324 Stat Lecture Notes

(4) Some Discrete Probability Distributions

(Book: Chapter 5, pg 143)



4.1 Discrete Uniform Distribution:

Discrete Uniform is not in the book, it should be studied from the notes

• If the random variable **X** assume the values with equal probabilities, then the discrete uniform distribution is given by:

$$P(X,K) = \frac{1}{K} , \quad X = x_1, x_2, \dots, x_K$$
$$= 0 \qquad elsewhere \qquad (1)$$





• The mean and variance of the discrete uniform distribution P(X, K) are:

$$\mu = \frac{\sum_{i=1}^{K} X_{i}}{K} , \quad \sigma^{2} = \frac{\sum_{i=1}^{K} (X - \mu)^{2}}{K}$$
(2)





• When a die is tossed once, each element of the sample space $S = \{1,2,3,4,5,6\}$ occurs with probability 1/6. Therefore we have a uniform distribution with:

$$P(X,6) = \frac{1}{6}, X = 1,2,3,4,5,6$$

- Find:
- |. $P(1 \le x < 4)$
- 2. P(x < 3)
- **3**.P(3 < x < 6)

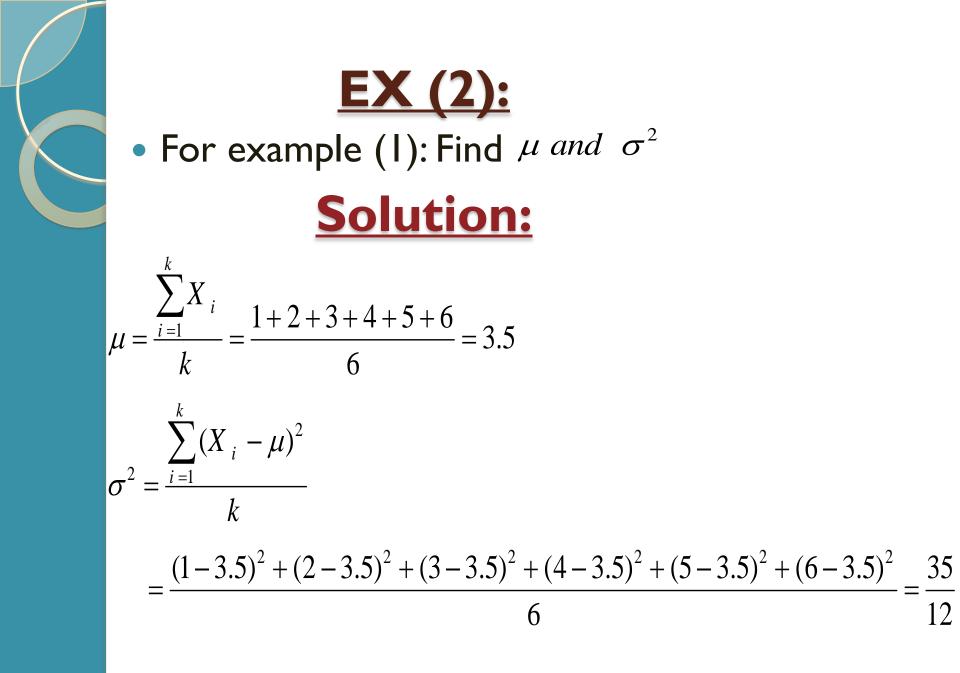


Solution:

• I. $P(1 \le x < 4)$ = $P(x = 1) + P(x = 2) + P(x = 3) = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{3}{6}$

• 2.
$$P(x < 3) = P(x = 1) + P(x = 2) = \frac{2}{6}$$

• **3**. $P(3 < x < 6) = P(x = 4) + P(x = 5) = \frac{2}{6}$





The Bernoulli Process:

- Bernoulli trials are an experiment with:
- I) Only two possible outcomes.
- 2) Labeled as success (S), failure (F).
- 3) Probability of success = p, probability of failure =q=1-p (p+q=1).



Binomial Distribution:

- * The **Binomial** trials must possess the following properties:
- I) The experiment consists of **n** repeated trials.
- 2) Each trial results in an outcome that may be classified as a success or a failure.
- 3) The probability of success denoted by **p** remains constant from trial to trial.
- 4) The repeated trials are independent.
- 5) The parameters of binomial are n,p.



Binomial Distribution:

• The probability distribution of the binomial random variable **X**, the number of successes in **n** independent trials is:

$$b(x,n,p) = \binom{n}{x} p^{X} q^{n-X} , \quad x = 0,1,2,...,n$$
 (5)



Theorem:

The mean and the variance of the binomial distribution b(x, n, p) are:

 $\mu = np \qquad and \qquad \sigma^2 = npq \qquad (6)$



EX (4):

- According to a survey by the Administrative Management society, 1/3 of U.S. companies give employees four weeks of vacation after they have been with the company for 15 years. Find the probability that among 6 companies surveyed at random, the number that gives employees 4 weeks of vacation after 15 years of employment is:
- a) anywhere from 2 to 5;
- b) fewer than 3;
- c) at most I;
- d) at least 5;
- e) greater than 2;
- f) calculate μ and σ^2



Solution:

$$n = 6, p = \frac{1}{3, q} = \frac{2}{3}$$

$$(a) P(2 \le x \le 5) = P(x = 2) + P(x = 3) + P(x = 4) + P(x = 5)$$

$$= \binom{6}{2} \left(\frac{1}{3}\right)^{2} \left(\frac{2}{3}\right)^{4} + \binom{6}{3} \left(\frac{1}{3}\right)^{3} \left(\frac{2}{3}\right)^{3} + \binom{6}{4} \left(\frac{1}{3}\right)^{4} \left(\frac{2}{3}\right)^{2} + \binom{6}{5} \left(\frac{1}{3}\right)^{5} \left(\frac{2}{3}\right)^{1}$$

$$= \frac{240}{729} + \frac{160}{729} + \frac{60}{729} + \frac{12}{729} = \frac{472}{729} = 0.647$$

$$(b) P(x < 3) = P(x = 0) + P(x = 1) + P(x = 2)$$

$$= \binom{6}{0} \left(\frac{1}{3}\right)^{0} \left(\frac{2}{3}\right)^{6} + \binom{6}{1} \left(\frac{1}{3}\right)^{1} \left(\frac{2}{3}\right)^{5} + \binom{6}{2} \left(\frac{1}{3}\right)^{2} \left(\frac{2}{3}\right)^{4}$$

$$= \frac{64}{729} + \frac{192}{729} + \frac{240}{729} = \frac{496}{729} = 0.68$$

$$(c) P(x \le 1) = P(x = 0) + p(x = 1)$$
$$= \binom{6}{0} \left(\frac{1}{3}\right)^0 \left(\frac{2}{3}\right)^6 + \binom{6}{1} \left(\frac{1}{3}\right)^1 \left(\frac{2}{3}\right)^5$$
$$= \frac{64}{729} + \frac{192}{729} = \frac{256}{729} = 0.351$$

 $(d) P(x \ge 5) = P(x = 5) + P(x = 6)$

 $= \binom{6}{5} \left(\frac{1}{3}\right)^{5} \left(\frac{2}{3}\right)^{1} + \binom{6}{6} \left(\frac{1}{3}\right)^{6} \left(\frac{2}{3}\right)^{0}$

 $=\frac{12}{729}+\frac{1}{729}=\frac{13}{729}=0.018$

$$(e) P(x > 2) = 1 - P(x \le 2) = P(x = 0) + P(x = 1) + P(x = 2)$$
$$= 1 - \left\{ \binom{6}{0} \left(\frac{1}{3}\right)^0 \left(\frac{2}{3}\right)^6 + \binom{6}{1} \left(\frac{1}{3}\right)^1 \left(\frac{2}{3}\right)^5 + \binom{6}{2} \left(\frac{1}{3}\right)^2 \left(\frac{2}{3}\right)^4 \right\}$$
$$= 1 - \left\{ \frac{64}{729} + \frac{192}{729} + \frac{240}{729} \right\} = 1 - \frac{496}{729} = \frac{233}{729} = 0.319$$

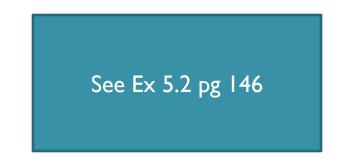
$$(f) \mu = np = (6)(\frac{1}{3}) = \frac{6}{3} = 2$$

$$\sigma^2 = npq = (6)(\frac{1}{3})(\frac{2}{3}) = \frac{12}{9} = 1.333$$



EX (5):

- The probability that a person suffering from headache will obtain relief with a particular drug is **0.9**. **Three** randomly selected sufferers from headache are given the drug. Find the probability that the number obtaining relief will be:
- a) exactly zero;
- b) at most one;
- c) more than one;
- d) two or fewer;
- e) Calculate μ and σ^2





$$n = 3, p = 0.9, q = 0.1$$

(a)
$$P(X = 0) = {\binom{3}{0}} (0.9)^0 (0.1)^3 = 0.001$$

 $(b) P(x \le 1) = P(x = 0) + P(x = 1)$

$$= \binom{3}{0} (0.9)^{0} (0.1)^{3} + \binom{3}{1} (0.9)^{1} (0.1)^{2}$$

= 0.001 + 0.027 = 0.028

$(c) P(X > 1) = 1 - p(x \le 1) = 1 - [P(x = 0) + P(x = 1)]$

$$=1-\left[\binom{3}{0}\left(0.9\right)^{0}\left(0.1\right)^{3}+\binom{3}{1}\left(0.9\right)^{1}\left(0.1\right)^{2}\right]$$

=1 - (0.001 + 0.027) = 1 - 0.028 = 0.972

$$(d) P(x \le 2) = P(x = 0) + P(x = 1) + P(x = 2)$$

$$= \binom{3}{0} (0.9)^0 (0.1)^3 + \binom{3}{1} (0.9)^1 (0.1)^2 + \binom{3}{2} (0.9)^2 (0.1)^1$$

= 0.001 + 0.027 + 0.243 = 0.271