# **Hypotheses Testing:**

- A hypothesis is a statement about one or more populations.
- A statistical hypothesis is a conjecture (or a statement) concerning the population which can be evaluated by appropriate statistical technique.
- We usually test the null hypothesis  $(H_0)$  against the alternative (or the research) hypothesis ( $H_A$  or  $H_1$ ) by choosing one of the following situations: Two-sided hypothesis:

 $H_0: \theta = \theta_0$  against  $H_A: \theta \neq \theta_0$ 

One-sided hypothesis:

- (i)  $H_0: \theta \ge \theta_0$  against  $H_A: \theta < \theta_0$
- (ii)  $H_0: \theta \leq \theta_0$  against  $H_A: \theta > \theta_0$
- There are 4 possible situations in testing a statistical hypothesis:

		Condition of Null Hypothesis $H_0$ (Nature/Reality)	
		$H_0$ is true $H_0$ is false	
Possible	Accepting $H_0$	Correct	Type II error
Action		Decision	(β)
(Decision)	Rejecting $H_0$	Type I error	Correct Decision
		(α)	

- There are two types of errors:
  - (i) Type I error = Rejecting  $H_0$  when  $H_0$  is true P(Type I error) = P(Rejecting  $H_0 | H_0$  is true) =  $\alpha$ Which is called the significance level of the test.
  - (ii) Type II error = Accepting  $H_0$  when  $H_0$  is false P(Type II error) = P(Accepting  $H_0 | H_0$  is false) =  $\beta$
- The test statistic has the following form:

Test Statistic =  $\frac{\text{estimate} - \text{hypotheized parameter}}{\text{standard error of the estimate}}$ 

### 1. Hypotheses Testing for the population Mean $(\mu)$ :

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Hypotheses	$H_0: \mu = \mu_0 \ \nu s$ $H_A: \mu \neq \mu_0$	$ \begin{array}{l} H_0: \mu \leq \mu_0 \ \nu s \\ H_A: \mu > \mu_0 \end{array} $	$ \begin{array}{l} H_0: \mu \geq \mu_0 \ vs \\ H_A: \mu < \mu_0 \end{array} $
Assumptions:	First Case: $\sigma^2$ is known; Normal or Non-Normal Distribution		
Test Statistic (T.S.)	$Z = \frac{\bar{X} - \mu_0}{\sigma / \sqrt{n}} \sim N(0, 1)$		
Rejection Region (R.R.) & Acceptance Region (A.R.) of $H_0$	$\begin{array}{c c} & 1 - \alpha \\ & \alpha/2 \\ R.R. \\ of H_0 - Z_{1-\frac{\alpha}{2}} \\ & Z_{1-\frac{\alpha}{2}} \\ \end{array} \begin{array}{c} \alpha/2 \\ R.R. \\ & R.R. \\ & \alpha/2 \\ \\$	$1 - \alpha$ A.R. of $H_0$ $Z_{1-\alpha}$ R.R. of $H_0$	$\begin{array}{c} \alpha \\ A.R. \text{ of } H_0 \\ \sigma f H_0 \\ -Z_{1-\alpha} \end{array}$
Critical Value	$-Z_{1-\frac{\alpha}{2}}$ and $Z_{1-\frac{\alpha}{2}}$	$Z_{1-\alpha}$	$-Z_{1-\alpha}$
Decision	We reject $H_0$ (and acce $Z < -Z_{1-\frac{\alpha}{2}}$ or $Z > Z_{1-\frac{\alpha}{2}}$	Ept $H_A$ ) at the significant $Z > Z_{1-\alpha}$	ce level $\alpha$ if: $Z < -Z_{1-\alpha}$
Assumptions:	Second Case: $\sigma^2$ is unknown; Normal Distribution		
Test Statistic (T.S.)	$T = \frac{\bar{x} - \mu_0}{S/\sqrt{n}} \sim t_{(n-1)};$ d. f = v = n - 1		
Rejection Region (R.R.) & Acceptance Region (A.R.) of $H_0$	$\begin{array}{c c} & 1-\alpha \\ \alpha/2 \\ R.R. \\ of H_0 - t_{1-\frac{\alpha}{2}} \\ t_{1-\frac{\alpha}{2}} \\ t_{1-\frac{\alpha}{2}} \\ t_{1-\frac{\alpha}{2}} \\ \end{array}$	$\begin{array}{c c} & & & \\ & & &$	$\begin{array}{c} 1 - \alpha \\ 1 - \alpha \\ A.R. \text{ of } H_0 \\ -t_{1-\alpha} \end{array}$
Critical Value	$-t_{1-\frac{\alpha}{2}}$ and $t_{1-\frac{\alpha}{2}}$	$t_{1-lpha}$	$-t_{1-lpha}$
We reject $H_0$ (and accept $H_A$ ) at the significance level $\alpha$ if:			ce level $\alpha$ if:
Decision	$T < -t_{1-\frac{\alpha}{2}}$ or $T > t_{1-\frac{\alpha}{2}}$	$T > t_{1-\alpha}$	$T < -t_{1-\alpha}$

## 2. Hypotheses Testing for the Difference Between Two Population Means $(\mu_1 - \mu_2)$ (Independent Populations):

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Hypotheses	$H_0: \mu_1 = \mu_2 \ \nu s$ $H_A: \mu_1 \neq \mu_2$	$H_0: \mu_1 \le \mu_2 \ \nu s \\ H_A: \mu_1 > \mu_2$	$ \begin{array}{l} H_0: \mu_1 \geq \mu_2 \ \nu s \\ H_A: \mu_1 < \mu_2 \end{array} $
Assumptions:	First Case: $\sigma_1^2$ and $\sigma_2^2$ are known		
Test Statistic (T.S.)	$Z = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \sim N(0, 1)$		
Rejection Region (R.R.) & Acceptance Region (A.R.) of $H_0$	$\begin{array}{c c} & 1-\alpha \\ & \alpha/2 \\ \text{R.R.} \\ \text{of } H_0 - Z_{1-\frac{\alpha}{2}} \\ \end{array} \begin{array}{c} \alpha/2 \\ \text{R.R.} \\ Z_{1-\frac{\alpha}{2}} \\ \end{array} \begin{array}{c} \alpha/2 \\ \text{R.R.} \\ \text{of } H_0 \end{array}$	$1 - \alpha$ A.R. of $H_0$ $Z_{1-\alpha}$ R.R. of $H_0$	$\begin{array}{c} \alpha \\ A.R. \text{ of } H_0 \\ -Z_{1-\alpha} \end{array}$
Critical Value	$-Z_{1-\frac{\alpha}{2}}$ and $Z_{1-\frac{\alpha}{2}}$	$Z_{1-\alpha}$	$-Z_{1-\alpha}$
Decision	We reject $H_0$ (and acceled $Z < -Z_{1-\frac{\alpha}{2}}$ or $Z > Z_{1-\frac{\alpha}{2}}$	$Ept H_A$ ) at the significant $Z > Z_{1-\alpha}$	the level $\alpha$ if: $Z < -Z_{1-\alpha}$
Assumptions:	Second Case: $\sigma_1^2$ and $\sigma_2^2$ are unknown but equal ( $\sigma_1^2 = \sigma_2^2 = \sigma^2$ )		
Test Statistic (T.S.)	$T = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{s_p^2}{n_1} + \frac{s_p^2}{n_2}}} \sim t(n_1 + n_2 - 2),  df = v = n_1 + n_2 - 2$ $s_p^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2}$		
Rejection Region (R.R.) & Acceptance Region (A.R.) of $H_0$	$\begin{array}{c c} \alpha/2 & 1-\alpha \\ \alpha/2 & A.R. \text{ of } H_0 \\ \text{of } H_0 & -t_{1-\frac{\alpha}{2}} & t_{1-\frac{\alpha}{2}} \\ \end{array} \\ \end{array} \\ \begin{array}{c} \alpha/2 \\ R.R. \\ \text{of } H_0 \\ \text{of } H_0 \end{array}$	$ \begin{array}{c} 1 - \alpha \\ A.R. \text{ of } H_0 \\ \hline t_{1-\alpha} \\ \end{array} \xrightarrow{\alpha} R.R. \\ \text{ of } H_0 \end{array} $	$\begin{array}{c} & 1 - \alpha \\ & \\ R.R. \\ of H_0 \\ -t_{1-\alpha} \end{array}$
Critical Value	$-t_{1-\frac{\alpha}{2}}$ and $t_{1-\frac{\alpha}{2}}$	$t_{1-lpha}$	$-t_{1-lpha}$
Decision	We reject $H_0$ (and acce $T < -t_{1-\frac{\alpha}{2}}$ or $T > t_{1-\frac{\alpha}{2}}$	ept $H_A$ ) at the significant $T > t_{1-\alpha}$	the level $\alpha$ if: $T < -t_{1-\alpha}$

#### 3. Confidence Interval and Hypotheses Testing for the Difference Between Two Population Means $(\mu_1 - \mu_2 = \mu_D)$ for Dependent (Related) Populations: Paired t-Test:

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Calculate the Quantities	<ul> <li>The differences (D-observations): D<sub>i</sub> = X<sub>i</sub> - Y<sub>i</sub>, i = 1,2,, n</li> <li>Sample Mean of the D-observations: D̄ = Σ<sup>n</sup><sub>i=1</sub>D<sub>i</sub>/n</li> <li>Sample Variance of the D-observations: S<sup>2</sup><sub>D</sub> = Σ<sup>n</sup><sub>i=1</sub>(D<sub>i</sub>-D̄)<sup>2</sup>/n-1</li> <li>Sample Standard Deviation of the D-observations: S<sub>D</sub> = √S<sup>2</sup><sub>D</sub></li> </ul>			
	Confidence Interv	val for $\mu_D = \mu_1 - \mu_2$		
$\begin{array}{c} 100(1-\alpha)\%\\ \text{Confidence}\\ \text{Interval for }\mu_D \end{array}$	$\overline{D} \pm t_{1-rac{lpha}{2}}rac{S_D}{\sqrt{n}}$ , $df = v = n-1$			
	Hypotheses Testing for $\mu_D = \mu_1 - \mu_2$			
Hypotheses	$H_A: \mu_1 \neq \mu_2$ or $H_0: \mu_D = 0 \ \nu s$	$H_0: \mu_1 \le \mu_2 \ vs$ $H_A: \mu_1 > \mu_2$ or $H_0: \mu_D \le 0 \ vs$ $H_A: \mu_D > 0$	$H_0: \mu_1 \ge \mu_2 \ vs$ $H_A: \mu_1 < \mu_2$ or $H_0: \mu_D \ge 0 \ vs$ $H_A: \mu_D < 0$	
Test Statistic (T.S.)	$\begin{array}{c c} H_A: \mu_D \neq 0 & H_A: \mu_D > 0 & H_A: \mu_D < 0 \\ \hline T = \frac{\overline{D}}{S_D / \sqrt{n}} \sim t(n-1) ,  df = v = n-1 \end{array}$			
Rejection Region (R.R.) & Acceptance Region (A.R.) of $H_0$	$\begin{array}{c c} & 1-\alpha \\ & \alpha/2 \\ R.R \\ \text{of } H_0 - t_{1-\frac{\alpha}{2}} \\ \end{array} \begin{array}{c} \alpha/2 \\ R.R. \\ f_{1-\frac{\alpha}{2}} \\ \end{array} \begin{array}{c} \alpha/2 \\ R.R. \\ f_{1-\frac{\alpha}{2}} \\ \end{array} \begin{array}{c} \alpha/2 \\ R.R. \\ f_{1-\frac{\alpha}{2}} \\ \end{array} \end{array}$	$ \begin{array}{c} 1 - \alpha \\ A.R. \text{ of } H_0 \\ t_{1-\alpha} \\ \end{array} \\ R.R. \\ of H_0 \end{array} $	$\begin{array}{c} \alpha \\ n.R. \\ of H_0 \\ -t_{1-\alpha} \end{array}$	
Critical Value	$-t_{1-\frac{\alpha}{2}}$ and $t_{1-\frac{\alpha}{2}}$	$t_{1-\alpha}$	$-t_{1-lpha}$	
Decision	We reject $H_0$ (and acceled $T < -t_{1-\frac{\alpha}{2}}$ or $T > t_{1-\frac{\alpha}{2}}$	$P(t H_A)$ at the significance $T > t_{1-\alpha}$	the level $\alpha$ if: $T < -t_{1-\alpha}$	

4. Hypotheses To	esting for	the Population	Proportion	( <b>p</b> ):
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Hypotheses	$H_0: p = p_0 \ vs$ $H_A: p \neq p_0$	$ \begin{array}{l} H_0: p \leq p_0 \ vs \\ H_A: p > p \end{array} $	$H_0: p \ge p_0 \ vs$ $H_A: p < p_0$
Test Statistic (T.S.)	$Z = \frac{1}{\sqrt{\underline{p}}}$	$\frac{\hat{p} - p_0}{\frac{0}{n} - p_0} \sim N(0, 1) ,$	$\hat{p} = \frac{X}{n}$
Rejection Region (R.R.) & Acceptance Region (A.R.) of $H_0$	$\begin{array}{c c} & 1-\alpha \\ \alpha/2 \\ R.R. \\ of H_0 - Z_{1-\frac{\alpha}{2}} \\ \end{array} \begin{array}{c} \alpha/2 \\ R.R. \\ A.R. of H_0 \\ Z_{1-\frac{\alpha}{2}} \\ \end{array} \begin{array}{c} \alpha/2 \\ R.R. \\ of H_0 \\ \end{array}$	$1 - \alpha$ A.R. of $H_0$ $Z_{1-\alpha}$ R.R. of $H_0$	$\begin{array}{c} \alpha \\ A.R. \text{ of } H_0 \\ end{tabular} \\ -Z_{1-\alpha} \end{array}$
Critical Value	$-Z_{1-\frac{\alpha}{2}}$ and $Z_{1-\frac{\alpha}{2}}$	$Z_{1-\alpha}$	$-Z_{1-\alpha}$
	We reject $H_0$ (and accept $H_A$ ) at the significance level $\alpha$ if:		
Decision	$Z < -Z_{1-\frac{\alpha}{2}}$ or $Z > Z_{1-\frac{\alpha}{2}}$	$Z > Z_{1-\alpha}$	$Z < -Z_{1-\alpha}$

## 5. Hypotheses Testing for the Difference Between Two Population Proportions $(p_1 - p_2)$ :

Hypotheses	$H_0: p_1 = p_2 \ vs$	$H_0: p_1 \le p_2 \ vs$	$H_0: p_1 \ge p_2 \ vs$
Trypotneses	$H_A: p_1 \neq p_2$	$H_A: p_1 > p_2$	$H_A: p_1 < p_2$
Test Statistic (T.S.)	$Z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\frac{\bar{p}(1-\bar{p})}{n_1} + \frac{\bar{p}(1-\bar{p})}{n_2}}} \sim N(0,1)$ $\hat{p}_1 = \frac{X_1}{n_1},  \hat{p}_2 = \frac{X_2}{n_2},  \bar{p} = \frac{X_1 + X_2}{n_1 + n_2}$		
Rejection Region (R.R.) & Acceptance Region (A.R.) of $H_0$	$\begin{array}{c} \alpha/2 \\ R.R. \\ of H_0 - Z_{1-\frac{\alpha}{2}} \\ \end{array} \begin{array}{c} \alpha/2 \\ R.R. \\ Z_{1-\frac{\alpha}{2}} \\ Z_{1-\frac{\alpha}{2}} \\ \end{array} \begin{array}{c} \alpha/2 \\ R.R. \\ \sigma f H_0 \\ R.R. \\ \sigma f H_0 \\ \end{array}$	$ \begin{array}{c} 1 - \alpha \\ A.R. \text{ of } H_0 \\ \hline Z_{1-\alpha} \\ \end{array} \\ \begin{array}{c} \alpha \\ R.R. \\ \text{ of } H_0 \end{array} $	$\begin{array}{c} & & 1 - \alpha \\ & & & \\ \alpha & & \\ \text{R.R.} & & \\ \text{of } H_0 & -Z_{1-\alpha} \end{array}$
Critical Value	$-Z_{1-\frac{\alpha}{2}}$ and $Z_{1-\frac{\alpha}{2}}$	$Z_{1-\alpha}$	$-Z_{1-\alpha}$
	We reject $H_0$ (and accept $H_A$ ) at the significance level $\alpha$ if:		
Decision	$Z < -Z_{1-\frac{\alpha}{2}}$ or $Z > Z_{1-\frac{\alpha}{2}}$	$Z > Z_{1-\alpha}$	$Z < -Z_{1-\alpha}$