Hypotheses Testing:

- A hypothesis is a statement about one or more populations.
- A statistical hypothesis is a conjecture (or a statement) concerning the population which can be evaluated by appropriate statistical technique.
- We usually test the null hypothesis (H_0) against the alternative (or the research) hypothesis (H_A or H_1) by choosing one of the following situations:

  Two-sided hypothesis:

  H_0: $\theta = \theta_0$ against H_A: $\theta \neq \theta_0$

  One-sided hypothesis:

  (i) H_0: $\theta \geq \theta_0$ against H_A: $\theta < \theta_0$

  (ii) H_0: $\theta \leq \theta_0$ against H_A: $\theta > \theta_0$
- There are 4 possible situations in testing a statistical hypothesis:

		Condition of Null Hypothesis H_0 (Nature/Reality)	
		H_0 is true	H_0 is false
Possible Action (Decision)	Accepting H_0	Correct Decision	Type II error (β)
	Rejecting H_0	Type I error (α)	Correct Decision

- There are two types of errors:

  (i) Type I error = Rejecting H_0 when H_0 is true

  P(Type I error) = P(Rejecting H_0 | H_0 is true) = α

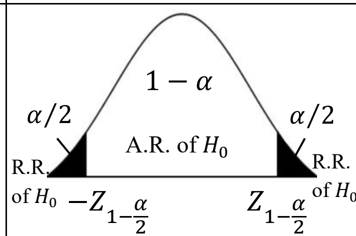
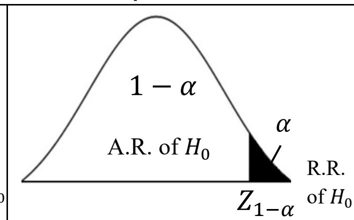
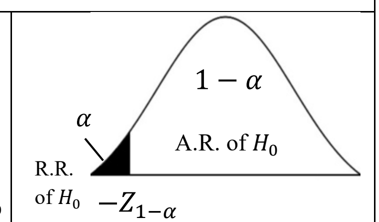
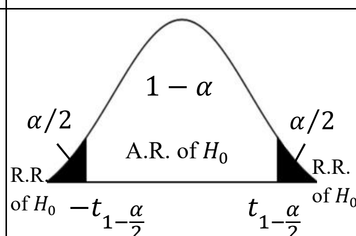
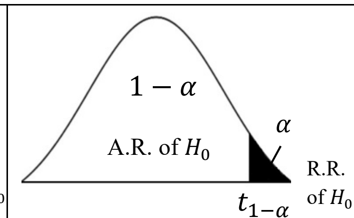
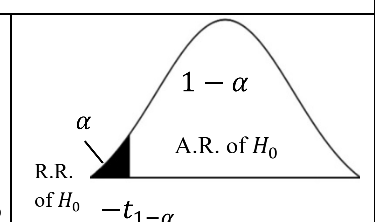
  Which is called the significance level of the test.

  (ii) Type II error = Accepting H_0 when H_0 is false

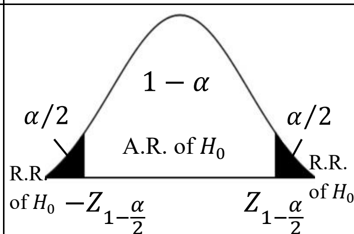
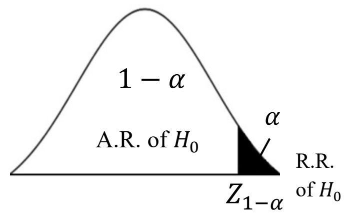
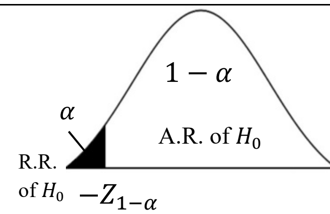
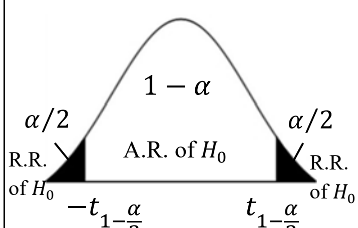
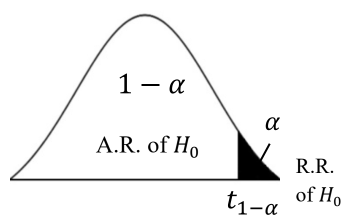
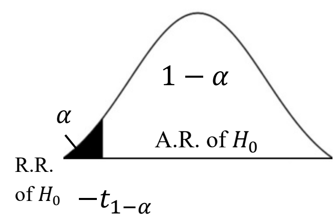
  P(Type II error) = P(Accepting H_0 | H_0 is false) = β
- The test statistic has the following form:

$$\text{Test Statistic} = \frac{\text{estimate} - \text{hypotheized parameter}}{\text{standard error of the estimate}}$$

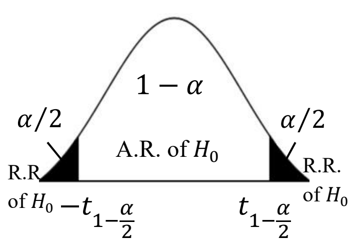
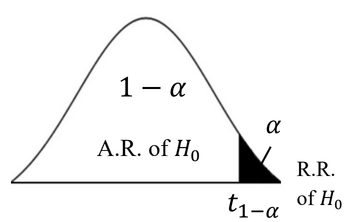
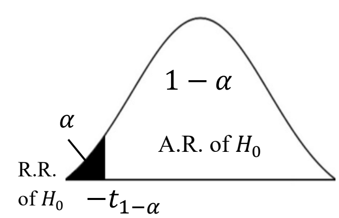
1. Hypotheses Testing for the population Mean (μ):

Hypotheses	$H_0: \mu = \mu_0 \;\; vs$ $H_A: \mu \neq \mu_0$	$H_0: \mu \leq \mu_0 \;\; vs$ $H_A: \mu > \mu_0$	$H_0: \mu \geq \mu_0 \;\; vs$ $H_A: \mu < \mu_0$
Assumptions:	First Case: σ^2 is known; Normal or Non-Normal Distribution		
Test Statistic (T.S.)	$Z = \dfrac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} \sim N(0,1)$		
Rejection Region (R.R.) & Acceptance Region (A.R.) of H_0			
Critical Value	$-Z_{1-\frac{\alpha}{2}}$ and $Z_{1-\frac{\alpha}{2}}$	$Z_{1-\alpha}$	$-Z_{1-\alpha}$
Decision	We reject H_0 (and accept H_A) at the significance level α if:		
	$Z < -Z_{1-\frac{\alpha}{2}}$ or $Z > Z_{1-\frac{\alpha}{2}}$	$Z > Z_{1-\alpha}$	$Z < -Z_{1-\alpha}$
Assumptions:	Second Case: σ^2 is unknown; Normal Distribution		
Test Statistic (T.S.)	$T = \frac{\bar{X}-\mu_0}{S/\sqrt{n}} \sim t_{(n-1)}$; $\quad$ d.f $= v = n - 1$		
Rejection Region (R.R.) & Acceptance Region (A.R.) of H_0			
Critical Value	$-t_{1-\frac{\alpha}{2}}$ and $t_{1-\frac{\alpha}{2}}$	$t_{1-\alpha}$	$-t_{1-\alpha}$
Decision	We reject H_0 (and accept H_A) at the significance level α if:		
	$T < -t_{1-\frac{\alpha}{2}}$ or $T > t_{1-\frac{\alpha}{2}}$	$T > t_{1-\alpha}$	$T < -t_{1-\alpha}$

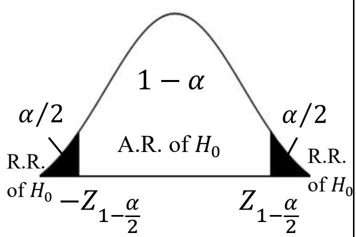
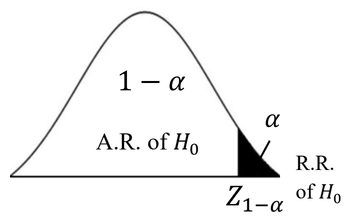
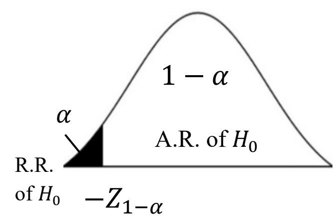
2. Hypotheses Testing for the Difference Between Two Population Means $(\mu_1 - \mu_2)$(Independent Populations):

<table>
<tr><td>Hypotheses</td><td>$H_0: \mu_1 = \mu_2 \ vs$
$H_A: \mu_1 \neq \mu_2$</td><td>$H_0: \mu_1 \leq \mu_2 \ vs$
$H_A: \mu_1 > \mu_2$</td><td>$H_0: \mu_1 \geq \mu_2 \ vs$
$H_A: \mu_1 < \mu_2$</td></tr>
<tr><td>Assumptions:</td><td colspan="3">First Case: σ_1^2 and σ_2^2 are known</td></tr>
<tr><td>Test Statistic (T.S.)</td><td colspan="3">$$Z = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \sim N(0,1)$$</td></tr>
<tr><td>Rejection Region (R.R.) & Acceptance Region (A.R.) of H_0</td><td>
</td><td>
</td><td>
</td></tr>
<tr><td>Critical Value</td><td>$-Z_{1-\frac{\alpha}{2}}$ and $Z_{1-\frac{\alpha}{2}}$</td><td>$Z_{1-\alpha}$</td><td>$-Z_{1-\alpha}$</td></tr>
<tr><td rowspan="2">Decision</td><td colspan="3">We reject H_0 (and accept H_A) at the significance level α if:</td></tr>
<tr><td>$Z < -Z_{1-\frac{\alpha}{2}}$
or $Z > Z_{1-\frac{\alpha}{2}}$</td><td>$Z > Z_{1-\alpha}$</td><td>$Z < -Z_{1-\alpha}$</td></tr>
<tr><td>Assumptions:</td><td colspan="3">Second Case: σ_1^2 and σ_2^2 are unknown but equal $(\sigma_1^2 = \sigma_2^2 = \sigma^2)$</td></tr>
<tr><td>Test Statistic (T.S.)</td><td colspan="3">$$T = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{S_p^2}{n_1} + \frac{S_p^2}{n_2}}} \sim t(n_1 + n_2 - 2), \ \ df = v = n_1 + n_2 - 2$$
$$S_p^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2}$$</td></tr>
<tr><td>Rejection Region (R.R.) & Acceptance Region (A.R.) of H_0</td><td>
</td><td>
</td><td>
</td></tr>
<tr><td>Critical Value</td><td>$-t_{1-\frac{\alpha}{2}}$ and $t_{1-\frac{\alpha}{2}}$</td><td>$t_{1-\alpha}$</td><td>$-t_{1-\alpha}$</td></tr>
<tr><td rowspan="2">Decision</td><td colspan="3">We reject H_0 (and accept H_A) at the significance level α if:</td></tr>
<tr><td>$T < -t_{1-\frac{\alpha}{2}}$
or $T > t_{1-\frac{\alpha}{2}}$</td><td>$T > t_{1-\alpha}$</td><td>$T < -t_{1-\alpha}$</td></tr>
</table>

3. Confidence Interval and Hypotheses Testing for the Difference Between Two Population Means $(\mu_1 - \mu_2 = \mu_D)$ for Dependent (Related) Populations: Paired t-Test:

Calculate the Quantities	• The differences (D-observations): $D_i = X_i - Y_i\,,\;\; i = 1,2,\dots,n$ • Sample Mean of the D-observations: $\overline{D} = \frac{\sum_{i=1}^{n} D_i}{n}$ • Sample Variance of the D-observations: $S_D^2 = \frac{\sum_{i=1}^{n}(D_i - \overline{D})^2}{n-1}$ • Sample Standard Deviation of the D-observations: $S_D = \sqrt{S_D^2}$		
Confidence Interval for $\mu_D = \mu_1 - \mu_2$			
$100(1-\alpha)\%$ Confidence Interval for μ_D	$\overline{D} \pm t_{1-\frac{\alpha}{2}} \frac{S_D}{\sqrt{n}}\,, \quad df = v = n-1$		
Hypotheses Testing for $\mu_D = \mu_1 - \mu_2$			
Hypotheses	$H_0: \mu_1 = \mu_2 \;\; vs$ $H_A: \mu_1 \neq \mu_2$ or $H_0: \mu_D = 0 \;\; vs$ $H_A: \mu_D \neq 0$	$H_0: \mu_1 \leq \mu_2 \;\; vs$ $H_A: \mu_1 > \mu_2$ or $H_0: \mu_D \leq 0 \;\; vs$ $H_A: \mu_D > 0$	$H_0: \mu_1 \geq \mu_2 \;\; vs$ $H_A: \mu_1 < \mu_2$ or $H_0: \mu_D \geq 0 \;\; vs$ $H_A: \mu_D < 0$
Test Statistic (T.S.)	$T = \frac{\overline{D}}{S_D/\sqrt{n}} \sim t(n-1)\,, \quad df = v = n-1$		
Rejection Region (R.R.) & Acceptance Region (A.R.) of H_0			
Critical Value	$-t_{1-\frac{\alpha}{2}}$ and $t_{1-\frac{\alpha}{2}}$	$t_{1-\alpha}$	$-t_{1-\alpha}$
Decision	We reject H_0 (and accept H_A) at the significance level α if:		
	$T < -t_{1-\frac{\alpha}{2}}$ or $T > t_{1-\frac{\alpha}{2}}$	$T > t_{1-\alpha}$	$T < -t_{1-\alpha}$

4. Hypotheses Testing for the Population Proportion (p):

<table>
<tr><td>Hypotheses</td><td>$H_0: p = p_0 \ \ vs$
$H_A: p \neq p_0$</td><td>$H_0: p \leq p_0 \ \ vs$
$H_A: p > p$</td><td>$H_0: p \geq p_0 \ \ vs$
$H_A: p < p_0$</td></tr>
<tr><td>Test Statistic (T.S.)</td><td colspan="3">$Z = \dfrac{\hat{p} - p_0}{\sqrt{\dfrac{p_0(1-p_0)}{n}}} \sim N(0,1)\,, \qquad \hat{p} = \dfrac{X}{n}$</td></tr>
<tr><td>Rejection Region (R.R.) & Acceptance Region (A.R.) of H_0</td><td>
</td><td>
</td><td>
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<tr><td>Critical Value</td><td>$-Z_{1-\frac{\alpha}{2}}$ and $Z_{1-\frac{\alpha}{2}}$</td><td>$Z_{1-\alpha}$</td><td>$-Z_{1-\alpha}$</td></tr>
<tr><td rowspan="2">Decision</td><td colspan="3">We reject H_0 (and accept H_A) at the significance level α if:</td></tr>
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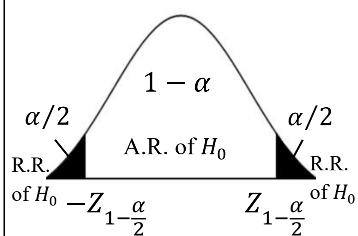
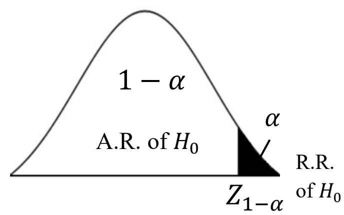
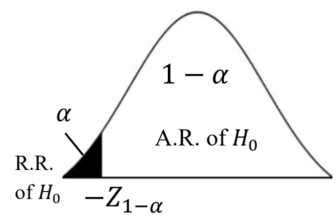
or $Z > Z_{1-\frac{\alpha}{2}}$</td><td>$Z > Z_{1-\alpha}$</td><td>$Z < -Z_{1-\alpha}$</td></tr>
</table>

5. Hypotheses Testing for the Difference Between Two Population Proportions $(p_1 - p_2)$:

<table>
<tr><td>Hypotheses</td><td>$H_0: p_1 = p_2 \ \ vs$
$H_A: p_1 \neq p_2$</td><td>$H_0: p_1 \leq p_2 \ \ vs$
$H_A: p_1 > p_2$</td><td>$H_0: p_1 \geq p_2 \ \ vs$
$H_A: p_1 < p_2$</td></tr>
<tr><td>Test Statistic
(T.S.)</td><td colspan="3">$$Z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\frac{\bar{p}(1-\bar{p})}{n_1} + \frac{\bar{p}(1-\bar{p})}{n_2}}} \sim N(0,1)$$
$$\hat{p}_1 = \frac{X_1}{n_1}, \quad \hat{p}_2 = \frac{X_2}{n_2}, \quad \bar{p} = \frac{X_1 + X_2}{n_1 + n_2}$$</td></tr>
<tr><td>Rejection Region
(R.R.) &
Acceptance
Region (A.R.) of
H_0</td><td>
</td><td>
</td><td>
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