

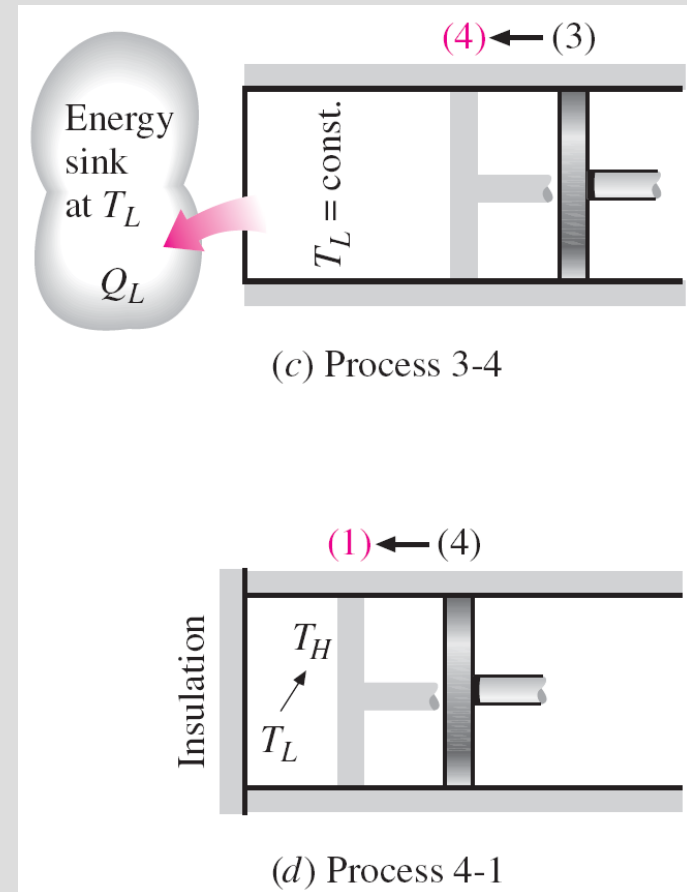
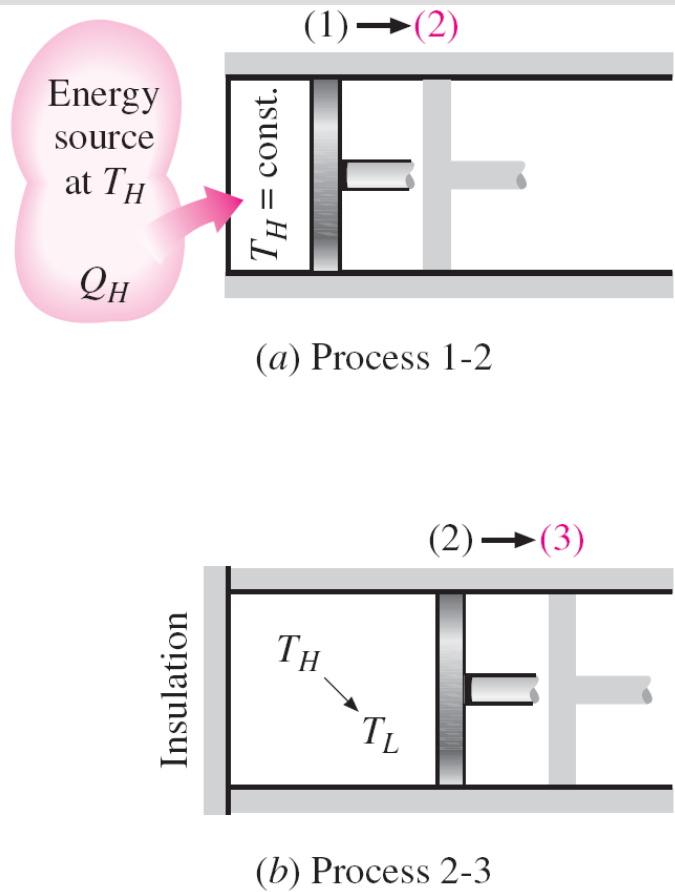
Chapter 7

ENTROPY

CLAUSIUS INEQUALITY

$$\oint \frac{\delta Q}{T} \leq 0$$

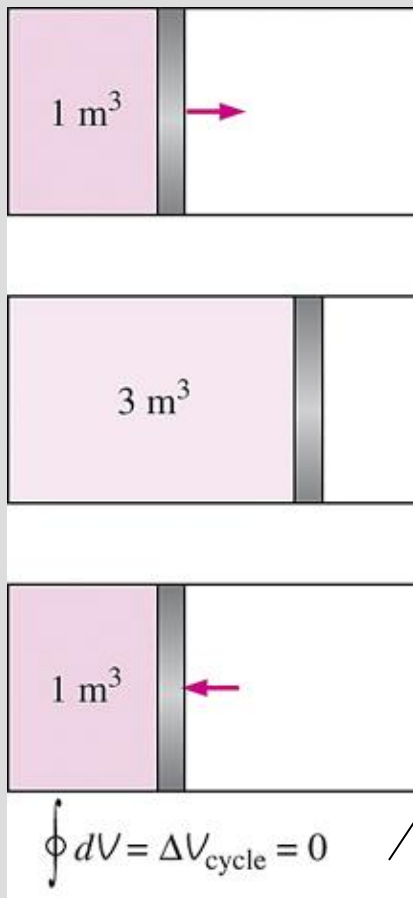
PROOF: In Classroom



RESULTS OF CLAUSIUS INEQUALITY

For internally reversible cycles

$$\oint \left(\frac{\delta Q}{T} \right)_{\text{int rev}} = 0$$



For irreversible cycles

$$\oint \left(\frac{\delta Q}{T} \right)_{\text{irr}} < 0$$

A quantity whose cyclic integral is zero is a property

$\oint \left(\frac{\delta Q}{T} \right)_{\text{int rev}}$ is a property

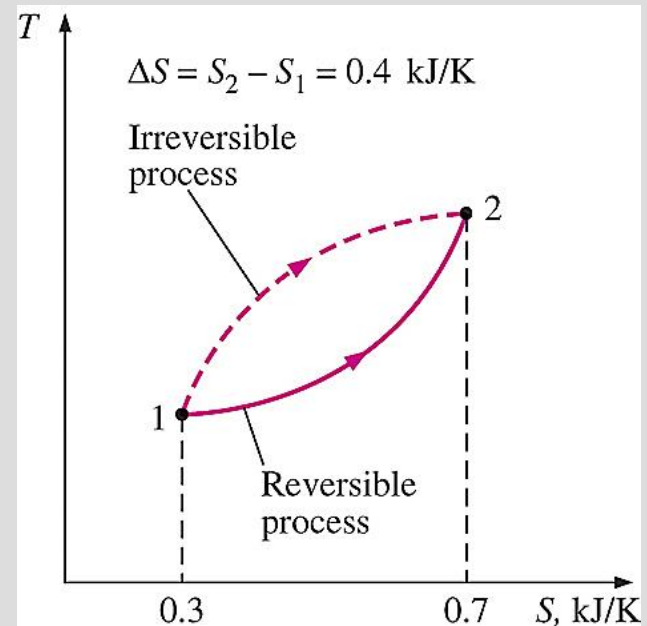
RESULTS OF CLAUSIUS INEQUALITY

$$dS = \left(\frac{\delta Q}{T} \right)_{\text{int rev}} \quad (\text{kJ/K})$$

Formal definition of **entropy**

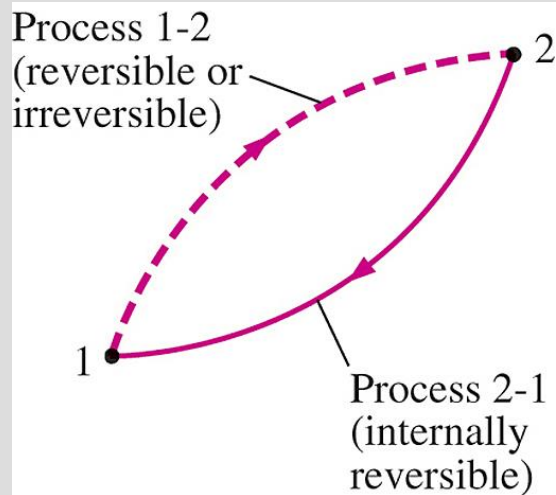
$$\Delta S = S_2 - S_1 = \int_1^2 \left(\frac{\delta Q}{T} \right)_{\text{int rev}}$$

Entropy is an extensive property of a system.



The entropy change between two specified states is the same whether the process is reversible or irreversible.

THE INCREASE OF ENTROPY PRINCIPLE



A cycle composed of a reversible and an irreversible process.

$$\oint \frac{\delta Q}{T} \leq 0 \quad \int_1^2 \frac{\delta Q}{T} + \int_2^1 \left(\frac{\delta Q}{T} \right)_{\text{int rev}} \leq 0$$

$$\int_1^2 \frac{\delta Q}{T} + S_1 - S_2 \leq 0 \quad S_2 - S_1 \geq \int_1^2 \frac{\delta Q}{T}$$

$$dS \geq \frac{\delta Q}{T}$$

The equality holds for an internally reversible process and the inequality for an irreversible process.

$$\Delta S_{\text{sys}} = S_2 - S_1 = \int_1^2 \frac{\delta Q}{T} + S_{\text{gen}}$$

Some entropy is *generated* or *created* during an irreversible process, and this generation is due entirely to the presence of irreversibilities.

The entropy generation S_{gen} is always a *positive* quantity or zero.

Can the entropy of a system during a process decrease?

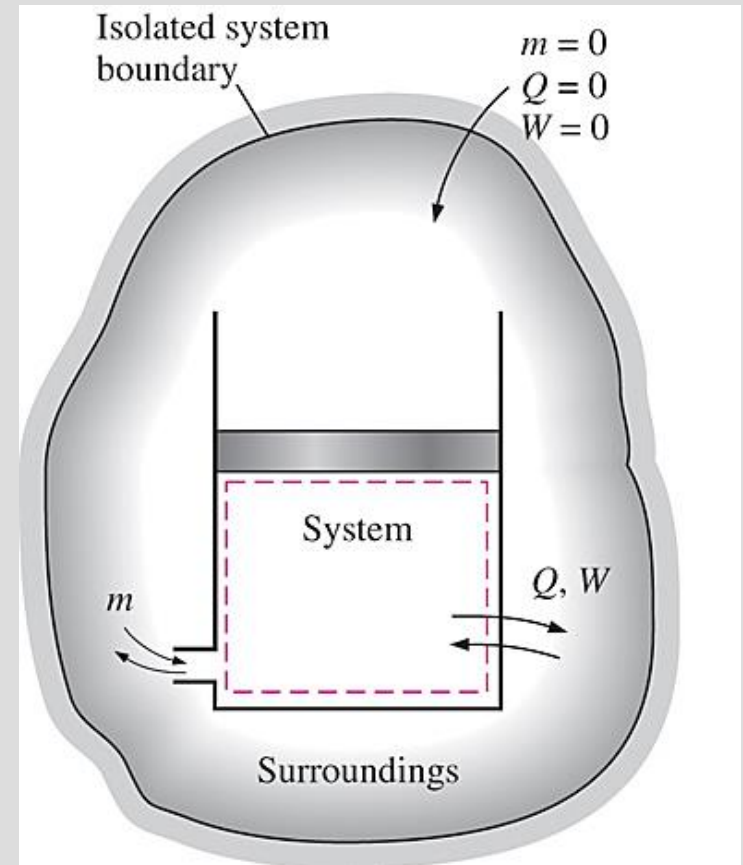
THE INCREASE OF ENTROPY PRINCIPLE

$$\Delta S_{\text{total}} = \Delta S_{\text{sys}} + \Delta S_{\text{surr}} = \int_1^2 \frac{\delta Q}{T} + S_{\text{gen}}$$

$$\Delta S_{\text{total}} \geq 0$$

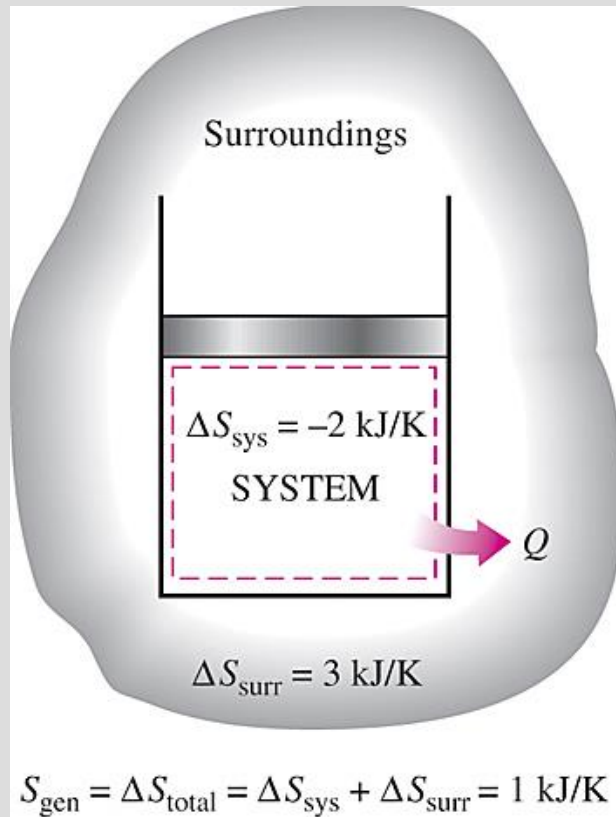
The increase
of entropy
principle

$$S_{\text{gen}} \begin{cases} > 0 & \text{Irreversible process} \\ = 0 & \text{Reversible process} \\ < 0 & \text{Impossible process} \end{cases}$$



A system and its surroundings form an isolated system.

Some Remarks about Entropy

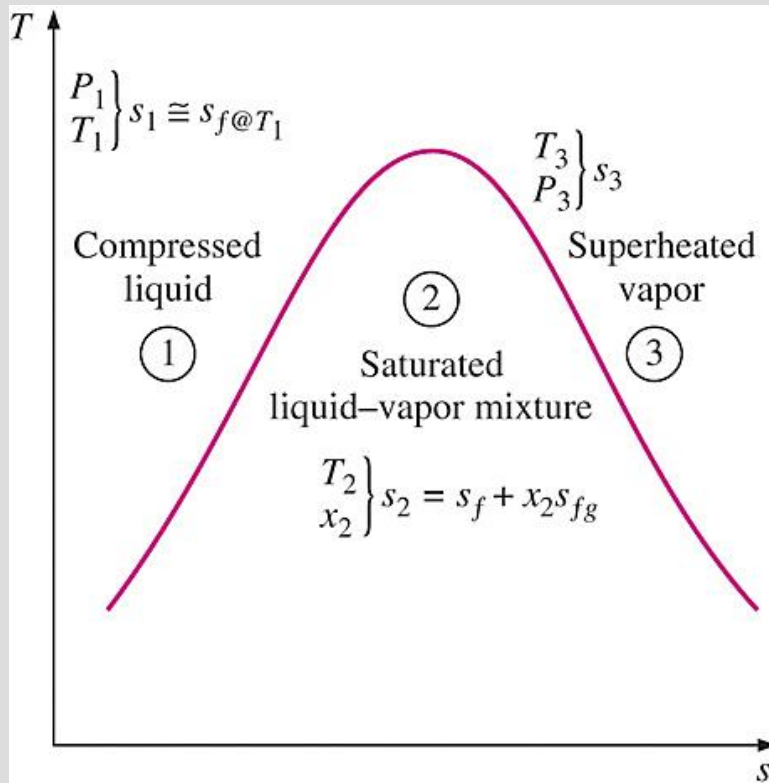


The entropy change of a system can be negative, but the entropy generation cannot.

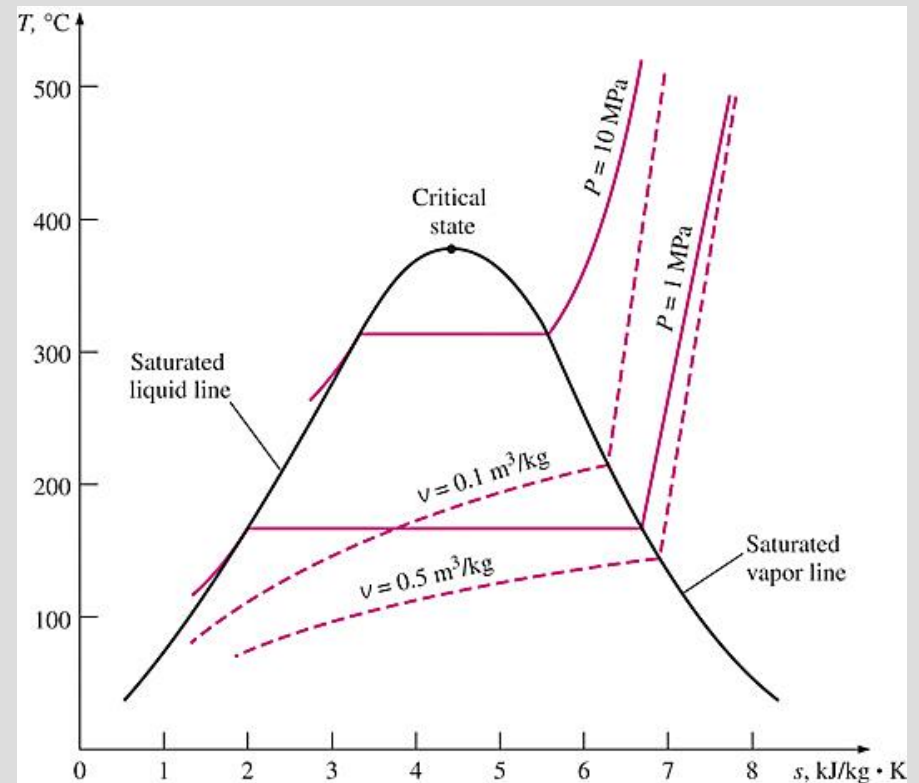
1. Processes can occur in a *certain* direction only, not in *any* direction. A process must proceed in the direction that complies with the increase of entropy principle, that is, $S_{\text{gen}} \geq 0$. A process that violates this principle is impossible.
2. Entropy is a *nonconserved property*, and there is *no* such thing as the *conservation of entropy principle*. Entropy is conserved during the idealized reversible processes only and increases during *all* actual processes.
3. The performance of engineering systems is degraded by the presence of irreversibilities, and *entropy generation* is a measure of the magnitudes of the irreversibilities during that process. It is also used to establish criteria for the performance of engineering devices.

ENTROPY CHANGE OF PURE SUBSTANCES

Entropy is a property, and thus the value of entropy of a system is fixed once the state of the system is fixed.



The entropy of a pure substance is determined from the tables (like other properties).



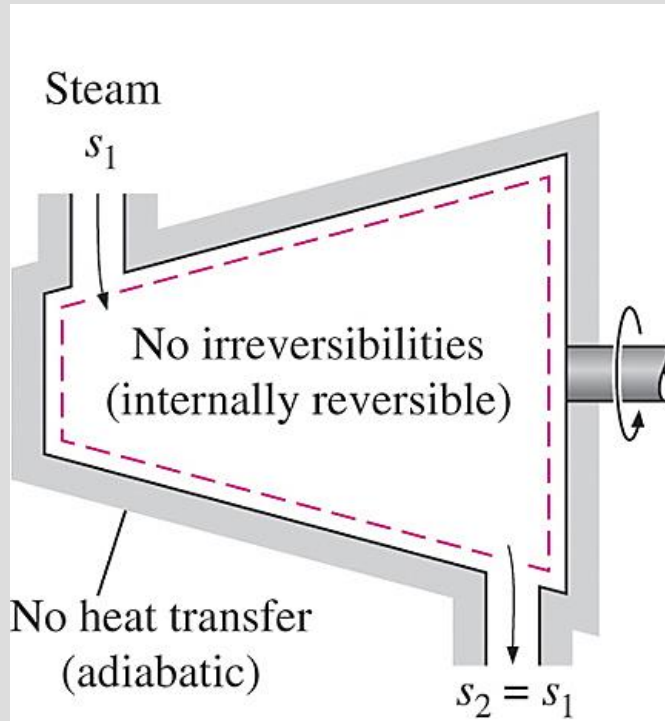
Schematic of the $T-s$ diagram for water.

Entropy change

$$\Delta S = m\Delta s = m(s_2 - s_1) \quad (\text{kJ/K})$$

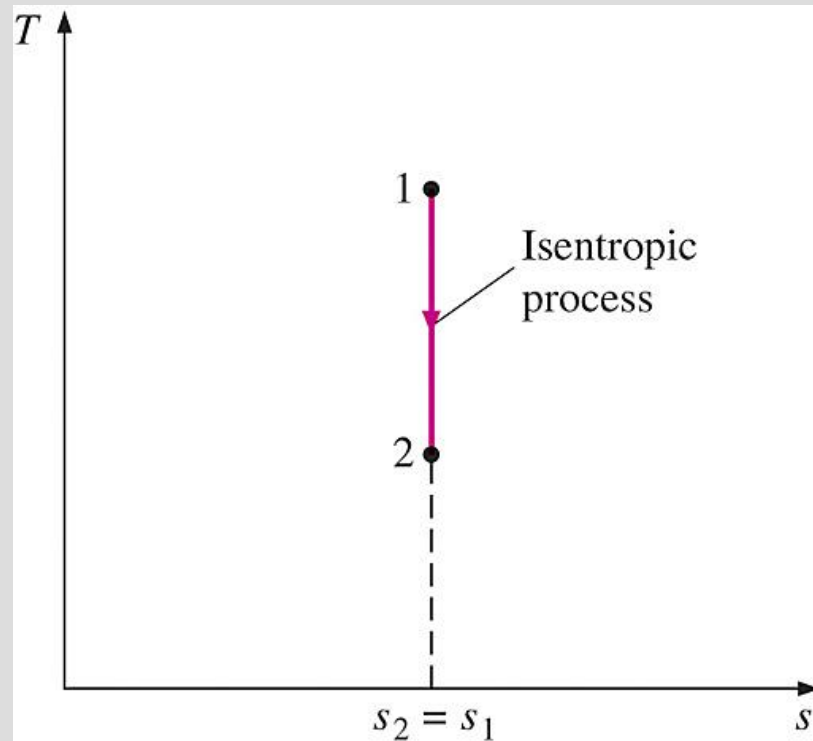
ISENTROPIC PROCESSES

A process during which the entropy remains constant is called an **isentropic process**.



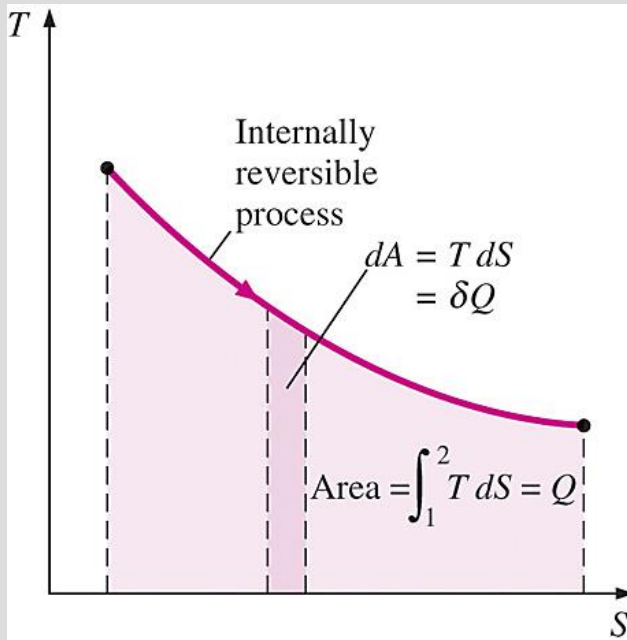
During an internally reversible, adiabatic (isentropic) process, the entropy remains constant.

$$\Delta s = 0 \quad \text{or} \quad s_2 = s_1 \quad (\text{kJ/kg} \cdot \text{K})$$

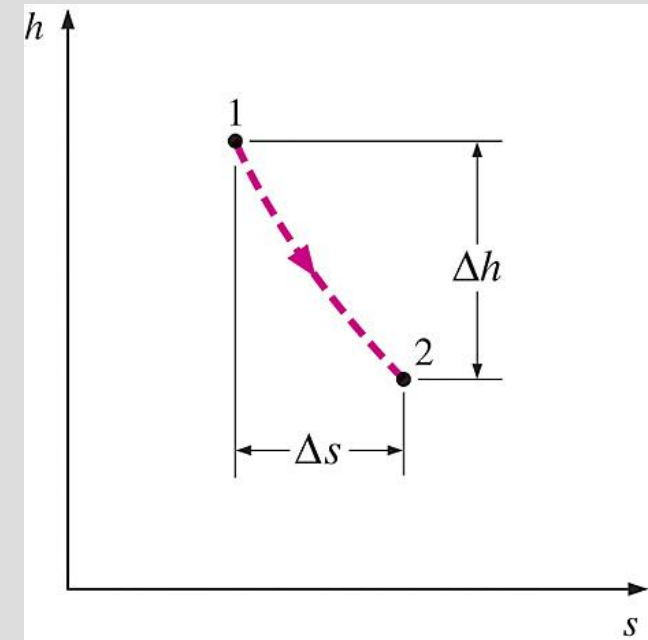


The isentropic process appears as a *vertical* line segment on a T - s diagram.

PROPERTY DIAGRAMS INVOLVING ENTROPY



On a T - S diagram, the area under the process curve represents the heat transfer for internally reversible processes.



For adiabatic steady-flow devices, the vertical distance Δh on an h - s diagram is a measure of work, and the horizontal distance Δs is a measure of irreversibilities.

$$\delta Q_{\text{int rev}} = T dS$$

$$Q_{\text{int rev}} = \int_1^2 T dS$$

$$\delta q_{\text{int rev}} = T ds$$

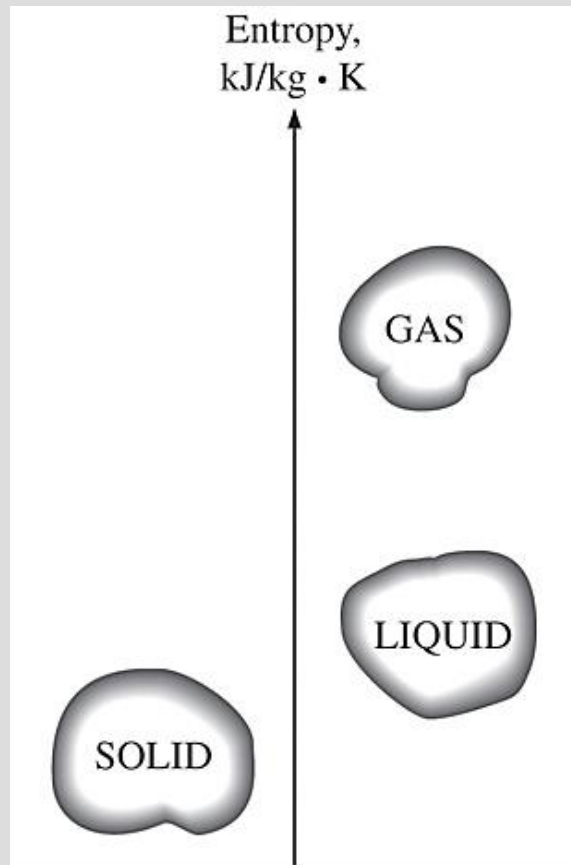
$$q_{\text{int rev}} = \int_1^2 T ds$$

$$Q_{\text{int rev}} = T_0 \Delta S$$

$$q_{\text{int rev}} = T_0 \Delta s$$

Mollier diagram: The h - s diagram

WHAT IS ENTROPY?

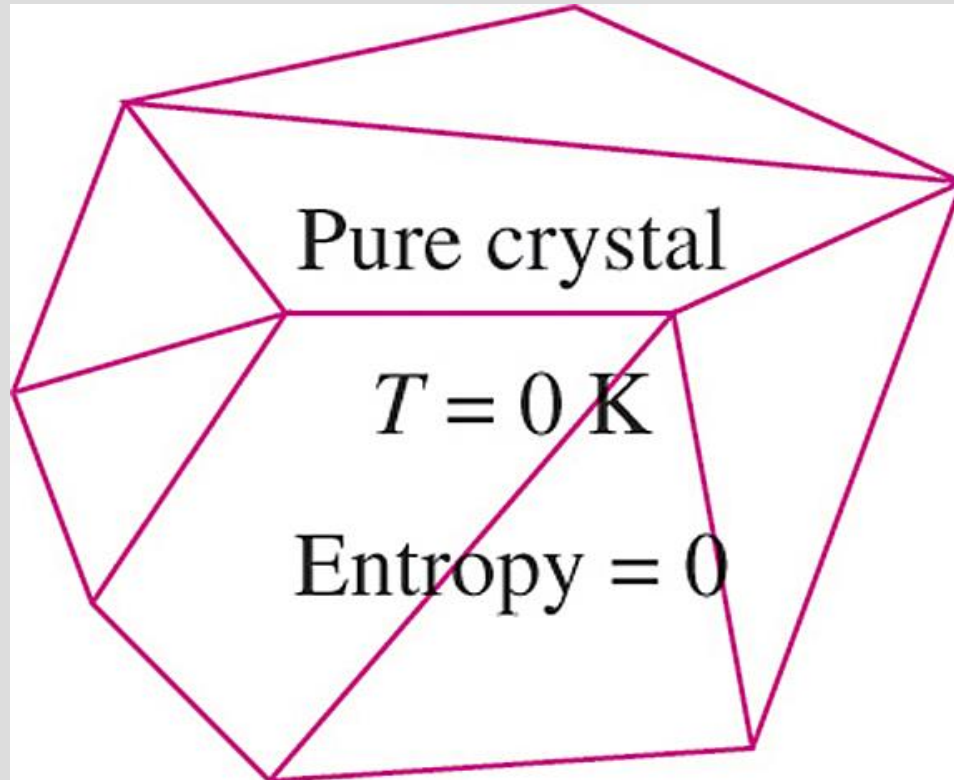


The level of molecular disorder (entropy) of a substance increases as it melts or evaporates.

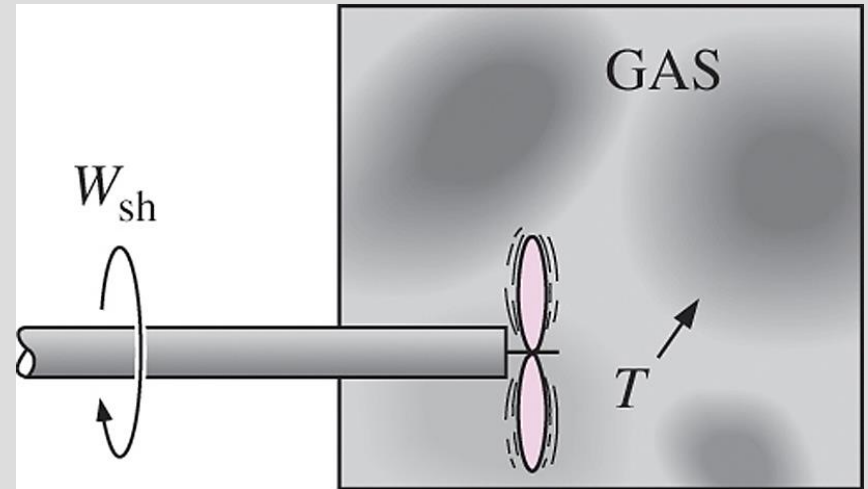
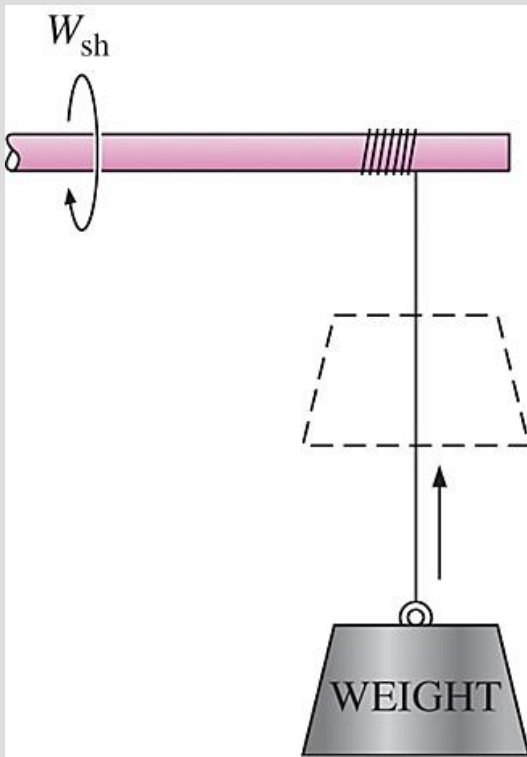


Disorganized energy does not create much useful effect, no matter how large it is.

WHAT IS ENTROPY?

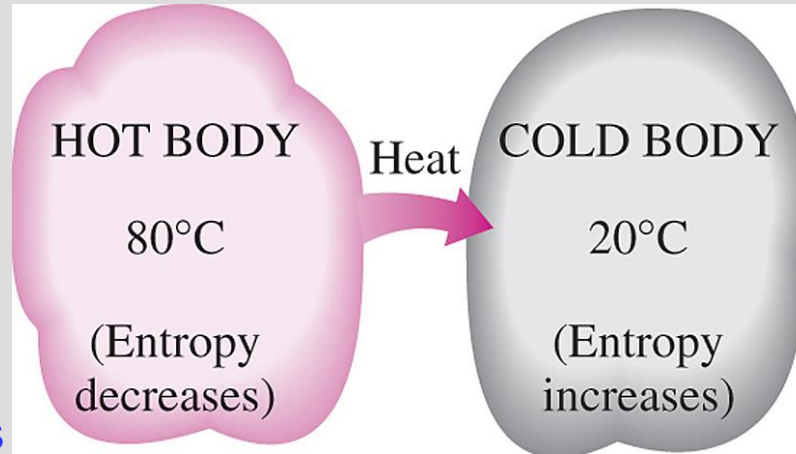


A pure crystalline substance at absolute zero temperature is in perfect order, and its entropy is zero (**the third law of thermodynamics**).



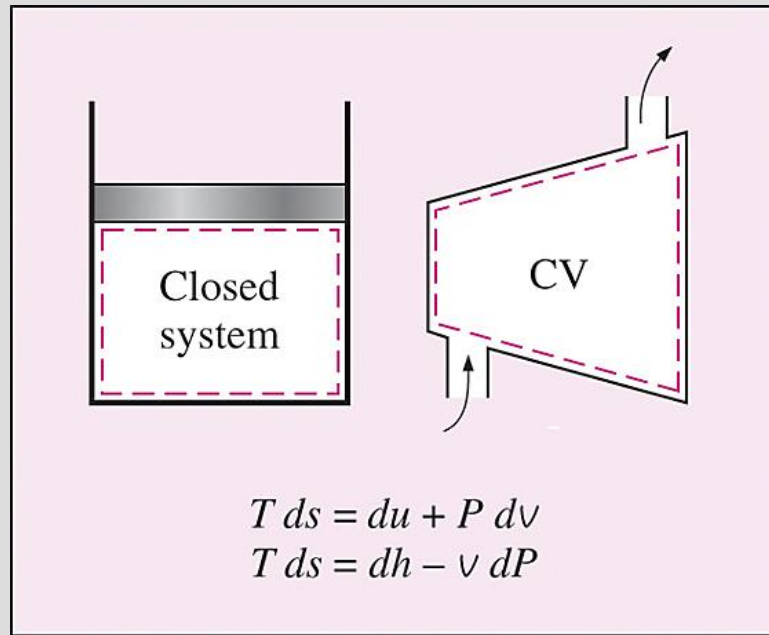
The paddle-wheel work done on a gas increases the level of disorder (entropy) of the gas, and thus energy is degraded during this process.

In the absence of friction, raising a weight by a rotating shaft does not create any disorder (entropy), and thus energy is not degraded during this process.



During a heat transfer process, the net entropy increases. (The increase in the entropy of the cold body more than offsets the decrease in the entropy of the hot body.)

THE $T ds$ RELATIONS



The $T ds$ relations are valid for both reversible and irreversible processes and for both closed and open systems.

$$\delta Q_{\text{int rev}} - \delta W_{\text{int rev,out}} = dU$$

$$\delta Q_{\text{int rev}} = T dS$$

$$\delta W_{\text{int rev,out}} = P dV$$

$$T dS = dU + P dV \quad (\text{kJ})$$

$$T ds = du + P dv \quad (\text{kJ/kg})$$

the first $T ds$, or Gibbs equation

$$h = u + Pv$$

$$\left. \begin{array}{l} dh = du + P dv + v dP \\ T ds = du + P dv \end{array} \right\} T ds = dh - v dP$$

the second $T ds$ equation

$$ds = \frac{du}{T} + \frac{P dv}{T}$$

$$ds = \frac{dh}{T} - \frac{v dP}{T}$$

Differential changes in entropy in terms of other properties

ENTROPY CHANGE OF LIQUIDS AND SOLIDS

$$ds = \frac{du}{T} + \frac{P dv}{T}$$

Since $dv \cong 0$ for liquids and solids

$$ds = \frac{du}{T} = \frac{c dT}{T}$$

since $c_p = c_v = c$ and $du = c dT$

Liquids and solids can be approximated as *incompressible substances* since their specific volumes remain nearly constant during a process.

$$\text{Liquids, solids:} \quad s_2 - s_1 = \int_1^2 c(T) \frac{dT}{T} \cong c_{\text{avg}} \ln \frac{T_2}{T_1} \quad (\text{kJ/kg} \cdot \text{K})$$

For an isentropic process of an incompressible substance

$$\text{Isentropic:} \quad s_2 - s_1 = c_{\text{avg}} \ln \frac{T_2}{T_1} = 0 \quad \rightarrow \quad T_2 = T_1$$

THE ENTROPY CHANGE OF IDEAL GASES

From the first $T ds$ relation

$$ds = \frac{du}{T} + \frac{P dv}{T} \quad \begin{matrix} du = c_v dT \\ P = RT/v \end{matrix}$$

$$ds = c_v \frac{dT}{T} + R \frac{dv}{v}$$

$$s_2 - s_1 = \int_1^2 c_v(T) \frac{dT}{T} + R \ln \frac{v_2}{v_1}$$

From the second $T ds$ relation

$$ds = \frac{dh}{T} - \frac{v dP}{T}$$

$$dh = c_p dT \quad v = RT/P$$

$$s_2 - s_1 = \int_1^2 c_p(T) \frac{dT}{T} - R \ln \frac{P_2}{P_1}$$

$$\begin{aligned} P v &= RT \\ du &= C_v dT \\ dh &= C_p dT \end{aligned}$$

A broadcast
from channel IG.

Constant Specific Heats (Approximate Analysis)

$$s_2 - s_1 = \int_1^2 c_v(T) \frac{dT}{T} + R \ln \frac{v_2}{v_1}$$



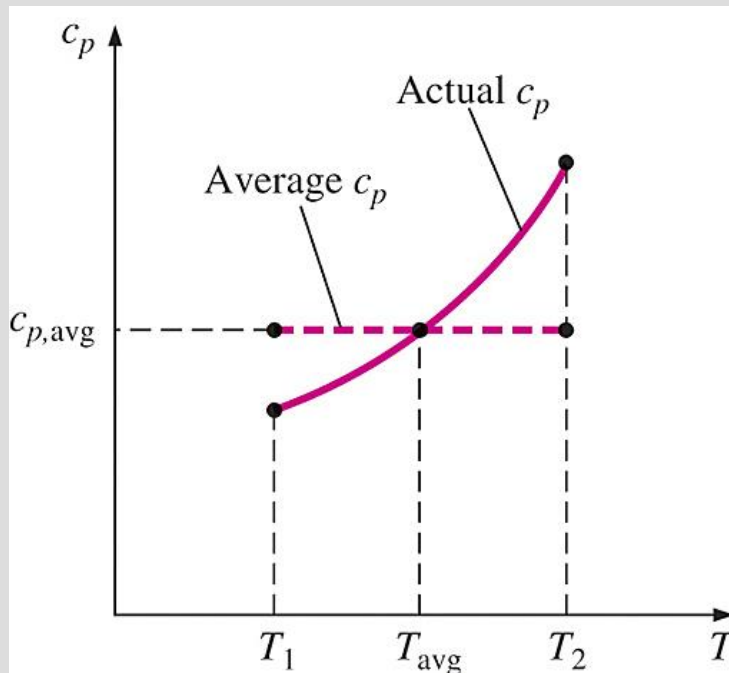
$$s_2 - s_1 = c_{v,\text{avg}} \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1}$$

$$s_2 - s_1 = \int_1^2 c_p(T) \frac{dT}{T} - R \ln \frac{P_2}{P_1}$$



$$s_2 - s_1 = c_{p,\text{avg}} \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}$$

(kJ/kg · K)



Entropy change of an ideal gas on a unit-mole basis

$$\bar{s}_2 - \bar{s}_1 = \bar{c}_{v,\text{avg}} \ln \frac{T_2}{T_1} + R_u \ln \frac{v_2}{v_1} \quad (\text{kJ/kmol} \cdot \text{K})$$

$$\bar{s}_2 - \bar{s}_1 = \bar{c}_{p,\text{avg}} \ln \frac{T_2}{T_1} - R_u \ln \frac{P_2}{P_1} \quad (\text{kJ/kmol} \cdot \text{K})$$

Under the constant-specific-heat assumption, the specific heat is assumed to be constant at some average value.

Isentropic Processes of Ideal Gases

Constant Specific Heats (Approximate Analysis)

$$s_2 - s_1 = c_{v,\text{avg}} \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1}$$

Setting this eq. equal to zero, we get

$$\ln \frac{T_2}{T_1} = -\frac{R}{c_v} \ln \frac{v_2}{v_1}$$

$$\ln \frac{T_2}{T_1} = \ln \left(\frac{v_1}{v_2} \right)^{R/c_v}$$

$$R = c_p - c_v, k = c_p/c_v$$

and thus $R/c_v = k - 1$

$$\left(\frac{T_2}{T_1} \right)_{s=\text{const.}} = \left(\frac{v_1}{v_2} \right)^{k-1}$$

$$\left(\frac{T_2}{T_1} \right)_{s=\text{const.}} = \left(\frac{P_2}{P_1} \right)^{(k-1)/k} \quad \left(\frac{P_2}{P_1} \right)_{s=\text{const.}} = \left(\frac{v_1}{v_2} \right)^k$$

$$\left(\frac{T_2}{T_1} \right)_{s=\text{const.}} = \left(\frac{P_2}{P_1} \right)^{(k-1)/k} = \left(\frac{v_1}{v_2} \right)^{k-1}$$

VALID FOR

- *ideal gas
- *isentropic process
- *constant specific heats

The isentropic relations of ideal gases are valid for the isentropic processes of ideal gases only.

$$TV^{k-1} = \text{constant}$$

$$TP^{(1-k)/k} = \text{constant}$$

$$PV^k = \text{constant}$$

REVERSIBLE STEADY-FLOW WORK

$$\delta q_{\text{rev}} - \delta w_{\text{rev}} = dh + dke + dpe$$

$$\left. \begin{array}{l} \delta q_{\text{rev}} = T ds \quad (\text{Eq. 7-16}) \\ T ds = dh - v dP \quad (\text{Eq. 7-24}) \end{array} \right\} \delta q_{\text{rev}} = dh - v dP$$

$$-\delta w_{\text{rev}} = v dP + dke + dpe$$

$$w_{\text{rev}} = - \int_1^2 v dP - \Delta ke - \Delta pe$$

$$w_{\text{rev}} = - \int_1^2 v dP \quad \text{When kinetic and potential energies are negligible}$$

$$w_{\text{rev,in}} = \int_1^2 v dP + \Delta ke + \Delta pe$$

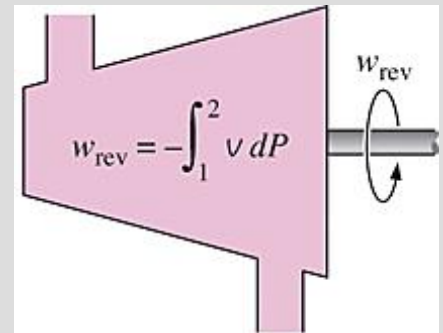
$$w_{\text{rev}} = -v(P_2 - P_1) - \Delta ke - \Delta pe$$

For the steady flow of a liquid through a device that involves no work interactions (such as a pipe section), the work term is zero (**Bernoulli equation**):

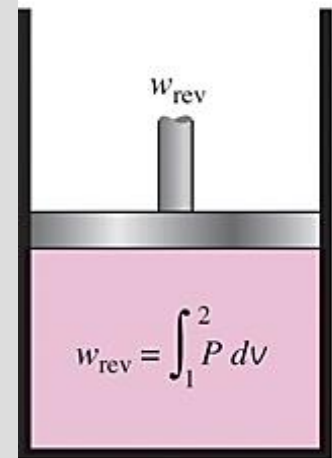
$$v(P_2 - P_1) + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1) = 0$$

$$\begin{aligned} w &= - \int_1^2 v dP \\ W &= - \int_1^2 V dP \\ W &= - \int_1^2 V dP \end{aligned}$$

The larger the specific volume, the greater the work produced (or consumed) by a steady-flow device.



(a) Steady-flow system



(b) Closed system

Reversible work relations for steady-flow and closed systems.

Proof that Steady-Flow Devices Deliver the Most and Consume the Least Work when the Process Is Reversible

Taking heat input and work output positive:

$$\delta q_{\text{act}} - \delta w_{\text{act}} = dh + dke + dpe \quad \text{Actual}$$

$$\delta q_{\text{rev}} - \delta w_{\text{rev}} = dh + dke + dpe \quad \text{Reversible}$$

$$\delta q_{\text{act}} - \delta w_{\text{act}} = \delta q_{\text{rev}} - \delta w_{\text{rev}}$$

$$\delta w_{\text{rev}} - \delta w_{\text{act}} = \delta q_{\text{rev}} - \delta q_{\text{act}}$$

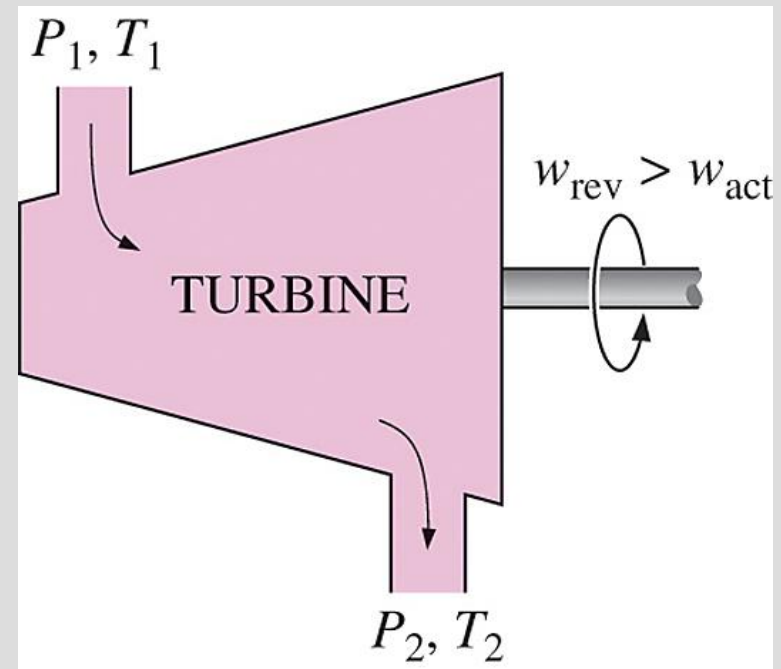
$$\delta q_{\text{rev}} = T ds \quad ds \geq \frac{\delta q_{\text{act}}}{T}$$

$$\frac{\delta w_{\text{rev}} - \delta w_{\text{act}}}{T} = ds - \frac{\delta q_{\text{act}}}{T} \geq 0$$

$$\delta w_{\text{rev}} \geq \delta w_{\text{act}}$$

$$w_{\text{rev}} \geq w_{\text{act}}$$

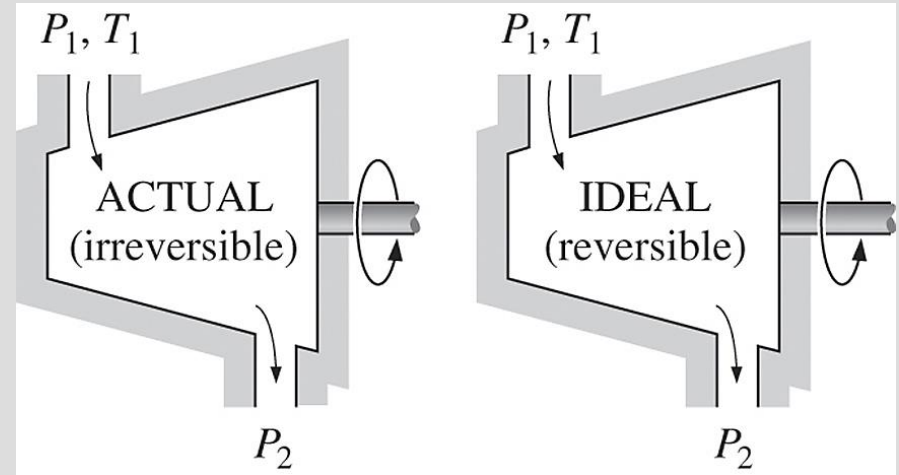
Work-producing devices such as turbines deliver more work, and work-consuming devices such as pumps and compressors require less work when they operate reversibly.



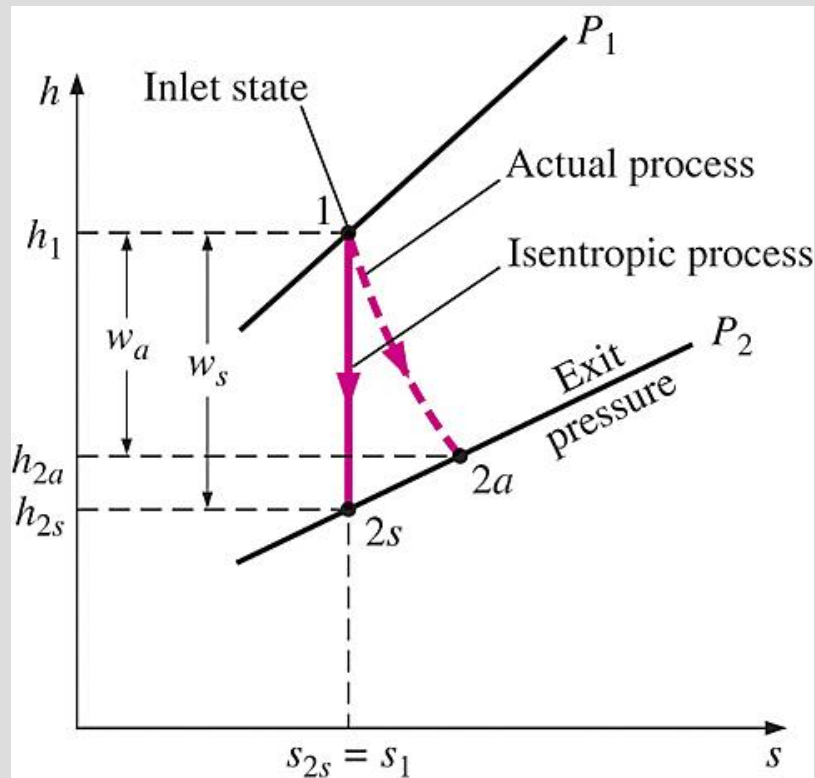
A reversible turbine delivers more work than an irreversible one if both operate between the same end states.

ISENTROPIC EFFICIENCIES OF STEADY-FLOW DEVICES

The isentropic process involves no irreversibilities and serves as the ideal process for **adiabatic devices**.



Isentropic Efficiency of Turbines



$$\eta_T = \frac{\text{Actual turbine work}}{\text{Isentropic turbine work}} = \frac{w_a}{w_s}$$

$$\eta_T \cong \frac{h_1 - h_{2a}}{h_1 - h_{2s}}$$

The h - s diagram for the actual and isentropic processes of an adiabatic turbine.

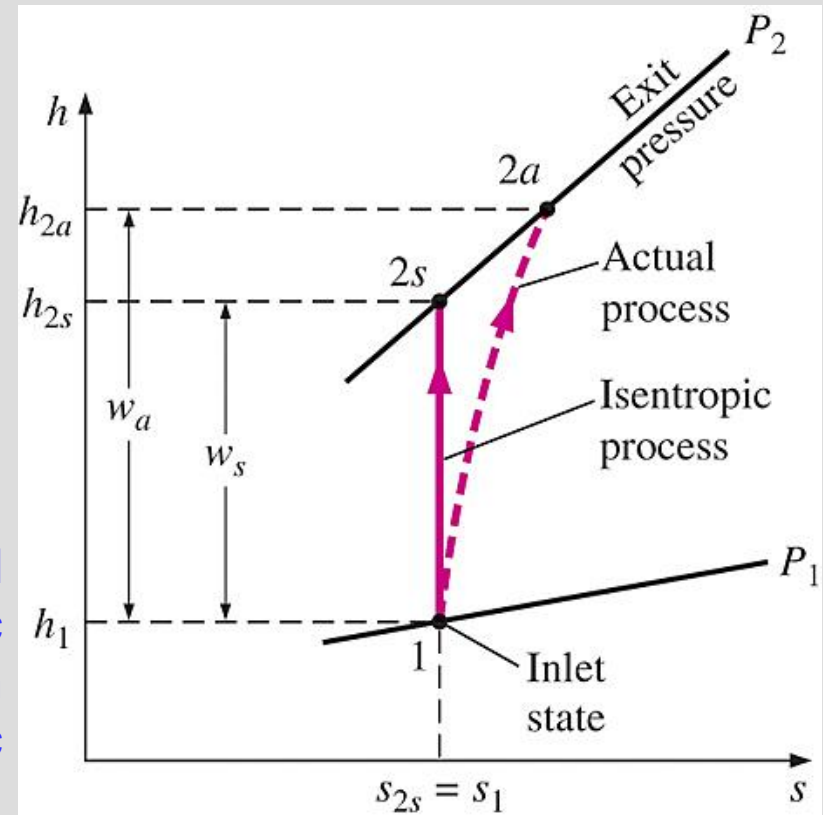
Isentropic Efficiencies of Compressors and Pumps

$$\eta_c = \frac{\text{Isentropic compressor work}}{\text{Actual compressor work}} = \frac{w_s}{w_a}$$

$$\eta_c \cong \frac{h_{2s} - h_1}{h_{2a} - h_1} \quad \text{When kinetic and potential energies are negligible}$$

$$\eta_P = \frac{w_s}{w_a} = \frac{v(P_2 - P_1)}{h_{2a} - h_1} \quad \text{For a pump}$$

The h - s diagram of the actual and isentropic processes of an adiabatic compressor.



Can you use isentropic efficiency for a non-adiabatic compressor?

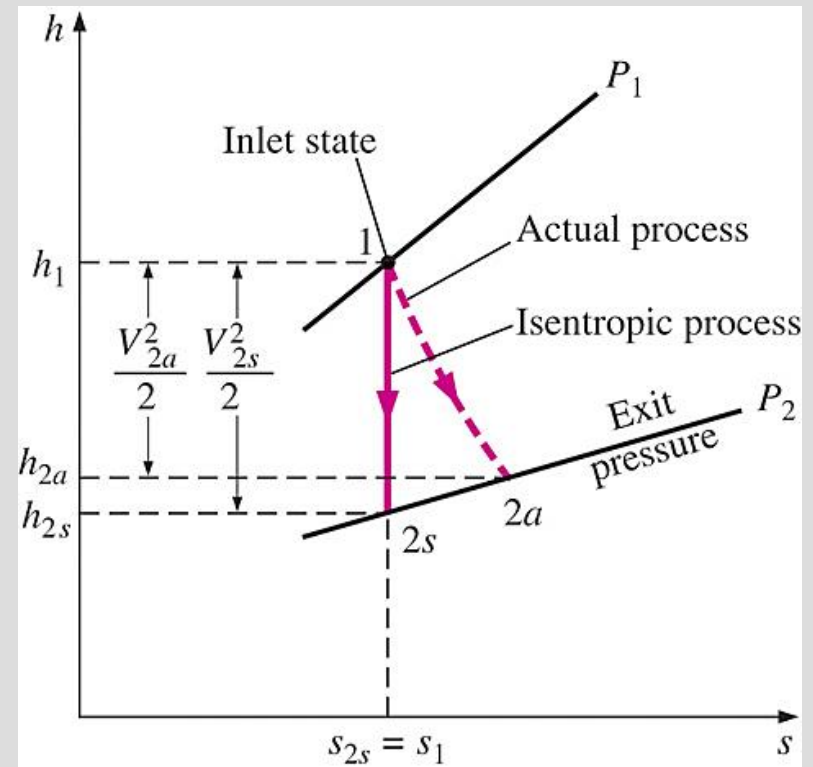
Isentropic Efficiency of Nozzles

$$\eta_N = \frac{\text{Actual KE at nozzle exit}}{\text{Isentropic KE at nozzle exit}} = \frac{V_{2a}^2}{V_{2s}^2}$$

If the inlet velocity of the fluid is small relative to the exit velocity, the energy balance is

$$h_1 = h_{2a} + \frac{V_{2a}^2}{2}$$

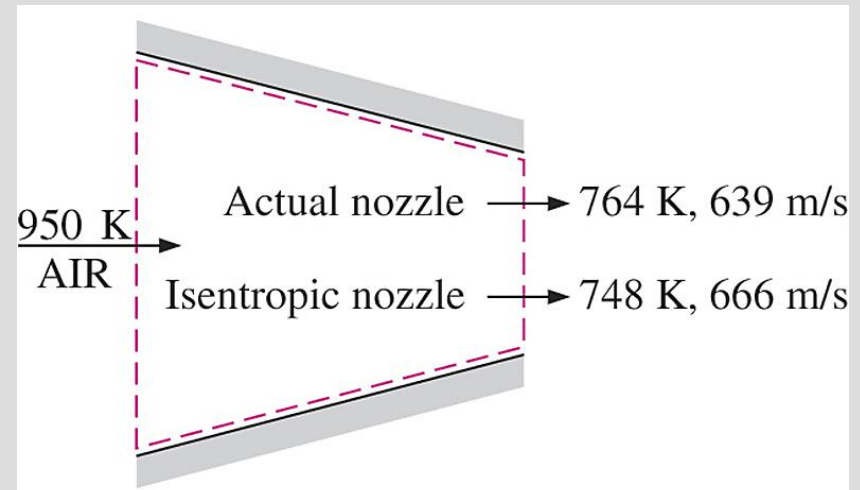
The h - s diagram of the actual and isentropic processes of an adiabatic nozzle.



Then,

$$\eta_T \cong \frac{h_1 - h_{2a}}{h_1 - h_{2s}}$$

A substance leaves actual nozzles at a higher temperature (thus a lower velocity) as a result of friction.



ENTROPY BALANCE

$$\left(\begin{array}{c} \text{Total} \\ \text{entropy} \\ \text{entering} \end{array} \right) - \left(\begin{array}{c} \text{Total} \\ \text{entropy} \\ \text{leaving} \end{array} \right) + \left(\begin{array}{c} \text{Total} \\ \text{entropy} \\ \text{generated} \end{array} \right) = \left(\begin{array}{c} \text{Change in the} \\ \text{total entropy} \\ \text{of the system} \end{array} \right)$$

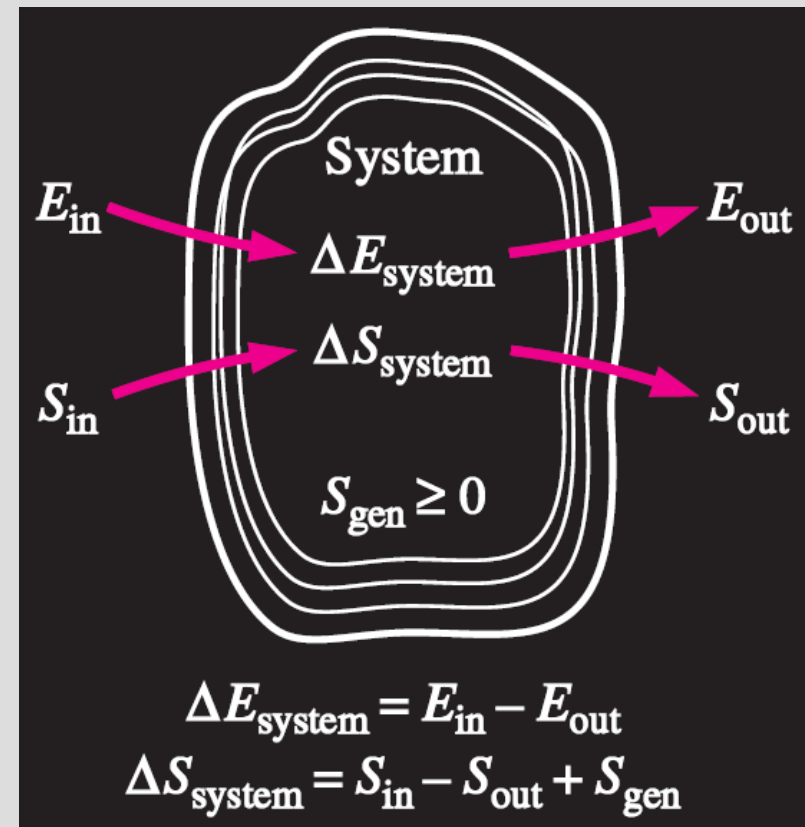
$$\underbrace{S_{\text{in}} - S_{\text{out}}}_{\text{Net entropy transfer by heat and mass}} + \underbrace{S_{\text{gen}}}_{\text{Entropy generation}} = \underbrace{\Delta S_{\text{system}}}_{\text{Change in entropy}}$$

$$\underbrace{\dot{S}_{\text{in}} - \dot{S}_{\text{out}}}_{\text{Rate of net entropy transfer by heat and mass}} + \underbrace{\dot{S}_{\text{gen}}}_{\text{Rate of entropy generation}} = \underbrace{dS_{\text{system}}/dt}_{\text{Rate of change in entropy}}$$

$$(S_{\text{in}} - S_{\text{out}}) + S_{\text{gen}} = \Delta S_{\text{system}}$$

Entropy Change of a System, ΔS_{system}

$$\Delta S_{\text{system}} = S_{\text{final}} - S_{\text{initial}} = S_2 - S_1$$

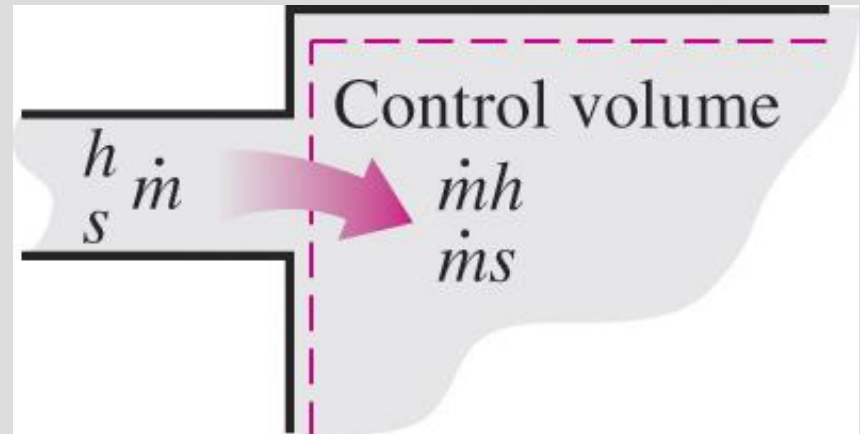


Mechanisms of Entropy Transfer, S_{in} and S_{out}

2 Mass Flow

Entropy transfer by mass:

$$S_{\text{mass}} = m s$$



Mass contains entropy as well as energy, and thus mass flow into or out of system is always accompanied by energy and entropy transfer.

Closed Systems

$$\text{Closed system:} \quad \sum \frac{Q_k}{T_k} + S_{\text{gen}} = \Delta S_{\text{system}} = S_2 - S_1 \quad (\text{kJ/K})$$

The entropy change of a closed system during a process is equal to the sum of the net entropy transferred through the system boundary by heat transfer and the entropy generated within the system boundaries.

$$\text{Adiabatic closed system:} \quad S_{\text{gen}} = \Delta S_{\text{adiabatic system}}$$

$$\text{System + Surroundings:} \quad S_{\text{gen}} = \sum \Delta S = \Delta S_{\text{system}} + \Delta S_{\text{surroundings}}$$

$$\Delta S_{\text{system}} = m(s_2 - s_1) \quad \Delta S_{\text{surr}} = Q_{\text{surr}}/T_{\text{surr}}$$

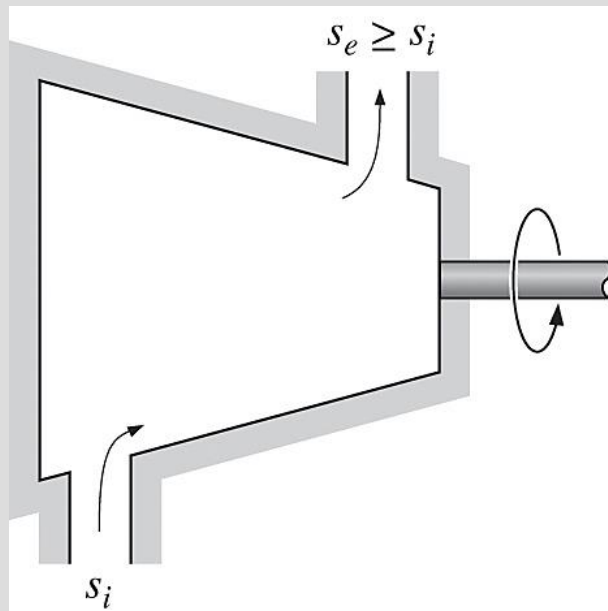
$$\sum \frac{Q_k}{T_k} + \sum m_i s_i - \sum m_e s_e + S_{\text{gen}} = (S_2 - S_1)_{\text{CV}} \quad (\text{kJ/K})$$

$$\sum \frac{\dot{Q}_k}{T_k} + \sum \dot{m}_i s_i - \sum \dot{m}_e s_e + \dot{S}_{\text{gen}} = dS_{\text{CV}}/dt \quad (\text{kW/K})$$

Steady-flow:
$$\dot{S}_{\text{gen}} = \sum \dot{m}_e s_e - \sum \dot{m}_i s_i - \sum \frac{\dot{Q}_k}{T_k}$$

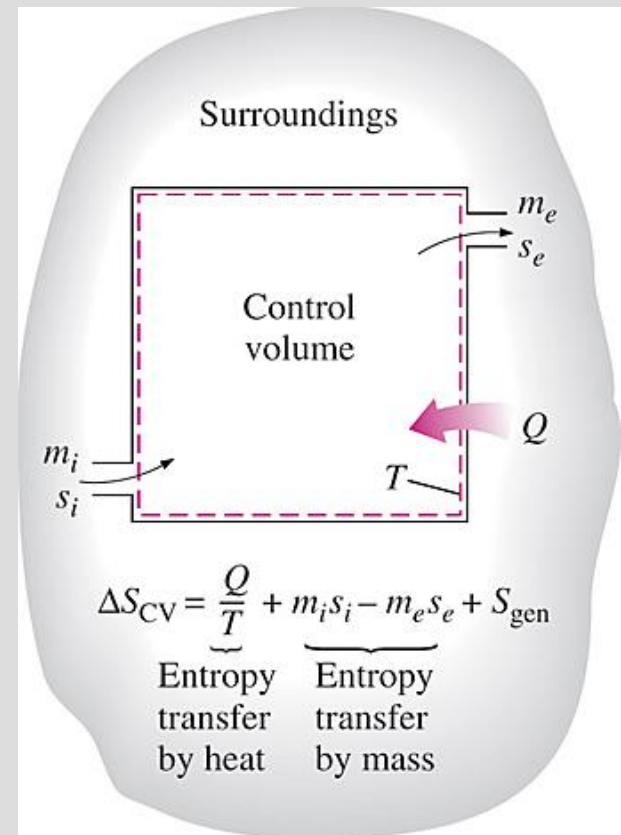
Steady-flow, single-stream:
$$\dot{S}_{\text{gen}} = \dot{m}(s_e - s_i) - \sum \frac{\dot{Q}_k}{T_k}$$

Steady-flow, single-stream, adiabatic:
$$\dot{S}_{\text{gen}} = \dot{m}(s_e - s_i)$$



The entropy of a substance always increases (or remains constant in the case of a reversible process) as it flows through a single-stream, adiabatic, steady-flow device.

Control Volumes



The entropy of a control volume changes as a result of mass flow as well as heat transfer.