Chapter 33

Alternating Current Circuits

Outline

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- 33.2 Resistors in an AC Circuit
- 33.3 Inductors in an AC Circuit
- 33.4 Capacitors in an AC Circuit
- 33.5 The RLC Series Circuit
- 33.6 Power in an AC Circuit
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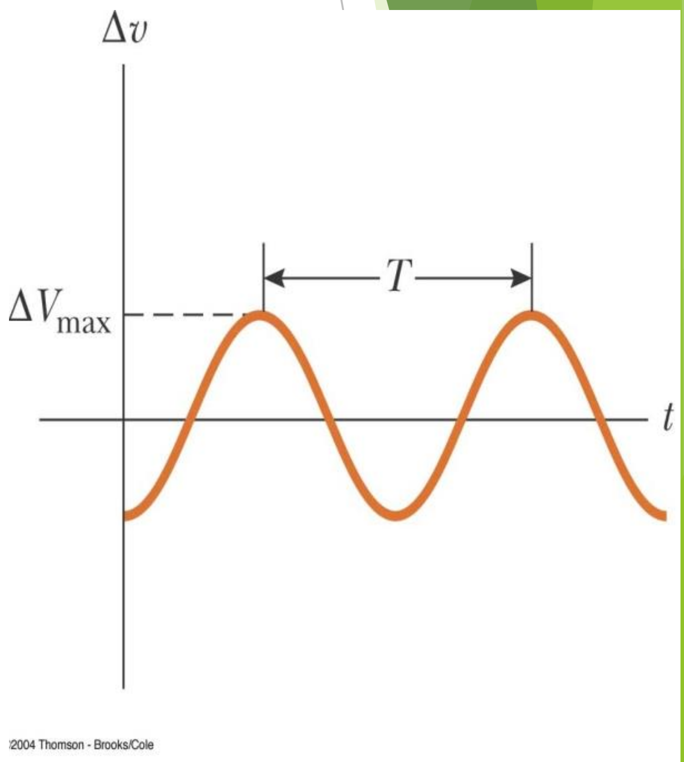
33.1 AC Sources

- **An AC circuit consists of circuit elements and a power source that provides an alternating voltage**

$$\Delta v = \Delta V_{max} \sin \omega t$$

Δv is the alternating voltage
ΔV_{max} is the maximum output voltage of the AC source
(the voltage amplitude)
ω is the angular frequency of the AC voltage



The angular frequency is

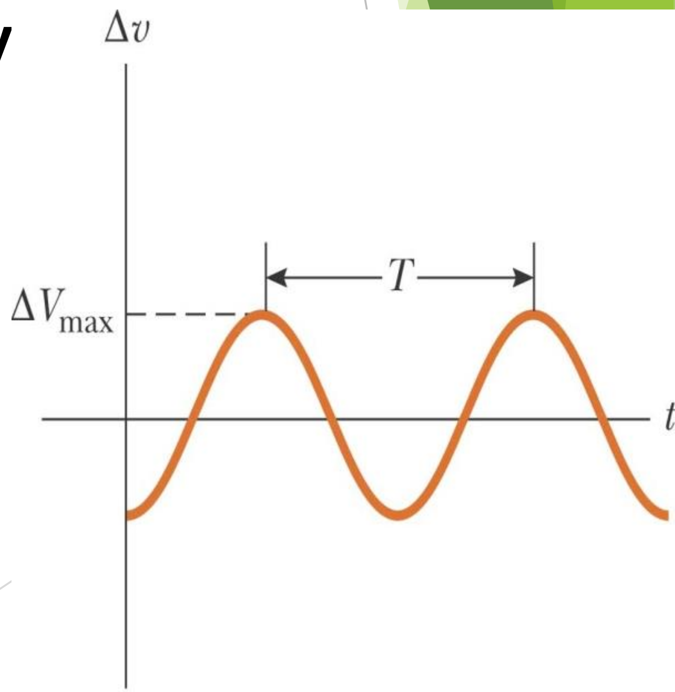
$$\omega = 2\pi f = \frac{2\pi}{T}$$

f is the frequency of the source
T is the period of the source

33.1 AC Sources

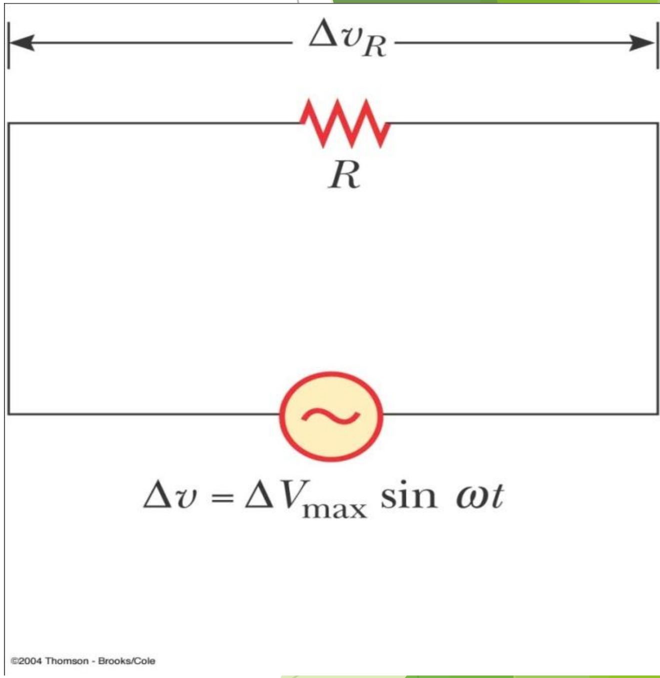
- **Because the output voltage of an AC source varies sinusoidally with time, the voltage is positive during one half of the cycle and negative during the other half**

- **The current in any circuit driven by source is an alternating current that varies sinusoidally with time**



33.2 Resistors in an AC Circuit

- Consider a circuit consisting of an AC source and a resistor



- $\Delta v_R = \Delta v = V_{max} \sin wt$

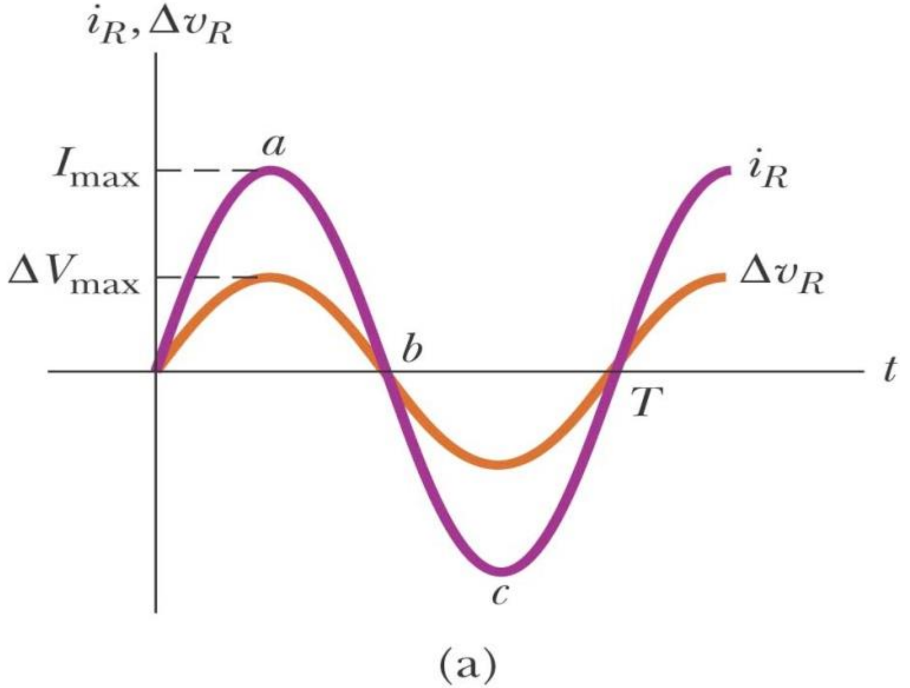
Δv_R is the instantaneous voltage across the resistor

- The *instantaneous current* in the resistor is

$$i_R = \frac{\Delta v_R}{R} = \frac{\Delta V_{max}}{R} \sin \omega t = I_{max} \sin \omega t$$

- The *instantaneous voltage* across the resistor is also given as $\Delta v_R = I_{max} R \sin \omega t$

33.2 Resistors in an AC Circuit

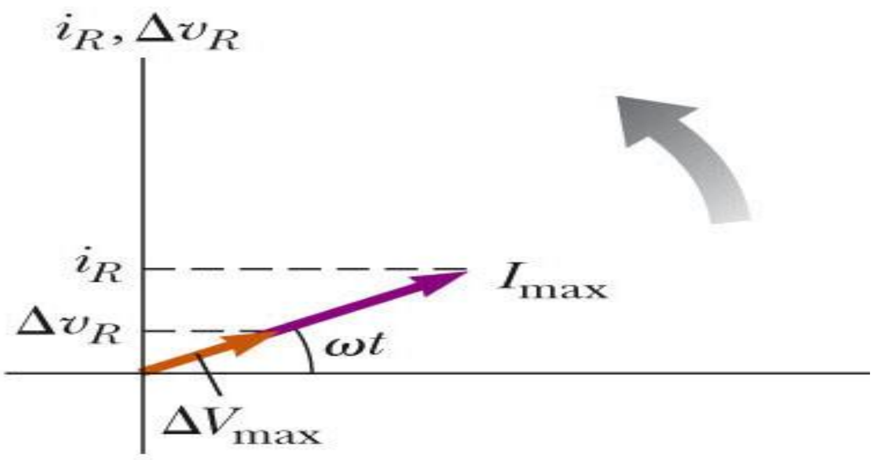


For a sinusoidal applied voltage, the current in a resistor is always *<u>in phase</u>* with the voltage across the resistor.

33.2 Resistors in an AC Circuit

- Resistors behave essentially the same way in both DC and AC circuits

<u>- phasor diagrams</u>



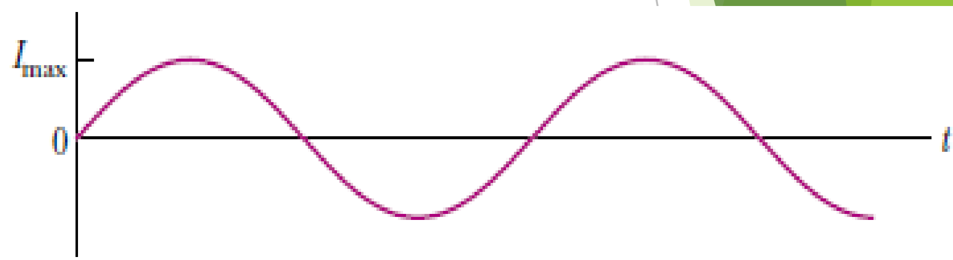
(b)

rms Current and Voltage

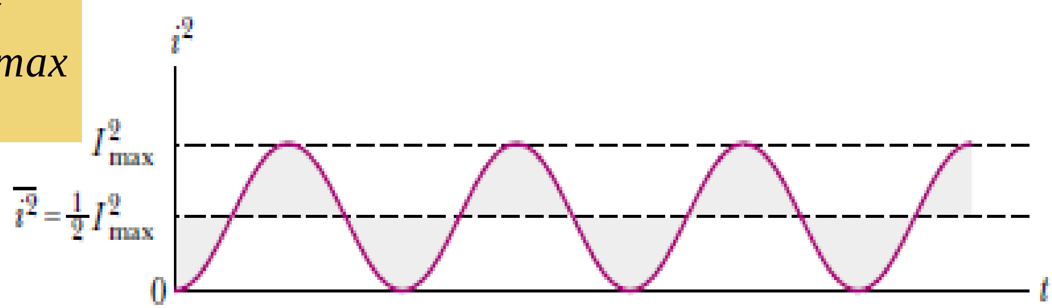
- The average value of the current over one cycle is zero.
- The rms current is the average value of current in an AC circuit
- rms stands for **root-mean-square (average)**

$$I_{rms} = \frac{I_{max}}{\sqrt{2}} = 0.707\, I_{max}$$

$$\Delta V_{rms} = \frac{\Delta V_{max}}{\sqrt{2}} = 0.707\, \Delta V_{max}$$



(a)



(b)

- rms values are used when discussing alternating currents and voltages because AC ammeters and voltmeters are designed to read rms values

- The average power delivered to a resistor that carries an alternating current is

$$\mathscr{P}_{\mathrm{av}} = I_{\mathrm{rms}}^2 R$$

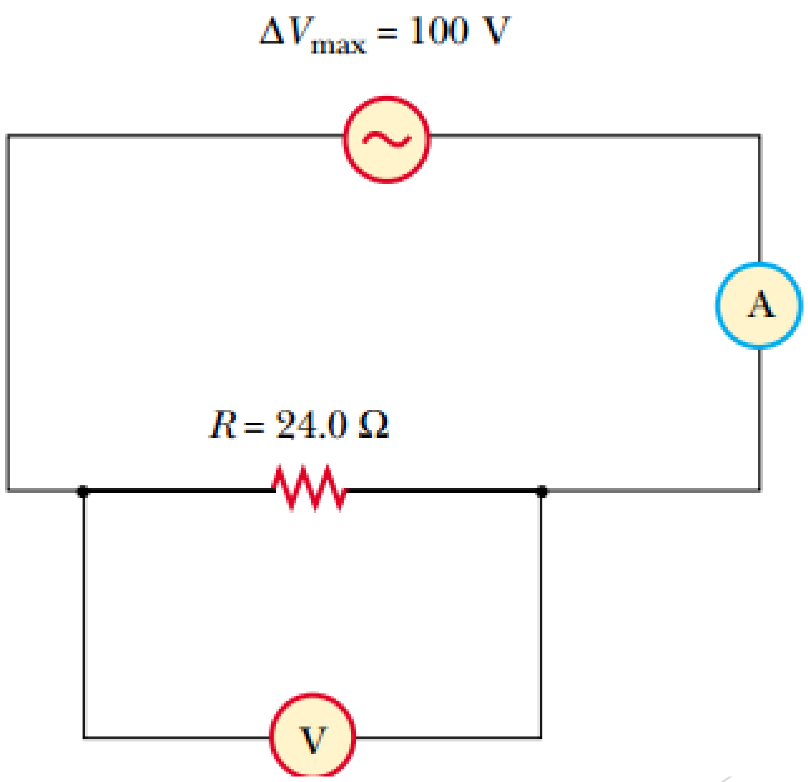
Example

1) The voltage output of an AC source is given by the expression $\Delta v = (200V)\sin \omega t$. Find the rms current in the circuit when this source is connected to a 100 Ωresistor

2- For a particular device, the house ac voltage is 120-V and the ac current is 10 A. What are their maximum values?

Problems:

3. An AC power supply produces a maximum voltage $\Delta V_{max} = 100$ V. This power supply is connected to a 24.0-Ω resistor, and the current and resistor voltage are measured with an ideal AC ammeter and voltmeter, as shown in Figure P33.3. What does each meter read? Note that an ideal ammeter has zero resistance and that an ideal voltmeter has infinite resistance.

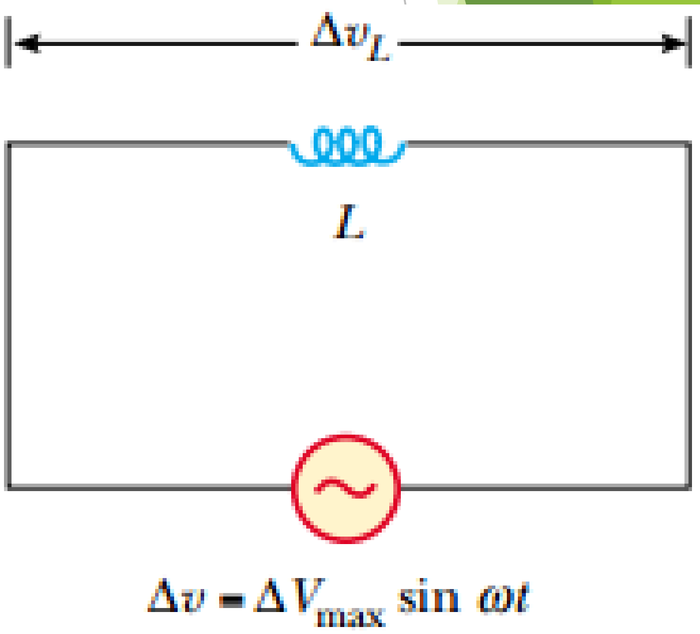


33.3 Inductors in an AC Circuit

- Consider a circuit consisting of an AC source and an inductor

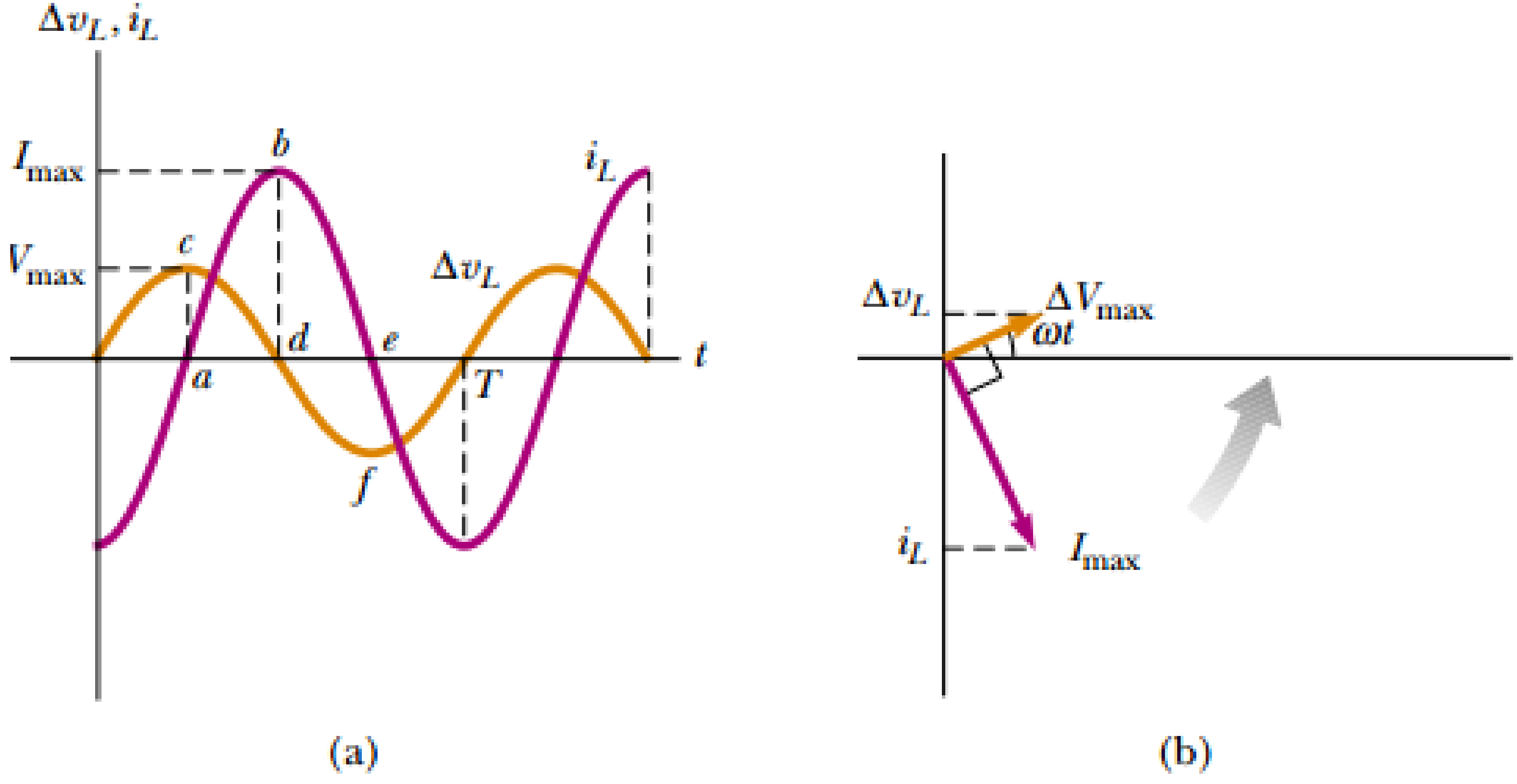
- The *instantaneous current* in the inductor

$$i_L = \frac{\Delta V_{\max}}{\omega L} \sin\left(\omega t - \frac{\pi}{2}\right)$$



The instantaneous current i_L in the inductor and the instantaneous voltage Δv_L across the inductor are *out of phase* by $(\frac{\pi}{2})$ rad= 90°.

33.3 Inductors in an AC Circuit



For a sinusoidal applied voltage, the current in an inductor always *lags behind the voltage* across the inductor by 90°

33.3 Inductors in an AC Circuit

- Maximum current in an inductor

$$I_{\max} = \frac{\Delta V_{\max}}{\omega L}$$

- The inductive reactance

$$X_L \equiv \omega L$$

$$I_{\max} = \frac{\Delta V_{\max}}{X_L}$$

- The *instantaneous voltage* in the inductor

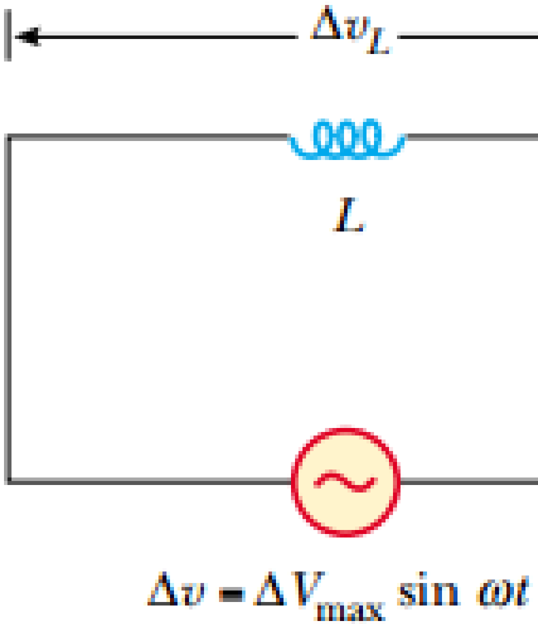
$$\Delta v_L = -L\,\frac{di}{dt} = -\Delta V_{\max}\sin\omega t = -I_{\max}X_L\sin\omega t$$

Example 33.2

In a purely inductive AC circuit (see Fig below), L= 25.0mH

and the rms voltage is 150V. Calculate the inductive reactance

and rms current in the circuit if the frequency is 60.0Hz

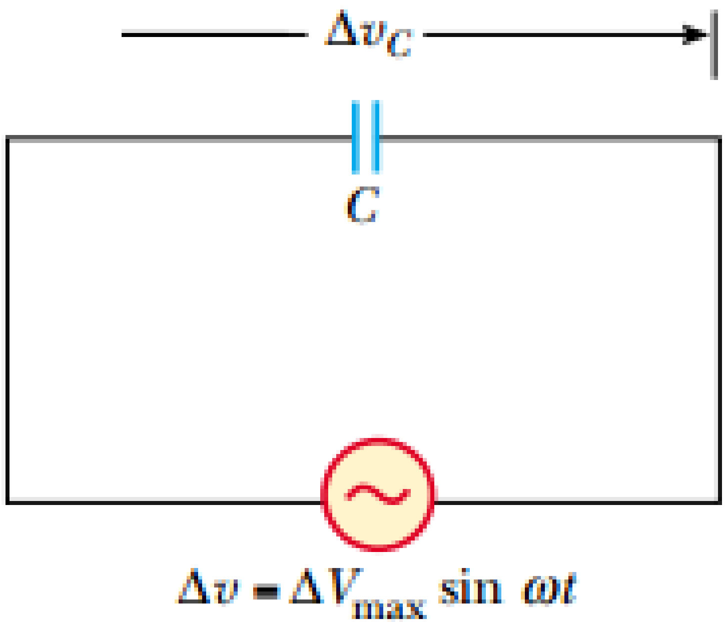


33.4 Capacitors in an AC Circuit

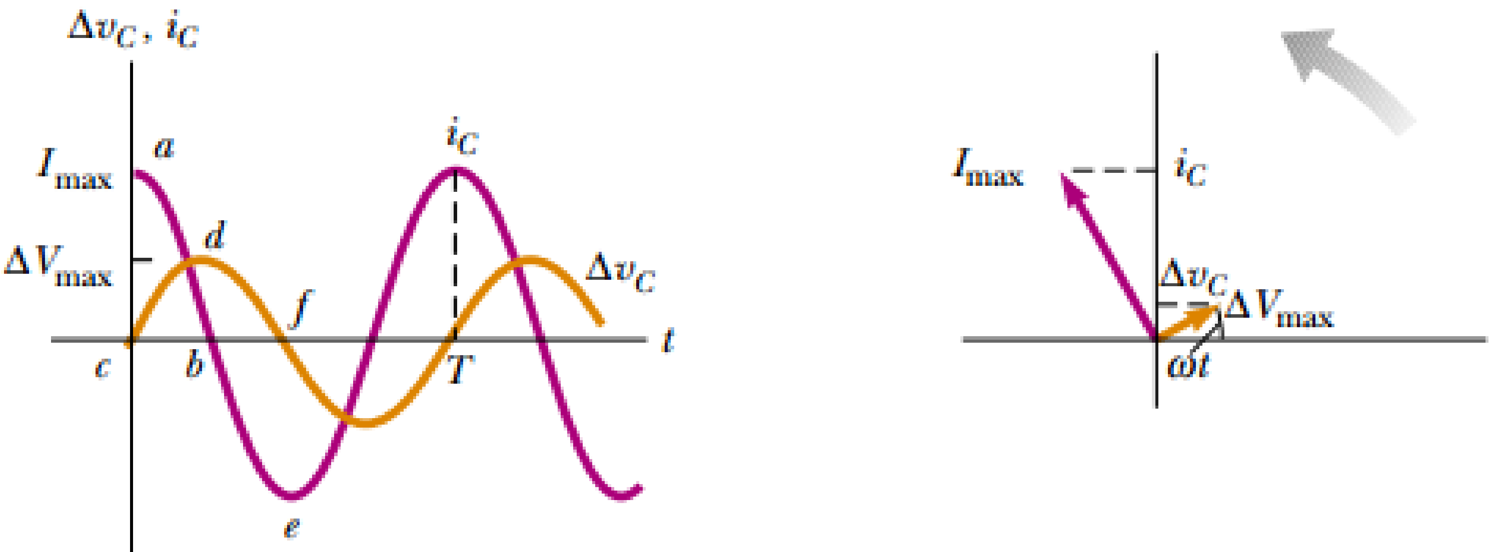
- Consider a circuit consisting of an AC source and a capacitor

- The *instantaneous current* in the capacitor

$$i_C = \omega C \Delta V_{max} \sin\left(\omega t + \frac{\pi}{2}\right)$$



33.4 Capacitors in an AC Circuit



(a) (b)

For a sinusoidally applied voltage, the current always *leads the* voltage across a capacitor by 90°

33.4 Capacitors in an AC Circuit

- Maximum current in a capacitor

$$I_{\max} = \omega C \Delta V_{\max} = \frac{\Delta V_{\max}}{(1/\omega C)}$$

- The capacitive reactance

$$X_C \equiv \frac{1}{\omega C}$$

$$I_{\max} = \frac{\Delta V_{\max}}{X_C}$$

- The *instantaneous voltage* in the capacitor

$$\Delta v_C = \Delta V_{\max} \sin \omega t = I_{\max} X_C \sin \omega t$$

Quick Quiz 33.6

Consider the AC circuit in the Figure below,

The frequency of the AC source is adjusted

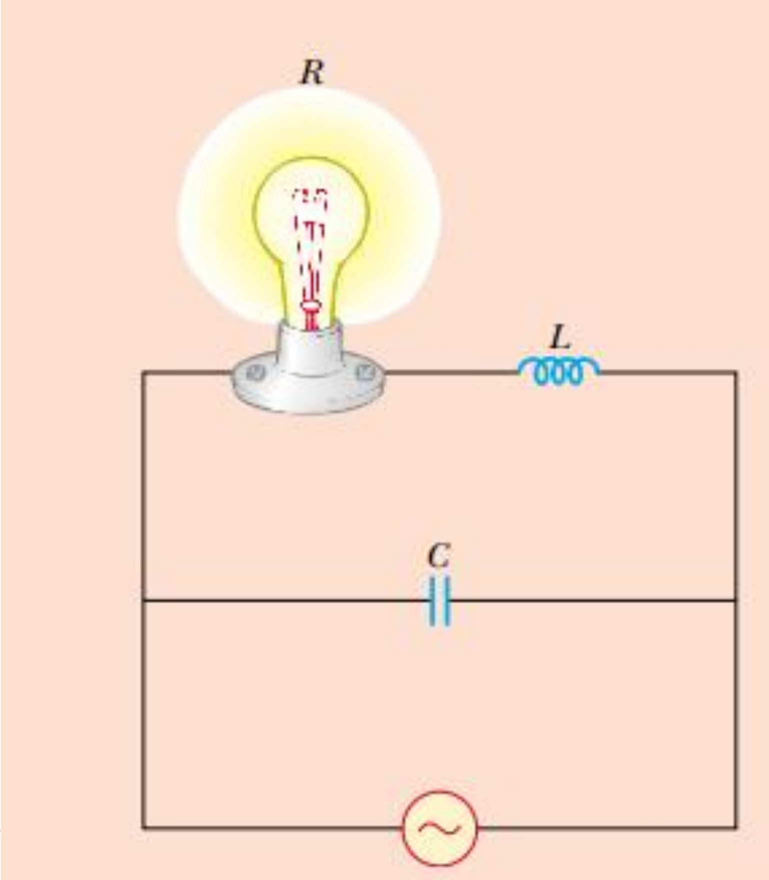
while its voltage amplitude is held constant.

The lightbulb will glow the brightest at

(a) high frequencies

(b) low frequencies

(c) The brightness will be same at all frequencies.

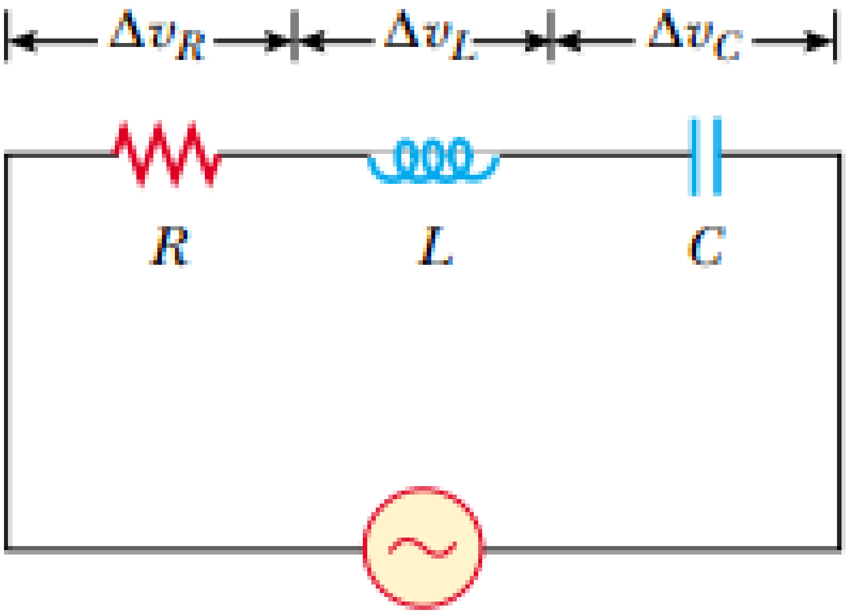


Example 33.3

An 8.00 μF capacitor is connected to the terminals of a 60.0-Hz AC source whose rms voltage is 150V. Find the capacitive reactance and the rms current in the circuit

33.5 The RLC Series Circuit

- **The current at all points in a series AC circuit has the same amplitude and phase.**



- **The instantaneous applied voltage**

$$\Delta v = \Delta V_{max} \sin \omega t$$

- **The current**

$$i = I_{\max} \sin(\omega t - \phi)$$

∅ is Phase Angle between current and voltage

33.5 The RLC Series Circuit

The *instantaneous voltage* across the three circuit elements

$$\Delta v_R = I_{\max} R \sin \omega t = \Delta V_R \sin \omega t$$

$$\Delta v_L = I_{\max} X_L \sin\left(\omega t + \frac{\pi}{2}\right) = \Delta V_L \cos \omega t$$

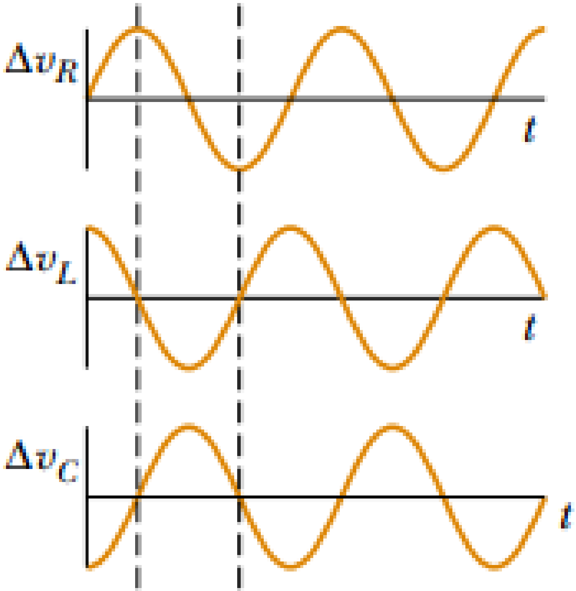
$$\Delta v_C = I_{\max} X_C \sin\left(\omega t - \frac{\pi}{2}\right) = -\Delta V_C \cos \omega t$$

- **The *maximum voltage* across the three circuit elements**

$$\Delta V_R = I_{max} R$$

$$\Delta V_L = I_{max} X_L$$

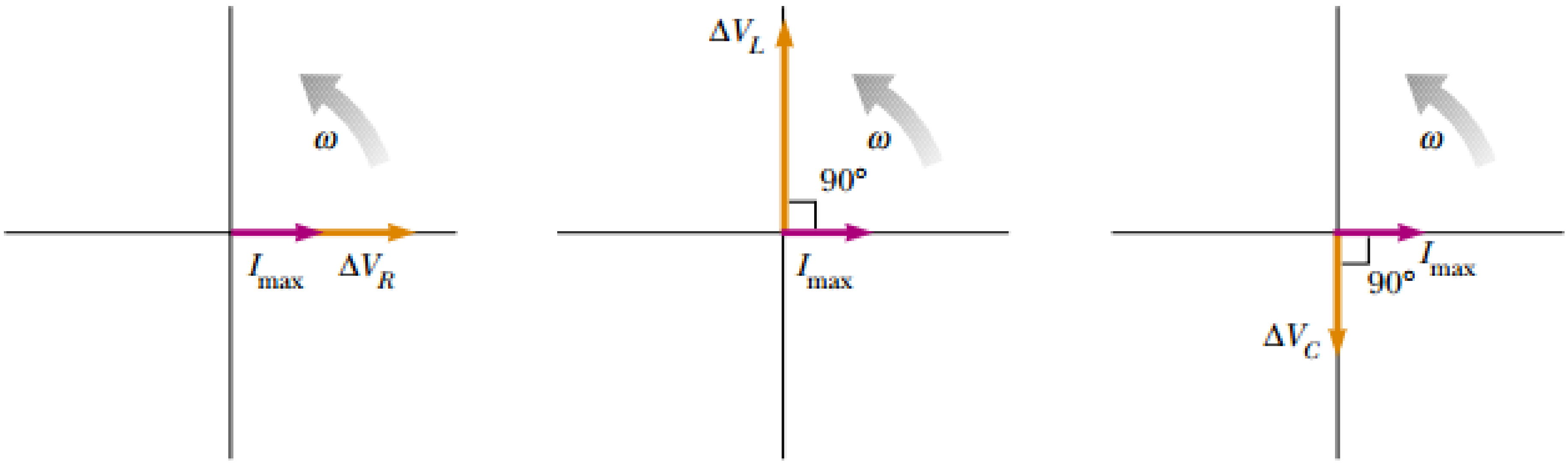
$$\Delta V_C = I_{max} X_c$$



33.5 The RLC Series Circuit

The *instantaneous voltage* across the three circuit elements

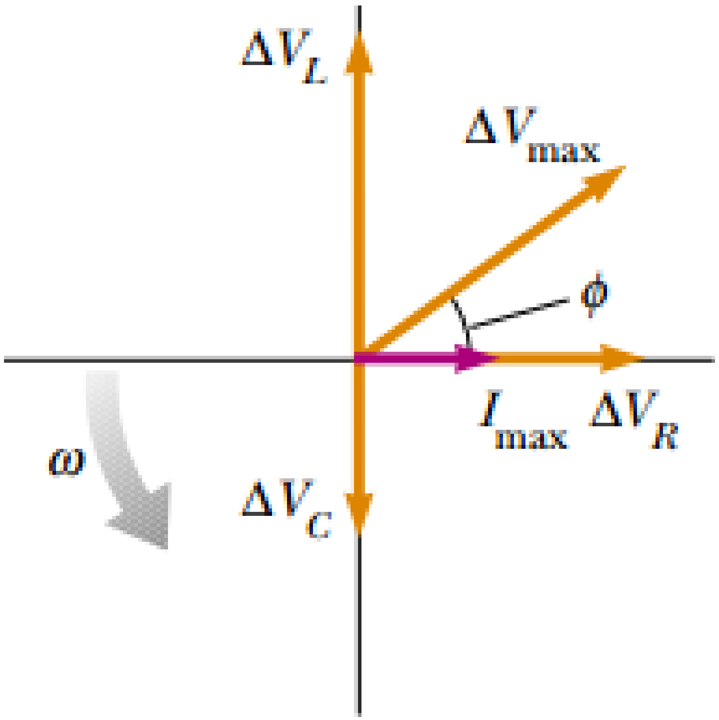
$$\Delta v = \Delta v_R + \Delta v_L + \Delta v_C$$

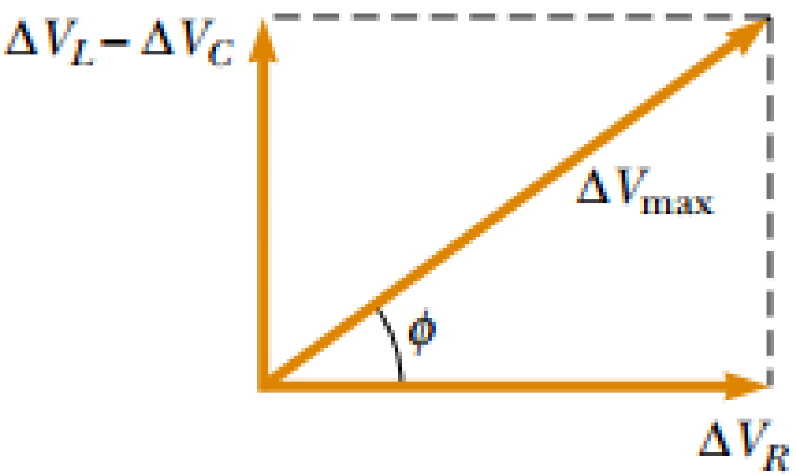


(a) Resistor (b) Inductor (c) Capacitor

33.5 The RLC Series Circuit

<u>Phasor diagram for the series RLCcircuit</u>





33.5 The RLC Series Circuit

From either one of the right triangles, we see that

$$\Delta V_{\max} = \sqrt{\Delta V_R{}^2 + (\Delta V_L - \Delta V_C)^2} = \sqrt{(I_{\max} R)^2 + (I_{\max} X_L - I_{\max} X_C)^2}$$

$$\Delta V_{\max} = I_{\max} \sqrt{R^2 + (X_L - X_C)^2}$$

We can write the maximum current in the RLC circuit as

$$I_{\max} = \frac{\Delta V_{\max}}{\sqrt{R^2 + (X_L - X_C)^2}}$$

Impedance Z of the circuit

$$Z \equiv \sqrt{R^2 + (X_L - X_C)^2}$$

33.5 The RLC Series Circuit

Now we can write the following equation

$$\Delta V_{max} = I_{max}\sqrt{R^2 + (X_L - X_C)^2}$$

as

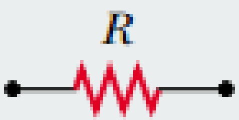
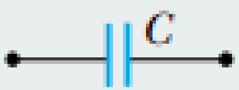
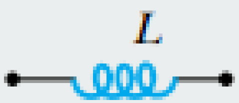
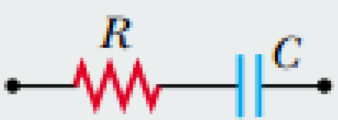
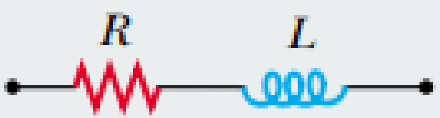
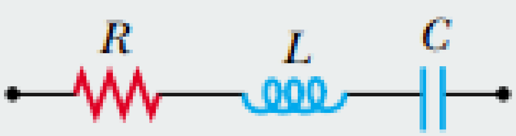
$$\Delta V_{max} = I_{max} Z$$

The Phase Angle between current and voltage

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

33.5 The RLC Series Circuit

Impedance Values and Phase Angles for Various Circuit-Element Combinations[a]

Circuit Elements	Impedance Z	Phase Angle ϕ
	R	$0°$
	X_C	$-90°$
	X_L	$+90°$
	$\sqrt{R^2 + X_C{}^2}$	Negative, between $-90°$ and $0°$
	$\sqrt{R^2 + X_L{}^2}$	Positive, between $0°$ and $90°$
	$\sqrt{R^2 + (X_L - X_C)^2}$	Negative if $X_C > X_L$ Positive if $X_C < X_L$

Example 33.5

A series RLC AC circuit has $R = 425\ \Omega$, $L = 1.25$ H, $C = 3.50\ \mu$F, $\omega = 377\ s^{-1}$, and $\Delta V_{max} = 150$ V.

(A) Determine the inductive reactance, the capacitive reactance, and the impedance of the circuit.

(B) Find the maximum current in the circuit.

(C) Find the phase angle between the current and voltage.

(D) Find both the maximum voltage and the instantaneous voltage across each element.

33.6 Power in an AC Circuit

- **Average power delivered to an RLC circuit is**

$$\mathcal{P}_{av} = \frac{1}{2} I_{max} \Delta V_{max} \cos\phi$$

$$\boxed{\mathcal{P}_{av} = I_{rms} \Delta V_{rms} \cos\phi}$$

$$\cos\emptyset = \frac{I_{max} R}{\Delta v_{max}}$$

$\cos\emptyset$ is *called power factor*

$$\boxed{\mathcal{P}_{av} = I_{rms}^2 R}$$

The average power delivered by the source is converted to internal energy in the resistor (as in DC CIRCUIT)

33.6 Power in an AC Circuit

- ***<u>Important Note</u>***

No power losses are associated with pure capacitors and pure inductors in an AC circuit

33.7 Resonance in a Series RLC Circuit

- A series RLC circuit is said to be in resonance when the ***current has its maximum value***.

- Because the impedance depends on the frequency of the source, the **current in the RLC circuit also depends on the frequency**

$$I_{\text{rms}} = \frac{\Delta V_{\text{rms}}}{\sqrt{R^2 + (X_L - X_C)^2}}$$

- ***Resonance frequency***

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

33.7 Resonance in a Series RLC Circuit

- ***The current in a series RLC circuit <u>reaches its maximum</u> value when the frequency of the applied voltage matches the natural oscillator frequency—which depends only on L and C***

Example 33.7

Consider a series RLC circuit for which $R = 150\ \Omega$, $L = 20.0$ mH, $\Delta V_{\text{rms}} = 20.0$ V, and $\omega = 5\,000\ \text{s}^{-1}$. Determine the value of the capacitance for which the current is a maximum.