where **M** is the **magnetization vector.** The magnetization vector is the magnetic moment per unit volume in the substance.

The effect of external currents on the magnetic field in a substance is described by the **magnetic field strength H** $= \mathbf{B}_0/\mu_0$. The magnetization vector is related to the magnetic field strength as follows:

$$\mathbf{M} = \chi \mathbf{H} \tag{30.32}$$

where χ is the **magnetic susceptibility.**

Substances can be classified into one of three categories that describe their magnetic behavior. **Diamagnetic** substances are those in which the magnetization is weak and opposite the field $\mathbf{B}_0$, so that the susceptibility is negative. **Paramagnetic** substances are those in which the magnetization is weak and in the same direction as the field $\mathbf{B}_0$, so that the susceptibility is positive. In **ferromagnetic** substances, interactions between atoms cause magnetic moments to align and create a strong magnetization that remains after the external field is removed.

QUESTIONS

1. Is the magnetic field created by a current loop uniform? Explain.
2. A current in a conductor produces a magnetic field that can be calculated using the Biot–Savart law. Because current is defined as the rate of flow of charge, what can you conclude about the magnetic field produced by stationary charges? What about that produced by moving charges?
3. Explain why two parallel wires carrying currents in opposite directions repel each other.
4. Parallel current-carrying wires exert magnetic forces on each other. What about perpendicular wires? Imagine two such wires oriented perpendicular to each other, and almost touching. Does a magnetic force exist between the wires?
5. Is Ampère's law valid for all closed paths surrounding a conductor? Why is it not useful for calculating **B** for all such paths?
6. Compare Ampère's law with the Biot–Savart law. Which is more generally useful for calculating **B** for a current-carrying conductor?
7. Is the magnetic field inside a toroid uniform? Explain.
8. Describe the similarities between Ampère's law in magnetism and Gauss's law in electrostatics.
9. A hollow copper tube carries a current along its length. Why is $\mathbf{B} = 0$ inside the tube? Is **B** nonzero outside the tube?
10. Describe the change in the magnetic field in the space enclosed by a solenoid carrying a steady current I if (a) the length of the solenoid is doubled but the number of turns remains the same and (b) the number of turns is doubled but the length remains the same.
11. A flat conducting loop is located in a uniform magnetic field directed along the x axis. For what orientation of the loop is the flux through it a maximum? A minimum?
12. What new concept did Maxwell's generalized form of Ampère's law include?
13. Many loops of wire are wrapped around a nail and the ends of the wire are connected to a battery. Identify the source of **M**, of **H**, and of **B**.
14. A magnet attracts a piece of iron. The iron can then attract another piece of iron. On the basis of domain alignment, explain what happens in each piece of iron.
15. Why does hitting a magnet with a hammer cause the magnetism to be reduced?
16. A Hindu ruler once suggested that he be entombed in a magnetic coffin with the polarity arranged so that he would be forever suspended between heaven and Earth. Is such magnetic levitation possible? Discuss.
17. Why is $\mathbf{M} = 0$ in a vacuum? What is the relationship between **B** and **H** in a vacuum?
18. Explain why some atoms have permanent magnetic dipole moments and others do not.
19. What factors contribute to the total magnetic dipole moment of an atom?
20. Why is the susceptibility of a diamagnetic substance negative?
21. Why can the effect of diamagnetism be neglected in a paramagnetic substance?
22. Explain the significance of the Curie temperature for a ferromagnetic substance.
23. Discuss the difference among ferromagnetic, paramagnetic, and diamagnetic substances.
24. A current in a solenoid having air in the interior creates a magnetic field $\mathbf{B} = \mu_0\mathbf{H}$. Describe qualitatively what happens to the magnitude of **B** as (a) aluminum, (b) copper, and (c) iron are placed in the interior.
25. What is the difference between hard and soft ferromagnetic materials?
26. Should the surface of a computer disk be made from a hard or a soft ferromagnetic substance?

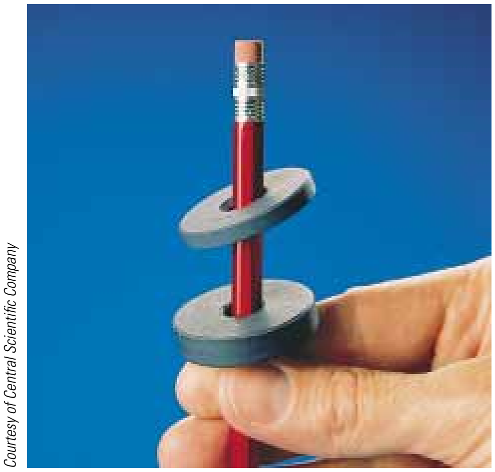
27. Explain why it is desirable to use hard ferromagnetic materials to make permanent magnets.

28. Would you expect the tape from a tape recorder to be attracted to a magnet? (Try it, but not with a recording you wish to save.)

29. Given only a strong magnet and a screwdriver, how would you first magnetize and then demagnetize the screwdriver?

30. Which way would a compass point if you were at the north magnetic pole of the Earth?

31. Figure Q30.31 shows two permanent magnets, each having a hole through its center. Note that the upper magnet is levitated above the lower one. (a) How does this occur? (b) What purpose does the pencil serve? (c) What can you say about the poles of the magnets from this observation? (d) If the upper magnet were inverted, what do you suppose would happen?



Courtesy of Central Scientific Company

Figure Q30.31

PROBLEMS

1, 2, 3 = straightforward, intermediate, challenging ☐ = full solution available in the *Student Solutions Manual and Study Guide*

= coached solution with hints available at http://www.pse6.com = computer useful in solving problem

= paired numerical and symbolic problems

Section 30.1 The Biot–Savart Law

1. In Niels Bohr's 1913 model of the hydrogen atom, an electron circles the proton at a distance of 5.29×10^{-11} m with a speed of 2.19×10^6 m/s. Compute the magnitude of the magnetic field that this motion produces at the location of the proton.

2. A lightning bolt may carry a current of 1.00×10^4 A for a short period of time. What is the resulting magnetic field 100 m from the bolt? Suppose that the bolt extends far above and below the point of observation.

3. (a) A conductor in the shape of a square loop of edge length $\ell = 0.400$ m carries a current $I = 10.0$ A as in Fig. P30.3. Calculate the magnitude and direction of the magnetic field at the center of the square. (b) **What If?** If this conductor is formed into a single circular turn and carries the same current, what is the value of the magnetic field at the center?

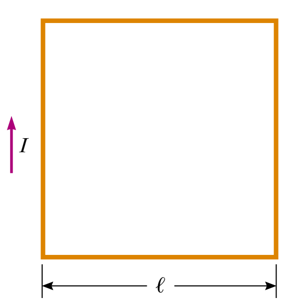


Figure P30.3

4. Calculate the magnitude of the magnetic field at a point 100 cm from a long, thin conductor carrying a current of 1.00 A.

5. Determine the magnetic field at a point P located a distance x from the corner of an infinitely long wire bent at a right angle, as shown in Figure P30.5. The wire carries a steady current I.

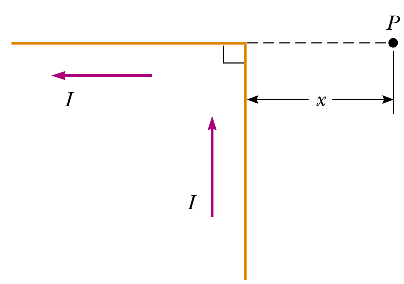


Figure P30.5

6. A conductor consists of a circular loop of radius R and two straight, long sections, as shown in Figure P30.6. The wire

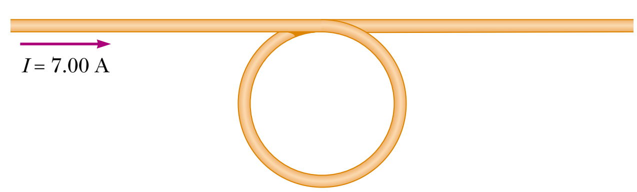


Figure P30.6

lies in the plane of the paper and carries a current I. Find an expression for the vector magnetic field at the center of the loop.

7. The segment of wire in Figure P30.7 carries a current of $I = 5.00$ A, where the radius of the circular arc is $R = 3.00$ cm. Determine the magnitude and direction of the magnetic field at the origin.

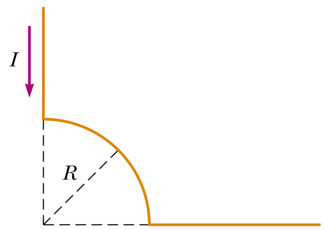


Figure P30.7

8. Consider a flat circular current loop of radius R carrying current I. Choose the x axis to be along the axis of the loop, with the origin at the center of the loop. Plot a graph of the ratio of the magnitude of the magnetic field at coordinate x to that at the origin, for $x = 0$ to $x = 5R$. It may be useful to use a programmable calculator or a computer to solve this problem.

9. Two very long, straight, parallel wires carry currents that are directed perpendicular to the page, as in Figure P30.9. Wire 1 carries a current I_1 into the page (in the $-z$ direction) and passes through the x axis at $x = +a$. Wire 2 passes through the x axis at $x = -2a$ and carries an unknown current I_2. The total magnetic field at the origin due to the current-carrying wires has the magnitude $2\mu_0 I_1/(2\pi a)$. The current I_2 can have either of two possible values. (a) Find the value of I_2 with the smaller magnitude, stating it in terms of I_1 and giving its direction. (b) Find the other possible value of I_2.

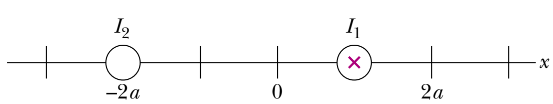


Figure P30.9

10. A very long straight wire carries current I. In the middle of the wire a right-angle bend is made. The bend forms an arc of a circle of radius r, as shown in Figure P30.10. Determine the magnetic field at the center of the arc.

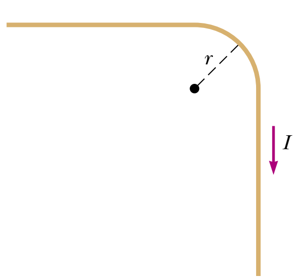


Figure P30.10

11. One very long wire carries current 30.0 A to the left along the x axis. A second very long wire carries current 50.0 A to the right along the line ($y = 0.280$ m, $z = 0$). (a) Where in the plane of the two wires is the total magnetic field equal to zero? (b) A particle with a charge of $-2.00\ \mu$C is moving with a velocity of $150\hat{\mathbf{i}}$ Mm/s along the line ($y = 0.100$ m, $z = 0$). Calculate the vector magnetic force acting on the particle. (c) **What If?** A uniform electric field is applied to allow this particle to pass through this region undeflected. Calculate the required vector electric field.

12. Consider the current-carrying loop shown in Figure P30.12, formed of radial lines and segments of circles whose centers are at point P. Find the magnitude and direction of **B** at P.

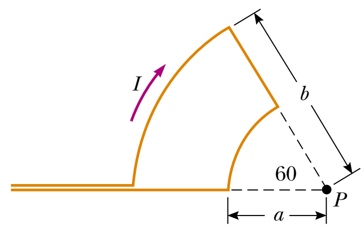


Figure P30.12

13. A wire carrying a current I is bent into the shape of an equilateral triangle of side L. (a) Find the magnitude of the magnetic field at the center of the triangle. (b) At a point halfway between the center and any vertex, is the field stronger or weaker than at the center?

14. Determine the magnetic field (in terms of I, a, and d) at the origin due to the current loop in Figure P30.14.

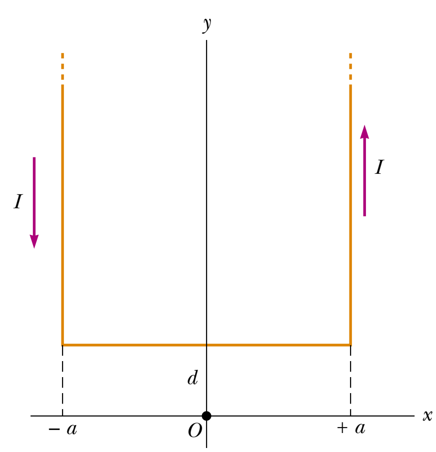


Figure P30.14

15. Two long, parallel conductors carry currents $I_1 = 3.00$ A and $I_2 = 3.00$ A, both directed into the page in Figure P30.15. Determine the magnitude and direction of the resultant magnetic field at P.

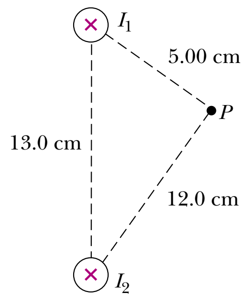


Figure P30.15

Section 30.2 The Magnetic Force Between Two Parallel Conductors

16. Two long, parallel conductors, separated by 10.0 cm, carry currents in the same direction. The first wire carries current $I_1 = 5.00$ A and the second carries $I_2 = 8.00$ A. (a) What is the magnitude of the magnetic field created by I_1 at the location of I_2? (b) What is the force per unit length exerted by I_1 on I_2? (c) What is the magnitude of the magnetic field created by I_2 at the location of I_1? (d) What is the force per length exerted by I_2 on I_1?

17. In Figure P30.17, the current in the long, straight wire is $I_1 = 5.00$ A and the wire lies in the plane of the rectangular loop, which carries the current $I_2 = 10.0$ A. The dimensions are $c = 0.100$ m, $a = 0.150$ m, and $\ell = 0.450$ m. Find the magnitude and direction of the net force exerted on the loop by the magnetic field created by the wire.

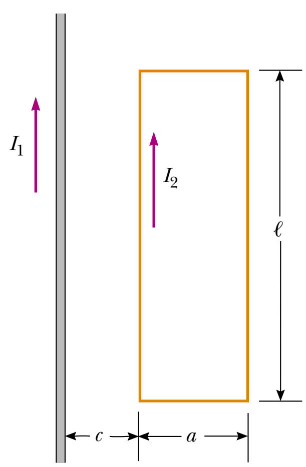


Figure P30.17

18. Two long, parallel wires are attracted to each other by a force per unit length of 320 μN/m when they are separated by a vertical distance of 0.500 m. The current in the upper wire is 20.0 A to the right. Determine the location of the line in the plane of the two wires along which the total magnetic field is zero.

19. Three long wires (wire 1, wire 2, and wire 3) hang vertically. The distance between wire 1 and wire 2 is 20.0 cm. On the left, wire 1 carries an upward current of 1.50 A. To the right, wire 2 carries a downward current of 4.00 A. Wire 3 is located such that when it carries a certain current, each wire experiences no net force. Find (a) the position of wire 3, and (b) the magnitude and direction of the current in wire 3.

20. The unit of magnetic flux is named for Wilhelm Weber. The practical-size unit of magnetic field is named for Johann Karl Friedrich Gauss. Both were scientists at Göttingen, Germany. Along with their individual accomplishments, together they built a telegraph in 1833. It consisted of a battery and switch, at one end of a transmission line 3 km long, operating an electromagnet at the other end. (André Ampère suggested electrical signaling in 1821; Samuel Morse built a telegraph line between Baltimore and Washington in 1844.) Suppose that Weber and Gauss's transmission line was as diagrammed in Figure P30.20. Two long, parallel wires, each having a mass per unit length of 40.0 g/m, are supported in a horizontal plane by strings 6.00 cm long. When both wires carry the same current I, the wires repel each other so that the angle θ between the supporting strings is 16.0°. (a) Are the currents in the same direction or in opposite directions? (b) Find the magnitude of the current.

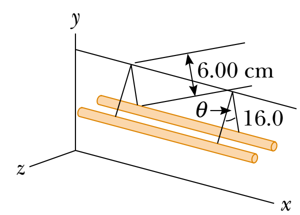


Figure P30.20

Section 30.3 Ampère's Law

21. Four long, parallel conductors carry equal currents of $I = 5.00$ A. Figure P30.21 is an end view of the conductors. The current direction is into the page at points A and B (indicated by the crosses) and out of the page at C and D (indicated by the dots). Calculate the magnitude and direction of the magnetic field at point P, located at the center of the square of edge length 0.200 m.

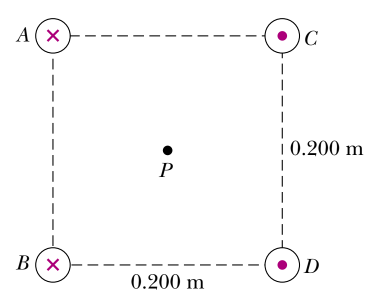


Figure P30.21

22. A long straight wire lies on a horizontal table and carries a current of 1.20 μA. In a vacuum, a proton moves parallel to the wire (opposite the current) with a constant speed of 2.30×10^4 m/s at a distance d above the wire. Determine the value of d. You may ignore the magnetic field due to the Earth.

23. Figure P30.23 is a cross-sectional view of a coaxial cable. The center conductor is surrounded by a rubber layer, which is surrounded by an outer conductor, which is surrounded by another rubber layer. In a particular application, the current in the inner conductor is 1.00 A out of the page and the current in the outer conductor is 3.00 A into the page. Determine the magnitude and direction of the magnetic field at points *a* and *b*.

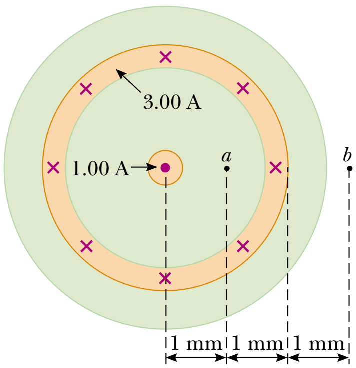


Figure P30.23

24. The magnetic field 40.0 cm away from a long straight wire carrying current 2.00 A is 1.00 μT. (a) At what distance is it 0.100 μT? (b) **What If?** At one instant, the two conductors in a long household extension cord carry equal 2.00-A currents in opposite directions. The two wires are 3.00 mm apart. Find the magnetic field 40.0 cm away from the middle of the straight cord, in the plane of the two wires. (c) At what distance is it one tenth as large? (d) The center wire in a coaxial cable carries current 2.00 A in one direction and the sheath around it carries current 2.00 A in the opposite direction. What magnetic field does the cable create at points outside?

25. A packed bundle of 100 long, straight, insulated wires forms a cylinder of radius $R = 0.500$ cm. (a) If each wire carries 2.00 A, what are the magnitude and direction of the magnetic force per unit length acting on a wire located 0.200 cm from the center of the bundle? (b) **What If?** Would a wire on the outer edge of the bundle experience a force greater or smaller than the value calculated in part (a)?

26. The magnetic coils of a tokamak fusion reactor are in the shape of a toroid having an inner radius of 0.700 m and an outer radius of 1.30 m. The toroid has 900 turns of large-diameter wire, each of which carries a current of 14.0 kA. Find the magnitude of the magnetic field inside the toroid along (a) the inner radius and (b) the outer radius.

27. Consider a column of electric current passing through plasma (ionized gas). Filaments of current within the column are magnetically attracted to one another. They can crowd together to yield a very great current density and a very strong magnetic field in a small region. Sometimes the current can be cut off momentarily by this *pinch effect.* (In a metallic wire a pinch effect is not important, because the current-carrying electrons repel one another with electric forces.) The pinch effect can be demonstrated by making an empty aluminum can carry a large current parallel to its axis. Let R represent the radius of the can and I the upward current, uniformly distributed over its curved wall. Determine the magnetic field (a) just inside the wall and (b) just outside. (c) Determine the pressure on the wall.

28. Niobium metal becomes a superconductor when cooled below 9 K. Its superconductivity is destroyed when the surface magnetic field exceeds 0.100 T. Determine the maximum current a 2.00-mm-diameter niobium wire can carry and remain superconducting, in the absence of any external magnetic field.

29. A long cylindrical conductor of radius R carries a current I as shown in Figure P30.29. The current density J, however, is not uniform over the cross section of the conductor but is a function of the radius according to $J = br$, where b is a constant. Find an expression for the magnetic field B (a) at a distance $r_1 < R$ and (b) at a distance $r_2 > R$, measured from the axis.

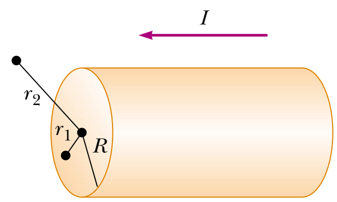


Figure P30.29

30. In Figure P30.30, both currents in the infinitely long wires are in the negative x direction. (a) Sketch the magnetic field pattern in the yz plane. (b) At what distance d along the z axis is the magnetic field a maximum?

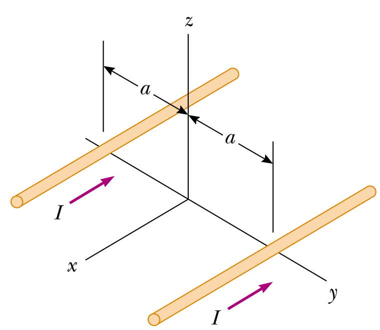


Figure P30.30

Section 30.4 The Magnetic Field of a Solenoid

31. What current is required in the windings of a long solenoid that has 1 000 turns uniformly distributed over a length of 0.400 m, to produce at the center of the solenoid a magnetic field of magnitude 1.00×10^{-4} T?

32. Consider a solenoid of length ℓ and radius R, containing N closely spaced turns and carrying a steady current I. (a) In terms of these parameters, find the magnetic field at a point along the axis as a function of distance a from the end of the solenoid. (b) Show that as ℓ becomes very long, B approaches $\mu_0 NI/2\ell$ at each end of the solenoid.

33. A single-turn square loop of wire, 2.00 cm on each edge, carries a clockwise current of 0.200 A. The loop is inside a solenoid, with the plane of the loop perpendicular to the magnetic field of the solenoid. The solenoid has 30 turns/cm and carries a clockwise current of 15.0 A. Find the force on each side of the loop and the torque acting on the loop.

Section 30.5 Magnetic Flux

34. Consider the hemispherical closed surface in Figure P30.34. The hemisphere is in a uniform magnetic field that makes an angle θ with the vertical. Calculate the magnetic flux through (a) the flat surface S_1 and (b) the hemispherical surface S_2.

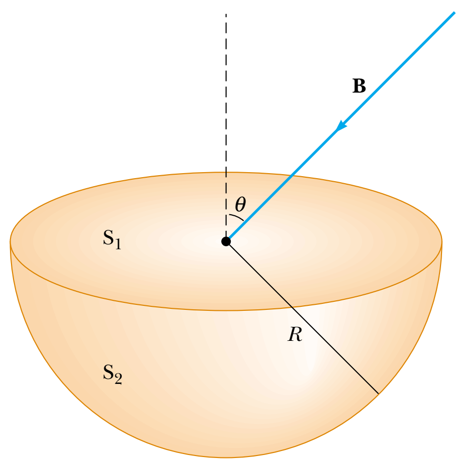


Figure P30.34

35. A cube of edge length ℓ = 2.50 cm is positioned as shown in Figure P30.35. A uniform magnetic field given by $\mathbf{B} = (5\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 3\hat{\mathbf{k}})$T exists throughout the region. (a) Calculate the flux through the shaded face. (b) What is the total flux through the six faces?

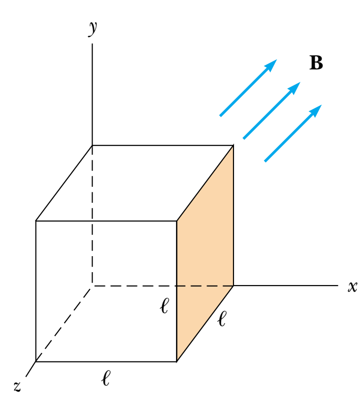


Figure P30.35

36. A solenoid 2.50 cm in diameter and 30.0 cm long has 300 turns and carries 12.0 A. (a) Calculate the flux through the surface of a disk of radius 5.00 cm that is positioned perpendicular to and centered on the axis of the solenoid, as shown in Figure P30.36a. (b) Figure P30.36b shows an enlarged end view of the same solenoid. Calculate the flux through the blue area, which is defined by an annulus that has an inner radius of 0.400 cm and outer radius of 0.800 cm.

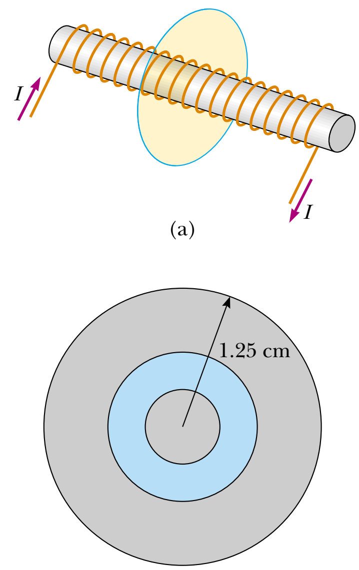


Figure P30.36

Section 30.7 Displacement Current and the General Form of Ampère's Law

37. A 0.100-A current is charging a capacitor that has square plates 5.00 cm on each side. The plate separation is 4.00 mm. Find (a) the time rate of change of electric flux between the plates and (b) the displacement current between the plates.

38. A 0.200-A current is charging a capacitor that has circular plates 10.0 cm in radius. If the plate separation is 4.00 mm, (a) what is the time rate of increase of electric field between the plates? (b) What is the magnetic field between the plates 5.00 cm from the center?

Section 30.8 Magnetism in Matter

39. In Bohr's 1913 model of the hydrogen atom, the electron is in a circular orbit of radius 5.29×10^{-11} m and its speed is 2.19×10^{6} m/s. (a) What is the magnitude of the magnetic moment due to the electron's motion? (b) If the electron moves in a horizontal circle, counterclockwise as seen from above, what is the direction of this magnetic moment vector?

40. A magnetic field of 1.30 T is to be set up in an iron-core toroid. The toroid has a mean radius of 10.0 cm, and magnetic permeability of 5 000 μ_0. What current is required if the winding has 470 turns of wire? The thickness of the iron ring is small compared to 10 cm, so the field in the material is nearly uniform.

41. A toroid with a mean radius of 20.0 cm and 630 turns (see Fig. 30.30) is filled with powdered steel whose magnetic susceptibility χ is 100. The current in the windings is 3.00 A. Find B (assumed uniform) inside the toroid.

42. A particular paramagnetic substance achieves 10.0% of its saturation magnetization when placed in a magnetic field of 5.00 T at a temperature of 4.00 K. The density of magnetic atoms in the sample is 8.00×10^{27} atoms/m^3, and the magnetic moment per atom is 5.00 Bohr magnetons. Calculate the Curie constant for this substance.

43. Calculate the magnetic field strength H of a magnetized substance in which the magnetization is 0.880×10^6 A/m and the magnetic field has magnitude 4.40 T.

44. At *saturation*, when nearly all of the atoms have their magnetic moments aligned, the magnetic field in a sample of iron can be 2.00 T. If each electron contributes a magnetic moment of 9.27×10^{-24} A·m^2 (one Bohr magneton), how many electrons per atom contribute to the saturated field of iron? Iron contains approximately 8.50×10^{28} atoms/m^3.

45. (a) Show that Curie's law can be stated in the following way: The magnetic susceptibility of a paramagnetic substance is inversely proportional to the absolute temperature, according to $\chi = C\mu_0/T$, where C is Curie's constant. (b) Evaluate Curie's constant for chromium.

Section 30.9 The Magnetic Field of the Earth

46. A circular coil of 5 turns and a diameter of 30.0 cm is oriented in a vertical plane with its axis perpendicular to the horizontal component of the Earth's magnetic field. A horizontal compass placed at the center of the coil is made to deflect 45.0° from magnetic north by a current of 0.600 A in the coil. (a) What is the horizontal component of the Earth's magnetic field? (b) The current in the coil is switched off. A "dip needle" is a magnetic compass mounted so that it can rotate in a vertical north–south plane. At this location a dip needle makes an angle of 13.0° from the vertical. What is the total magnitude of the Earth's magnetic field at this location?

47. The magnetic moment of the Earth is approximately 8.00×10^{22} A·m^2. (a) If this were caused by the complete magnetization of a huge iron deposit, how many unpaired electrons would this correspond to? (b) At two unpaired electrons per iron atom, how many kilograms of iron would this correspond to? (Iron has a density of 7 900 kg/m^3, and approximately 8.50×10^{28} iron atoms/m^3.)

Additional Problems

48. The magnitude of the Earth's magnetic field at either pole is approximately 7.00×10^{-5} T. Suppose that the field fades away, before its next reversal. Scouts, sailors, and conservative politicians around the world join together in a program to replace the field. One plan is to use a current loop around the equator, without relying on magnetization of any materials inside the Earth. Determine the current that would generate such a field if this plan were carried out. (Take the radius of the Earth as $R_E = 6.37 \times 10^6$ m.)

49. A very long, thin strip of metal of width w carries a current I along its length as shown in Figure P30.49. Find the magnetic field at the point P in the diagram. The point P is in the plane of the strip at distance b away from it.

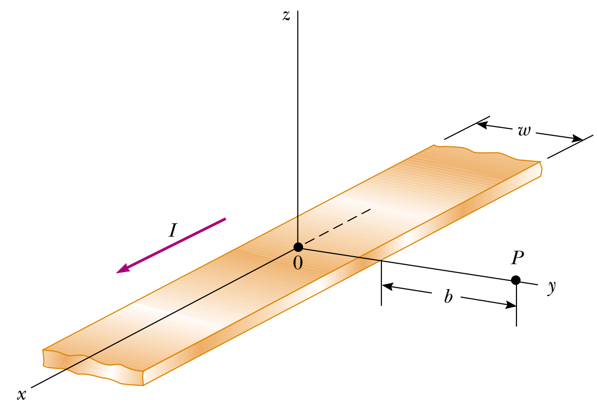


Figure P30.49

50. Suppose you install a compass on the center of the dashboard of a car. Compute an order-of-magnitude estimate for the magnetic field at this location produced by the current when you switch on the headlights. How does it compare with the Earth's magnetic field? You may suppose the dashboard is made mostly of plastic.

51. For a research project, a student needs a solenoid that produces an interior magnetic field of 0.030 0 T. She decides to use a current of 1.00 A and a wire 0.500 mm in diameter. She winds the solenoid in layers on an insulating form 1.00 cm in diameter and 10.0 cm long. Determine the number of layers of wire needed and the total length of the wire.

52. A thin copper bar of length $\ell = 10.0$ cm is supported horizontally by two (nonmagnetic) contacts. The bar carries current $I_1 = 100$ A in the $-x$ direction, as shown in Figure P30.52. At a distance $h = 0.500$ cm below one end of the bar, a long straight wire carries a current $I_2 = 200$ A in the z direction. Determine the magnetic force exerted on the bar.

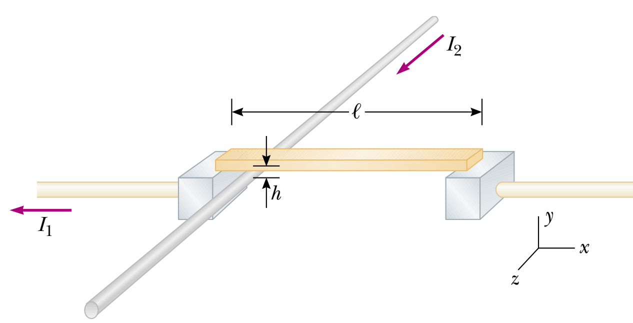


Figure P30.52

53. A nonconducting ring of radius 10.0 cm is uniformly charged with a total positive charge 10.0 μC. The ring rotates at a constant angular speed 20.0 rad/s about an

axis through its center, perpendicular to the plane of the ring. What is the magnitude of the magnetic field on the axis of the ring 5.00 cm from its center?

54. A nonconducting ring of radius R is uniformly charged with a total positive charge q. The ring rotates at a constant angular speed ω about an axis through its center, perpendicular to the plane of the ring. What is the magnitude of the magnetic field on the axis of the ring a distance $R/2$ from its center?

55. Two circular coils of radius R, each with N turns, are perpendicular to a common axis. The coil centers are a distance R apart. Each coil carries a steady current I in the same direction, as shown in Figure P30.55. (a) Show that the magnetic field on the axis at a distance x from the center of one coil is

$$B = \frac{N\mu_0 IR^2}{2}\left[\frac{1}{(R^2 + x^2)^{3/2}} + \frac{1}{(2R^2 + x^2 - 2Rx)^{3/2}}\right]$$

(b) Show that dB/dx and d^2B/dx^2 are both zero at the point midway between the coils. This means the magnetic field in the region midway between the coils is uniform. Coils in this configuration are called *Helmholtz coils.*

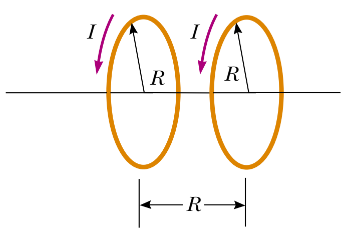
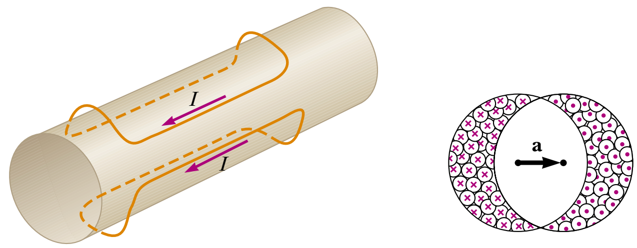


Figure P30.55 Problems 55 and 56.

56. Two identical, flat, circular coils of wire each have 100 turns and a radius of 0.500 m. The coils are arranged as a set of Helmholtz coils (see Fig. P30.55), parallel and with separation 0.500 m. Each coil carries a current of 10.0 A. Determine the magnitude of the magnetic field at a point on the common axis of the coils and halfway between them.

57. We have seen that a long solenoid produces a uniform magnetic field directed along the axis of a cylindrical region. However, to produce a uniform magnetic field directed parallel to a *diameter* of a cylindrical region, one can use the *saddle coils* illustrated in Figure P30.57. The loops are wrapped over a somewhat flattened tube. Assume the straight sections of wire are very long. The end view of the tube shows how the windings are applied. The overall current distribution is the superposition of two overlapping circular cylinders of uniformly distributed current, one toward you and one away from you. The current density J is the same for each cylinder. The position of the axis of one cylinder is described by a position vector $\mathbf{a}$ relative to the other cylinder. Prove that the magnetic field inside the hollow tube is $\mu_0 Ja/2$ downward. *Suggestion:* The use of vector methods simplifies the calculation.



(a) (b)

Figure P30.57 (a) General view of one turn of each saddle coil. (b) End view of the coils carrying current into the paper on the left and out of the paper on the right.

58. A very large parallel-plate capacitor carries charge with uniform charge per unit area $+\sigma$ on the upper plate and $-\sigma$ on the lower plate. The plates are horizontal and both move horizontally with speed v to the right. (a) What is the magnetic field between the plates? (b) What is the magnetic field close to the plates but outside of the capacitor? (c) What is the magnitude and direction of the magnetic force per unit area on the upper plate? (d) At what extrapolated speed v will the magnetic force on a plate balance the electric force on the plate? Calculate this speed numerically.

59. Two circular loops are parallel, coaxial, and almost in contact, 1.00 mm apart (Fig. P30.59). Each loop is 10.0 cm in radius. The top loop carries a clockwise current of 140 A. The bottom loop carries a counterclockwise current of 140 A. (a) Calculate the magnetic force exerted by the bottom loop on the top loop. (b) The upper loop has a mass of 0.021 0 kg. Calculate its acceleration, assuming that the only forces acting on it are the force in part (a) and the gravitational force. *Suggestion:* Think about how one loop looks to a bug perched on the other loop.

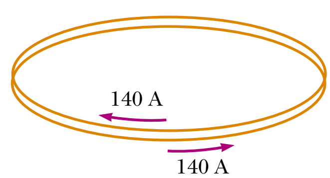


Figure P30.59

60. What objects experience a force in an electric field? Chapter 23 gives the answer: any electric charge, stationary or moving, other than the charge that created the field. What creates an electric field? Any electric charge, stationary or moving, as you studied in Chapter 23. What objects experience a force in a magnetic field? An electric current or a moving electric charge, other than the current or charge that created the field, as discussed in Chapter 29. What creates a magnetic field? An electric current, as you studied in Section 30.1, or a moving electric charge, as shown in this problem. (a) To display how a moving charge creates a magnetic field, consider a charge q moving with velocity $\mathbf{v}$. Define the vector $\mathbf{r} = r\hat{\mathbf{r}}$

to lead from the charge to some location. Show that the magnetic field at that location is

$$\mathbf{B} = \frac{\mu_0}{4\pi} \frac{q\mathbf{v} \times \hat{\mathbf{r}}}{r^2}$$

(b) Find the magnitude of the magnetic field 1.00 mm to the side of a proton moving at 2.00×10^7 m/s. (c) Find the magnetic force on a second proton at this point, moving with the same speed in the opposite direction. (d) Find the electric force on the second proton.

61. *Rail guns* have been suggested for launching projectiles into space without chemical rockets, and for ground-to-air antimissile weapons of war. A tabletop model rail gun (Fig. P30.61) consists of two long parallel horizontal rails 3.50 cm apart, bridged by a bar *BD* of mass 3.00 g. The bar is originally at rest at the midpoint of the rails and is free to slide without friction. When the switch is closed, electric current is quickly established in the circuit *ABCDEA*. The rails and bar have low electric resistance, and the current is limited to a constant 24.0 A by the power supply. (a) Find the magnitude of the magnetic field 1.75 cm from a single very long straight wire carrying current 24.0 A. (b) Find the magnitude and direction of the magnetic field at point *C* in the diagram, the midpoint of the bar, immediately after the switch is closed. *Suggestion:* Consider what conclusions you can draw from the Biot–Savart law. (c) At other points along the bar *BD*, the field is in the same direction as at point *C*, but larger in magnitude. Assume that the average effective magnetic field along *BD* is five times larger than the field at *C*. With this assumption, find the magnitude and direction of the force on the bar. (d) Find the acceleration of the bar when it is in motion. (e) Does the bar move with constant acceleration? (f) Find the velocity of the bar after it has traveled 130 cm to the end of the rails.

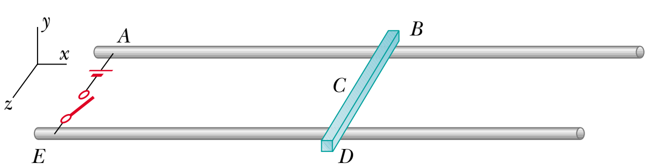


Figure P30.61

62. Fifty turns of insulated wire 0.100 cm in diameter are tightly wound to form a flat spiral. The spiral fills a disk surrounding a circle of radius 5.00 cm and extending to a radius 10.00 cm at the outer edge. Assume the wire carries current *I* at the center of its cross section. Approximate each turn of wire as a circle. Then a loop of current exists at radius 5.05 cm, another at 5.15 cm, and so on. Numerically calculate the magnetic field at the center of the coil.

63. Two long, parallel conductors carry currents in the same direction as shown in Figure P30.63. Conductor A carries a current of 150 A and is held firmly in position. Conductor B carries a current I_B and is allowed to slide freely up and down (parallel to A) between a set of nonconducting guides. If the mass per unit length of conductor B is 0.100 g/cm, what value of current I_B will result in equilibrium when the distance between the two conductors is 2.50 cm?

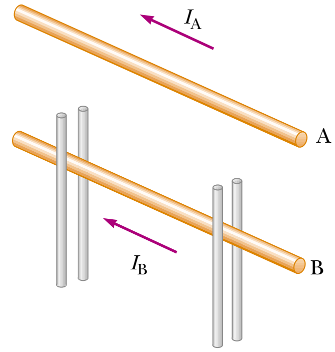


Figure P30.63

64. Charge is sprayed onto a large nonconducting belt above the left-hand roller in Figure P30.64. The belt carries the charge with a uniform surface charge density σ as it moves with a speed v between the rollers as shown. The charge is removed by a wiper at the right-hand roller. Consider a point just above the surface of the moving belt. (a) Find an expression for the magnitude of the magnetic field **B** at this point. (b) If the belt is positively charged, what is the direction of **B**? (Note that the belt may be considered as an infinite sheet.)

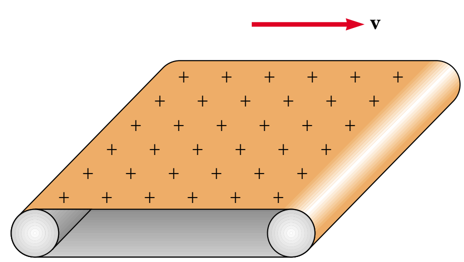


Figure P30.64

65. An infinitely long straight wire carrying a current I_1 is partially surrounded by a loop as shown in Figure P30.65.

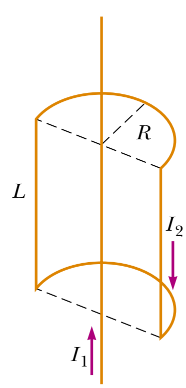


Figure P30.65

The loop has a length L, radius R, and carries a current I_2. The axis of the loop coincides with the wire. Calculate the force exerted on the loop.

66. Measurements of the magnetic field of a large tornado were made at the Geophysical Observatory in Tulsa, Oklahoma, in 1962. The tornado's field was measured to be $B = 1.50 \times 10^{-8}$ T pointing north when the tornado was 9.00 km east of the observatory. What current was carried up or down the funnel of the tornado, modeled as a long straight wire?

67. A wire is formed into the shape of a square of edge length L (Fig. P30.67). Show that when the current in the loop is I, the magnetic field at point P, a distance x from the center of the square along its axis is

$$B = \frac{\mu_0 I L^2}{2\pi(x^2 + L^2/4)\sqrt{x^2 + L^2/2}}$$

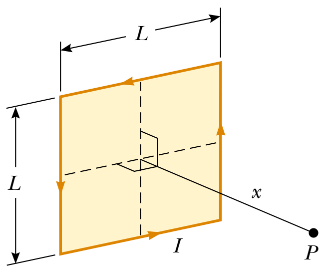


Figure P30.67

68. The force on a magnetic dipole $\boldsymbol{\mu}$ aligned with a nonuniform magnetic field in the x direction is given by $F_x = |\boldsymbol{\mu}|dB/dx$. Suppose that two flat loops of wire each have radius R and carry current I. (a) The loops are arranged coaxially and separated by a variable distance x, large compared to R. Show that the magnetic force between them varies as $1/x^4$. (b) Evaluate the magnitude of this force if $I = 10.0$ A, $R = 0.500$ cm, and $x = 5.00$ cm.

69. A wire carrying a current I is bent into the shape of an exponential spiral, $r = e^{\theta}$, from $\theta = 0$ to $\theta = 2\pi$ as suggested in Figure P30.69. To complete a loop, the ends of the spiral are connected by a straight wire along the x axis. Find the magnitude and direction of $\mathbf{B}$ at the origin. *Suggestions:* Use the Biot–Savart law. The angle β between a radial line and its tangent line at any point on the curve $r = f(\theta)$ is related to the function in the following way:

$$\tan \beta = \frac{r}{dr/d\theta}$$

Thus in this case $r = e^{\theta}$, $\tan \beta = 1$ and $\beta = \pi/4$. Therefore, the angle between $d\mathbf{s}$ and $\hat{\mathbf{r}}$ is $\pi - \beta = 3\pi/4$. Also

$$ds = \frac{dr}{\sin(\pi/4)} = \sqrt{2}\, dr$$

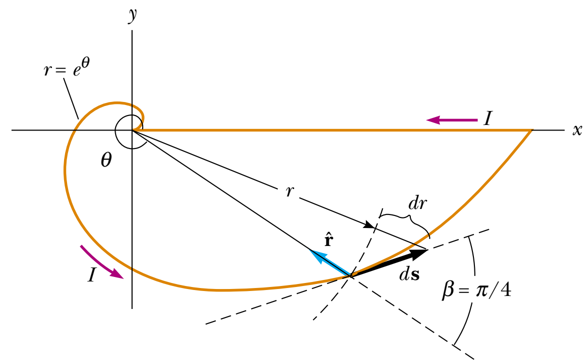


Figure P30.69

70. Table P30.70 contains data taken for a ferromagnetic material. (a) Construct a magnetization curve from the data. Remember that $\mathbf{B} = \mathbf{B}_0 + \mu_0\mathbf{M}$. (b) Determine the ratio B/B_0 for each pair of values of B and B_0, and construct a graph of B/B_0 versus B_0. (The fraction B/B_0 is called the relative permeability, and it is a measure of the induced magnetic field.)

Table P30.70

B (T)	B_0 (T)
0.2	4.8×10^{-5}
0.4	7.0×10^{-5}
0.6	8.8×10^{-5}
0.8	1.2×10^{-4}
1.0	1.8×10^{-4}
1.2	3.1×10^{-4}
1.4	8.7×10^{-4}
1.6	3.4×10^{-3}
1.8	1.2×10^{-1}

71. A sphere of radius R has a uniform volume charge density ρ. Determine the magnetic field at the center of the sphere when it rotates as a rigid object with angular speed ω about an axis through its center (Fig. P30.71).

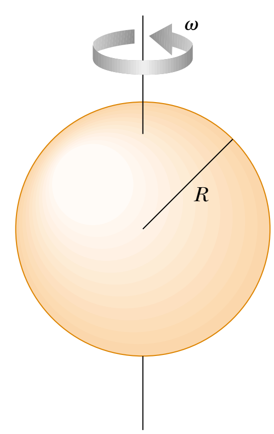


Figure P30.71 Problems 71 and 72.

72. A sphere of radius R has a uniform volume charge density ρ. Determine the magnetic dipole moment of the sphere when it rotates as a rigid body with angular speed ω about an axis through its center (Fig. P30.71).

73. A long cylindrical conductor of radius a has two cylindrical cavities of diameter a through its entire length, as shown in Figure P30.73. A current I is directed out of the page and is uniform through a cross section of the conductor. Find the magnitude and direction of the magnetic field in terms of μ_0, I, r, and a at (a) point P_1 and (b) point P_2.

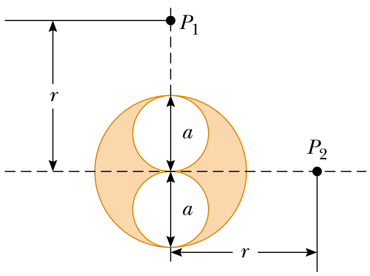


Figure P30.73

Answers to Quick Quizzes

30.1 *B*, *C*, *A*. Point *B* is closest to the current element. Point *C* is farther away and the field is further reduced by the $\sin\theta$ factor in the cross product $d\mathbf{s} \times \hat{\mathbf{r}}$. The field at *A* is zero because $\theta = 0$.

30.2 (c). $F_1 = F_2$ as required by Newton's third law. Another way to arrive at this answer is to realize that Equation 30.11 gives the same result whether the multiplication of currents is (2 A)(6 A) or (6 A)(2 A).

30.3 (a). The coils act like wires carrying parallel currents in the same direction and hence attract one another.

30.4 *b*, *d*, *a*, *c*. Equation 30.13 indicates that the value of the line integral depends only on the net current through each closed path. Path *b* encloses 1 A, path *d* encloses 3 A, path *a* encloses 4 A, and path *c* encloses 6 A.

30.5 *b*, then $a = c = d$. Paths *a*, *c*, and *d* all give the same nonzero value $\mu_0 I$ because the size and shape of the paths do not matter. Path *b* does not enclose the current, and hence its line integral is zero.

30.6 (c). The magnetic field in a very long solenoid is independent of its length or radius. Overwrapping with an additional layer of wire increases the number of turns per unit length.

30.7 (b). There can be no conduction current because there is no conductor between the plates. There is a time-varying electric field because of the decreasing charge on the plates, and the time-varying electric flux represents a displacement current.

30.8 (c). There is a time-varying electric field because of the decreasing charge on the plates. This time-varying electric field produces a magnetic field.

30.9 (a). The loop that looks like Figure 30.32a is better because the remanent magnetization at the point corresponding to point *b* in Figure 30.31 is greater.

30.10 (b). The lines of the Earth's magnetic field enter the planet in Hudson Bay and emerge from Antarctica; thus, the field lines resulting from the current would have to go in the opposite direction. Compare Figure 30.7a with Figure 30.36.