chapter 2 :- unifor and exponential distribution

6.3 : $X \sim U(7,10)$, $f(x) = \frac{1}{3}$; $7 < X \leq 10$

(a) $P(X \leq 8.8) = \int_{7}^{8.8} \frac{1}{3}\,dx = \frac{8.8-7}{3} = 0.6$

(b) $P(7.4 < X < 9.5) = \int_{7.4}^{9.5} \frac{1}{3}\,dx = \frac{9.5-7.4}{3} = 0.7$

(c) $P(X > 8.5) = \int_{8.5}^{10} \frac{1}{3}\,dx = \frac{10-8.5}{3} = 0.5$

6.4 : X waiting time

$X \sim U(0,10)$

$\therefore f(x) = \frac{1}{10}$; $0 < X < 10$

(a) $P(X > 7) = \frac{10-7}{10} = 0.3$

(b) $P(2 < X < 7) = \frac{7-2}{10} = 0.5$

6.45 : $X \sim Exp(\frac{1}{4})$, $E(X) = \frac{1}{\lambda} = 4$ thus, $\lambda = \frac{1}{4}$

First, calculate the Probability of served less than 3 min.

$P(X<3) = \int_{0}^{3} \frac{1}{4} e^{-\frac{x}{4}}\,dx = -e^{-\frac{x}{4}}\Big|_{0}^{3} = 0.5276$

consider "served less than 3" is success thus, $P = 0.5276$

Now, let Y be the number of success in 6 days.

$Y \sim Bin(6, 0.5276)$

$P(Y \geq 4) = f(4) + f(5) + f(6) = \sum_{y=4}^{6} \binom{6}{y} (0.5276)^{y} (0.4724)^{6-y} = 0.3986$

Chapter 3 :- Two - means

8.28 :

$n_1 = 25$, $\mu_1 = 80$, $\sigma_1 = 5$

$n_2 = 36$, $\mu_2 = 75$, $\sigma_2 = 3$

$P(3.4 < \bar{X}_1 - \bar{X}_2 < 5.9) = ?$

$\mu_{\bar{X}_1 - \bar{X}_2} = \mu_1 - \mu_2 = 80 - 75 = 5$

$\sigma_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} = \sqrt{\frac{5^2}{25} + \frac{3^2}{36}} = 1.118$

$\bar{X}_1 - \bar{X}_2 \sim N(5, 1.118)$

$P\left(\frac{3.4 - 5}{1.118} < Z < \frac{5.9 - 5}{1.118}\right) = P(-1.43 < Z < 0.81)$

$= 0.791 - 0.0764 = 0.8946$

8.29 :

$\mu_1 = 72$, $\sigma_1 = 10$, $n_1 = 64$

$\mu_2 = 28$, $\sigma_2 = 5$, $n_2 = 100$

$\mu_{\bar{X}_1 - \bar{X}_2} = \mu_1 - \mu_2 = 44$

$\sigma_{\bar{X}_1 - \bar{X}_2} = \sqrt{\frac{10^2}{64} + \frac{5^2}{100}} = 1.3463$

$\therefore \bar{X}_1 - \bar{X}_2 \sim N(44, 1.3463)$

$P(\bar{X}_1 - \bar{X}_2 \leq 44.2) = P\left(Z \leq \frac{44.2 - 44}{1.3463}\right) = P(Z \leq 0.15) = 0.5596$

chapter 4:

9.2: Population normal and $\sigma = 40$ "Known"

$n = 30$, $\bar{X} = 780$

96% C.I for μ is:

$$\bar{X} \pm Z_{1-\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$$

$Z_{1-\frac{0.04}{2}} = Z_{0.98} = 2.05$

Thus, $780 \pm (2.05) \frac{40}{\sqrt{30}}$

$\Rightarrow 780 \pm 14.9711$

$\Rightarrow \mu \in (765.03, 794.971)$

9.4:

$n = 50$, $\bar{X} = 174.5$, $S = 6.9$

a) 98% C.I for μ is:

$$\bar{X} \pm Z_{1-\frac{\alpha}{2}} \frac{S}{\sqrt{n}}$$

$Z_{0.99} = 2.33$

Thus, $174.5 \pm (2.33) \frac{6.9}{\sqrt{50}} \Rightarrow 174.5 \pm 2.2736$

$\therefore \mu \in (172.23, 176.77)$

b) the error will not exceed $Z_{1-\frac{\alpha}{2}} \frac{S}{\sqrt{n}} = 2.2736$.

9.5 :

$n=100, \quad \bar{x}=23500, \quad S=3900$

a) 99% C.I for μ is:

$$\bar{X} \pm Z_{1-\frac{\alpha}{2}} \frac{S}{\sqrt{n}}$$

$Z_{0.995} = 2.575$

thus, $23500 \pm 2.575 \frac{(3900)}{\sqrt{100}}$

$\Rightarrow 23500 \pm 1004.25$

$\Rightarrow 22495.75 < \mu < 24504.25$

b) The error will not exceeds 1004.25

9.6 :

$$n = \left(\frac{Z_{1-\frac{\alpha}{2}} \sigma}{e} \right)^2$$

$\sigma = 40, \quad Z_{0.98} = 2.05, \quad e = 10$

$$\therefore \quad n = \left(\frac{2.05(40)}{10} \right)^2 = 67.24 \simeq 68$$

" we always rounded the number up "

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9.35 :

$n_1 = 25$, $\sigma_1 = 5$, $\overline{X}_1 = 80$

$n_2 = 36$, $\sigma_2 = 3$, $\overline{X}_2 = 75$

94 % C.I for $\mu_1 - \mu_2$ is :

$$(\overline{X}_1 - \overline{X}_2) \pm Z_{1-\frac{\alpha}{2}} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

$Z_{0.97} = 1.88$

$$(80 - 75) \pm (1.88) \sqrt{\frac{5^2}{25} + \frac{3^2}{36}}$$

$$5 \pm 2.1019$$

$$2.8981 < \mu_1 - \mu_2 < 7.1019$$

9.38 :-

$n_1 = 12$, $\overline{X}_1 = 85$, $S_1 = 4$

$n_2 = 10$, $\overline{X}_2 = 81$, $S_2 = 5$

σ_1 and σ_2 unknown but equal.

90 % C.I for $\mu_1 - \mu_2$ is :

$$(\overline{X}_1 - \overline{X}_2) \pm t_{\frac{\alpha}{2}, n_1+n_2-2} \, S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

$$S_p = \sqrt{\frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1+n_2-2}} = \sqrt{\frac{11(16) + (9)25}{20}} = 4.4777$$

$t_{0.05,20} = 1.725$

thus, $(85 - 81) \pm (1.725) \sqrt{\frac{1}{12} + \frac{1}{10}}$

$$4 \pm 3.3072$$

$$\Rightarrow \quad 0.693 < \mu_1 - \mu_2 < 7.307$$

9.41 :

$n_1 = 14$, $\bar{X}_1 = 17$, $S_1^2 = 1.5$

$n_2 = 16$, $\bar{X}_2 = 19$, $S_2^2 = 1.8$

σ_1 and σ_2 unknown but equal :

$$(\bar{X}_2 - \bar{X}_1) \pm t_{\frac{\alpha}{2}, n_1+n_2-2} \, S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

$$\Rightarrow (19-17) \pm (2.763)(1.3336) \sqrt{\frac{1}{14} + \frac{1}{16}}$$

$$\Rightarrow 2 \pm 1.348$$

$$\Rightarrow 0.65 < \mu_2 - \mu_1 < 3.35$$

9.44 : $\sigma_1 \neq \sigma_2$ (unknown)

$\therefore$ We use $(\bar{X}_1 - \bar{X}_2) \pm t_{\frac{\alpha}{2}, n_1+n_2-2} \sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}$

From the data, we calculate :

$\bar{X}_1 = 37250$, $S_1 = 6546.76$

$\bar{X}_2 = 38362.5$, $S_2 = 6181.06$

$t_{0.005, 14} = 2.977$

thus, $(37250 - 38362.5) \pm (2.977) \sqrt{\frac{(6546.76)^2}{8} + \frac{(6181.06)^2}{8}}$

$$\Rightarrow -112.5 \pm 20546.9$$

$$\Rightarrow -21659.4 < \mu_1 - \mu_2 < 19434.4$$

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9.53

(a) $n = 200$, $x = 114$

$\therefore \hat{P} = \frac{114}{200} = 0.57$

96% C.I for P is:

$\hat{P} \pm Z_{1-\frac{\alpha}{2}} \sqrt{\frac{\hat{P}\hat{q}}{n}}$

$Z_{0.98} = 2.05$, $\hat{q} = 1 - \hat{P} = 0.43$

thus, $0.57 \pm (2.05)\sqrt{\frac{0.57(0.43)}{200}}$

$\Rightarrow 0.57 \pm 0.0718$

$\Rightarrow 0.51 < P < 0.64$

(b) The error will not exceeds 0.0718

9.56: $n = 100$, $x = 24$, $\hat{P} = \frac{24}{100} = 0.24 \Rightarrow \hat{q} = 0.76$

(a) 99% C.I for P is

$\hat{P} \pm Z_{1-\frac{\alpha}{2}} \sqrt{\frac{\hat{P}\hat{q}}{n}}$; $Z_{0.995} = 2.575$

$\Rightarrow 0.24 \pm (2.575)\sqrt{\frac{(0.24)(0.76)}{100}}$

0.24 ± 0.0427

$\Rightarrow 0.2 < P < 0.28$

(b) The error will not exceeds 0.0427

9.67: $X_1 = 120$, $X_2 = 98$, $n_1 = n_2 = 500$

$\therefore \hat{P}_1 = \frac{120}{500} = 0.24 \Rightarrow \hat{q}_1 = 0.76$, $\hat{P}_2 = \frac{98}{500} = 0.196 \Rightarrow \hat{q}_2 = 0.804$

$Z_{0.95} = 1.645$

$\therefore$ 90% C.I for $P_1 - P_2$ is

$(\hat{P}_1 - \hat{P}_2) \pm Z_{1-\frac{\alpha}{2}} \sqrt{\frac{\hat{P}_1\hat{q}_1}{n_1} + \frac{\hat{P}_2\hat{q}_2}{n_2}} \Rightarrow (0.24 - 0.196) \pm (1.645)\sqrt{\frac{0.24(0.76)}{500} + \frac{0.196(0.804)}{500}}$

0.044 ± 0.0428

$\Rightarrow 0.001 < P_1 - P_2 < 0.0869$