Chapter 30

Sources of the Magnetic Field

Outline

- 30.1 The Biot-Savart Law
- 30.2 The Magnetic Force Between Two Parallel Conductors
- 30.3 Ampère’s Law
- 30.4 The magnetic Field of a Solenoid
- 30.5 Magnetic Flux
- 30.6 Gauss’s Law in Magnetism

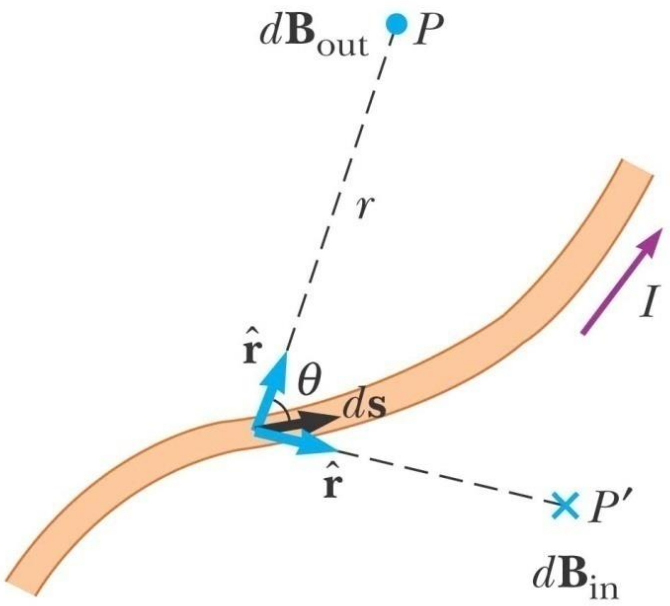
Introduction

- **In this chapter, we show how to use the law of Biot and Savart to calculate the magnetic field produced at some point in space by a small current element**

30.1 The Biot–Savart Law

- Jean-Baptiste Biot (1774–1862) and Félix Savart (1791–1841) performed quantitative experiments on the force exerted by an electric current on a nearby magnet. From their experimental results, Biot and Savart arrived at a mathematical expression that gives the magnetic field at some point in space in terms of the current that produces the field

30.1 The Biot–Savart Law



The magnetic field dB at a point due to the current I through a length element ds is given by the Biot–Savart law. The direction of the field is out of the page at P and into the page at P.

- The vector dB is perpendicular both to ds (which points in the direction of the current) and to the unit vector directed from ds to P.
- The magnitude of dB is inversely proportional to r 2, where r is the distance from ds to P.
- The magnitude of dB is proportional to the current and to the magnitude ds of the length element ds.
- The magnitude of dB is proportional to sin , where is the angle between the vectors ds and ˆr.

30.1 The Biot–Savart Law

These observations are summarized in the mathematical formula known today as the **Biot–Savart law:**

$$d\mathbf{B} = \frac{\mu_0}{4\pi} \frac{I\, d\mathbf{s} \times \hat{\mathbf{r}}}{r^2} \qquad (30.1)$$

where μ_0 is a constant called the **permeability of free space:**

$$\mu_0 = 4\pi \times 10^{-7}\ \mathrm{T \cdot m/A} \qquad (30.2)$$

To find the total magnetic field B created at some point by a current of finite size, we must sum up contributions from all current elements *Ids that make up the current. That is, we* must evaluate B by integrating Equation 30.1

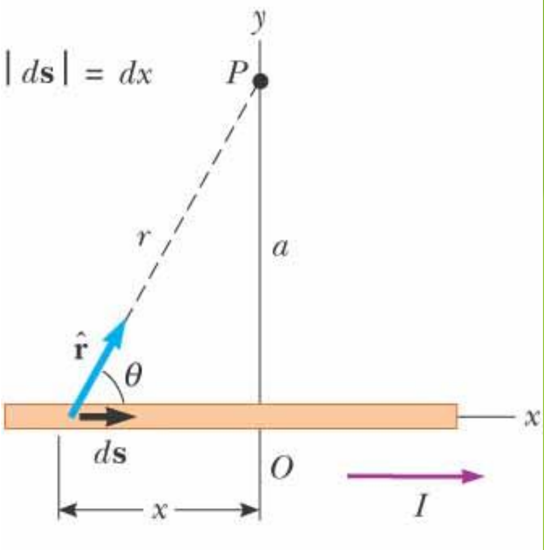
$$\vec{\mathbf{B}} = \frac{\mu_o}{4\pi} \int \frac{I d\vec{\mathbf{s}} \times \hat{\mathbf{r}}}{r^2}$$

Magnetic Field Surrounding a Thin, Straight Conductor

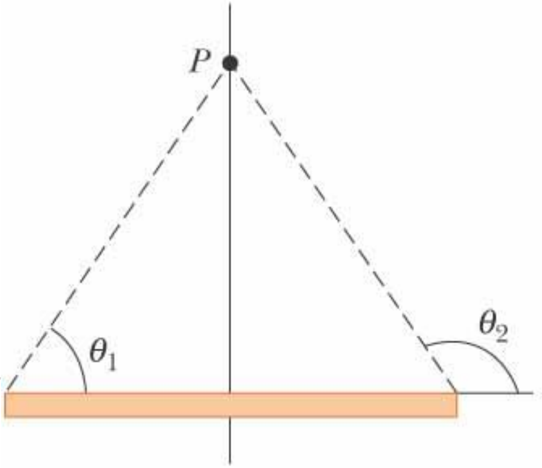
Consider a thin, straight wire carrying a constant current I and placed along the x axis as shown in the Figure

Determine the magnitude and direction of the magnetic field at point P due to this current

$$B = \frac{\mu_o I}{2\pi a}$$



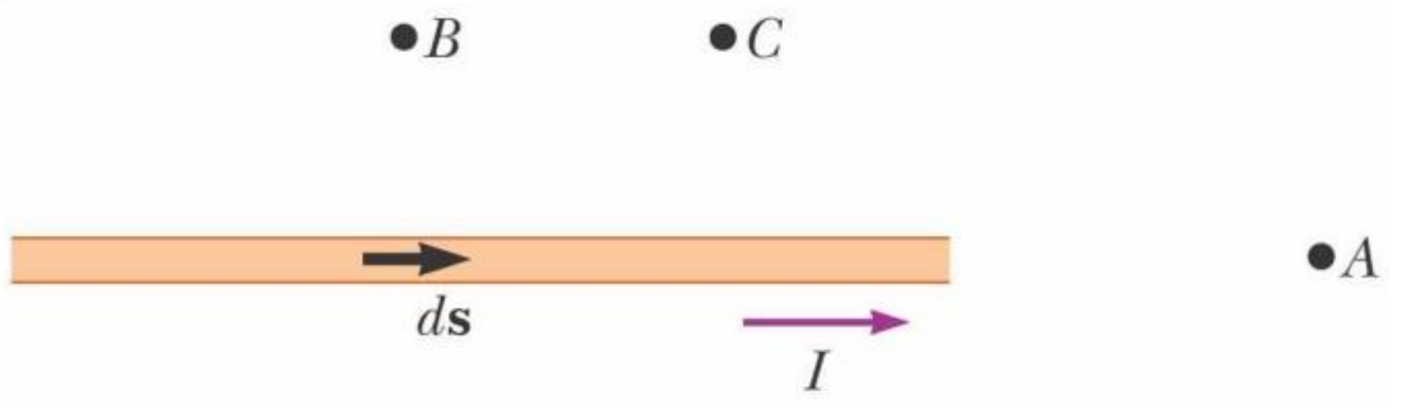
(a)



(b)

Quick Quiz 30.1

Consider the current in the length of wire shown in the figure below. Rank the points *A*, *B*, and *C* in terms of magnitude of the magnetic field due to the current in the length element shown, from greatest to least.



(a) A, B, C

(b) B, C, A

(c) C, B, A

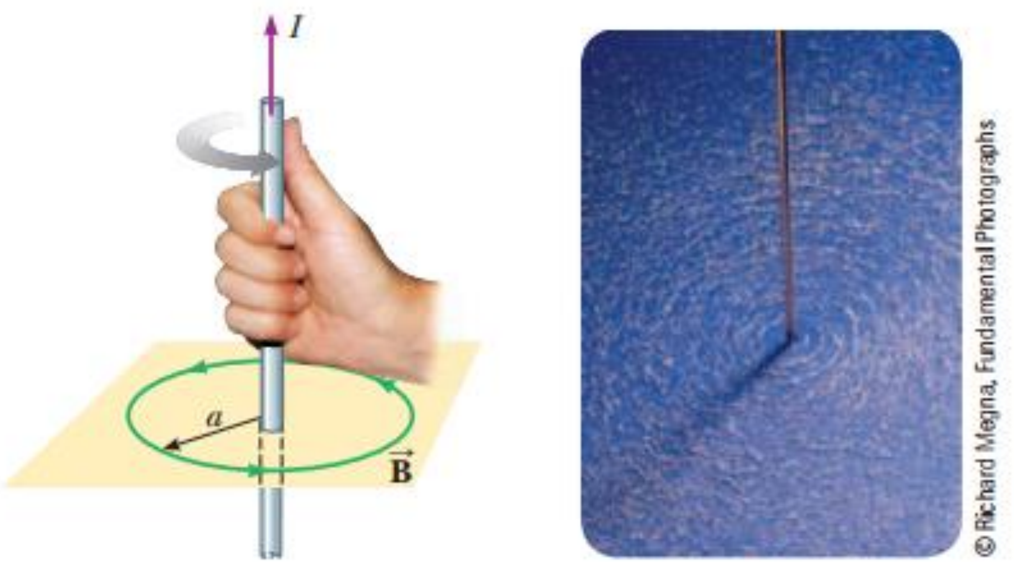
(d) C, A, B

(e) An equal field applies at all these points.

The right-hand rule for determining the direction of the magnetic field surrounding a long, straight wire carrying a current

Point the thumb of your right hand along a wire in the direction of positive current, as in Figure 19.24a. Your fingers then naturally curl in the direction of the magnetic field $\vec{\mathbf{B}}$.

When the current is reversed, the filings in Figure 19.24b also reverse.



(a) (b)

Example

A long, straight wire carries a current of 5.00 A. At one instant, a proton, 4.00 mm from the wire, travels at a speed of 1.50×10^3 m/s parallel to the wire and in the same direction as the current.

(a) Find the magnitude and direction of the magnetic field created by the wire.

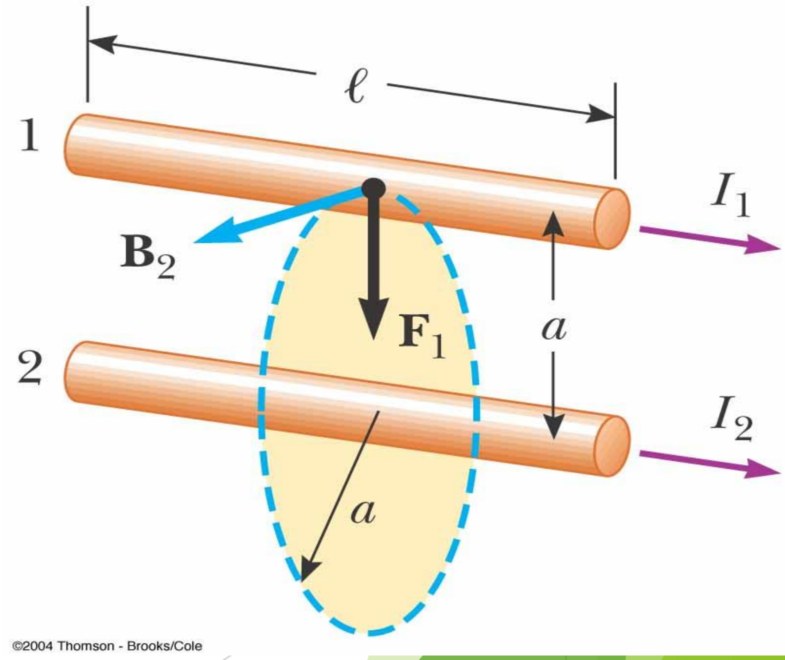
(b)Find the magnitude and direction of the magnetic force the wire's magnetic field exerts on the proton.

30.2 The Magnetic Force Between Two Parallel Conductors

Two current-carrying conductors exert magnetic forces on each other.

$$F_1 = I_1 \ell B_2 = I_1 \ell \left(\frac{\mu_0 I_2}{2\pi a} \right) = \frac{\mu_0 I_1 I_2}{2\pi a} \ell$$

Parallel conductors carrying currents in the same direction *attract each other*, and parallel conductors carrying currents in opposite directions *repel each other.*



30.2 The Magnetic Force Between Two Parallel Conductors

Because the magnitudes of the forces are the same on both wires, we denote the magnitude of the magnetic force between the wires as simply F_B .

$$\frac{F_B}{\ell} = \frac{\mu_o I_1 I_2}{2\pi a}$$

Definition of the Ampere

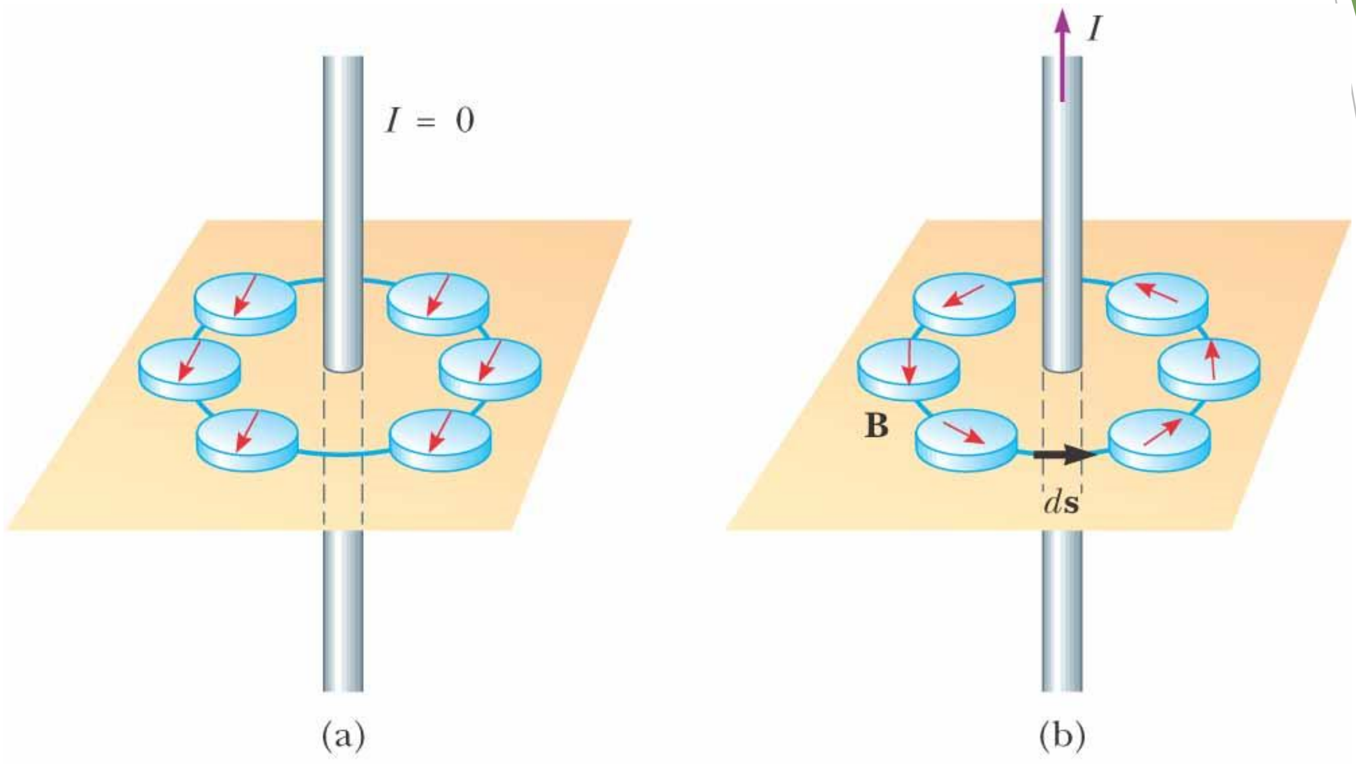
- **When the magnitude of the force per unit length between two long parallel wires that carry identical currents and are separated by 1 m is 2 x 10^{-7} N/m, the current in each wire is defined to be 1 A**

30.2 The Magnetic Force Between Two Parallel Conductors

Definition of the Coulomb in terms of Amperes

When a conductor carries a steady current of 1 A, the quantity of charge that flows through a cross section of the conductor in 1 s is 1 C

30.3 Ampere's Law



(a) When no current is present in the wire, *all compass needles point in the same direction* (toward the Earth's north pole).
(b) When the wire carries a strong current, the compass needles *deflect in a direction tangent to the circle*, which is the direction of the magnetic field created by the current.

30.3 Ampere's Law

$$\oint \mathbf{B} \cdot d\mathbf{s} = B \oint ds = \frac{\mu_0 I}{2\pi r}(2\pi r) = \mu_0 I$$

The line integral of $\mathbf{B} \cdot d\mathbf{s}$ around any closed path equals $\mu_0 I$, where I is the total continuous current passing through any surface bounded by the closed path.

$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I \qquad (30.13)$$

Ampere's law

Ampere's law describes the creation of magnetic fields by all continuous current configurations but at our mathematical level it is useful only for calculating the magnetic field of current configurations having a high degree of symmetry.

30.3 Ampere's Law

- ***Quick Quiz 30.4***

Rank the magnitudes of $\oint B \cdot ds$ for the closed paths in the figure below, from least to greatest.

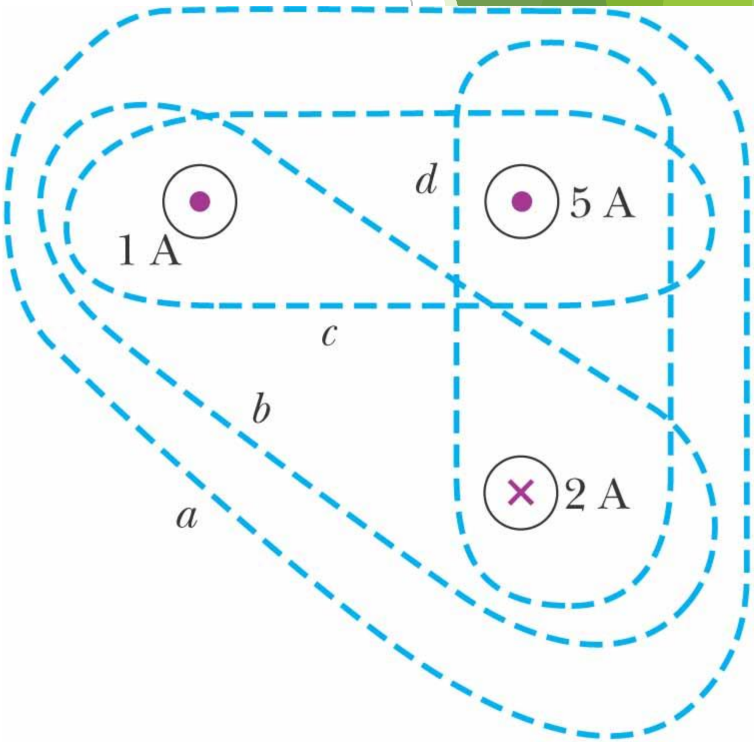
1/ a, b, c, d

2/ b, d, a, c

3/ c, d, b, a

4/ c, b, a, d

5/ d, c, a, b

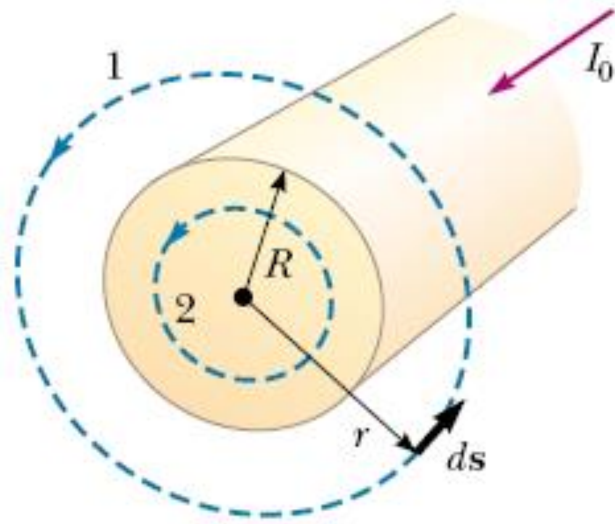


Example: The Magnetic Field Created by a Long Current-Carrying Wire

A long, straight wire of radius R carries a steady current I_0 that is uniformly distributed through the cross-section of the wire (Fig. 30.11). Calculate the magnetic field a distance r from the center of the wire in the regions $r \geq R$ and $r < R$.

- Because the wire has a high degree of symmetry, we categorize this as an Ampère's law problem

- Let us choose for our path of integration circle 1 in the figure. From symmetry, B must be constant in magnitude and parallel to *ds at every* point on this circle. Because the total current passing through the plane of the circle is I_0, *Ampère's law gives*

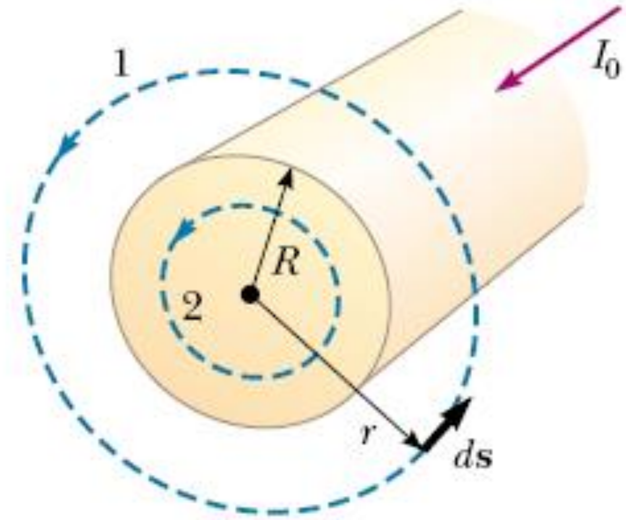


$$\oint \mathbf{B} \cdot d\mathbf{s} = B \oint ds = B(2\pi r) = \mu_0 I_0$$

$$B = \frac{\mu_0 I_0}{2\pi r} \qquad (\text{for } r \geq R)$$

Example: The Magnetic Field Created by a Long Current-Carrying Wire

Now consider the interior of the wire, where r <R. Here the current I passing through the plane of circle 2 is less than the total current I_0 . Because the current is uniform over the cross-section of the wire, the fraction of the current enclosed by circle 2 must equal the ratio of the area r^2 enclosed by circle 2 to the cross-sectional area R^2 of the wire:



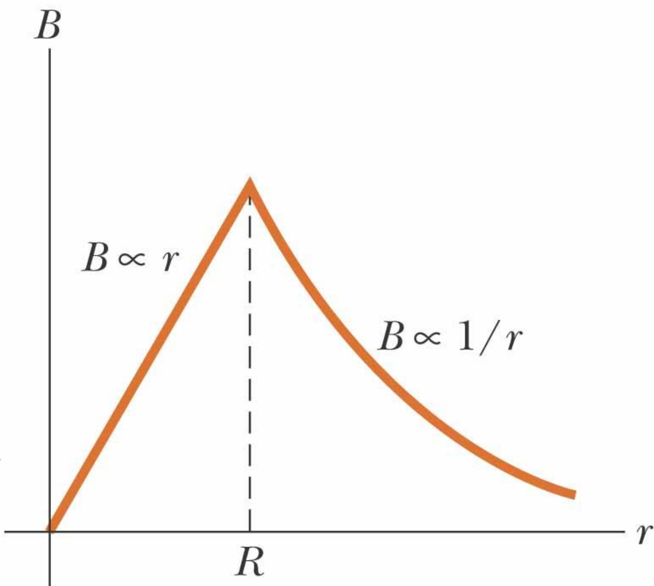
$$\frac{I}{I_0} = \frac{\pi r^2}{\pi R^2}$$

$$\oint \mathbf{B} \cdot d\mathbf{s} = B(2\pi r) = \mu_0 I = \mu_0\left(\frac{r^2}{R^2} I_0\right)$$

$$I = \frac{r^2}{R^2} I_0$$

$$B = \left(\frac{\mu_0 I_0}{2\pi R^2}\right) r \qquad (\text{for } r < R)$$

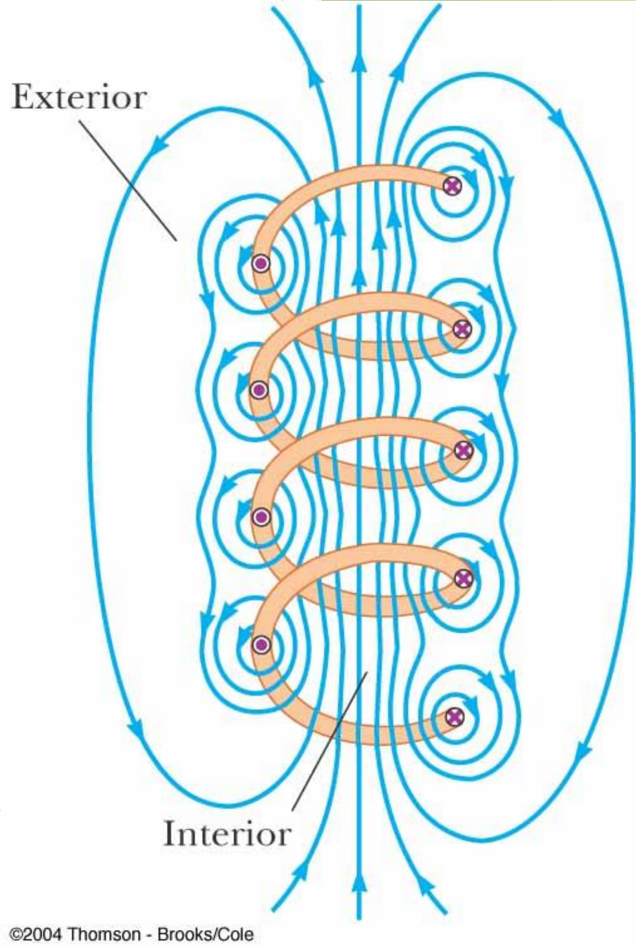
$$B = \frac{\mu_0 I_0}{2\pi r} \qquad (\text{for } r \geq R)$$



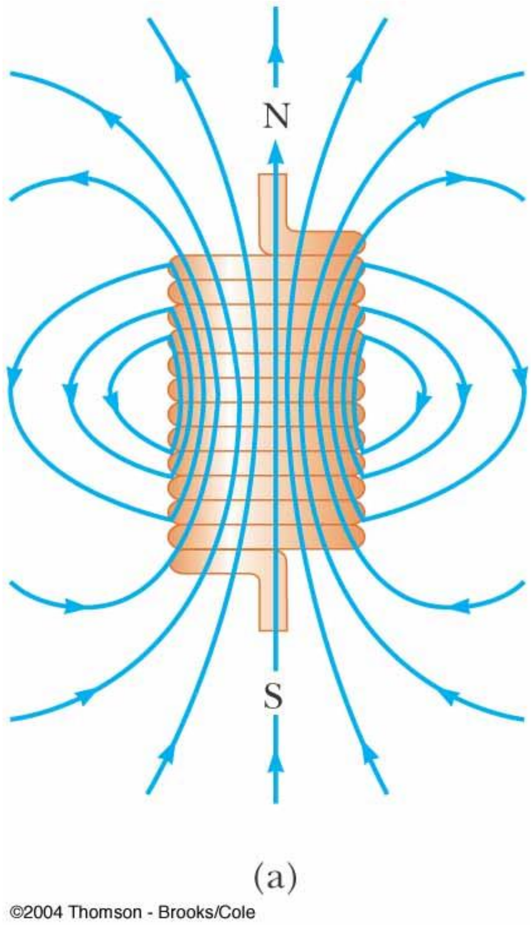
30.4 The magnetic Field of a Solenoid

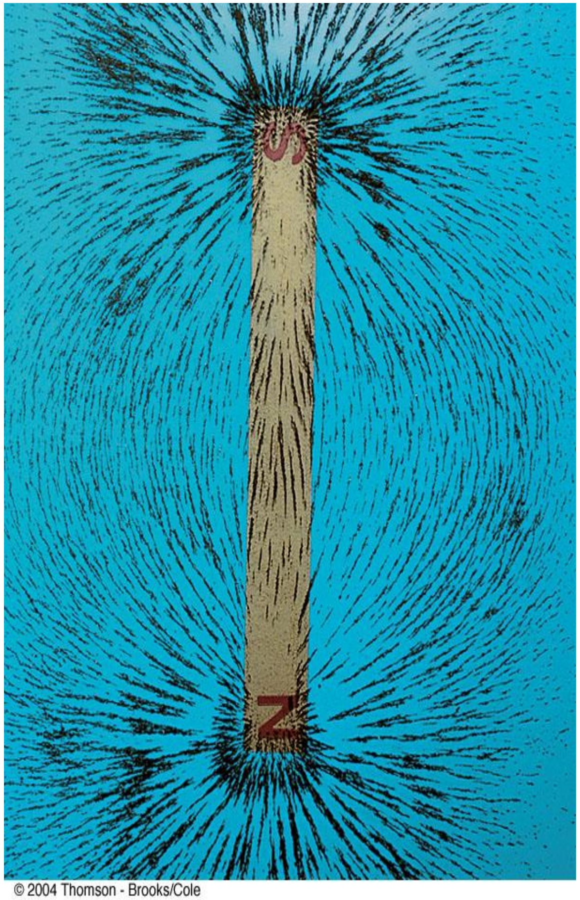
A Solenoid is a long wire wound in the form of a helix. With this configuration, a reasonably uniform magnetic field can be produced in the space surrounded by the turns of wire

- **The field lines in the interior are nearly parallel to one another, are uniformly distributed, and are close together, *indicating that the field in this space is uniform and strong*.**



30.4 The magnetic Field of a Solenoid





(a) Magnetic field lines for a tightly wound solenoid of finite length, carrying a steady current. The field in the interior space is strong and nearly uniform. Note that the field lines resemble those of a bar magnet, meaning that the solenoid effectively has north and south poles. (b) The magnetic field pattern of a bar magnet, displayed with small iron filings on a sheet of paper.

30.4 The magnetic Field of a Solenoid

- **In *ideal solenoid,* the turns are closely spaced and the length is much greater than the radius of the turns. In this case, the *external field is zero*, and the interior field is uniform over a great volume.**

30.4 The magnetic Field of a Solenoid

- **Consider the rectangular path of length and width w shown in the figure. We can apply Ampere's law to this path by evaluating the integral of B.*ds* over each side of the rectangle.**

$$\oint \vec{B}.\vec{ds} = \int_1 \vec{B}.\vec{ds} + \int_2 \vec{B}.\vec{ds} + \int_3 \vec{B}.\vec{ds} + \int_4 \vec{B}.\vec{ds}$$

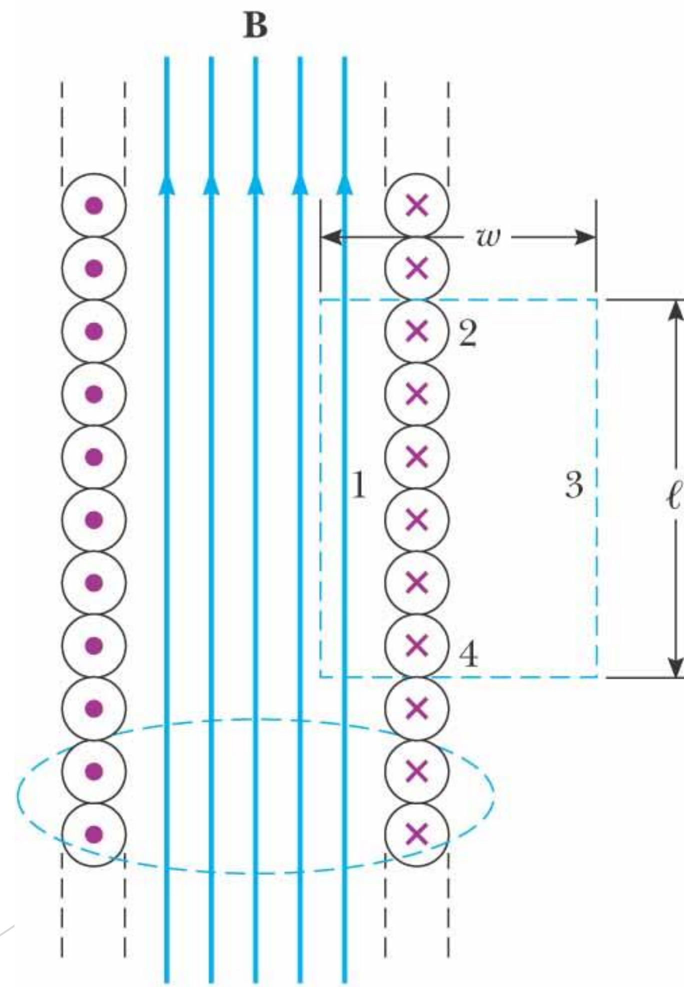
$$\oint \vec{B}.\vec{ds} = Bl + 0 + 0 + 0 = \mu_0 I_{enclosed}$$

$$Bl = \mu_0 NI$$

$$n = \frac{N}{l}$$

n= N/L is the number of turns per unit length

$$B = \mu_0 \frac{N}{\ell} I = \mu_0 n I$$



Example

A certain solenoid consists of 100 turns of wire and has a length of 10.0 cm.

(a) Find the magnitude of the magnetic field inside the solenoid when it carries a current of 0.500 A.

(b) Approximately how much wire would be needed to build this solenoid? Assume the solenoid's radius is 5.00 cm.

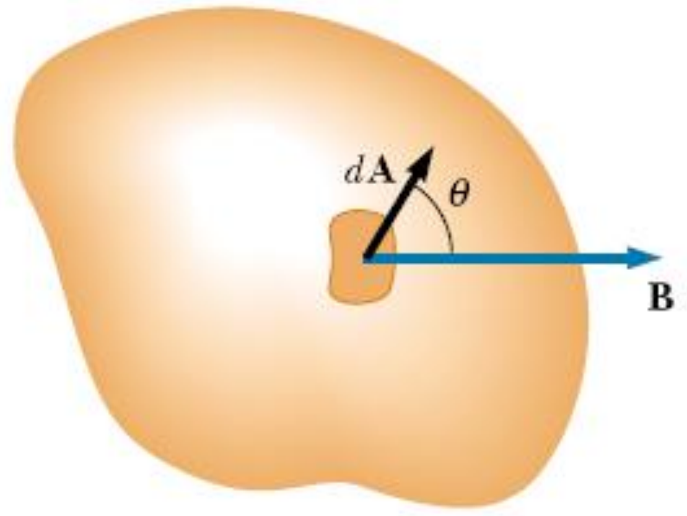
30.5 Magnetic Flux

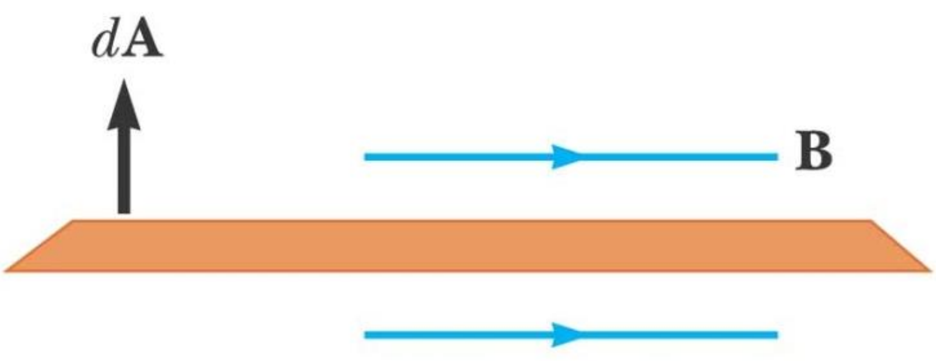
$$\Phi = \int \vec{B}.\vec{ds}$$

Unit: T.m^2 (Wb)

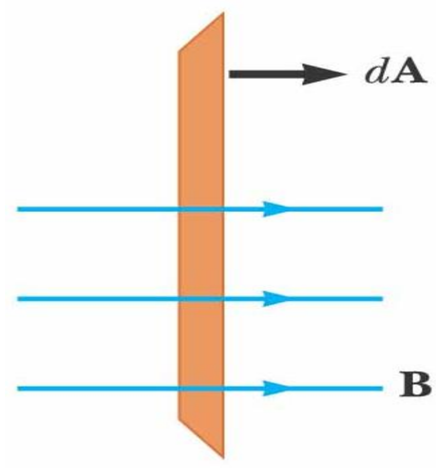
$$for\ \text{constant B and A}$$

$$\Phi = \text{BA} \cos\theta$$





(a)



(b)

$\theta = 90^0$ $\Phi = 0$

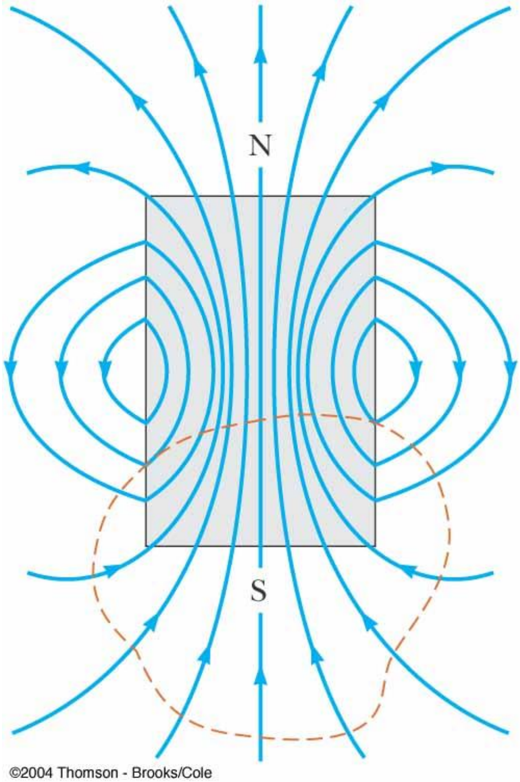
$\theta = 0$ $\Phi = BA$

30.6 Gauss's Law in Magnetism

Gauss's law in magnetism states that

the net magnetic flux through any closed surface is always zero:

$$\oint \mathbf{B} \cdot d\mathbf{A} = 0$$



Problems

1) Two long, parallel conductors, separated by 10.0cm, carry currents in the same direction. The first wire carries current I_1=5.00A and the second carries I_2= 8.00A.

(a) What is the magnitude of the magnetic field created by I_1 at the location of I_2?

(b) What is the force per unit length exerted by I_1 on I_2?

(c) What is the magnitude of the magnetic field created by I_2 at the location of I_1?

(d)What is the force per length exerted by I_2 on I_1?