

# LECTURE # 11



## **Chapter 4 (Session #1):** **Energy Analysis of Closed Systems**

# **CHAPTER 4**



## **ENERGY ANALYSIS OF CLOSED SYSTEMS**

# CHAPTER 4

## ENERGY ANALYSIS OF CLOSED SYSTEMS



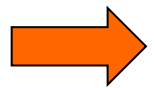
### OUTCOME:

- Develop and Apply the conservation of energy principle (the First Law) to a closed system.
- Determine the work associated with a moving boundary.
- Define the constant pressure and constant volume specific heats and relate them to changes in internal energy and enthalpy.
- Solve energy balance problems for closed systems that involve heat and work interactions for pure substances, ideal gases and incompressible substances.

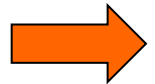
# The First Law of Thermodynamics

## Energy is Conserved

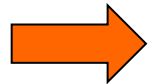
$$E_{\text{in}} - E_{\text{out}} = \Delta E_{\text{system}}$$



$$(Q_{\text{in}} + W_{\text{in}} + E_{\text{mass,in}}) - (Q_{\text{out}} + W_{\text{out}} + E_{\text{mass,out}}) = \Delta E_{\text{system}}$$



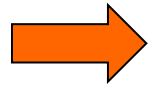
$$(Q_{\text{in}} + W_{\text{in}} + E_{\text{mass,in}}) - (Q_{\text{out}} + W_{\text{out}} + E_{\text{mass,out}}) = \Delta U + \Delta KE + \Delta PE$$



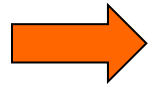
$$(Q_{\text{in}} + W_{\text{in}} + E_{\text{mass,in}}) - (Q_{\text{out}} + W_{\text{out}} + E_{\text{mass,out}}) = m(u_2 - u_1) + \frac{1}{2} m (V_2^2 - V_1^2) + m g (z_2 - z_1)$$

# The First Law of Thermodynamics for a CLOSED SYSTEM

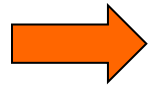
**No mass crosses the boundary of the system:**



$$(Q_{\text{in}} + W_{\text{in}}) - (Q_{\text{out}} + W_{\text{out}}) = \\ m (u_2 - u_1) + \frac{1}{2} m (V_2^2 - V_1^2) + m g (z_2 - z_1)$$



$$(Q_{\text{in}} - Q_{\text{out}}) + (W_{\text{in}} - W_{\text{out}}) = \\ m (u_2 - u_1) + \frac{1}{2} m (V_2^2 - V_1^2) + m g (z_2 - z_1)$$



$$Q_{\text{net,in}} - W_{\text{net,out}} = \\ m (u_2 - u_1) + \frac{1}{2} m (V_2^2 - V_1^2) + m g (z_2 - z_1)$$

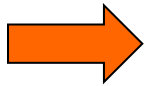
# The First Law of Thermodynamics for a CLOSED SYSTEM

$$Q_{\text{net,in}} - W_{\text{net,out}} =$$

$$m (u_2 - u_1) + \frac{1}{2} m (V_2^2 - V_1^2) + m g (z_2 - z_1)$$

$$Q = Q_{\text{in}} - Q_{\text{out}} = Q_{\text{net,in}}$$

$$W = W_{\text{out}} - W_{\text{in}} = W_{\text{net,out}}$$



$$Q - W =$$

$$m (u_2 - u_1) + \frac{1}{2} m (V_2^2 - V_1^2) + m g (z_2 - z_1)$$

If subscripts indicating directions of Q and W are removed; the sign convention must be used:

- ☐ heat input and work output are positive
- ☐ heat output and work input are negative

# The First Law of Thermodynamics for a **STATIONARY** CLOSED SYSTEM

$$Q - W = m(u_2 - u_1) + \cancel{\frac{1}{2} m (V_2^2 - V_1^2)} + m g (\cancel{z_2 - z_1})$$

$$Q - W = m(u_2 - u_1)$$

$$Q - W = \Delta U$$

# STATIONARY CLOSED SYSTEM UNDERGOING A CYCLE

$$Q - W = m (u_2 - u_1)$$

$$Q = W$$

- For a closed system operating continuously in a cycle, above equation can be written as a rate equation

$$\dot{Q} = \dot{W}$$

$$\dot{Q}_{net, in} = \dot{W}_{net, out}$$



# The First Law of Thermodynamics for a CLOSED SYSTEM

## □ Other Forms:

General  $Q - W = \Delta E$

Stationary systems  $Q - W = \Delta U$

Per unit mass  $q - w = \Delta e$

Differential form  $\delta q - \delta w = de$

# The First Law of Thermodynamics for a CLOSED SYSTEM

$$(Q_{\text{in}} + W_{\text{in}}) - (Q_{\text{out}} + W_{\text{out}}) = \\ m (u_2 - u_1) + \frac{1}{2} m (V_2^2 - V_1^2) + m g (z_2 - z_1)$$

- “ $W$ ” can be calculated from equations given earlier for different types of work (shaft, electrical, etc.)
  - Only one type of work has not been covered yet (moving boundary work) – will be covered in this Chapter
- “ $u$ ” can be obtained from tables for pure substances
  - Evaluation of “ $u$ ” for ideal gases will be covered in this Chapter

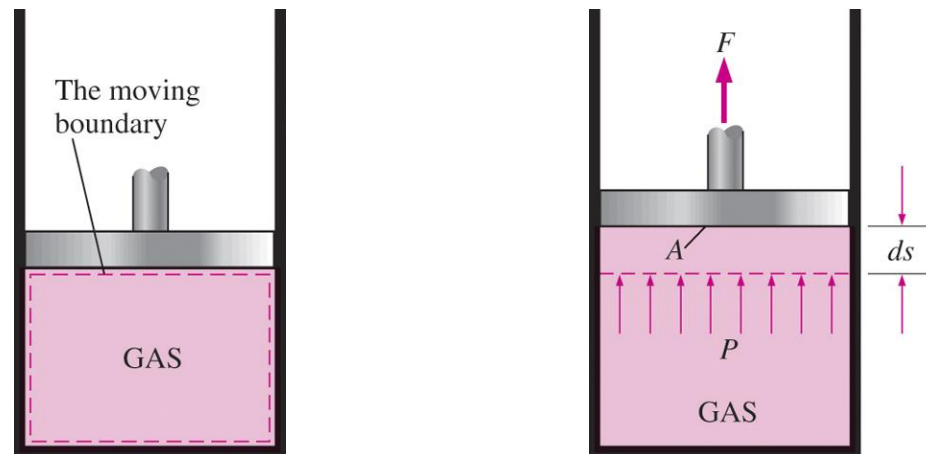
# Moving Boundary Work



- ❑ The work associated with a moving boundary (due to expansion or contraction of the system) is called *boundary work*,  $W_b$
- ❑ Work is the result of a force ( $F$ ) acting on an object for a distance ( $s$ ):

# Moving Boundary Work

- Consider gas in cylinder/piston system
- To expand, gas will have to **do** work
  - $\delta W_b = F ds = P A ds = P dV$



# Moving Boundary Work



- ❑ If the system is expanding, the fluid inside the system is pushing **out** the boundary
  - ❑ The system is exerting work on the surroundings
  - ❑ Work is going **out** of the system
  
- ❑ If the system is being compressed, the surroundings are pushing the boundary **in**
  - ❑ The surroundings are exerting work on the system
  - ❑ Work is going **into** the system

# Moving Boundary Work

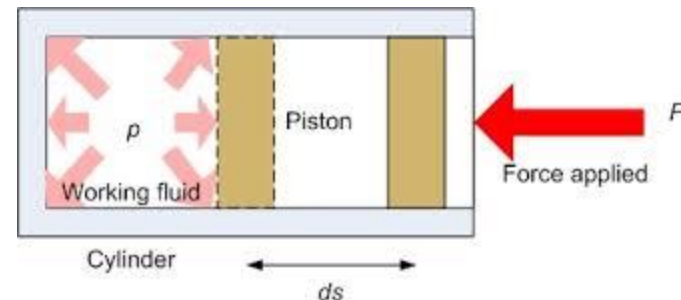
$$\delta W_b = P dV$$

- **Expansion:**  $dV > 0 \Rightarrow \delta W_b > 0$

[work done **by** the system (gas) **on** the surroundings;  
work output]

- **Compression:**  $dV < 0 \Rightarrow \delta W_b < 0$

[work done **on** the system (gas) **by** the surroundings;  
work input]

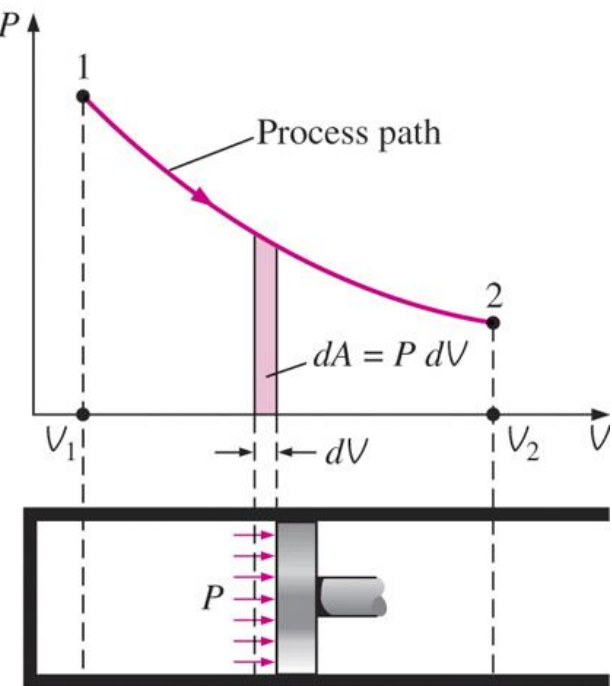


# Moving Boundary Work

- If process can be drawn on P-V diagram (sequence of equilibrium states):

- $W_b = \int_1^2 P \, dV$

- Area under the P-V diagram for a quasi-equilibrium process represents boundary work done during the process
- Expansion (positive work, i.e. work out)
- Compression (negative work, i.e. work in)



# Moving Boundary Work

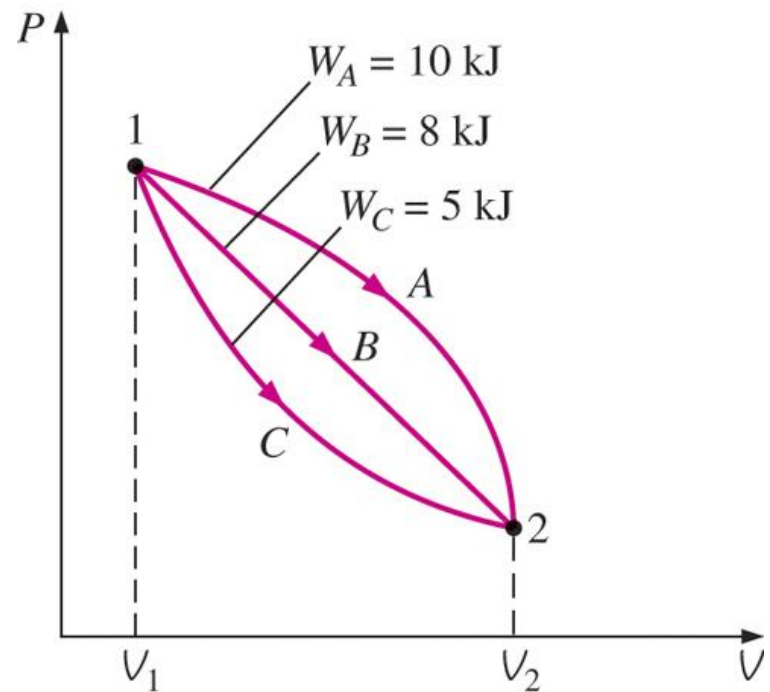
- System can follow different paths from state "1" to state "2"

- Area under the curve will depend on the path

$$W_A \neq W_B \neq W_C$$

- Work is a "Path Function"

- The boundary work done during a process depends on the path followed as well as the beginning and end states.





# Moving Boundary Work -- Examples

- Constant Pressure (Isobaric) process

$$W_b = \int_1^2 P dV = P_0 \int_1^2 dV = P_0(V_2 - V_1)$$

- Constant volume process

➤  $W_b = \text{zero}$  ( $V$  is constant)

# Moving Boundary Work -- Examples

- Isothermal process for an ideal gas

- $P V = m R T = C \text{ (constant)}$

- $$\begin{aligned} W_b &= \int_1^2 P \, dV \\ &= \int_1^2 [C / V] \, dV \\ &= C \ln (V_2 / V_1) = m R T \ln (V_2 / V_1) \\ &= P_1 V_1 \ln (V_2 / V_1) = P_2 V_2 \ln (V_2 / V_1) \end{aligned}$$

# Moving Boundary Work -- Examples

- “Polytropic” process for an ideal gas

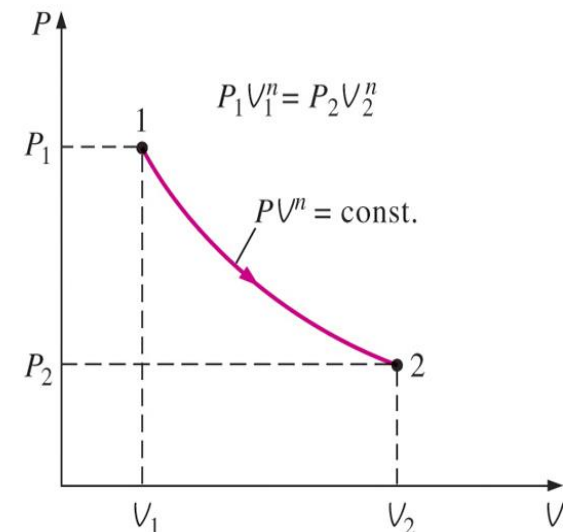
- $P V^n = \text{constant} = C \quad (n \neq 1)$

$$W_b = \int_1^2 P \, dV = \int_1^2 [C \cdot V^{-n}] \, dV$$

$$= C [(V_2)^{-n+1} - (V_1)^{-n+1}] / (-n + 1)$$

$$= [P_2 V_2 - P_1 V_1] / (1 - n)$$

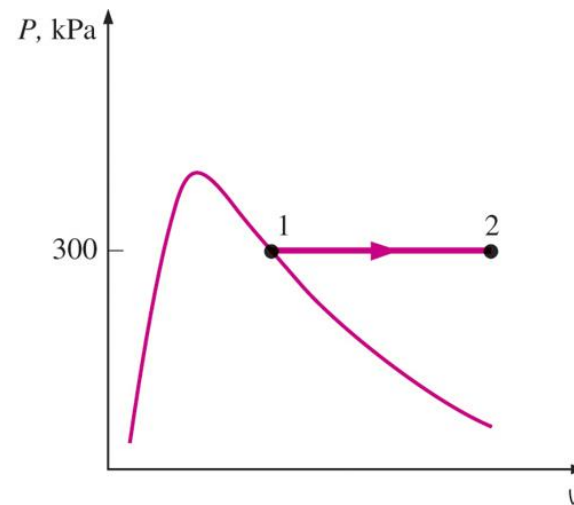
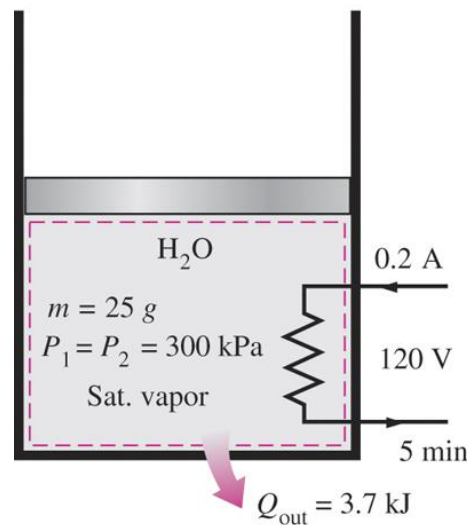
$$= m R (T_2 - T_1) / (1 - n)$$



# Application of the First Law – Closed Stationary System Undergoing an Isobaric Process

$$(Q_{\text{in}} + W_{\text{in}}) - (Q_{\text{out}} + W_{\text{out}}) =$$

$$m(u_2 - u_1) + \cancel{\frac{1}{2} m (V_2^2 - V_1^2)} + m g (\cancel{z_2 - z_1})$$



# Application of the First Law – Closed Stationary System Undergoing an Isobaric Process

$$\square (Q_{\text{in}} - Q_{\text{out}}) + W_{\text{in}} - W_{\text{out}} = \Delta U$$

$$W_{\text{out}} = W_b = \int_1^2 P \, dV \quad \Rightarrow \quad \boxed{W_{\text{out}} = P (V_2 - V_1)}$$

$$(Q_{\text{in}} - Q_{\text{out}}) + W_{\text{in}} = P (V_2 - V_1) + (U_2 - U_1)$$

$$(Q_{\text{in}} - Q_{\text{out}}) + W_{\text{in}} = H_2 - H_1$$

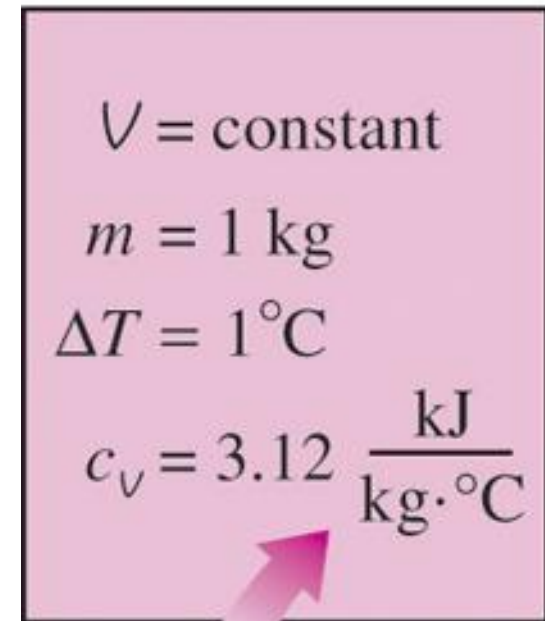
# Specific Heat



- **Energy required to raise the temperature of a unit mass of a substance by one degree**
  - **In general, depends on how the process is executed**
- Specific heat at constant volume,  $C_v$
- Specific heat at constant pressure,  $C_p$

# Specific Heat at Constant Volume

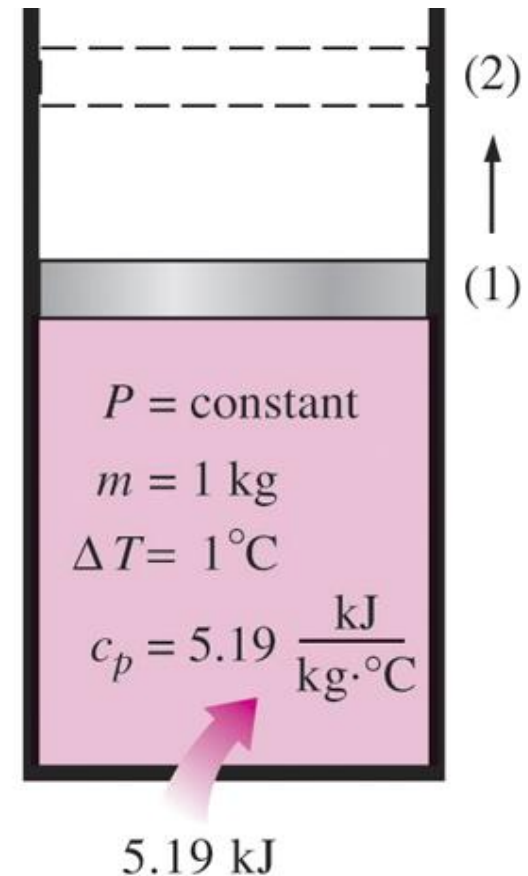
- ❑ ***Specific heat at constant volume ( $C_v$ )*** is the energy required to raise the temperature of a unit mass of a substance by one degree **as the volume is maintained constant.**


$$V = \text{constant}$$
$$m = 1 \text{ kg}$$
$$\Delta T = 1^\circ\text{C}$$
$$c_v = 3.12 \frac{\text{kJ}}{\text{kg} \cdot ^\circ\text{C}}$$

3.12 kJ

# Specific Heat at Constant Pressure

- ***Specific heat at constant pressure ( $C_p$ )*** is the energy required to raise the temperature of a unit mass of a substance by one degree **as the pressure is maintained constant.**





# Formal Definition of $C_v$ and $C_p$

$$C_v = \left( \frac{\partial u}{\partial T} \right)_v \qquad C_p = \left( \frac{\partial h}{\partial T} \right)_p$$

- Relations result directly from application of first law to a closed stationary system
  - At constant volume, the amount of energy added to the system equals the change in internal energy
  - At constant pressure, the amount of energy added to the system equals the change in enthalpy (see example on slide #20)

# LECTURE # 12



## **Chapter 4 (Session #2):** **Energy Analysis of Closed Systems**

# Relation Between $C_v$ and Specific Internal Energy

$$Q_{\text{net,in}} - \cancel{W_{\text{net,out}}^0} = m(u_2 - u_1) + \cancel{\frac{1}{2} m(V_2^2 - V_1^2)} + m g(\cancel{z_2 - z_1}^0)$$

For a stationary closed system at constant volume,  $W_b$  is zero

$$Q_{\text{in}} = m(u_2 - u_1) \quad \text{or} \quad q_{\text{in}} = u_2 - u_1$$

$$C_v \equiv (\delta q_{\text{in}}/dT)_v = (\partial u/\partial T)_v$$

$V = \text{constant}$   
 $m = 1 \text{ kg}$   
 $\Delta T = 1^\circ\text{C}$   
 $c_v = 3.12 \frac{\text{kJ}}{\text{kg}\cdot^\circ\text{C}}$

3.12 kJ

# Relation Between $C_p$ and Specific Enthalpy

$$Q_{\text{net,in}} - W_{\text{net,out}} = m(u_2 - u_1) + \cancel{\frac{1}{2} m (V_2^2 - V_1^2)} + m g (\cancel{z_2 - z_1})$$

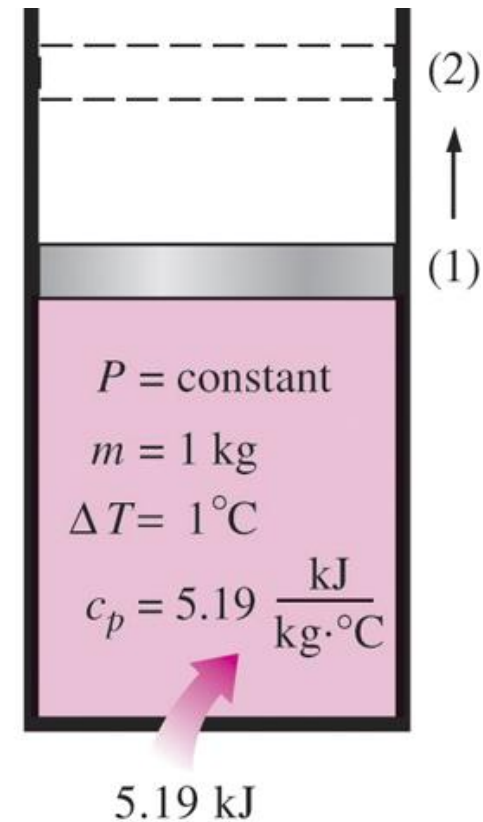
For a stationary closed system at constant pressure, the boundary work  $W_b = P (V_2 - V_1)$

$$Q_{\text{in}} - P (V_2 - V_1) = m (u_2 - u_1)$$

$$Q_{\text{in}} - m P (v_2 - v_1) = m (u_2 - u_1)$$

$$Q_{\text{in}} = m [ (u_2 + P v_2) - (u_1 + P v_1) ] = m (h_2 - h_1)$$

$$C_p \equiv (\delta q_{\text{in}}/dT)_p = (\partial h/\partial T)_p$$



# The Specific Heats $C_v$ and $C_p$

- From energy conservation (First Law) we showed that:
  - $C_v = (\partial u / \partial T)_v$  and  $C_p = (\partial h / \partial T)_p$
- Since  $(u, T \text{ \& } v)$  and  $(h, T \text{ \& } P)$  are properties, the Specific heats ( $C_v$  and  $C_p$ ) are also **properties**, i.e. they have unique values at each equilibrium state [point functions]

# Notes on $C_v$ and $C_p$

- ❑  $C_v$  and  $C_p$  are properties.
- ❑  $C_p$  is always larger than  $C_v$
- ❑ Recall the State Postulate: The properties of matter are functionally related -- The specific heat of a substance can be determined from any two independent intensive properties

# Ideal Gas Specific Heats

- In 1843, Joule showed that the internal energy of an ideal gas is a **function of temperature ONLY**;
- **$u = u(T)$ ; therefore:**
  - **$C_v = (\partial u / \partial T)_v = (du/dT) = f(T)$  only**
- By definition:  $h = u + P v = u + R T$ ;  
therefore:  $h = h(T)$ ;
  - **$C_p = (\partial h / \partial T)_p = (dh/dT) = f(T)$  only**

# Ideal Gas Internal Energy & Enthalpy

- Since  $C_v$  and  $C_p$  for an ideal gas are functions of Temperature only

$$du = c_v(T) dT$$

$$\Delta u = u_2 - u_1 = \int_1^2 c_v(T) dT$$

$$dh = c_p(T) dT$$

$$\Delta h = h_2 - h_1 = \int_1^2 c_p(T) dT$$



# Ideal Gas Specific Heats

- Real gases behave as ideal gases in the “zero-density limit” (i.e. zero pressure)
- Specific heats of real gases at low pressures [denoted  $C_{p0}(T)$  and  $C_{v0}(T)$ ] are called the Ideal Gas Specific Heats
- Analytical expressions of  $C_{p0}$  and  $C_{v0}$  for different gases are given in Table A-2c
  - Can Integrate to determine  $\Delta u$  and  $\Delta h$  between  $T_1$  &  $T_2$  (Cumbersome)
  - Values of  $u$  and  $h$  are tabulated relative to a reference of Zero K

# Ideal Gas Internal Energy & Enthalpy

- For an ideal gas,  $u = u(T)$  &  $h = h(T)$
- There are tables showing values of  $u$  and  $h$  for different temperatures (Reference Temperature = zero K):

TABLE A-17

Ideal-gas properties of air

$T$ K	$h$ kJ/kg	$P_r$	$u$ kJ/kg	$v_r$	$s^\circ$ kJ/kg·K	$T$ K	$h$ kJ/kg	$P_r$	$u$ kJ/kg	$v_r$	$s^\circ$ kJ/kg·K
200	199.97	0.3363	142.56	1707.0	1.29559	580	586.04	14.38	419.55	115.7	2.37348
210	209.97	0.3987	149.69	1512.0	1.34444	590	596.52	15.31	427.15	110.6	2.39140
220	219.97	0.4690	156.82	1346.0	1.39105	600	607.02	16.28	434.78	105.8	2.40902
230	230.02	0.5477	164.00	1205.0	1.43557	610	617.53	17.30	442.42	101.2	2.42644
240	240.02	0.6355	171.13	1084.0	1.47824	620	628.07	18.36	450.09	96.92	2.44356
250	250.05	0.7329	178.28	979.0	1.51917	630	638.63	19.84	457.78	92.84	2.46048
260	260.09	0.8405	185.45	887.8	1.55848	640	649.22	20.64	465.50	88.99	2.47716
270	270.11	0.9590	192.60	808.0	1.59634	650	659.84	21.86	473.25	85.34	2.49364
280	280.13	1.0889	199.75	738.0	1.63279	660	670.47	23.13	481.01	81.89	2.50985
285	285.14	1.1584	203.33	706.1	1.65055	670	681.14	24.46	488.81	78.61	2.52589

# Ideal Gas Internal Energy & Enthalpy

- An alternative is to find  $C_v$  and  $C_p$  and apply:

$$\Delta u = u_2 - u_1 = \int_1^2 c_v(T) dT \quad \Delta h = h_2 - h_1 = \int_1^2 c_p(T) dT$$

- There are tables showing the values of  $C_v$  and  $C_p$  for different gases at different temperatures:

TABLE A-2

Ideal-gas specific heats of various common gases (Continued)

(b) At various temperatures

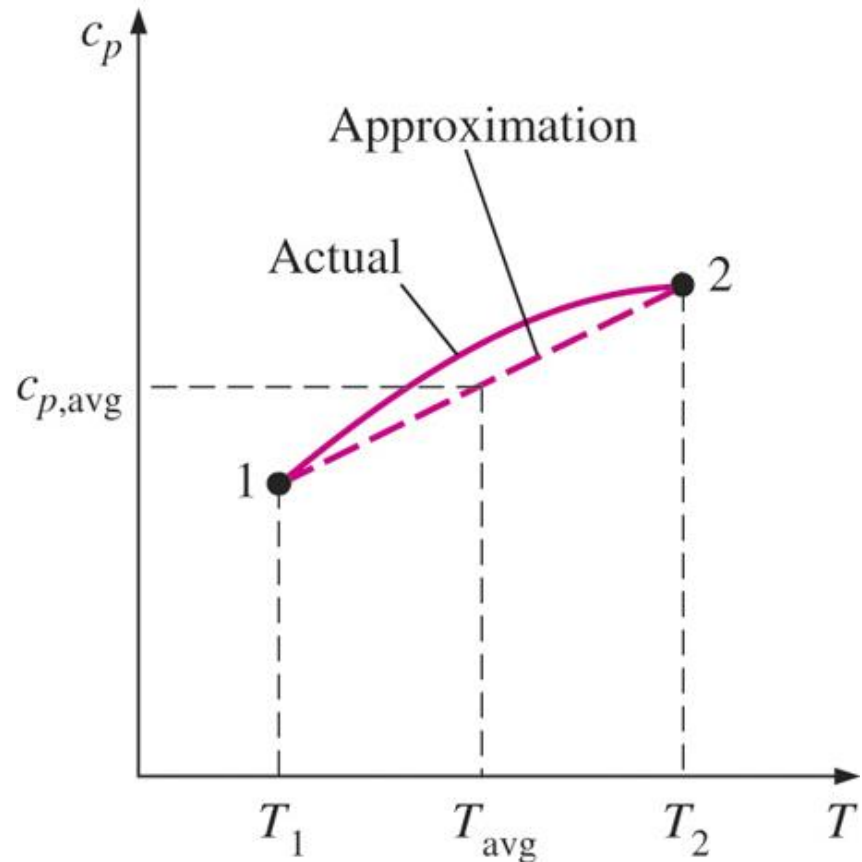
Temperature, K	$c_p$ kJ/kg·K	$c_v$ kJ/kg·K	$k$	$c_p$ kJ/kg·K	$c_v$ kJ/kg·K	$k$	$c_p$ kJ/kg·K	$c_v$ kJ/kg·K	$k$
	<i>Air</i>			<i>Carbon dioxide, CO<sub>2</sub></i>			<i>Carbon monoxide, CO</i>		
250	1.003	0.716	1.401	0.791	0.602	1.314	1.039	0.743	1.400
300	1.005	0.718	1.400	0.846	0.657	1.288	1.040	0.744	1.399
350	1.008	0.721	1.398	0.895	0.706	1.268	1.043	0.746	1.398
400	1.013	0.726	1.395	0.939	0.750	1.252	1.047	0.751	1.395

# Ideal Gas Internal Energy & Enthalpy

- ❑ Yet another alternative is to use average values of  $C_v$  and  $C_p$  and consider them constant.
- ❑ For small temperature intervals, this assumption is acceptable.

$$u_2 - u_1 = c_{v,\text{avg}}(T_2 - T_1)$$

$$h_2 - h_1 = c_{p,\text{avg}}(T_2 - T_1)$$



# Specific Heat Relations for an Ideal Gas

□ By definition:  $h = u + P v$

□ For an Ideal Gas:  $h = u + R T$

➤  $dh/dT = du/dT + R$  [h & u are functions of T only]

$$c_p = c_v + R \quad (\text{kJ/kg K})$$

OR  $\bar{C}_p = \bar{C}_v + R_u \quad (\text{kJ/kmol K})$

# Ideal Gas Specific Heat Relations

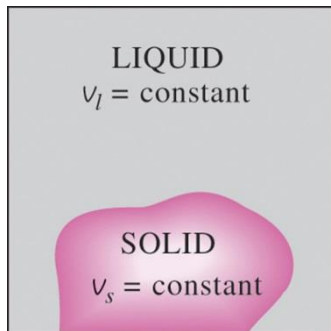
## ● Define: Specific heat ratio

$$k = \frac{c_p}{c_v}$$

- $k$  is a “weak” function of temperature
- For monatomic gases :  $k = 1.667$
- For diatomic gases:  $k \approx 1.4$  (at  $T_{\text{amb}}$ )

# Internal Energy, Enthalpy and Specific Heats of Solids & Liquids

**Incompressible substance:** A substance whose specific volume (or density) is constant. **Solids and liquids** can be approximated as incompressible substances.



The specific volumes of incompressible substances remain constant during a process.



The  $c_v$  and  $c_p$  values of incompressible substances are identical and are denoted by  $c$ .

# Specific Heats – Incompressible Substances



- **For incompressible substances:**

- $C_p = C_v = C$

- $C$  is a function of temperature only



# Internal Energy – Incompressible Substances

- $\Delta u = u_2 - u_1 = \int_1^2 C(T) dT \quad (\text{kJ/kg})$
- For small temperature intervals:
  - $\Delta u = u_2 - u_1 \approx C_{\text{avg}} (T_2 - T_1) \quad (\text{kJ/kg})$

# Enthalpy -- Incompressible substances

- $h = u + P v \Rightarrow dh = du + P dv + v dP$ 
  - $v = \text{constant} \Rightarrow dh = du + v dP$
  - $\Delta h = \Delta u + v \Delta P$
  - **For solids:**  $v \Delta P$  insignificant  $\Rightarrow \Delta h = \Delta u \approx C_{\text{avg}} \Delta T$
  - **For Liquids,** two cases are encountered:
    - Constant Pressure process (e.g. heaters)  
 $\Rightarrow \Delta h = \Delta u \approx C_{\text{avg}} \Delta T$
    - Constant temperature process (e.g. pumps)  
 $\Rightarrow \Delta h = v \Delta P$

# CHAPTER 4

## ENERGY ANALYSIS OF CLOSED SYSTEMS



### OUTCOME:

- ✓ Develop and Apply the conservation of energy principle (the First Law) to a closed system.
- ✓ Determine the work associated with a moving boundary.
- ✓ Define the constant pressure and constant volume specific heats and relate them to changes in internal energy and enthalpy.
- ✓ Solve energy balance problems for closed systems that involve heat and work interactions for pure substances, ideal gases and incompressible substances.