LECTURE # 13

Chapter 5 (Session #1):

Mass and Energy Analysis of Control Volumes

CHAPTER 5

MASS AND ENERGY ANALYSIS OF CONTROL VOLUMES

CHAPTER 5 -- Mass and Energy Analysis of Control Volumes

OUTCOME:

- Develop and Apply the conservation of mass principle to both steady and unsteady control volumes.
- Identify the energy carried by a fluid stream crossing a control surface.
- Develop and Apply the conservation of energy principle to control volumes
- Solve energy balance problems for steady flow devices.
- Apply energy balance to unsteady flow processes with emphasis on the uniform-flow process.

CONSERVATION OF MASS

- **Mass is conserved -- it cannot be created or destroyed**
  - Relativistic effects are ignored since minute changes in mass are beyond precision of engineering measurements

Conservation of Mass

- For a control volume, the conservation of mass principle can be expressed as:

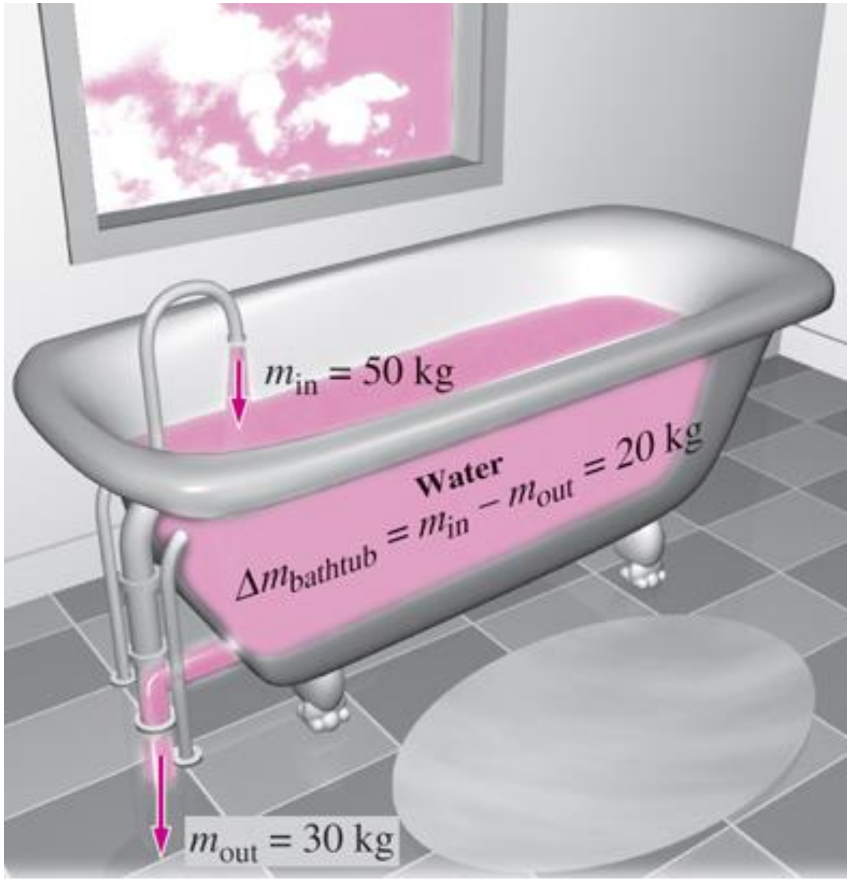
$$\boxed{\begin{pmatrix}\text{Total mass entering}\\ \text{the CV during } \Delta t\end{pmatrix} - \begin{pmatrix}\text{Total mass leaving}\\ \text{the CV during } \Delta t\end{pmatrix} = \begin{pmatrix}\text{Net change in mass}\\ \text{within the CV during } \Delta t\end{pmatrix}}$$

CONSERVATION OF MASS

- In mathematical form, the conservation of mass principle for a control volume can be expressed as:
- $\Delta \mathbf{m}_{\mathbf{CV}} = \mathbf{m}_{\mathbf{in}} - \mathbf{m}_{\mathbf{out}}$
  - Δm_{CV} = Net change in mass within the control volume (kg)
  - m_{in} = Total mass entering the system (kg)
  - m_{out} = Total mass leaving the system (kg)

Conservation of Mass

$$m_{in} - m_{out} = \Delta m_{CV}$$



CONSERVATION OF MASS

- Control Volume with multiple inlets and outlets undergoing a process between initial state "1" and final state "2"
- $\Delta m_{CV} = (m_2 - m_1)_{CV} = \Sigma m_{in} - \Sigma m_{out}$
- summation sign "Σ" indicates that all inlets "in" and outlets "out" are to be included

CONSERVATION OF MASS

- Equation can expressed in a rate form:

$$\boxed{\dot{m}_{\text{in}} - \dot{m}_{\text{out}} = dm_{\text{CV}}/dt}$$

$$\boxed{\sum \dot{m}_{in} - \sum \dot{m}_{out} = dm_{\text{CV}}/dt}$$

  - (dm_{CV}/dt) = rate of change of system (CV) mass (kg/s)
  - $\dot{m}_{in}$ = inlet mass flow rate (kg/s)
  - $\dot{m}_{out}$ = exit mass flow rate (kg/s)

Control Volume Mass

- **For uniform density**
  - $m_{CV} = \rho\ V$

- **For non-uniform density**
  - $m_{CV} = \int_{CV} \rho\ dV$

Mass Flow Rate

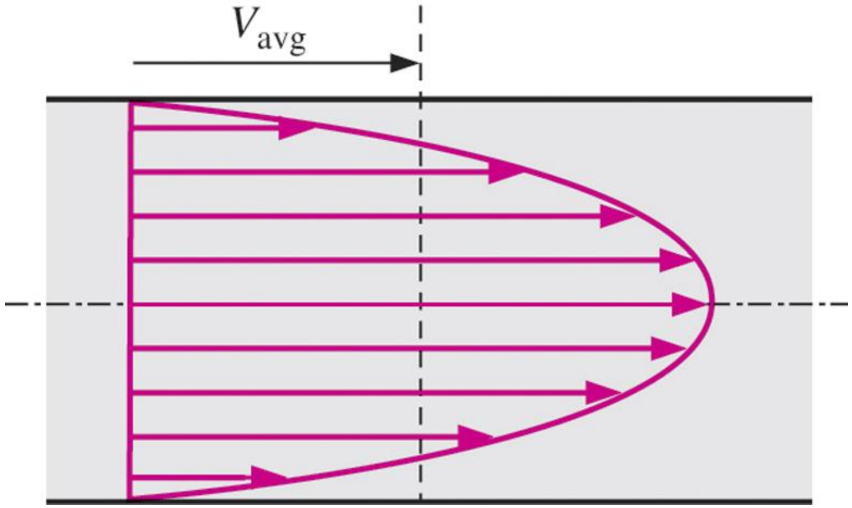
- For uniform Velocity (and density)
  - $\dot{m} = \rho \, V_n \, A_c$ (kg/s)
    - V_n = velocity component normal to A_c (m/s)
    - A_c = Cross sectional area normal to flow direction

Mass Flow Rate

- For Non-Uniform velocity:

$$\delta \dot{m} = \rho V_n \, dA_c$$

$$\dot{m} = \int_{A_c} \delta \dot{m} = \int_{A_c} \rho V_n \, dA_c$$



For uniform density this can be written as

$$\boxed{\dot{m} = \rho V_{avg} A_c \qquad (\text{kg/s})}$$

Where V_{avg} is the average velocity

Volumetric Flow Rate

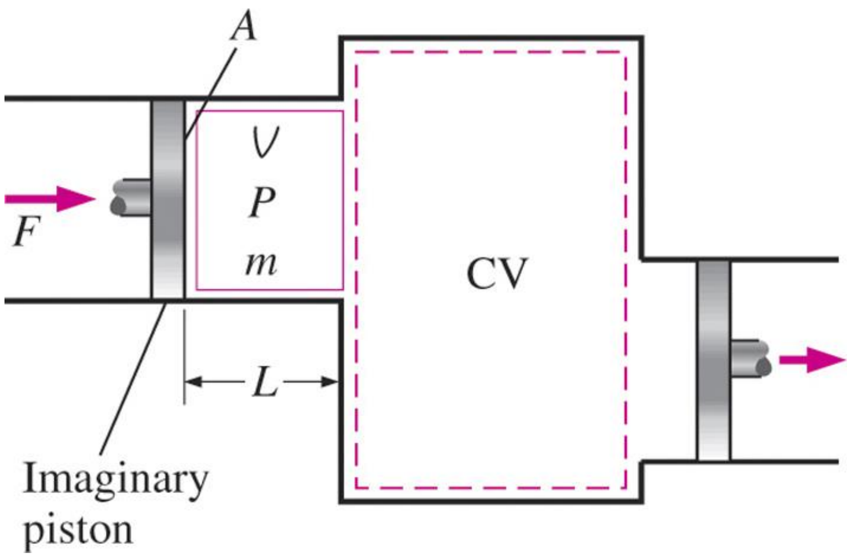
- For uniform Velocity
  - $\dot{V} = V_n A_c$ (m^3/s) $= \dot{m} / \rho = \dot{m}\, v$
- For non-uniform Velocity
  - $\dot{V} = \int_{Ac} V_n \, dA_c$ (m^3/s)
  - $\dot{V} = V_{avg} A_c$
    - V_{avg} (m/s) = Average fluid velocity normal to A
    - A_c (m^2) = cross sectional area normal to flow direction

Total Energy of a Flowing Fluid

- **For a closed system**: the energy per unit mass: $e = u + V^2/2 + g\,z$ (kJ/kg)
- **For an open system:** mass crosses the boundary
  - Each unit mass crossing the boundary has total energy: $e = u + V^2/2 + g\,z$ (kJ/kg)
  - In addition, when mass crosses a system boundary, work is done to move that mass across the boundary. This is referred to as "Flow work"

FLOW WORK

- **Flow work, or flow energy** is the work (or energy) required to "push" the mass into or out of the control volume.
- This work is necessary for maintaining a continuous flow through a control volume.

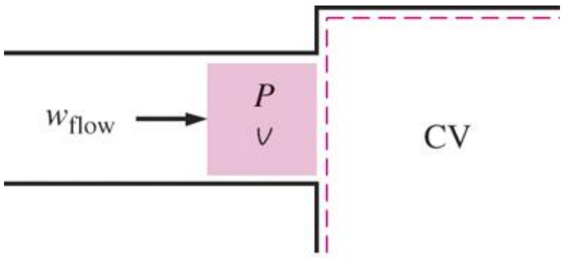


$$W_{\text{flow}} = FL = PAL = PV$$

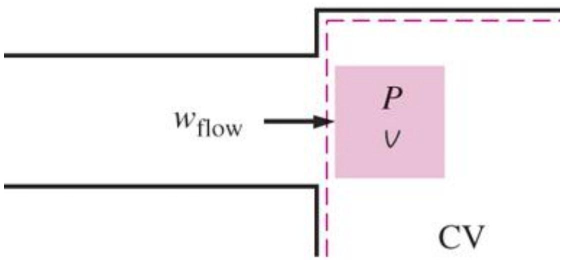
$$\boxed{w_{\text{flow}} = Pv}$$ Flow Work per unit mass

Total Energy of a Flowing Fluid

- As the fluid crosses the boundary, the *energy* contained in it enters the system. Per unit mass, this energy is:
- $e = u + \mathbb{V}^2/2 + g\,z$ (kJ/kg)
- Also, the "flow work", i.e. work done by (on) the surroundings on (by) the system to "push" the fluid into (out of) the CV represents energy added to (removed from) the CV.
- **Sum of energy per unit mass "e" and the flow work per unit mass represents the total energy added (removed) to (from) the system per unit mass entering (exiting) the open system.**



(*a*) Before entering



(*b*) After entering

Total Energy of a Flowing Fluid

- **Total energy of a flowing fluid per unit mass:**
- θ = e + flow work per unit mass [kJ/kg]
  - $\theta = (u + \mathbb{V}^2/2 + g\,z) + (P\,v)$ [kJ/kg]
  - $\theta = (u + P\,v) + \mathbb{V}^2/2 + g\,z$ [kJ/kg]
  - $\theta = h + \mathbb{V}^2/2 + g\,z$ [kJ/kg]

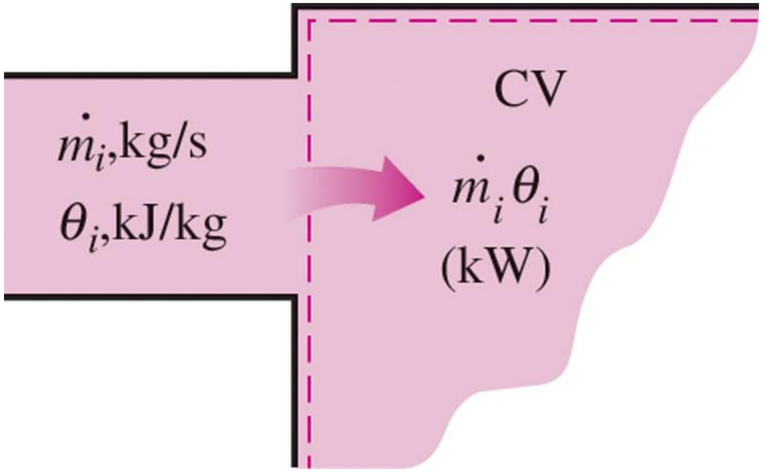
ENERGY TRANSPORT BY MASS

- The total energy transferred by mass is denoted by E_{mass} and is equal to:

$$E_{\text{mass}} = m\theta = m\left(h + \frac{V^2}{2} + gz\right)$$

- In rate form:

$$\dot{E}_{\text{mass}} = \dot{m}\theta = \dot{m}\left(h + \frac{V^2}{2} + gz\right)$$



The First Law of Thermodynamics for Open Systems

$$\left[\dot{Q}_{\text{in}} + \dot{W}_{\text{in}} + \sum\nolimits_{\text{in}} \dot{m}\left(h + \frac{V^2}{2} + gz\right)\right] -$$

$$\left[\dot{Q}_{\text{out}} + \dot{W}_{\text{out}} + \sum\nolimits_{\text{out}} \dot{m}\left(h + \frac{V^2}{2} + gz\right)\right] =$$

$$= \frac{dU}{dt} + \frac{dKE}{dt} + \frac{dPE}{dt}$$

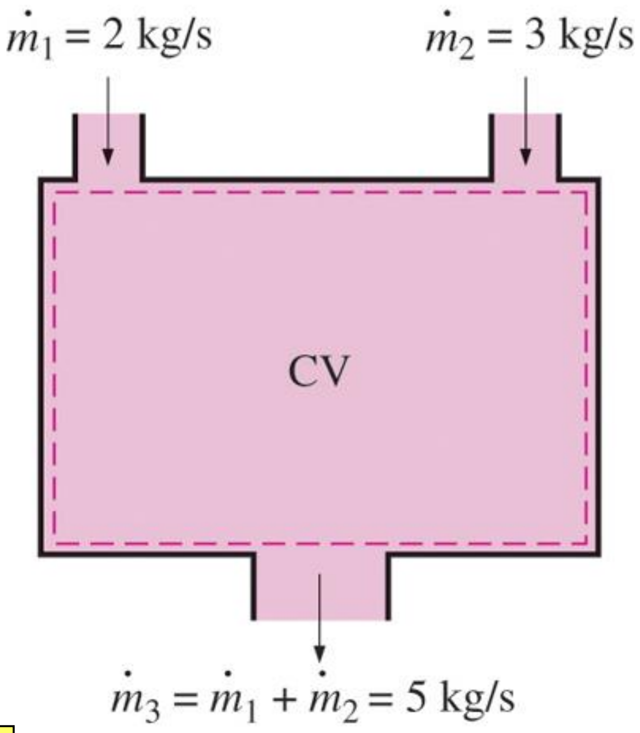
Mass Balance for Steady-Flow Processes

- Steady flow ≡ **no change with time**
  - **$(dm_{CV}/dt) = 0$**
- Conservation of mass reduces to:

$$\boxed{\sum_{\text{in}} \dot{m} = \sum_{\text{out}} \dot{m}}$$

- If there is only a single inlet and single outlet

$$\boxed{\dot{m}_1 = \dot{m}_2} \Rightarrow \boxed{\rho_1 V_1 A_1 = \rho_2 V_2 A_2}$$



Mass Balance for Steady Incompressible Flow Processes

- ***Incompressible Flow***: A flow in which the specific volume (and density) remain constant.
- The conservation of mass relations can be simplified even further when the fluid is incompressible, which is usually the case for liquids.

$$\sum_{\text{in}} \dot{V} = \sum_{\text{out}} \dot{V} \qquad (\text{m}^3/\text{s})$$

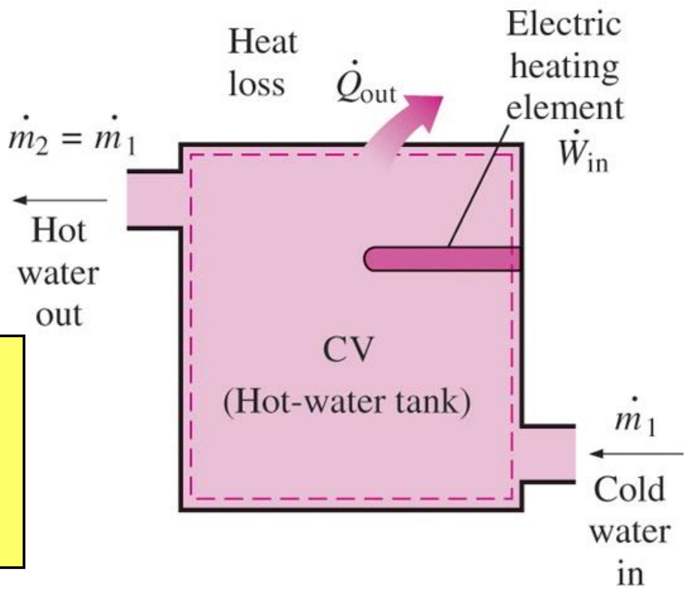
Steady, incompressible Flow

$$\dot{V}_1 = \dot{V}_2 \rightarrow V_1 A_1 = V_2 A_2$$

Steady, incompressible Flow (Single Stream)

Energy Balance for Steady-Flow Processes

- Steady flow ≡ **no change with time**
  - ➢ **$(dE_{CV}/dt) = 0$**
- Conservation of energy reduces to:



$$\underbrace{\dot{E}_{in}}_{\substack{\text{Rate of net energy transfer in}\\ \text{by heat, work, and mass}}} = \underbrace{\dot{E}_{out}}_{\substack{\text{Rate of net energy transfer out}\\ \text{by heat, work, and mass}}}$$

$$\dot{Q}_{in} + \dot{W}_{in} + \sum_{in} \underbrace{\dot{m}\left(h + \frac{V^2}{2} + gz\right)}_{\text{for each inlet}} = \dot{Q}_{out} + \dot{W}_{out} + \sum_{out} \underbrace{\dot{m}\left(h + \frac{V^2}{2} + gz\right)}_{\text{for each exit}}$$

Some Steady-Flow Engineering Devices

- Many engineering devices operate steadily for long periods of time -- **Conditions at different points in the system are different but they do not change with time.**

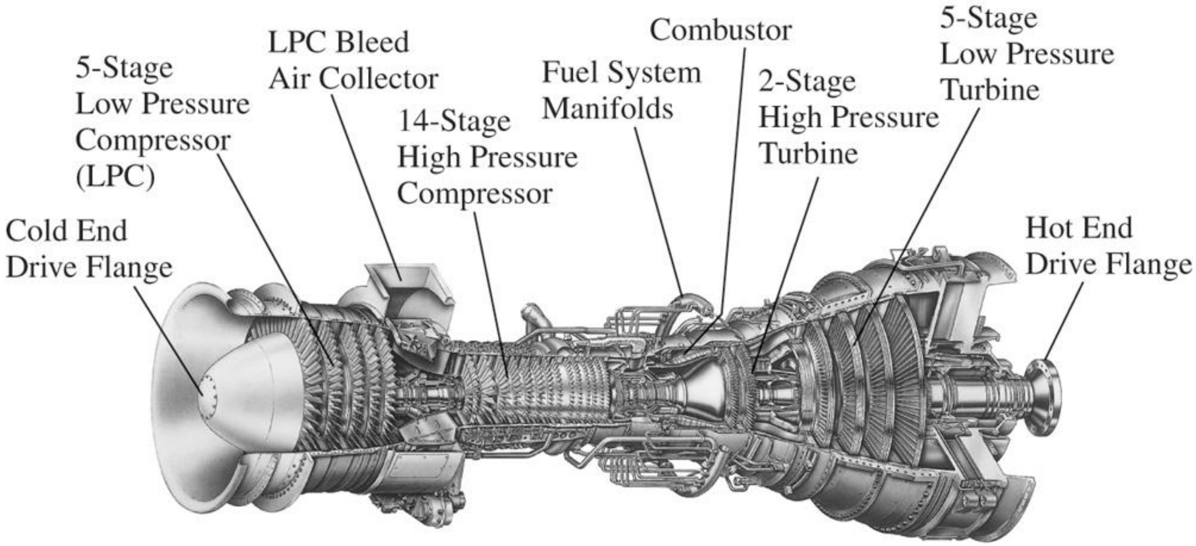
- Examples: turbines, compressors, nozzles, heat exchangers, pumps.

- These devices can be conveniently analyzed as steady-flow devices.

- Application of the first law (energy balance) to some of these devices will be presented:

  - **Turbines and compressors**
  - **Throttling valves**
  - **Heat exchangers**

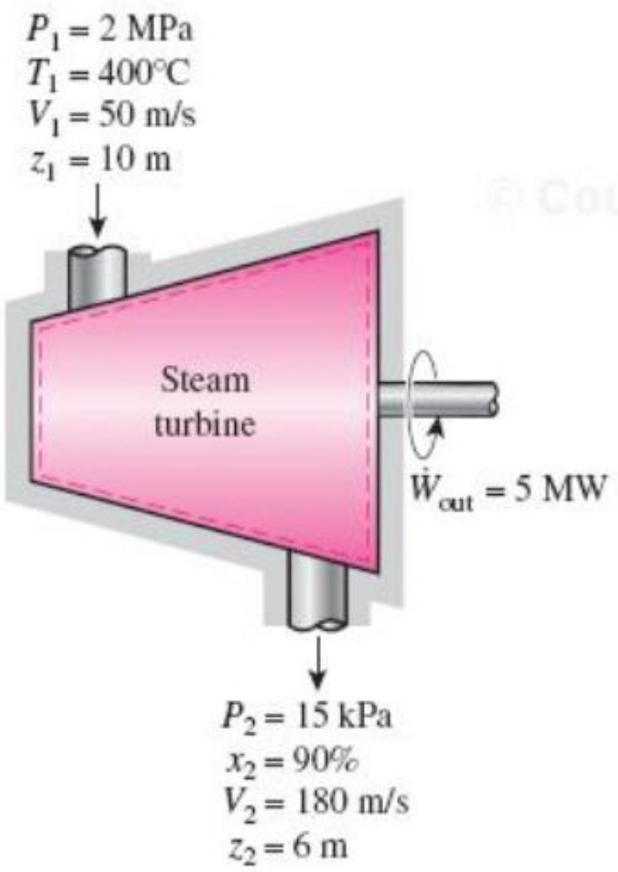
Steady-Flow Engineering Devices -- Example



- A modern land-based gas turbine used for electric power production. This is a General Electric LM5000 turbine. It produces 55.2 MW at 3600 rpm with steam injection.
- Entire Device can be treated as a Steady-Flow System -- Conditions at different points in the system are different but they do not change with time.

Turbines

- **Turbine** drives the electric generator in steam, gas, or hydroelectric power plants.
- The working fluid passes through the turbine.
- Force acting on the blades, which are attached to the shaft, exert a torque on the shaft
- The shaft rotates, and the turbine produces work.



Turbines –
Common Assumptions

- Heat transfer is often neglected (well insulated)
- Changes in kinetic and potential energy of the entering and leaving fluid are sometimes neglected (small compared to change in enthalpy)
- No work input

$$\cancel{\dot{Q}_{\text{in}}} + \cancel{\dot{W}_{\text{in}}} + \underbrace{\sum_{\text{in}} \dot{m}\left(h + \cancel{\frac{V^2}{2}} + \cancel{gz}\right)}_{\text{for each inlet}} = \cancel{\dot{Q}_{\text{out}}} + \dot{W}_{\text{out}} + \underbrace{\sum_{\text{out}} \dot{m}\left(h + \cancel{\frac{V^2}{2}} + \cancel{gz}\right)}_{\text{for each exit}}$$

$$\dot{m}h_1 - \dot{W}_{\text{out}} = \dot{m}h_2 \quad \Rightarrow \quad \boxed{\dot{W}_{\text{out}} = \dot{m}(h_1 - h_2)}$$

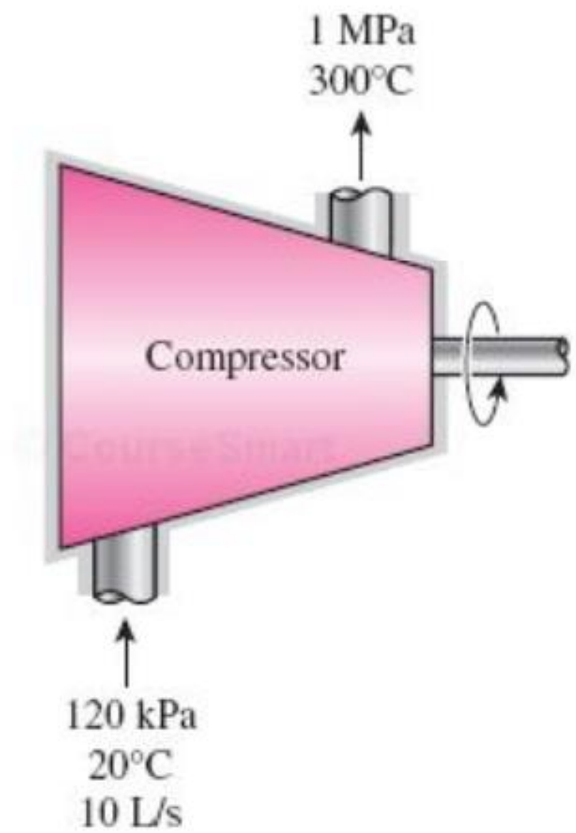
LECTURE # 14

Chapter 5 (Session #2):

Mass and Energy Analysis of Control Volumes

Compressors

- **Compressors** are used to increase the pressure of a gas.
- Work is supplied from an external source through a rotating shaft.
- **Pumps** are similar to compressors except that they handle liquids instead of gases.



Compressors – Common Assumptions

- Heat transfer is often neglected (well insulated)
- Changes in kinetic and potential energy of the entering and leaving fluid are sometimes neglected
- No work output

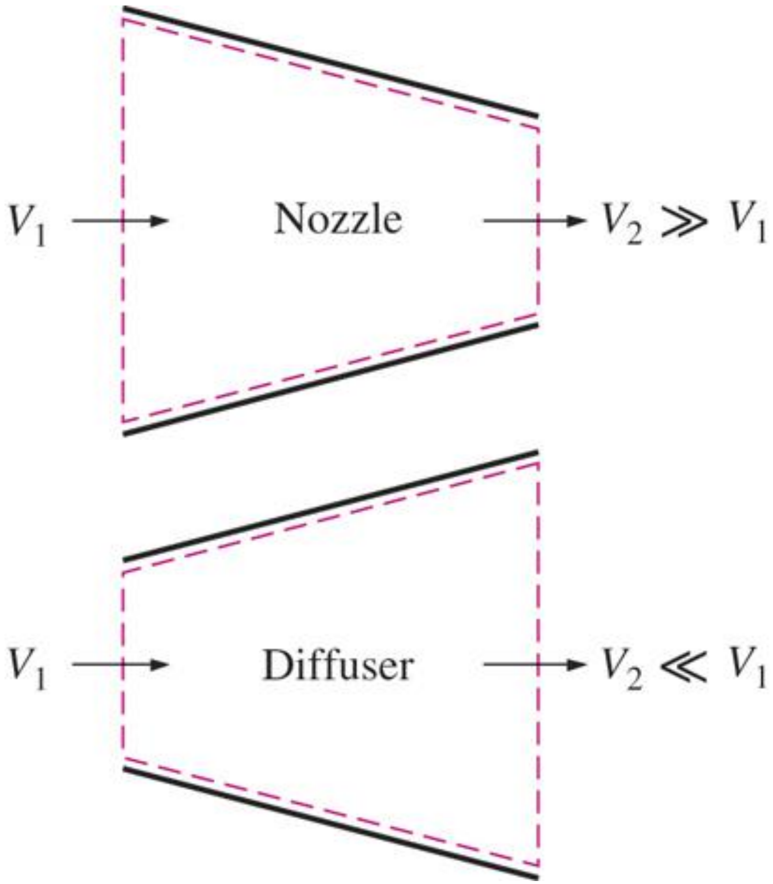
$$\cancel{\dot{Q}_{\text{in}}} + \dot{W}_{\text{in}} + \sum_{\text{in}} \underbrace{\dot{m}\left(h + \cancel{\frac{V^2}{2}} + \cancel{gz}\right)}_{\text{for each inlet}} = \cancel{\dot{Q}_{\text{out}}} + \cancel{\dot{W}_{\text{out}}} + \sum_{\text{out}} \underbrace{\dot{m}\left(h + \cancel{\frac{V^2}{2}} + \cancel{gz}\right)}_{\text{for each exit}}$$

$$\dot{W}_{\text{in}} + \dot{m}h_1 = \dot{m}h_2$$

$$\boxed{\dot{W}_{\text{in}} = \dot{m}(h_2 - h_1)}$$

Nozzles and Diffusers

- **Nozzles** increase the velocity of a fluid while pressure decreases.
- **Diffusers** increase the pressure of a fluid as the velocity decreases.
- The cross-sectional area of a nozzle decreases in the flow direction (incompressible; compressible subsonic flow); reverse is true for diffusers.



Nozzles and Diffusers –
Common Assumptions

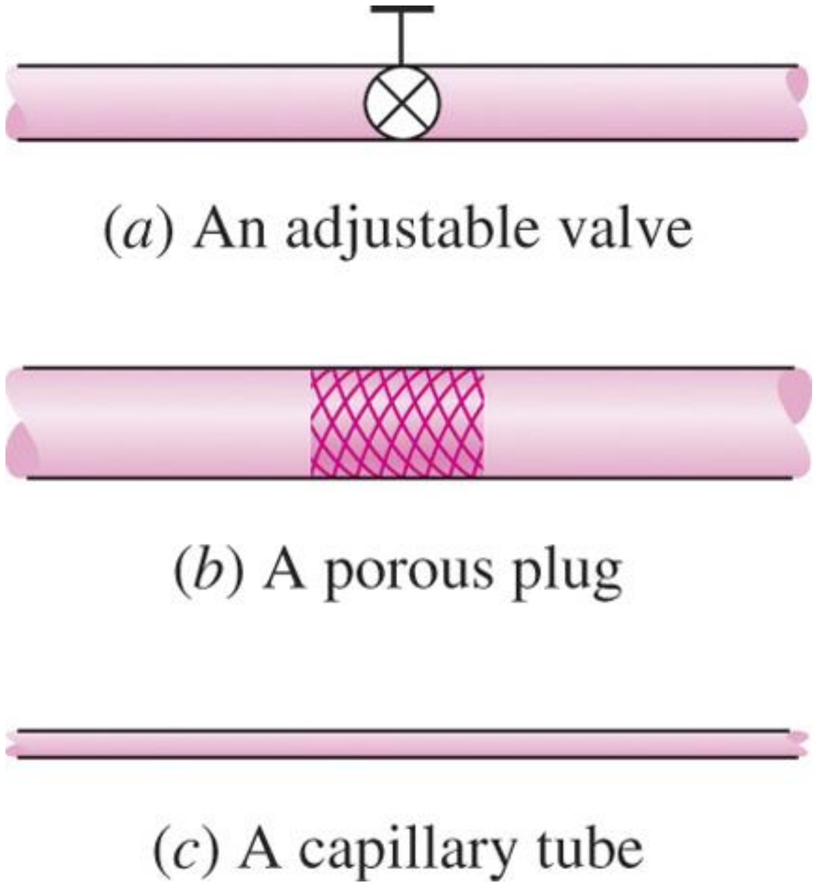
- ❑ Heat transfer is often neglected
- ❑ Changes in potential energy are neglected
- ❑ No work input or output

$$\cancel{\dot{Q}_{\text{in}}} + \cancel{\dot{W}_{\text{in}}} + \sum_{\text{in}} \underbrace{\dot{m}\left(h + \frac{V^2}{2} + \cancel{gz}\right)}_{\text{for each inlet}} = \cancel{\dot{Q}_{\text{out}}} + \cancel{\dot{W}_{\text{out}}} + \sum_{\text{out}} \underbrace{\dot{m}\left(h + \frac{V^2}{2} + \cancel{gz}\right)}_{\text{for each exit}}$$

$$\boxed{\dot{m}\left(h_1 + \frac{V_1^2}{2}\right) = \dot{m}\left(h_2 + \frac{V_2^2}{2}\right)}$$

Throttling Valves

- **Throttling valves** are devices restricting the flow, causing significant pressure drop in the fluid.
- Pressure drop is often accompanied by a large drop in temperature.
- Throttling devices are commonly used in refrigeration and air-conditioning applications.



Throttling Valves – Common Assumptions

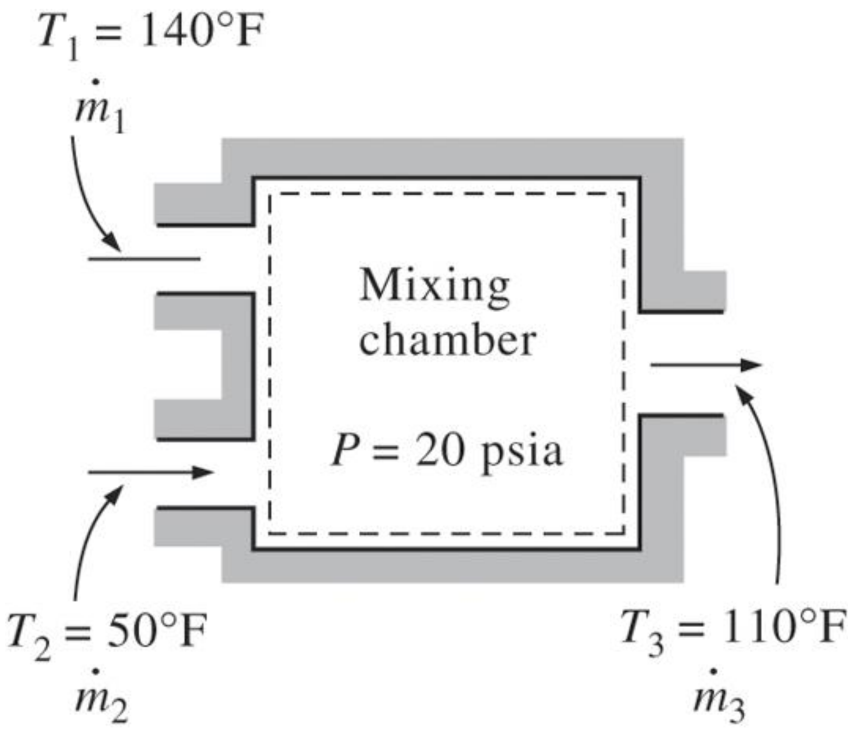
- heat transfer is often neglected
- Changes in kinetic and potential energy of the entering and leaving fluid are usually neglected
- No work input or output

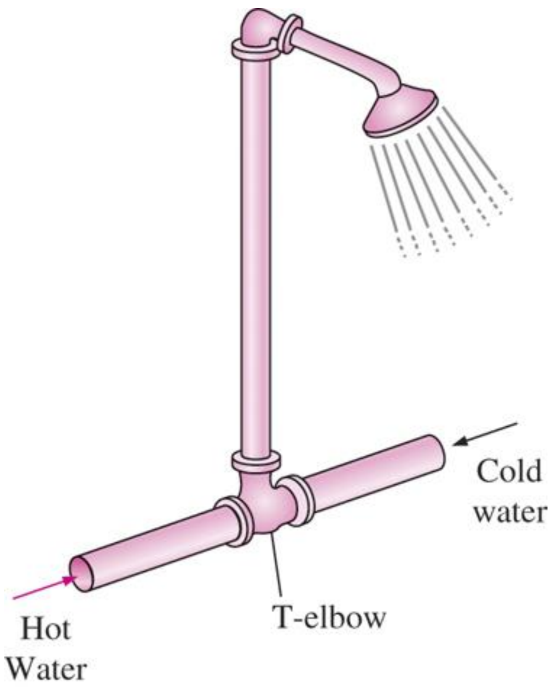
$$\boxed{\cancel{\dot{Q}_{\text{in}}} + \cancel{\dot{W}_{\text{in}}} + \underbrace{\sum_{\text{in}} \dot{m}\left(h + \cancel{\frac{V^2}{2}} + \cancel{gz}\right)}_{\text{for each inlet}} = \cancel{\dot{Q}_{\text{out}}} + \cancel{\dot{W}_{\text{out}}} + \underbrace{\sum_{\text{out}} \dot{m}\left(h + \cancel{\frac{V^2}{2}} + \cancel{gz}\right)}_{\text{for each exit}}}$$

$$\boxed{h_2 \cong h_1}$$

Isenthalpic Process

Mixing Chambers





In Engineering applications, the section where the mixing process of two (or more) streams takes place is referred to as a mixing chamber

Mixing Chambers –
Common Assumptions

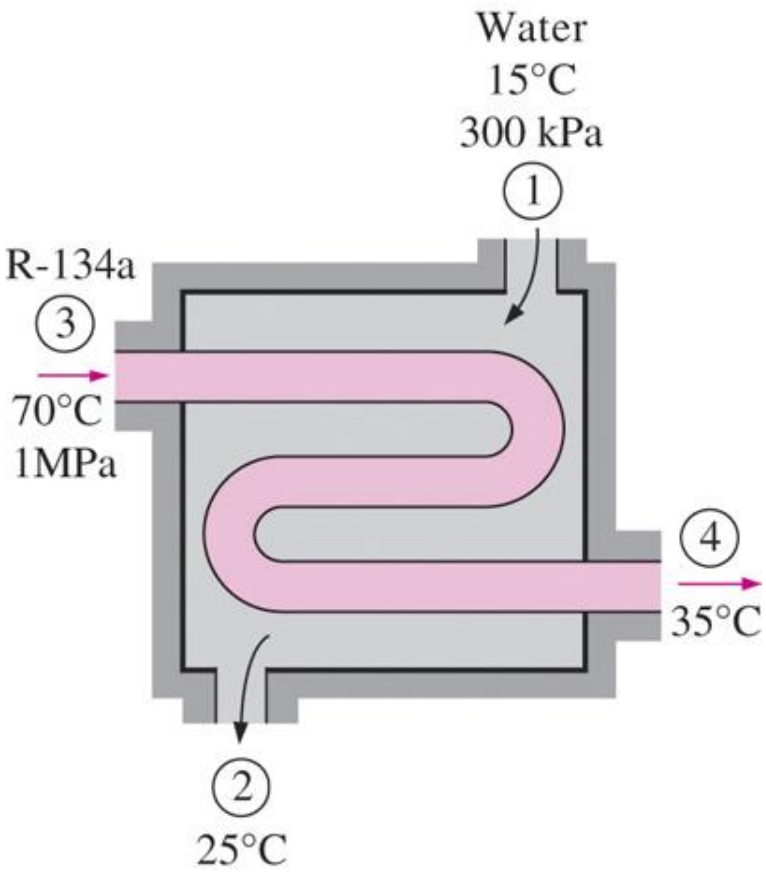
- Heat transfer is often neglected
- Changes in kinetic and potential energy usually neglected
- No work input or output

$$\cancel{\dot{Q}_{\text{in}}} + \cancel{\dot{W}_{\text{in}}} + \sum_{\text{in}} \underbrace{\dot{m}\left(h + \cancel{\frac{V^2}{2}} + \cancel{gz}\right)}_{\text{for each inlet}} = \cancel{\dot{Q}_{\text{out}}} + \cancel{\dot{W}_{\text{out}}} + \sum_{\text{out}} \underbrace{\dot{m}\left(h + \cancel{\frac{V^2}{2}} + \cancel{gz}\right)}_{\text{for each exit}}$$

$$\boxed{\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3}$$

Heat Exchangers

- **Heat exchangers** are devices where two moving fluid streams exchange heat without mixing.
- Heat exchangers are widely used in various industries.
- In refrigeration, the condenser and evaporator are heat exchangers.



Heat Exchangers –
Common Assumptions

- Changes in kinetic and potential energy of the entering and leaving fluid are usually neglected.
- No work input or output

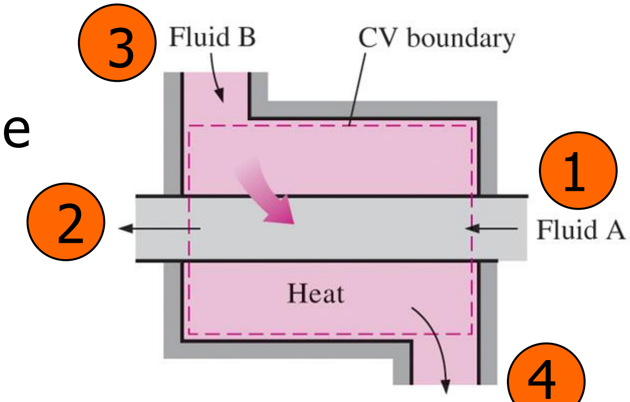
➢ ***What about heat transfer?***

$$\underset{?}{\dot{Q}_{\text{in}}} + \cancel{\dot{W}_{\text{in}}} + \sum_{\text{in}} \underbrace{\dot{m}\left(h + \cancel{\frac{V^2}{2}} + \cancel{gz}\right)}_{\text{for each inlet}} = \underset{?}{\dot{Q}_{\text{out}}} + \cancel{\dot{W}_{\text{out}}} + \sum_{\text{out}} \underbrace{\dot{m}\left(h + \cancel{\frac{V^2}{2}} + \cancel{gz}\right)}_{\text{for each exit}}$$

Heat Exchangers – Common Assumptions

- If the entire heat exchanger is selected as the control volume (case a); heat exchange with the surroundings is negligible

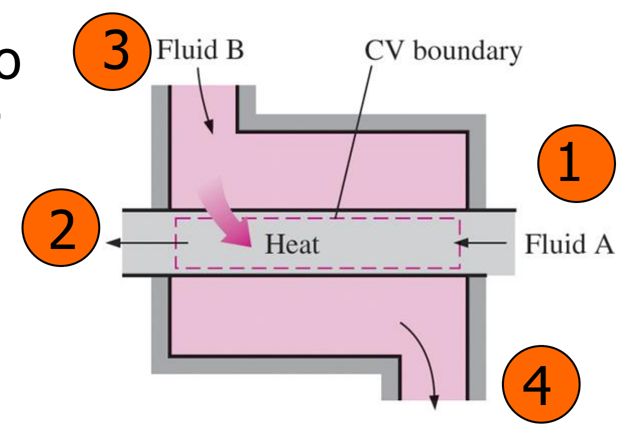
$$\boxed{\dot{m}_1 h_1 + \dot{m}_3 h_3 = \dot{m}_2 h_2 + \dot{m}_4 h_4}$$



(*a*) System: Entire heat exchanger ($Q_{CV} = 0$)

- In Case (b), the tube(s) alone is selected as the control volume; heat exchange cannot be neglected; heat is transferred from (to) fluid B to (from) fluid A depending on which fluid is hotter

$$\boxed{\dot{Q}_{\text{in}} = \dot{m}_1 (h_2 - h_1)}$$

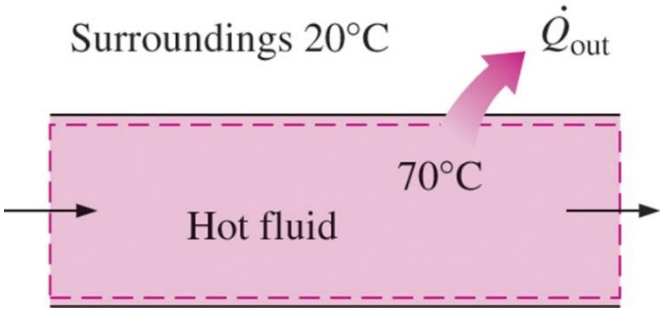


(*b*) System: Fluid A ($Q_{CV} \neq 0$)

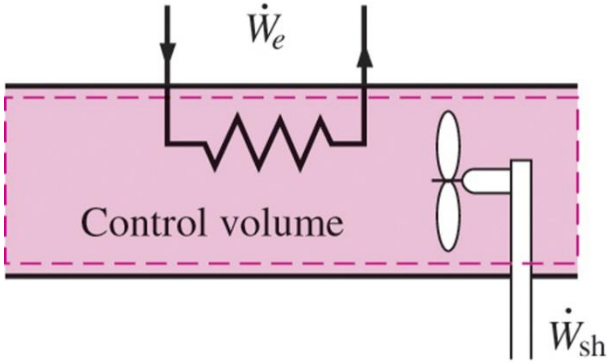
- What is $\dot{Q}_{out}$ for fluid B?

Pipe and Duct Flow

- Transport of liquids or gases in pipes and ducts is important in many engineering applications.
- Flow through a pipe or a duct usually satisfies the steady-flow conditions. Changes in KE and PE can often be neglected



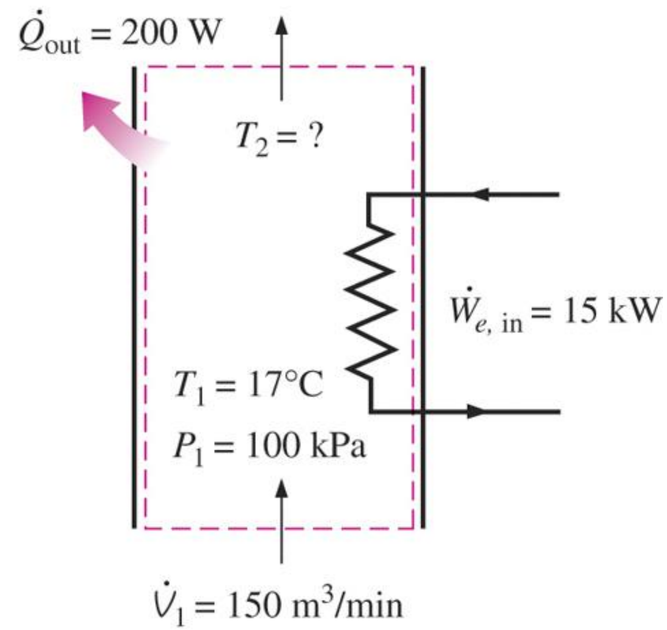
Heat loss from fluid flowing in an uninsulated pipe can be significant



More than one form of work can be involved at the same time

Pipe and Duct Flow Example

$$\cancel{\dot{Q}_{\text{in}}} + \dot{W}_{\text{in}} + \sum_{\text{in}} \underbrace{\dot{m}\left(h + \cancel{\frac{V^2}{2}} + \cancel{gz}\right)}_{\text{for each inlet}} = \dot{Q}_{\text{out}} + \cancel{\dot{W}_{\text{out}}} + \sum_{\text{out}} \underbrace{\dot{m}\left(h + \cancel{\frac{V^2}{2}} + \cancel{gz}\right)}_{\text{for each exit}}$$

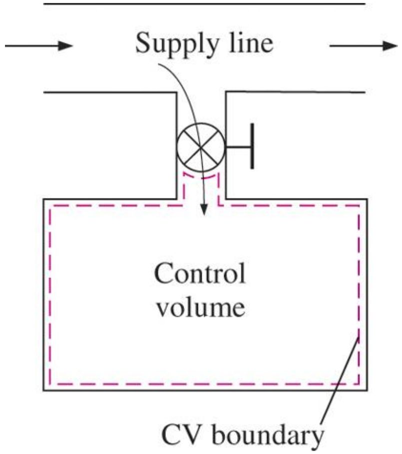


$$\dot{W}_{e,\text{in}} + \dot{m}h_1 = \dot{Q}_{\text{out}} + \dot{m}h_2$$

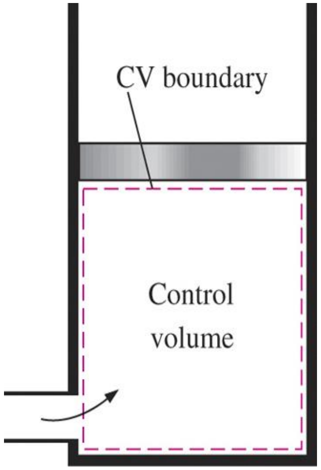
$$\dot{W}_{e,\text{in}} - \dot{Q}_{\text{out}} = \dot{m}c_p(T_2 - T_1)$$

Energy Analysis of Unsteady Flow Processes

- Many processes of interest involve *changes* within the control volume with time. Such processes are called *unsteady-flow,* or *transient-flow,* processes.



Charging of a tank from a supply line is an unsteady-flow process since conditions within the control volume change with time.



The shape and size of a control volume may change during an unsteady-flow process.

Unsteady, Uniform-Flow, Process

- Many unsteady-flow processes can be represented reasonably well by the *uniform-flow process* approximation.
- **Uniform-flow process:** The fluid flow at any inlet or exit is uniform and steady, and thus the fluid properties do not change with time or position over the cross section of an inlet or exit. If they do, they are averaged and treated as constants for the entire process.

Mass & Energy Balances – Unsteady Flow Processes

- Mass Balance:

$$m_{\text{in}} - m_{\text{out}} = \Delta m_{\text{system}}$$

$$\Delta m_{\text{system}} = m_{\text{final}} - m_{\text{initial}}$$

$$m_i - m_e = (m_2 - m_1)_{\text{CV}}$$

i = inlet, e = exit, 1 = initial state, and 2 = final state

Mass & Energy Balances – Unsteady Flow Processes

- Energy Balance

$$\underbrace{E_{\text{in}} - E_{\text{out}}}_{\substack{\text{Net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,} \\ \text{potential, etc., energies}}}$$

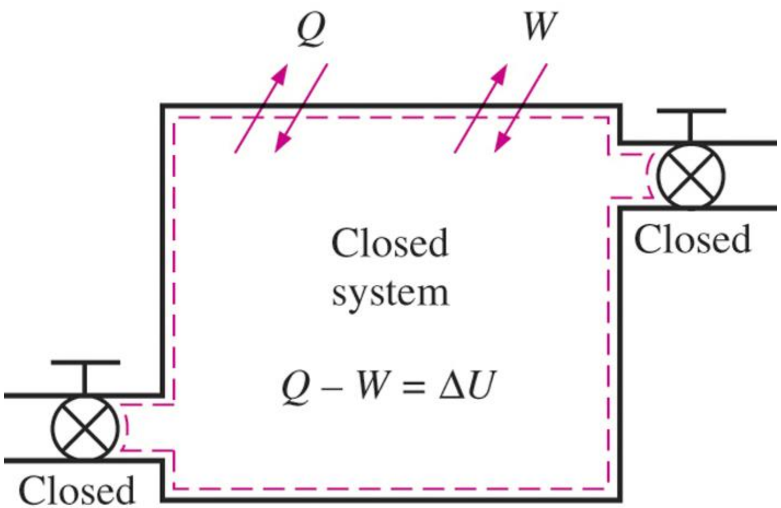
- For a Uniform-Flow Process

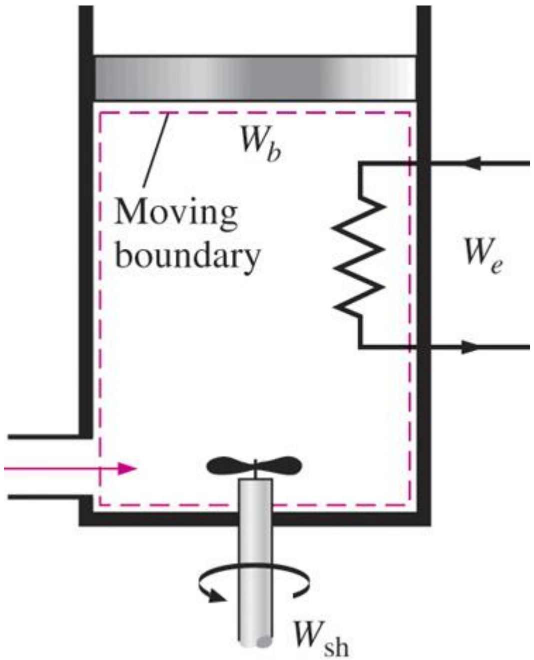
$$\left(Q_{\text{in}} + W_{\text{in}} + \sum_{\text{in}} m\theta\right) - \left(Q_{\text{out}} + W_{\text{out}} + \sum_{\text{out}} m\theta\right) = (m_2 e_2 - m_1 e_1)_{\text{system}}$$

$$\theta = h + \text{ke} + \text{pe} \qquad e = u + \text{ke} + \text{pe}$$

  - θ_{in} and θ_{out} do not change with time

Uniform-Flow Processes





The energy equation of a uniform-flow system reduces to that of a closed system when all the inlets and exits are closed.

A uniform-flow system may involve electrical, shaft, and boundary work all at once

CHAPTER 5 -- Mass and Energy Analysis of Control Volumes

OUTCOME:

- ✓ Develop and Apply the conservation of mass principle to both steady and unsteady control volumes.
- ✓ Identify the energy carried by a fluid stream crossing a control surface.
- ✓ Develop and Apply the conservation of energy principle to control volumes
- ✓ Solve energy balance problems for steady flow devices.
- ✓ Apply energy balance to unsteady flow processes with emphasis on the uniform-flow process.