

Chapter 4

ENERGY ANALYSIS OF CLOSED SYSTEMS

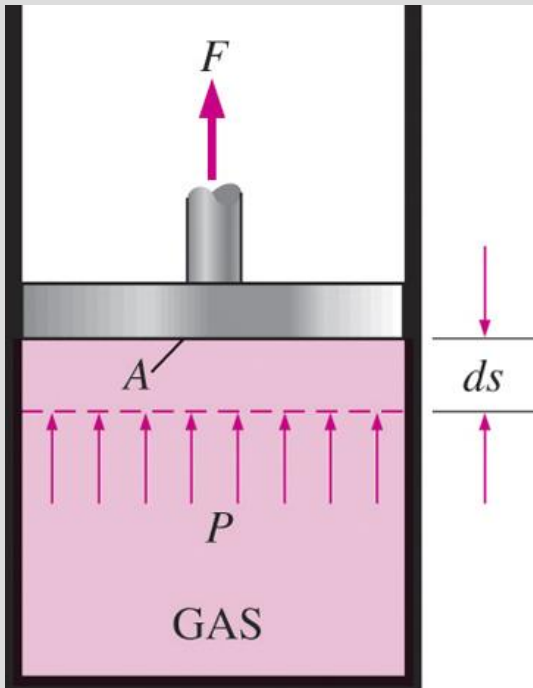
MOVING BOUNDARY WORK

Moving boundary work ($P dV$ work):

The expansion and compression work in a piston-cylinder device.

$$\delta W_b = F ds = PA ds = P dV$$

$$W_b = \int_1^2 P dV \quad (\text{kJ})$$



A gas does a differential amount of work δW_b as it forces the piston to move by a differential amount ds .

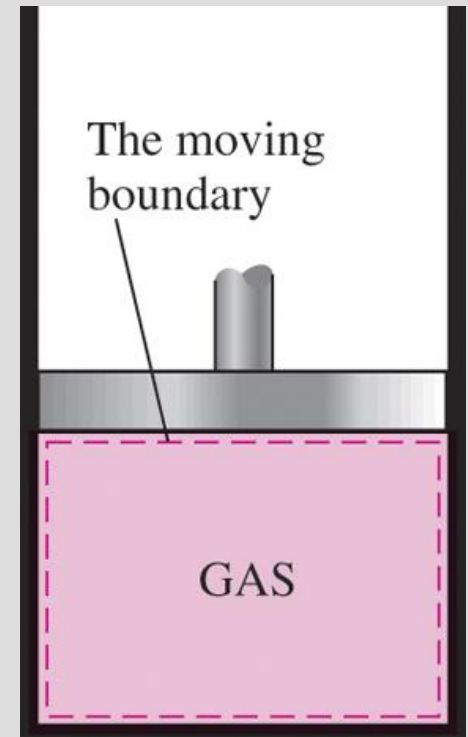
The work associated with a moving boundary is called *boundary work*.

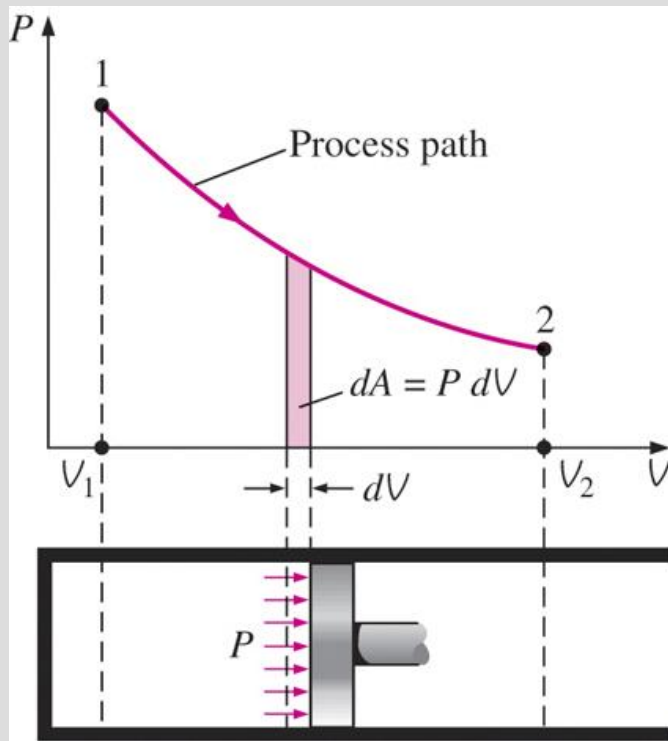
Quasi-equilibrium process:

A process during which the system remains nearly in equilibrium at all times.

W_b is positive \rightarrow for expansion

W_b is negative \rightarrow for compression

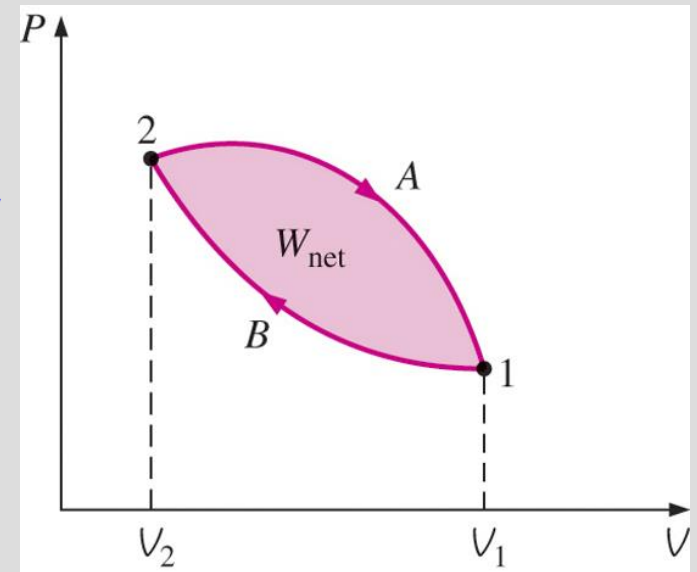




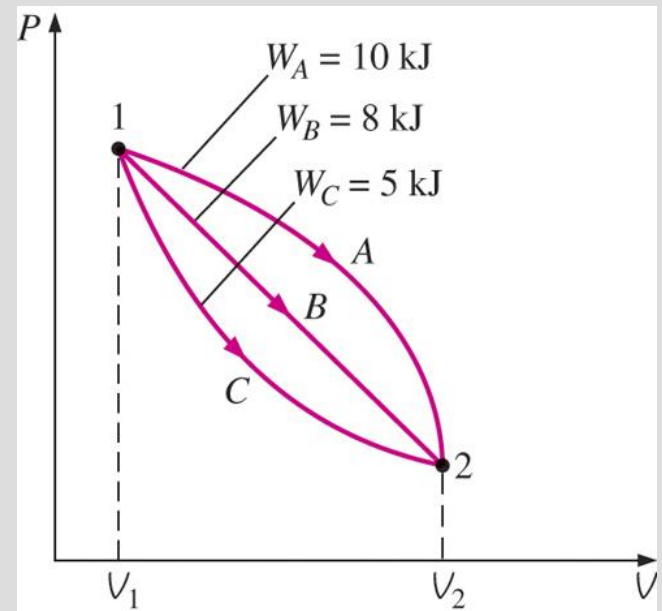
The area under the process curve on a P - V diagram represents the boundary work.

$$\text{Area} = A = \int_1^2 dA = \int_1^2 P dV$$

The boundary work done during a process depends on the path followed as well as the end states.



The net work done during a cycle is the difference between the work done by the system and the work done on the system.



Polytropic, Isothermal, and Isobaric processes

$P = CV^{-n}$ Polytropic process: C, n (polytropic exponent) constants

$$W_b = \int_1^2 P dV = \int_1^2 CV^{-n} dV = C \frac{V_2^{-n+1} - V_1^{-n+1}}{-n + 1} = \frac{P_2 V_2 - P_1 V_1}{1 - n}$$

Polytropic process

$$W_b = \frac{mR(T_2 - T_1)}{1 - n}$$

Polytropic and for ideal gas

$$W_b = \int_1^2 P dV = \int_1^2 CV^{-1} dV = PV \ln\left(\frac{V_2}{V_1}\right)$$

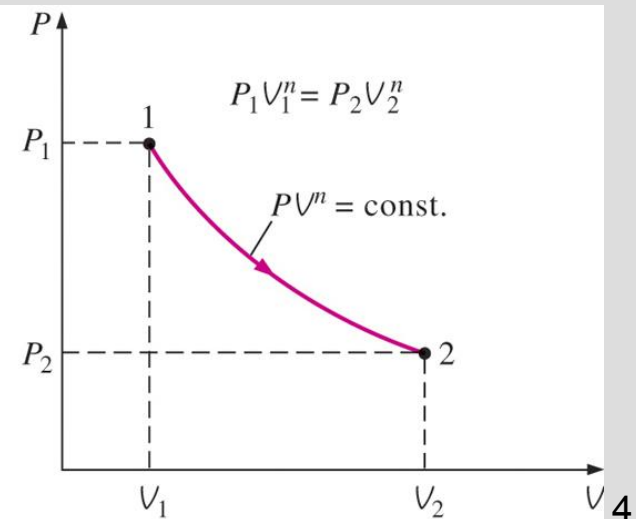
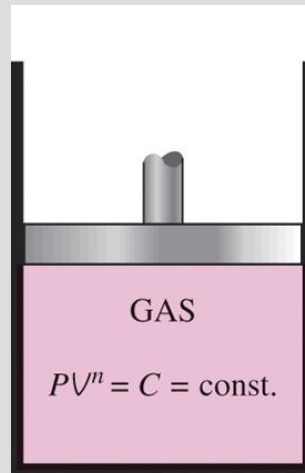
When $n = 1$
(isothermal process)

$$W_b = \int_1^2 P dV = P_0 \int_1^2 dV = P_0(V_2 - V_1)$$

Constant pressure process

What is the boundary work for a constant-volume process?

Schematic and P - V diagram for a polytropic process.



ENERGY BALANCE FOR CLOSED SYSTEMS

$$\underbrace{E_{\text{in}} - E_{\text{out}}}_{\text{Net energy transfer by heat, work, and mass}} = \underbrace{\Delta E_{\text{system}}}_{\text{Change in internal, kinetic, potential, etc., energies}} \quad (\text{kJ})$$

Energy balance for any system undergoing any process

$$\underbrace{\dot{E}_{\text{in}} - \dot{E}_{\text{out}}}_{\text{Rate of net energy transfer by heat, work, and mass}} = \underbrace{dE_{\text{system}}/dt}_{\text{Rate of change in internal, kinetic, potential, etc., energies}} \quad (\text{kW})$$

Energy balance in the rate form

The total quantities are related to the quantities per unit time is

$$Q = \dot{Q} \Delta t, \quad W = \dot{W} \Delta t, \quad \text{and} \quad \Delta E = (dE/dt) \Delta t \quad (\text{kJ})$$

$$e_{\text{in}} - e_{\text{out}} = \Delta e_{\text{system}} \quad (\text{kJ/kg})$$

Energy balance per unit mass basis

$$\delta E_{\text{in}} - \delta E_{\text{out}} = dE_{\text{system}} \quad \text{or} \quad \delta e_{\text{in}} - \delta e_{\text{out}} = de_{\text{system}}$$

Energy balance in differential form

$$W_{\text{net,out}} = Q_{\text{net,in}} \quad \text{or} \quad \dot{W}_{\text{net,out}} = \dot{Q}_{\text{net,in}}$$

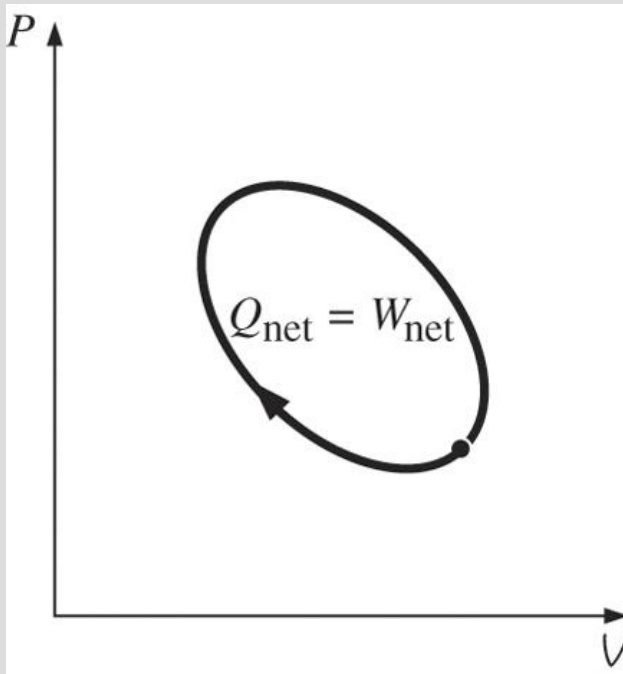
Energy balance for a cycle

$$Q_{\text{net,in}} - W_{\text{net,out}} = \Delta E_{\text{system}} \quad \text{or} \quad Q - W = \Delta E$$

$$Q = Q_{\text{net,in}} = Q_{\text{in}} - Q_{\text{out}}$$

$$W = W_{\text{net,out}} = W_{\text{out}} - W_{\text{in}}$$

Energy balance when sign convention is used (i.e., heat input and work output are positive; heat output and work input are negative).



For a cycle $\Delta E = 0$, thus $Q = W$.

General $Q - W = \Delta E$

Stationary systems $Q - W = \Delta U$

Per unit mass $q - w = \Delta e$

Differential form $\delta q - \delta w = de$

Various forms of the first-law relation for closed systems when sign convention is used.

The first law cannot be proven mathematically, but no process in nature is known to have violated the first law, and this should be taken as sufficient proof.

Energy balance for a constant-pressure expansion or compression process

General analysis for a closed system undergoing a quasi-equilibrium constant-pressure process. Q is *to* the system and W is *from* the system.

$$\underbrace{E_{\text{in}} - E_{\text{out}}}_{\text{Net energy transfer by heat, work, and mass}} = \underbrace{\Delta E_{\text{system}}}_{\text{Change in internal, kinetic, potential, etc., energies}}$$

$$Q - W = \Delta U + \cancel{\Delta KE}^0 + \cancel{\Delta PE}^0$$

$$Q - W_{\text{other}} - W_b = U_2 - U_1$$

$$Q - W_{\text{other}} - P_0(V_2 - V_1) = U_2 - U_1$$

$$Q - W_{\text{other}} = (U_2 + P_2 V_2) - (U_1 + P_1 V_1)$$

$$H = U + PV$$

$$Q - W_{\text{other}} = H_2 - H_1$$

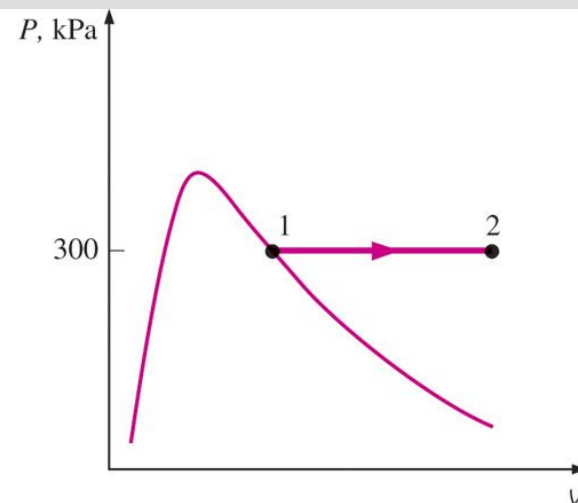
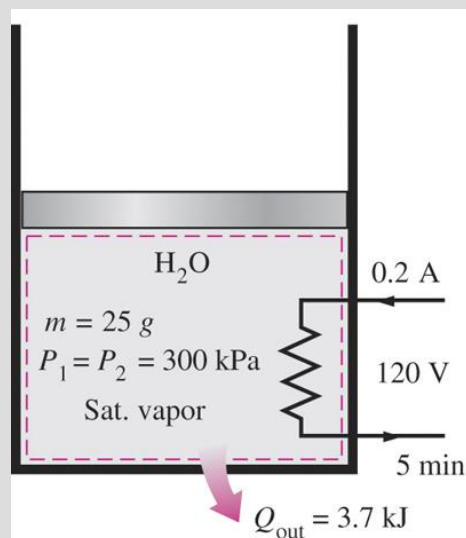
For a constant-pressure expansion or compression process:

$$\Delta U + W_b = \Delta H$$

An example of constant-pressure process

$$W_{e,\text{in}} - Q_{\text{out}} - W_b = \Delta U$$

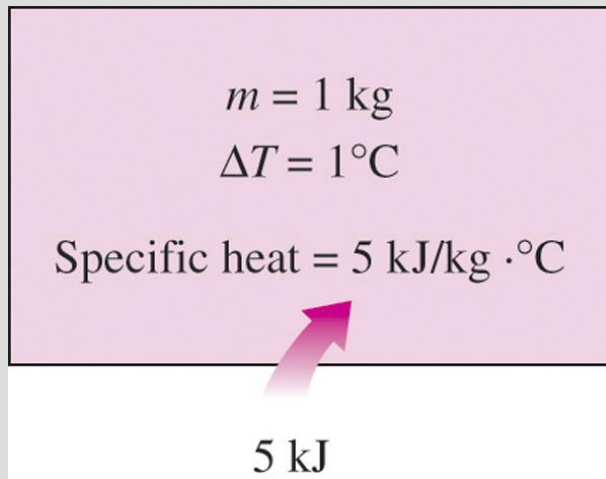
$$W_{e,\text{in}} - Q_{\text{out}} = \Delta H = m(h_2 - h_1)$$



SPECIFIC HEATS

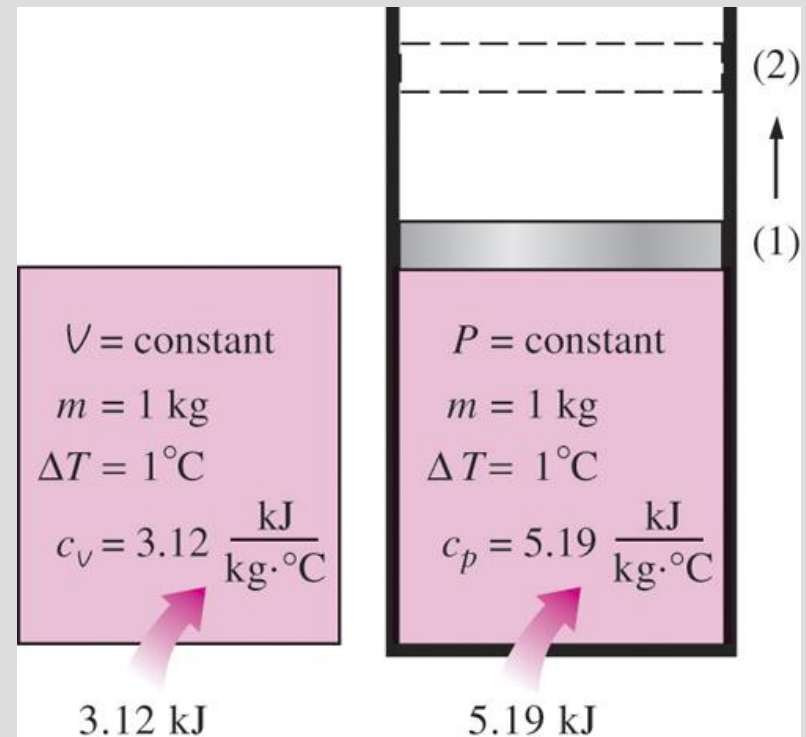
Specific heat at constant volume, c_v : The energy required to raise the temperature of the unit mass of a substance by one degree as the volume is maintained constant.

Specific heat at constant pressure, c_p : The energy required to raise the temperature of the unit mass of a substance by one degree as the pressure is maintained constant.

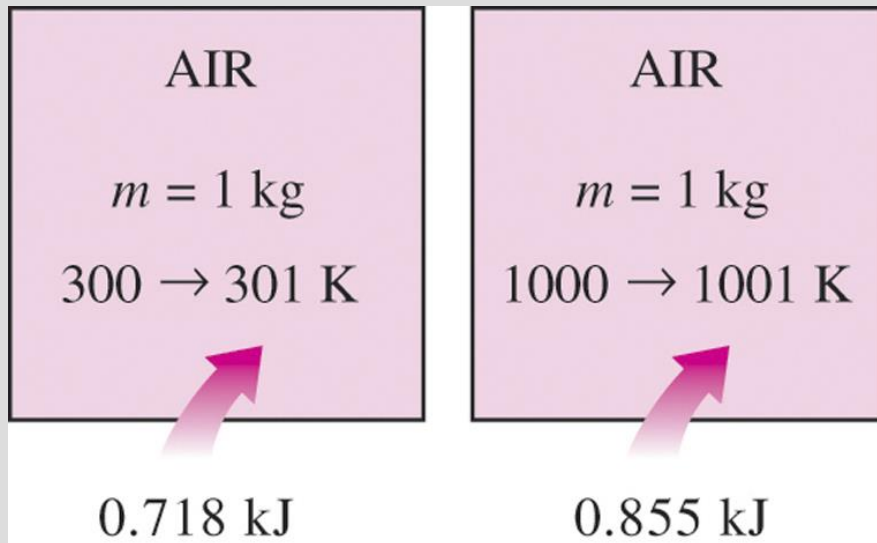


Specific heat is the energy required to raise the temperature of a unit mass of a substance by one degree in a specified way.

Constant-volume and constant-pressure specific heats c_v and c_p (values are for helium gas).



- The equations in the figure are valid for *any* substance undergoing *any* process.
- c_v and c_p are properties.
- c_v is related to the changes in *internal energy* and c_p to the changes in *enthalpy*.
- A common unit for specific heats is kJ/kg · °C or kJ/kg · K. **Are these units identical?**



The specific heat of a substance changes with temperature.

True or False?

c_p is always greater than c_v .

$$c_v = \left(\frac{\partial u}{\partial T} \right)_v$$

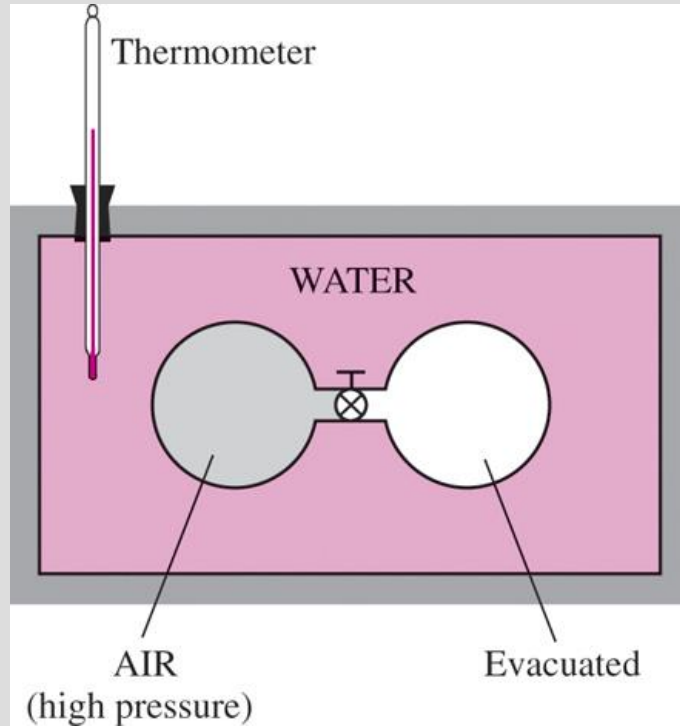
= the change in internal energy with temperature at constant volume

Formal definitions of c_v and c_p .

$$c_p = \left(\frac{\partial h}{\partial T} \right)_p$$

= the change in enthalpy with temperature at constant pressure

INTERNAL ENERGY, ENTHALPY, AND SPECIFIC HEATS OF IDEAL GASES



Joule showed
using this
experimental
apparatus that
 $u = u(T)$

$$\begin{aligned} u &= u(T) \\ h &= h(T) \\ c_v &= c_v(T) \\ c_p &= c_p(T) \end{aligned}$$

For ideal gases,
 u , h , c_v , and c_p
vary with
temperature only.

$$\left. \begin{aligned} h &= u + Pv \\ Pv &= RT \end{aligned} \right\} h = u + RT$$

$$u = u(T) \quad h = h(T)$$

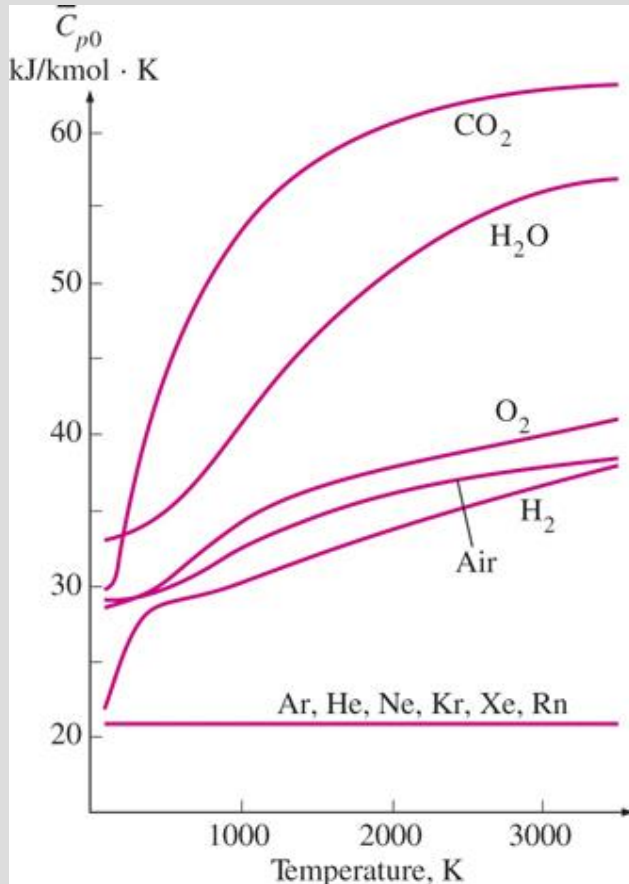
$$du = c_v(T) dT \quad dh = c_p(T) dT$$

$$\Delta u = u_2 - u_1 = \int_1^2 c_v(T) dT$$

$$\Delta h = h_2 - h_1 = \int_1^2 c_p(T) dT$$

Internal energy and
enthalpy change of
an ideal gas

- At low pressures, all real gases approach ideal-gas behavior, and therefore their specific heats depend on temperature only.
- The specific heats of real gases at low pressures are called *ideal-gas specific heats*, or *zero-pressure specific heats*, and are often denoted c_{p0} and c_{v0} .



Ideal-gas constant-pressure specific heats for some gases (see Table A-2c for c_p equations).

- u and h data for a number of gases have been tabulated.
- These tables are obtained by choosing an arbitrary reference point and performing the integrations by treating state 1 as the reference state.

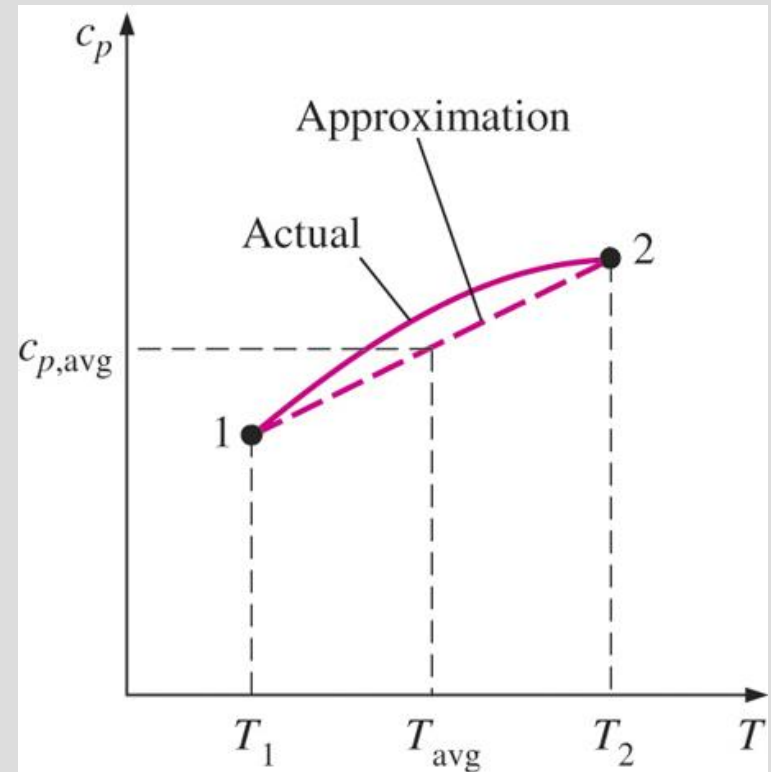
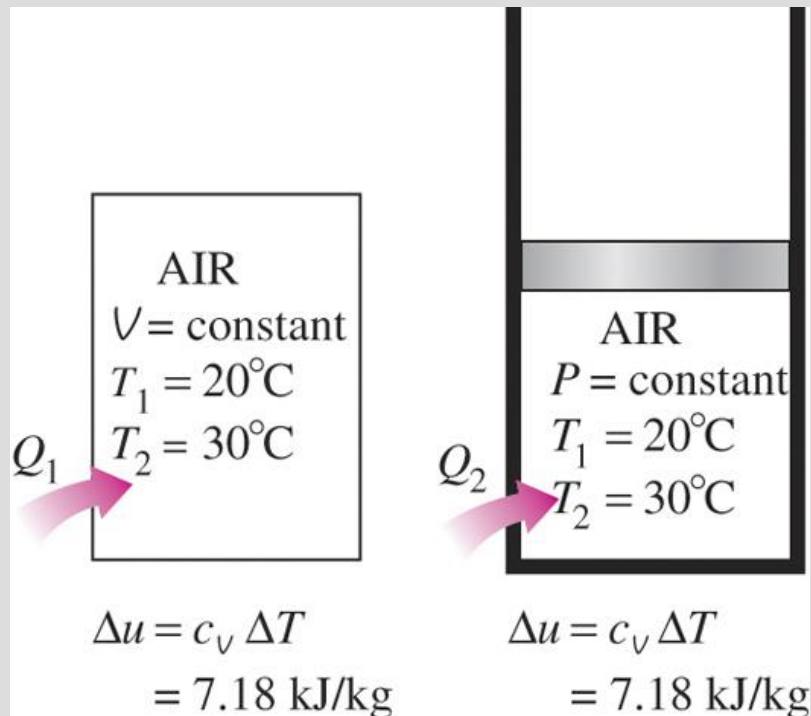
AIR		
T, K	$u, \text{kJ/kg}$	$h, \text{kJ/kg}$
0	0	0
.	.	.
.	.	.
300	214.07	300.19
310	221.25	310.24
.	.	.
.	.	.

In the preparation of ideal-gas tables, 0 K is chosen as the reference temperature.

Internal energy and enthalpy change when specific heat is taken constant at an average value

$$u_2 - u_1 = c_{v,\text{avg}}(T_2 - T_1) \quad (\text{kJ/kg})$$

$$h_2 - h_1 = c_{p,\text{avg}}(T_2 - T_1)$$



For small temperature intervals, the specific heats may be assumed to vary linearly with temperature.

The relation $\Delta u = c_v \Delta T$ is valid for *any* kind of process, constant-volume or not.

Three ways of calculating Δu and Δh

1. By using the tabulated u and h data. This is the easiest and **most accurate** way when tables are readily available.
2. By using the c_v or c_p relations (Table A-2c) as a function of temperature and performing the integrations. This is very inconvenient for hand calculations but quite desirable for computerized calculations. The results obtained are **very accurate**.
3. By using average specific heats. This is very simple and certainly very convenient when property tables are not available. The results obtained are **reasonably accurate** if the temperature interval is not very large.

$$\Delta u = u_2 - u_1 \text{ (table)}$$

$$\Delta u = \int_1^2 c_v(T) dT$$

$$\Delta u \cong c_{v,\text{avg}} \Delta T$$

Three ways of calculating Δu .

Specific Heat Relations of Ideal Gases

$$\left. \begin{aligned} h &= \tilde{u} + RT, \\ dh &= du + R dT \\ dh &= c_p dT \text{ and } du = c_v dT \end{aligned} \right\} \longrightarrow \begin{aligned} c_p &= c_v + R && (\text{kJ/kg} \cdot \text{K}) \\ \text{On a molar basis} \\ \bar{c}_p &= \bar{c}_v + R_u && (\text{kJ/kmol} \cdot \text{K}) \end{aligned}$$

AIR at 300 K

$$\left. \begin{aligned} c_v &= 0.718 \text{ kJ/kg} \cdot \text{K} \\ R &= 0.287 \text{ kJ/kg} \cdot \text{K} \end{aligned} \right\} c_p = 1.005 \text{ kJ/kg} \cdot \text{K}$$

or

$$\left. \begin{aligned} \bar{c}_v &= 20.80 \text{ kJ/kmol} \cdot \text{K} \\ R_u &= 8.314 \text{ kJ/kmol} \cdot \text{K} \end{aligned} \right\} \bar{c}_p = 29.114 \text{ kJ/kmol} \cdot \text{K}$$

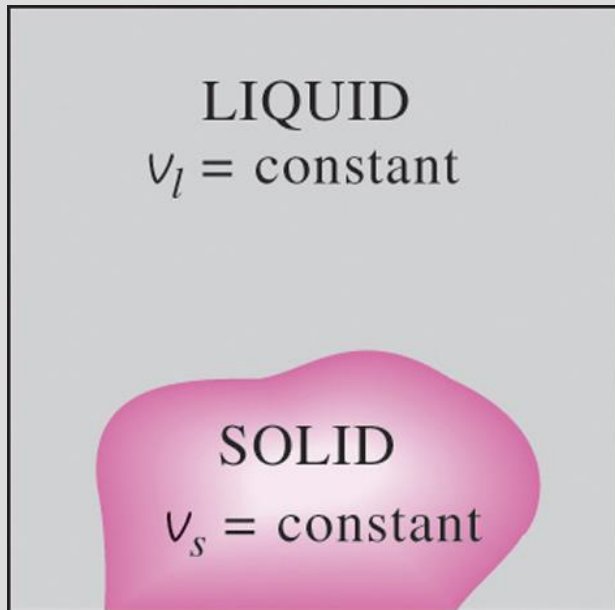
The c_p of an ideal gas can be determined from a knowledge of c_v and R .

$$k = \frac{c_p}{c_v} \quad \text{Specific heat ratio}$$

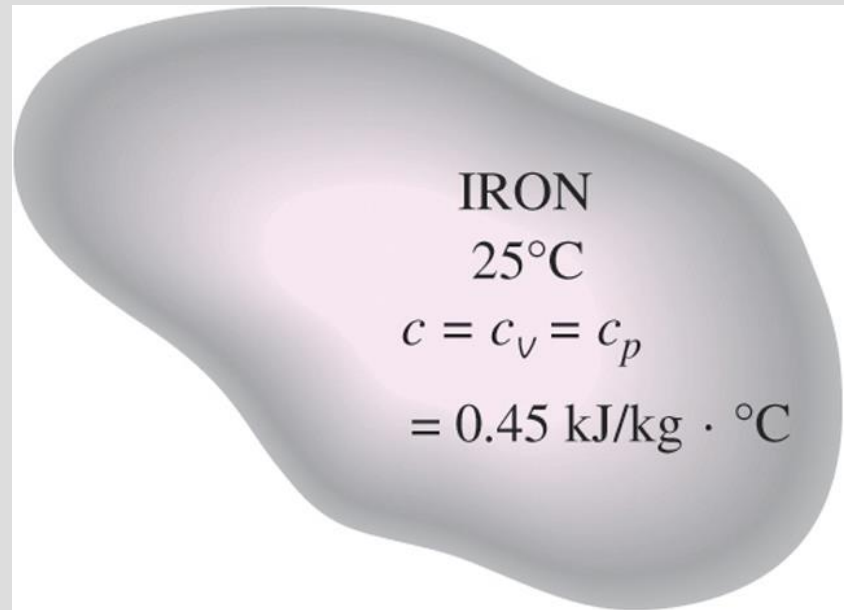
- The specific ratio varies with temperature, but this variation is very mild.
- For monatomic gases (helium, argon, etc.), its value is essentially constant at 1.667.
- Many diatomic gases, including air, have a specific heat ratio of about 1.4 at room temperature.

INTERNAL ENERGY, ENTHALPY, AND SPECIFIC HEATS OF SOLIDS AND LIQUIDS

Incompressible substance: A substance whose specific volume (or density) is constant. **Solids and liquids** are incompressible substances.



The specific volumes of incompressible substances remain constant during a process.



The c_v and c_p values of incompressible substances are identical and are denoted by c .

Internal Energy Changes

$$du = c_v dT = c(T) dT$$

$$\Delta u = u_2 - u_1 = \int_1^2 c(T) dT \quad (\text{kJ/kg})$$

$$\Delta u \cong c_{\text{avg}}(T_2 - T_1) \quad (\text{kJ/kg})$$

Enthalpy Changes

$$h = u + Pv$$

$$dh = du + v dP + P dv = du + v dP$$

$$\Delta h = \Delta u + v \Delta P \cong c_{\text{avg}} \Delta T + v \Delta P \quad (\text{kJ/kg})$$

For *solids*, the term $v \Delta P$ is insignificant and thus $\Delta h = \Delta u \cong c_{\text{avg}} \Delta T$. For *liquids*, two special cases are commonly encountered:

1. *Constant-pressure processes*, as in heaters ($\Delta P = 0$): $\Delta h = \Delta u \cong c_{\text{avg}} \Delta T$
2. *Constant-temperature processes*, as in pumps ($\Delta T = 0$): $\Delta h = v \Delta P$

$$h_{@P,T} \cong h_{f@T} + v_{f@T}(P - P_{\text{sat}@T})$$

The enthalpy of a compressed liquid

A more accurate relation than $h_{@P,T} \cong h_{f@T}$