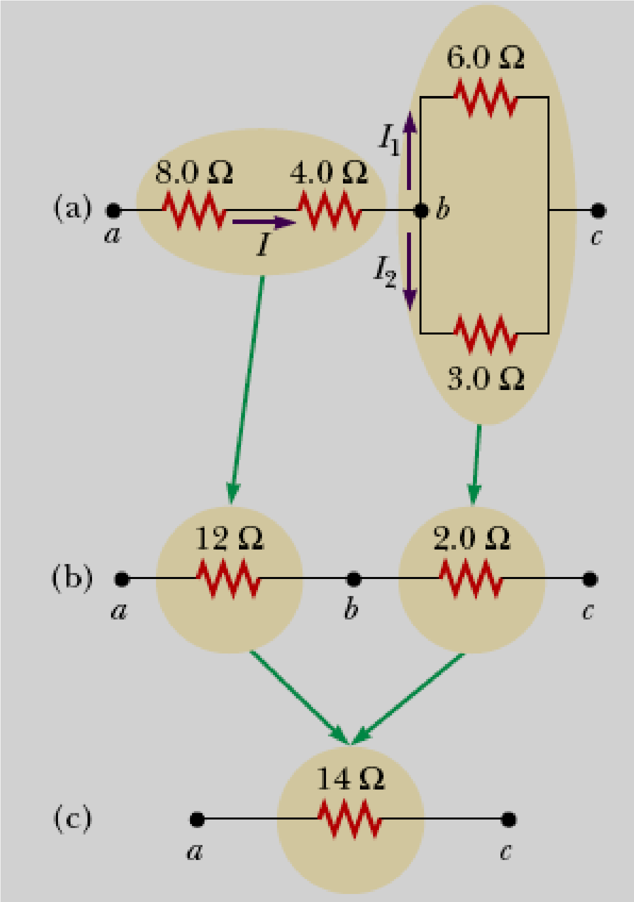


Example 28.4

(a) Find the equivalent resistance between points a and c.

(b) What is the current in each resistor if a potential difference of 42V is maintained between a and c ?

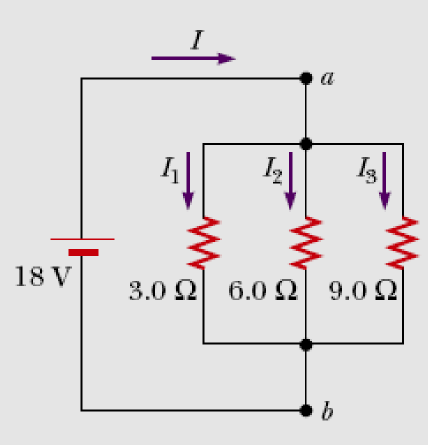
$$I = \frac{\Delta V_{ac}}{R_{eq}} = \frac{42\ \text{V}}{14\ \Omega} = 3.0\ \text{A}$$



Example 28.6

Three resistors are connected in parallel as shown in Figure 28.7. A potential difference of 18 V is maintained between points a and b.

(a) Find the current in each resistor.



$$I_1 = \frac{\Delta V}{R_1} = \frac{18\text{ V}}{3.0\ \Omega} = \boxed{6.0\text{ A}}$$

$$I_2 = \frac{\Delta V}{R_2} = \frac{18\text{ V}}{6.0\ \Omega} = \boxed{3.0\text{ A}}$$

$$I_3 = \frac{\Delta V}{R_3} = \frac{18\text{ V}}{9.0\ \Omega} = \boxed{2.0\text{ A}}$$

(b) Calculate the power delivered to each resistor and the total power delivered to the combination of resistors.

$$\mathcal{P}_1 = \frac{\Delta V^2}{R_1} = \frac{(18\text{ V})^2}{3.0\ \Omega} = \boxed{110\text{ W}}$$

$$\mathcal{P}_2 = \frac{\Delta V^2}{R_2} = \frac{(18\text{ V})^2}{6.0\ \Omega} = \boxed{54\text{ W}}$$

$$\mathcal{P}_3 = \frac{\Delta V^2}{R_3} = \frac{(18\text{ V})^2}{9.0\ \Omega} = \boxed{36\text{ W}}$$

Example 28.6

(c) Calculate the equivalent resistance of the circuit.

$$\frac{1}{R_{\mathrm{eq}}} = \frac{1}{3.0\ \Omega} + \frac{1}{6.0\ \Omega} + \frac{1}{9.0\ \Omega}$$

$$= \frac{6}{18\ \Omega} + \frac{3}{18\ \Omega} + \frac{2}{18\ \Omega} = \frac{11}{18\ \Omega}$$

$$R_{\mathrm{eq}} = \frac{18\ \Omega}{11} = \boxed{1.6\ \Omega}$$

Exercise :Use R_{eq} to calculate the total power delivered by the battery.

Answer :200 W.

28.3 Kirchhoff's Rules

Very often, it is not possible to reduce a circuit to a single loop.
The procedure for analyzing more complex circuits is greatly simplified if we use two principles called Kirchhoff's rules:

1. **Junction rule.** The sum of the currents entering any junction in a circuit must equal the sum of the currents leaving that junction:

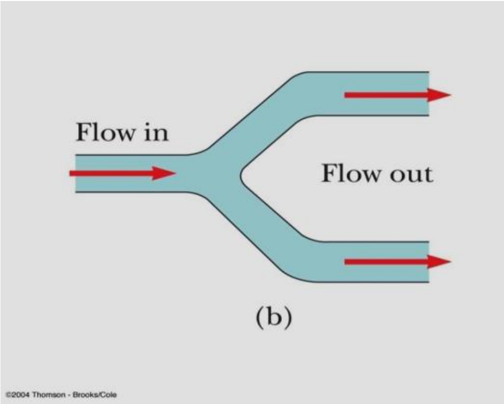
$$\sum I_{\text{in}} = \sum I_{\text{out}} \tag{28.9}$$

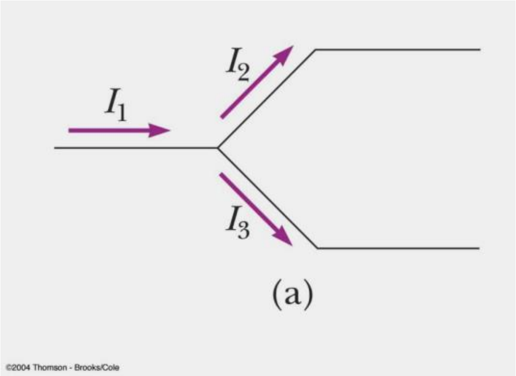
2. **Loop rule.** The sum of the potential differences across all elements around any closed circuit loop must be zero:

$$\sum_{\substack{\text{closed}\\ \text{loop}}} \Delta V = 0 \tag{28.10}$$

28.3 Kirchhoff's Rules

Kirchhoff's first rule is a statement of conservation of electric charge. All charges that enter a given point in a circuit must leave that point because charge cannot build up at a point.





$$I_1 = I_2 + I_3$$

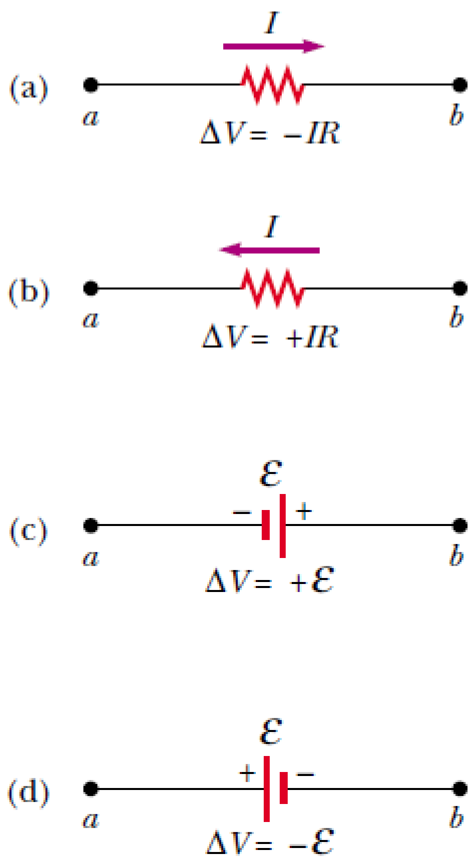
28.3 Kirchhoff’s Rules

Kirchhoff’s second rule follows from the law of conservation of energy. Imagine moving a charge around a closed loop of a circuit. When the charge returns to the starting point, the charge–circuit system must have the same total energy as it had before the charge was moved. The sum of the increases in energy as the charge passes through some circuit elements must equal the sum of the decreases in energy as it passes through other elements. The potential energy decreases whenever the charge moves through a potential drop -IR across a resistor or whenever it moves in the reverse direction through a source of emf. The potential energy increases whenever the charge passes through a battery from the negative terminal to the positive terminal. When applying Kirchhoff’s second rule in practice, we imagine traveling around the loop and consider changes in electric potential, rather than the changes in potential energy.

28.3 Kirchhoff's Rules

Note the following sign conventions when using the second rule:

- Traveling around the loop from a to b
- In a, the resistor is transversed in the direction of the current, the potential across the resistor is –IR.
- In b, the resistor is transversed in the direction opposite of the current, the potential across the resistor is +IR.
- In c, the source of emf is transversed in the direction of the emf (from – to +), the change in the electric potential is +ε
- In d, the source of emf is transversed in the direction opposite of the emf (from + to -), the change in the electric potential is –ε.



28.3 Kirchhoff's Rules

PROBLEM-SOLVING HINTS

Kirchhoff's Rules

- Draw a circuit diagram, and label all the known and unknown quantities. You must assign a *direction* to the current in each branch of the circuit. Although the assignment of current directions is arbitrary, you must adhere rigorously to the assigned directions when applying Kirchhoff's rules.
- Apply the junction rule to any junctions in the circuit that provide new relationships among the various currents.
- Apply the loop rule to as many loops in the circuit as are needed to solve for the unknowns. To apply this rule, you must correctly identify the potential difference as you imagine crossing each element while traversing the closed loop (either clockwise or counterclockwise). Watch out for errors in sign!
- Solve the equations simultaneously for the unknown quantities. Do not be alarmed if a current turns out to be negative; *its magnitude will be correct and the direction is opposite to that which you assigned.*

Example 28.8

(a) Find the current in the circuit. (Neglect the internal resistances of the batteries.)

Traversing the circuit in the clockwise direction, starting at a, we see that a →b represents a potential change of $+\varepsilon_1$, b → c represents a potential change of $-IR_1$, c → d represents a potential change of $-\varepsilon_2$, and d → a represents a potential change of $-IR_2$. Applying Kirchhoff's loop rule gives

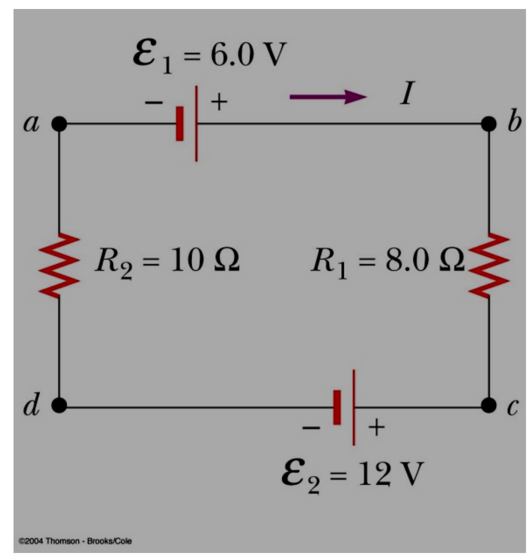
$$\sum \Delta V = 0$$

$$\mathcal{E}_1 - IR_1 - \mathcal{E}_2 - IR_2 = 0$$

Solving for I and using the values given in Figure 28.13, we obtain

$$I = \frac{\mathcal{E}_1 - \mathcal{E}_2}{R_1 + R_2} = \frac{6.0\text{ V} - 12\text{ V}}{8.0\ \Omega + 10\ \Omega} = \boxed{-0.33\text{ A}}$$

The negative sign for I indicates that the direction of the current is opposite the assumed direction.



Example 28.8

(b) What power is delivered to each resistor? What power is delivered by the 12-V battery?

Traversing the circuit in the clockwise direction, starting at a, we see that a →b represents a potential change of $+\varepsilon_1$, b → c represents a potential change of $-IR_1$, c → d represents a potential change of $-\varepsilon_2$, and d → a represents a potential change of $-IR_2$. Applying Kirchhoff's loop rule gives

$$\mathcal{P}_1 = I^2R_1 = (0.33\text{ A})^2(8.0\ \Omega) = \boxed{0.87\text{ W}}$$

$$\mathcal{P}_2 = I^2R_2 = (0.33\text{ A})^2(10\ \Omega) = \boxed{1.1\text{ W}}$$

Hence, the total power delivered to the resistors is $\mathcal{P}_1 + \mathcal{P}_2 = 2.0\text{ W}$.

The 12-V battery delivers power Iε2. Half of this power is delivered to the two resistors, as we just calculated. The other half is delivered to the 6-V battery, which is being charged by the 12-V battery.

Example 28.9

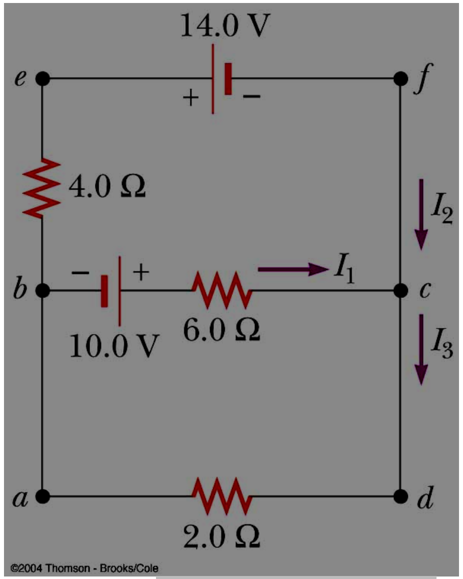
Find the currents I_1 , I_2 , and I_3 in the circuit.

We arbitrarily choose the directions of the currents as labeled in Figure.

(1) $I_1 + I_2 = I_3$

(2) *abcda* $10\text{ V} - (6\ \Omega)I_1 - (2\ \Omega)I_3 = 0$

(3) *befcb* $-14\text{ V} + (6\ \Omega)I_1 - 10\text{ V} - (4\ \Omega)I_2 = 0$



Substituting Equation (1) into Equation (2) gives

$$10\text{ V} - (6\ \Omega)I_1 - (2\ \Omega)\ (I_1 + I_2) = 0$$

(4) $10\text{ V} = (8\ \Omega)I_1 + (2\ \Omega)I_2$

Dividing each term in Equation (3) by 2 and rearranging gives (5) $-12\text{ V} = -(3\ \Omega)I_1 + (2\ \Omega)I_2$

Subtracting Equation (5) from Equation (4) eliminates I_2, giving

$$22\text{ V} = (11\ \Omega)I_1$$

$$I_1 = 2\text{ A}$$

Using this value of I_1 in Equation (5) gives a value for I_2:

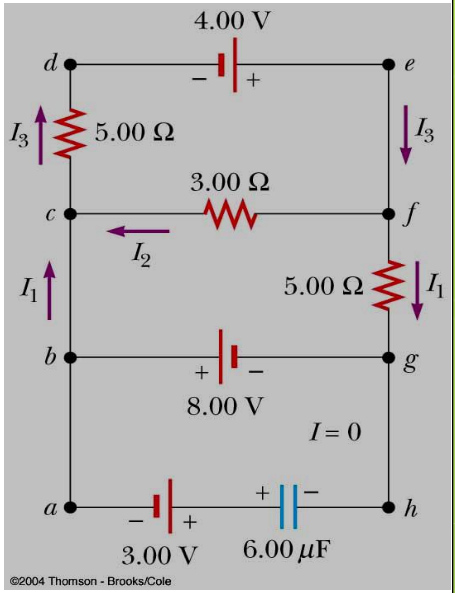
$$(2\ \Omega)I_2 = (3\ \Omega)I_1 - 12\text{ V} = (3\ \Omega)\ (2\text{ A}) - 12\text{ V} = -6\text{ V}$$

$$I_2 = -3\text{ A}$$

and $I_3 = I_1 + I_2 = -1\text{ A}$

Example 28.10

Under steady-state conditions, find the unknown currents I_1, I_2, and I_3 in the multiloop circuit shown in Figure.



(1) $I_1 + I_2 = I_3$

(2) *defcd* $4.00\text{ V} - (3.00\ \Omega)I_2 - (5.00\ \Omega)I_3 = 0$

(3) *cfgbc* $(3.00\ \Omega)I_2 - (5.00\ \Omega)I_1 + 8.00\text{ V} = 0$

(4) $(8.00\ \Omega)I_2 - (5.00\ \Omega)I_3 + 8.00\text{ V} = 0$

$$I_2 = -\frac{4.00\text{ V}}{11.0\ \Omega} = \boxed{-0.364\text{ A}}$$

Because our value for I_2 is negative, we conclude that the direction of I_2 is from c to f in the 3.00-Ω resistor. Despite this interpretation of the direction, however, we must continue to use this negative value for I_2 in subsequent calculations because our equations were established with our original choice of direction.

Using $I_2 = -0.364$ A in Equations (3) and (1) gives

$$I_1 = \boxed{1.38\text{ A}} \qquad I_3 = \boxed{1.02\text{ A}}$$

Example 28.10

(B) What is the charge on the capacitor?

Solution We can apply Kirchhoff's loop rule to loop *bghab* (or any other loop that contains the capacitor) to find the potential difference ΔV_{cap} across the capacitor. We use this potential difference in the loop equation without reference to a sign convention because the charge on the capacitor depends only on the magnitude of the potential difference. Moving clockwise around this loop, we obtain

$$-8.00\text{ V} + \Delta V_{\text{cap}} - 3.00\text{ V} = 0$$

$$\Delta V_{\text{cap}} = 11.0\text{ V}$$

Because $Q = C\,\Delta V_{\text{cap}}$ (see Eq. 26.1), the charge on the capacitor is

$$Q = (6.00\ \mu\text{F})(11.0\text{ V}) = \boxed{66.0\ \mu\text{C}}$$