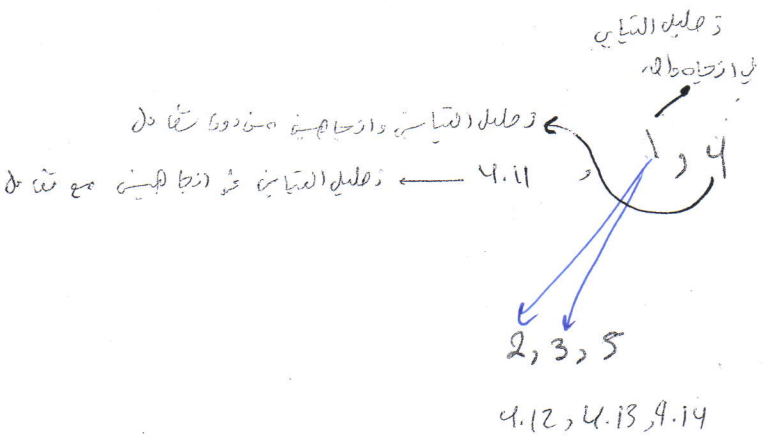
الفصل 6

حل



2, 3, 5

4.12, 4.13, 4.14

واجب

تحليل التباين

يهتم بالمقارنة بين عدة متوسطات تتجاوز عدد هذه المتوسطات 2 ، وعلى افتراض أن المتغير له توزيع طبيعي وتباينات متجانسة ومتساوية والعينات مستقلة

تحليل التباين له عدة أنواع

تحليل التباين من اتجاهين

وهو دراسة أثر عاملين (عامل A ، عامل B) على المتغير y ، بحيث مستويات العامل A تسمى بالمعالجات ومستويات العامل B تسمى بالقطاعات blocks.

السؤال عليه يكون: هل يوجد فروقات معنوية difference بين المعالجات في التأثير على متوسط y ؟!
هل ~ ~ ~ ~ القطاعات ~ ~ ~ ~ ؟!

- مع عدم التفاعل بين العاملين
- بدون التفاعل بين العاملين

تحليل التباين من اتجاه واحد

وهو دراسة أثر عامل واحد A على المتغير y ، بحيث هذا العامل A له عدة أنواع (مستويات) مختلفة وتسمى بالمعالجات treat.

السؤال عليه يكون: هل يوجد فروقات معنوية difference بين المعالجات في التأثير على متوسط y ؟!

Factor A or treatments of factor A (عامل / معالجات)

	1	2		a	
	y_{11}	y_{21}		y_{a1}	
	y_{1n_1}	y_{2n_2}		y_{an_a}	
total	$T_{1.}$	$T_{2.}$		$T_{a.}$	$T_{..}$

$$T_{..} = \sum_i \sum_j y_{ij} = \sum_i T_{i.}$$

$$T_{i.} = \sum_j y_{ij}$$

عدد المشاهدات الكلية $= N = n_1 + n_2 + \cdots + n_a$

مصدر الاختلاف Source	درجات الحرية DF	مجموع المربعات SS	متوسط مجموع المربعات MS	F	P-value
Factor A	$a-1$	SSA or $SSTr$	$MSA = \frac{SSA}{a-1}$ or $MSTr$	$F = \frac{MSA}{MSE}$	P
الخطأ Error	$N-a$	SSE	$MSE = \frac{SSE}{N-a}$		
Total	$N-1$	SST			

$SST = SSA + SSE$

$SSE = SST - SSA$

$$\sum_i \sum_j y_{ij}^2 - \frac{T_{..}^2}{N} \qquad \sum_i \frac{T_{i.}^2}{n_i} - \frac{T_{..}^2}{N}$$

على الأقل أحد المتوسطات يختلف

$H_0: \mu_1 = \mu_2 = \cdots = \mu_a$ vs $H_1:$ not all μ_i are equal

$$F = \frac{MSA}{MSE} \sim F_{a-1, N-a}$$

عملياً: بنفس الجدول السابق ثم

stat → ANOVA → one-way (unstacked)

confidence level عند $1-\alpha$ أو $(1-\alpha)100\%$

ثم ok

Factor A or treatments of Factor A

Factor B	1	2	...	i	...	a	total
1	y_{11}	y_{21}				y_{a1}	$T_{\cdot 1}$
2	y_{12}	y_{22}				y_{a2}	$T_{\cdot 2}$
⋮							
j							
⋮							
b	y_{1b}	y_{2b}				y_{ab}	$T_{\cdot b}$
total	$T_{1\cdot}$	$T_{2\cdot}$				$T_{a\cdot}$	$T_{\cdot\cdot}$

Factor B or قطاعات blocks of Factor B

y_{ij}

$$T_{\cdot\cdot} = \sum_i \sum_j y_{ij} = \sum_i T_{i\cdot} = \sum_j T_{\cdot j}$$

$$T_{i\cdot} = \sum_j y_{ij}$$

$$T_{\cdot j} = \sum_i y_{ij}$$

$N = a \times b$ = عدد المشاهدات الكلية

Source	DF	SS	MS	F	P-value
Factor A	$a-1$	SSA or SST_r	$MSA = \frac{SSA}{a-1}$ or MST_r	$F_A = \frac{MSA}{MSE}$	P_A
Factor B	$b-1$	SSB or $SSbk$	$MSB = \frac{SSB}{b-1}$ or $MSbk$	$F_B = \frac{MSB}{MSE}$	P_B
Error	$(a-1)(b-1)$	SSE	$MSE = \frac{SSE}{(a-1)(b-1)}$		
Total	$N-1 = ab-1$	SST			

$SST = SSA + SSB + SSE$

$SSE = SST - (SSA + SSB)$

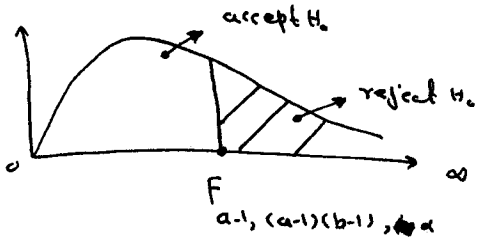
$$\sum_i \sum_j y_{ij}^2 - \frac{T_{\cdot\cdot}^2}{N}$$

$$\sum_i \frac{T_{i\cdot}^2}{b} - \frac{T_{\cdot\cdot}^2}{N}$$

$$\sum_j \frac{T_{\cdot j}^2}{a} - \frac{T_{\cdot\cdot}^2}{N}$$

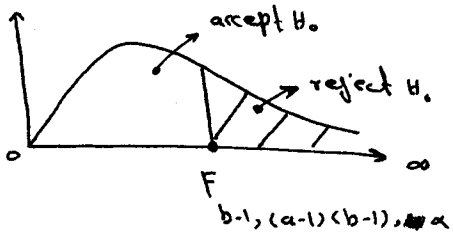
For Factor A: $H_0: \mu_1 = \mu_2 = \dots = \mu_a$ vs H_1: not all μ_i are equal (على الأقل إثنان متوسطاتهما يختلف)

$F_A \sim F_{a-1,\,(a-1)(b-1)}$



For Factor B: $H_0: \mu_1 = \mu_2 = \dots = \mu_b$ vs H_1: not all μ_j are equal (على الأقل إثنان متوسطاتهما يختلف)

$F_B \sim F_{b-1,\,(a-1)(b-1)}$



عملياً: لو كان لدينا الجدول التالي:

B \ A	1	2	3
1	①	③	⑤
2	②	④	⑥

يصبح على النحو التالي:

X	A	B
①	1	1
②	1	2
③	2	1
④	2	2
⑤	3	1
⑥	3	2

ثم

stat → ANOVA → Two way 14, 16

انقل X إلى response

وانقل A إلى row Factor

وانقل B إلى columns Factor

ثم اكتب عند $1-\alpha$ → confidence level

Factor A

Factor B	1	2	...	a
1	y_{111} y_{112} ⋮ y_{11n}	y_{211} y_{212} ⋮ y_{21n}		
⋮				
b				

كل خلية تحتوي على n قراءات

$N = abn$ = عدد المشاهدات الكلي

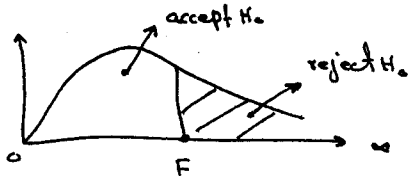
حيث n هي عدد القراءات في كل خلية

source	DF	SS	MS	F	P-value
Factor A	$a-1$	SSA	$MSA = \frac{SSA}{a-1}$	$F_A = \frac{MSA}{MSE}$	P_A
Factor B	$b-1$	SSB	$MSB = \frac{SSB}{b-1}$	$F_B = \frac{MSB}{MSE}$	P_B
interaction AB	$(a-1)(b-1)$	SSAB	$MSAB = \frac{SSAB}{(a-1)(b-1)}$	$F_{AB} = \frac{MSAB}{MSE}$	P_{AB}
error	$ab(n-1)$	SSE	$MSE = \frac{SSE}{ab(n-1)}$		
total	$abn-1 = N-1$	SST			

For interaction AB : H_0: There is no interaction between A and B Vs

H_1: " " X " " " "

$F_{AB} \sim F_{(a-1)(b-1),\, ab(n-1)}$

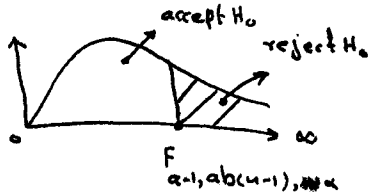


ملاحظة: إذا رفضنا H_0 نتوقف هنا

وإذا قبلنا H_0 نكمل التالي

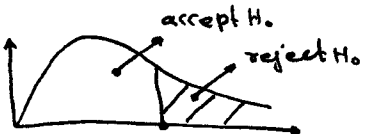
For Factor A : H_0: $M_1 = M_2 = \dots = M_a$ Vs H_1: not all M_i are equal

$F_A \sim F_{a-1,\, ab(n-1)}$



For Factor B : H_0: $M_1 = M_2 = \dots = M_b$ Vs H_1: not all M_j are equal

$F_B \sim F_{b-1,\, ab(n-1)}$



مثال: لو كان لدينا الجدول التالي:

B \ A	1	2
1	① ⑤	⑨ ⑬
2	③ ⑦	⑪ ⑮

يصبح على النحو التالي:

X	A	B
①	1	1
②	1	1
③	1	2
④	1	2
⑤	2	1
⑥	2	1
⑦	2	2
⑧	2	2

ثم

stat → ANOVA → two way

انقل X إلى response

وانقل A إلى raw factor

وانقل B إلى Columns Factor

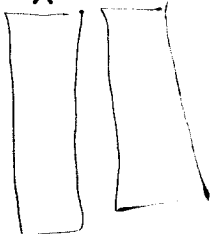
ثم اكتب من confidence level $1-\alpha$ → $(1-\alpha)100\%$

ثم ok

→ minitab 14
16

Stat → ANOVA → one Way

Response | Factor



Response data are in one column for all factor levels

Response data are in a separate column for each factor level

→ in minitab 17

Stat → ANOVA → General linear model → Fit General linear model

X → responses

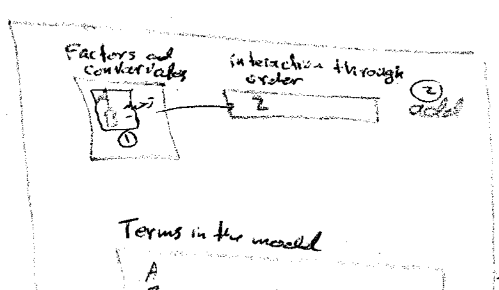
A , B → Factors

\+ option [illegible] (1-α)

→ ok إذا لم يوجد تداخل

في حالة وجود تداخل

Model



حل الأسئلة

treatments of Factor A : host plant types

[1]

	1	2	3	4 = a	
	$y_{11}=225$	$y_{21}=377$	$y_{31}=310$	$y_{41}=363$	
	$y_{12}=209$	$y_{22}=391$	$y_{32}=303$	$y_{42}=354$	
	$y_{13}=215$	$y_{23}=385$	$y_{33}=321$	$y_{43}=347$	
	$y_{14}=199$	$y_{24}=364$	$y_{34}=291$	$y_{4.}=373$	
	$y_{15}=206$	$y_{25}=388$	$y_{35}=313$	$y_{45}=365$	
total	$T_{1.}=1054$	$T_{2.}=1905$	$T_{3.}=1538$	$T_{4.}=1802$	$T_{..}=6299$

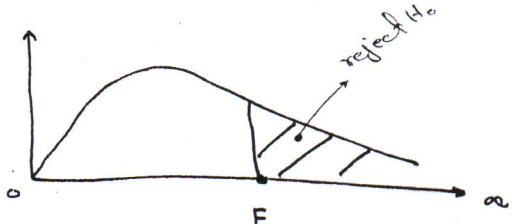
Y: # of eggs deposited by female melon ladybird beetles.

$\alpha = .05$

$N = 5+5+5+5 = 20$

$H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4$ Vs H_1: not all $\mu_{i.}$ are equal

$F = 260.61 \sim F_{3,16}$



$F_{4-1,20-4,.05} = F_{3,16,.05} = 3.23887$

$\therefore$ as $F = 260.61 > 3.23887$

so, we reject H_0

Source	dF	SS	MS	F	p
Factor A	$a-1$ = 3	86648	$\frac{SSA}{a-1}$ = 28883	$\frac{MSA}{MSE}$ = 260.61	0
error	$N-a$ = 16	1773	$\frac{SSE}{N-a}$ = 111		
total	19	88421			

$\sum_i \sum_j y_{ij}^2 = 2072291$

$SST = 2072291 - \frac{(6299)^2}{20} = 88420.95 \simeq 88421$

$SSA = \left[\frac{(1054)^2}{5} + \frac{(1905)^2}{5} + \frac{(1538)^2}{5} + \frac{(1802)^2}{5} \right] - \frac{(6299)^2}{20} = 86647.75 \simeq 86648$

$SSE = 88421 - 86648 = 1773$

$\left\{ \sum_i \sum_j y_{ij}^2 - \frac{(\sum T)^2}{N} \right.$

طريقة الري

(4) treatments of Factor A: irrigation method

blocks of Factor B: varieties of lettuce أصناف أنواع الخس	1	2	3	4 = a	total
1	$y_{11}=8.14$	$y_{21}=9.24$	$y_{31}=16.36$	$y_{41}=4.79$	$T_{.1}=38.53$
b = 2	$y_{12}=4.59$	$y_{22}=6.56$	$y_{32}=15.37$	$y_{42}=4.18$	$T_{.2}=30.7$
total	$T_{1.}=12.73$	$T_{2.}=15.8$	$T_{3.}=31.73$	$T_{4.}=8.97$	$T_{..}=69.23$

الوزن الطازج للخس
Y: Fresh weight of lettuce.

$\alpha = .05$

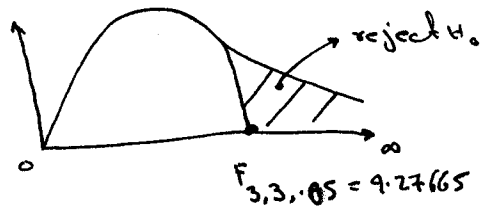
$N = ab = 8$

* assume that there is no interaction

For Factor A:

$H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4$ Vs H_1: not all μ_i are equal

$F_A = 51.77 \sim F_{3,3}$



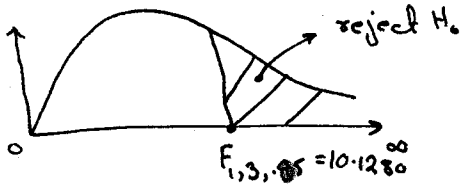
∴ as $F_A = 51.77 > 9.27665$

so, we reject H_0

For Factor B:

$H_0: \mu_1 = \mu_2$ Vs H_1: not all μ_j are equal

$F_B = 7.91 \sim F_{1,3}$



∴ as $F_B = 7.91 < 10.1280$

so, we accept H_0

source	dF	SS	MS	F	P
Factor A	3	150.374	50.1247	51.77	.004
Factor B	1	7.664	7.6636	7.91	.067
error	3	2.905	.9683		
total	7	160.943			

$\sum_i \sum_j y_{ij}^2 = 760.042$

$SST = \sum_i \sum_j y_{ij}^2 - \frac{T_{..}^2}{N} = 160.943$

$SSA = \sum_i \frac{T_{i.}^2}{b} - \frac{T_{..}^2}{N} = 150.374$

$SSB = \sum_j \frac{T_{.j}^2}{a} - \frac{T_{..}^2}{N} = 7.664$

$SSE = SST - (SSA + SSB) = 2.905$

4.11

مستوى الأسمدة

treatments of Factor A: Fertilizer level

	1	2 = a
1	$y_{111} = 16.01$ $y_{112} = 16.78$ $y_{113} = 16.44$	$y_{211} = 15.89$ $y_{212} = 16.23$ $y_{213} = 16.18$
b = 2	$y_{121} = 13.42$ $y_{122} = 13.25$ $y_{123} = 13.32$	$y_{221} = 13.32$ $y_{222} = 13.47$ $y_{223} = 13.26$

blocks of Factor B: Pacings between Plants — المسافة بين النباتات

وزن محصول البطاطس

Y: weight of potatoes in the yield

$N = abn = (2)(2)(3) = 12$, $n = 3$

For interaction AB: H_0: There is no interaction between A and B vs

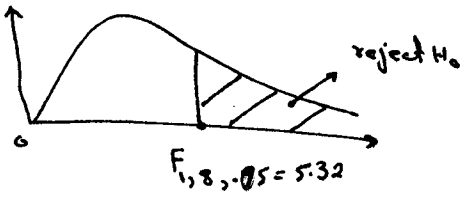
H_1: " " X " " "

$F_{AB} = 1.62$ ~ $F_{1,8}$ ← من الجدول نجد أن

∴ as $F_{AB} < 5.32$

So, we accept H_0

إذن نستنتج إحدى الاختبارات التالية

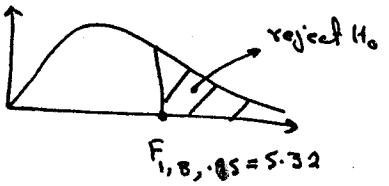


For Factor A: $H_0: \mu_1 = \mu_2$ Vs H_1: not all μ_i are equal $\equiv \mu_1 \neq \mu_2$

$F_A = 1.25$ ~ $F_{1,8}$ ← من الجدول نجد أن

∴ as $F_A < 5.32$

So, we accept H_0



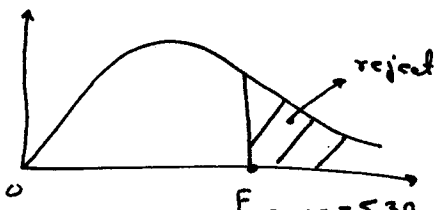
For Factor B

$H_0: \mu_1 = \mu_2$ Vs H_1: not all μ_j are equal $\equiv \mu_1 \neq \mu_2$

$F_B = 505.78$ ~ $F_{1,8}$ ← من الجدول نجد أن

∴ as $F_B > 5.32$

So, we reject H_0



يعطي هذا الجدول من البرنامج Minitab :

Source	DF	SS	MS	F	P-Value
Factor A	1	.0631	.0631	$F_A = 1.25$	$P_A = .296$
Factor B	1	25.4917	25.4917	$F_B = 505.79$	$P_B = 0$
interaction AB	1	.0817	.0817	$F_{AB} = 1.62$	$P_{AB} = .239$
error	8				

melon one way anova

1)

In a study on the growth of melon ladybird beetles, females were randomly assigned to 4 different vegetable leaf diets - cucumber, snake cucumber, squash and watermelon leaves. The number of eggs deposited by each female was recorded [Based on Ali and El-Saeady (1980)]:

Host Plant Type			
Cucumber	Snake Cucumber	Squash	Watermelon
225	377	310	363
209	391	303	354
215	386	321	347
199	364	291	373
206	388	313	365
Total $T_1 = 1054$	$T_2 = 1905$	$T_3 = 1538$	$T_4 = 1802$

$T_{..} = 6299$

a) Test whether there is a difference in host plant types on the average number of eggs deposited by female melon ladybird beetles. Use $\alpha = 0.05$.

b) If needed, make mean separation. Interpret.

2)

Samples were collected from three types of figs and the calcium (as a percent) was measured [Based on Saad et al. (1979)]:

Fig Type		
1	2	3
0.594	0.561	0.569
0.632	0.573	0.585
0.626	0.580	0.605
0.587	0.559	0.583
0.592	0.593	0.552
0.587	0.608	0.562

a) Test whether the three fig types have different average levels of calcium. Use $\alpha = 0.01$.

b) If needed, make mean separation. Interpret.

EXERCISES

4.11. In a study on fertilizer levels and spacings between plants, plots were assigned to combinations and the yield of potatoes (in kg/plot) was measured [Based on Samman and Khalil (1979)]:

		Fertilizer level (in tons/ha) 1	2
Spacing Between Plants	25 cm	16.01 16.78 16.44	15.89 16.23 16.18
	33 cm	13.42 13.25 13.32	13.32 13.47 13.26

Make all appropriate tests and use mean separation as needed. Use $\alpha = 0.05$.

4.12. In a study, it was desired to know the effect of 4 species of a certain type of plant and 3 levels of water depletion on the forage yield (in kg). Twenty-four homogeneous plots were randomly assigned to the combinations and the following forage yields were obtained [Based on Abohassan and Habib (1985)]:

	Water Depletion			
Species	20%	40%	60%	Total
1	13.77 14.17	11.37 11.71	12.21 11.63	74.86
2	18.53 19.09	14.41 13.75	13.43 12.89	92.10
3	16.52 15.80	10.53 11.09	9.89 10.27	74.10
4	16.12 16.60	11.21 11.95	11.37 11.99	79.24

Make all appropriate tests and use mean separation as needed. Use $\alpha = 0.01$.

4.13. In a study on a particular type of forage plant, plants were grown in 2 types of soil and given one of 4 nutrition treatments. The fresh weight (in g) at harvest gave the following [Modified from Zahran and Maghraby (1980)]:

Soil	Control	Complete Nutrition	Nutrient Minus Nitrogen	Nutrient Minus Phosphorus
Sandy	2.34	3.51	4.36	4.72
	2.59	3.64	2.70	2.48
	1.87	4.16	3.14	3.44
	4.13	3.90	9.12	2.58
	1.95	3.79	2.02	2.40
Silty	9.22	4.50	7.72	6.59
	3.72	4.78	8.18	6.76
	3.78	2.30	4.42	4.38
	5.16	3.86	5.12	8.56
	3.34	2.40	7.92	3.54

Make all appropriate tests and use mean separation if necessary. Use $\alpha = 0.05$.

4.14. In a study on alfalfa, two varieties of alfalfa and three harvest schedules (harvest every 20 days, every 30 days, and every 40 days) were used. Plots were randomly assigned to the combinations and the forage yield (in tons/hectare) was measured [Based on Ghandorah et al. (1986)]:

	Time Between Harvests		
Variety	20	30	40
1	32.57 34.21 33.87	42.07 41.31 39.89	47.95 49.14 48.83
2	29.42 28.04 30.62	36.98 34.23 35.20	44.17 43.09 45.40

Make all appropriate tests and use mean separation if necessary. Use $\alpha = 0.05$.