Chapter 7: Test Hypotheses

1. <u>Test Hypotheses for the population Mean (μ):</u>

Test Procedures:

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Hypotheses	$H_0: \mu = \mu_0 \ \nu s$ $H_A: \mu \neq \mu_0$	$ \begin{aligned} H_0: \mu &\leq \mu_0 \ \nu s \\ H_A: \mu &> \mu_0 \end{aligned} $	$ \begin{array}{l} H_0: \mu \geq \mu_0 \ \nu s \\ H_A: \mu < \mu_0 \end{array} $
First Case	σ^2 is known	; Normal or Non-norma	1 Distribution
Test Statistic (T.S.)	$Z = \frac{\bar{X} - \mu_0}{\sigma / \sqrt{n}} \sim N(0, 1)$		
Rejection Region (R.R.) & Acceptance Region (A.R.) of H_0	$\begin{array}{c c} & 1-\alpha \\ \alpha/2 \\ R.R. \\ of H_0 - Z_{1-\frac{\alpha}{2}} \\ \end{array} \begin{array}{c} \alpha/2 \\ R.R. \\ Z_{1-\frac{\alpha}{2}} \\ \end{array} \begin{array}{c} \alpha/2 \\ R.R. \\ R.R. \\ \alpha/2 \\ R.R. \\ R.R. \\ \alpha/2 \\ R.R.$	$1 - \alpha$ A.R. of H_0 $Z_{1-\alpha}$ R.R. of H_0	$\begin{array}{c} \alpha \\ 1 - \alpha \\ A.R. \text{ of } H_0 \\ -Z_{1-\alpha} \end{array}$
Critical Value	$-Z_{1-\frac{\alpha}{2}}$ and $Z_{1-\frac{\alpha}{2}}$	$Z_{1-\alpha}$	$-Z_{1-\alpha}$
	We reject H_0 (and accept H_A) at the significance level α if:		
Decision	$Z < -Z_{1-\frac{\alpha}{2}} \text{ or}$ $Z > Z_{1-\frac{\alpha}{2}}$	$Z > Z_{1-\alpha}$	$Z < -Z_{1-\alpha}$
Second Case	σ^2 is u	unknown; Normal Distri	ibution
Test Statistic (T.S.)	$T = \frac{\bar{X} - \mu_0}{S/\sqrt{n}} \sim t(n-1), \ df = v = n-1$		
Rejection Region (R.R.) & Acceptance Region (A.R.) of H_0	$\begin{array}{c c} & 1 - \alpha \\ & \alpha/2 \\ R.R. \\ of H_0 - t_{1-\frac{\alpha}{2}} \\ & t_{1-\frac{\alpha}{2}} \\ \end{array} \begin{array}{c} \alpha/2 \\ R.R. \\ & r.R$	$\begin{array}{c c} & & & \\ & & &$	$\begin{array}{c} & 1 - \alpha \\ 1 - \alpha \\ A.R. \text{ of } H_0 \\ -t_{1-\alpha} \end{array}$
Critical Value	$-t_{1-\frac{\alpha}{2}}$ and $t_{1-\frac{\alpha}{2}}$	$t_{1-\alpha}$	$-t_{1-\alpha}$
	We reject H_0 (and accept H_A) at the significance level α if:		
Decision	$\begin{array}{c c} T < -t_{1-\frac{\alpha}{2}} & \text{or} \\ T > t_{1-\frac{\alpha}{2}} \end{array}$	$T > t_{1-\alpha}$	$T < -t_{1-\alpha}$

Test Procedur	es:		
Hypotheses	$H_0: \mu_1 = \mu_2 \ vs$ $H_A: \mu_1 \neq \mu_2$	$\begin{array}{c} H_0: \mu_1 \leq \mu_2 \ vs \\ H_A: \mu_1 > \mu_2 \end{array}$	$\begin{array}{l} H_0: \mu_1 \ge \mu_2 \ vs \\ H_A: \mu_1 < \mu_2 \end{array}$
First Case		σ_1^2 and σ_2^2 are known	
Test Statistic (T.S.)	$Z = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \sim N(0, 1)$		
Rejection Region (R.R.) & Acceptance Region (A.R.) of H_0	$\begin{array}{c} 1 - \alpha \\ \alpha/2 \\ A.R. \text{ of } H_0 \\ \text{of } H_0 - Z_{1-\frac{\alpha}{2}} \\ \end{array} \begin{array}{c} \alpha/2 \\ R.R. \\ Z_{1-\frac{\alpha}{2}} \\ \end{array} \begin{array}{c} \alpha/2 \\ R.R. \\ T_{1-\frac{\alpha}{2}} \\ \end{array}$	$ \begin{array}{c} 1 - \alpha \\ A.R. \text{ of } H_0 \\ \hline Z_{1-\alpha} \\ \end{array} R.R. $	$\begin{array}{c} \alpha \\ A.R. \text{ of } H_0 \\ -Z_{1-\alpha} \end{array}$
Critical Value	$-Z_{1-\frac{\alpha}{2}}$ and $Z_{1-\frac{\alpha}{2}}$	$Z_{1-\alpha}$	$-Z_{1-\alpha}$
	We reject H_0 (and acce	ept H_A) at the significant	ce level α if:
Decision	$Z < -Z_{1-\frac{\alpha}{2}} \text{ or}$ $Z > Z_{1-\frac{\alpha}{2}}$	$Z > Z_{1-\alpha}$	$Z < -Z_{1-\alpha}$
Second Case	σ_1^2 and σ_2^2 a	re known but equal (σ_1^2	$=\sigma_2^2=\sigma^2$)
Test Statistic (T.S.)	$T = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{s_p^2}{n_1} + \frac{s_p^2}{n_2}}} \sim t(n_1 + n_2 - 2), df = v = n_1 + n_2 - 2$ $s_p^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2}$		
Rejection Region (R.R.) & Acceptance Region (A.R.) of H_0	$\begin{array}{c c} & 1-\alpha \\ & \alpha/2 \\ \text{R.R.} \\ \text{of } H_0 - t_{1-\frac{\alpha}{2}} \\ \end{array} \begin{array}{c} \alpha/2 \\ \text{R.R.} \\ t_{1-\frac{\alpha}{2}} \\ \end{array} \begin{array}{c} \alpha/2 \\ \text{R.R.} \\ \text{of } H_0 \end{array}$	$ \begin{array}{c} 1 - \alpha \\ A.R. \text{ of } H_0 \\ \hline t_{1-\alpha} \\ \end{array} R.R. \\ \text{of } H_0 \end{array} $	$\begin{array}{c} & 1 - \alpha \\ & \\ R.R. \\ of H_0 \\ -t_{1-\alpha} \end{array}$
Critical Value	$-t_{1-\frac{\alpha}{2}}$ and $t_{1-\frac{\alpha}{2}}$	t_{1-lpha}	$-t_{1-lpha}$
	We reject H_0 (and accept H_A) at the significance level α if:		
Decision	$T < -t_{1-\frac{\alpha}{2}} \text{ or}$ $T > t_{1-\frac{\alpha}{2}}$	$T > t_{1-\alpha}$	$T < -t_{1-\alpha}$

2. <u>Test Hypotheses for the Difference Between Two Population</u> <u>Means $(\mu_1 - \mu_2)$ (Independent Populations):</u>

3. <u>Confidence Interval and Test Hypotheses for the Difference</u> <u>Between Two Population Means $(\mu_1 - \mu_2 = \mu_D)$ </u> <u>(Dependent/Related Populations):</u>

The Procedure:

Calculate the Quantities	 The differences (D-observations): D_i = X_i - Y_i, i = 1,2,, n Sample Mean of the D-observations: D ^{Σⁿ_{i=1}D_i} ⁿ ^{Σⁿ_{i=1}D_i ⁿ ^{Σⁿ_{i=1}D_i ⁿ ^{(D)=D̄)² ²}}} 			
	• Sample Variance of the D-observations: $S_D^2 = \frac{Z_{l=1}(D_l - D)}{n-1}$			
	• Sample Standard Deviation of the D-observations: $S_D = \sqrt{S_D}$			
	Confidence Interval for $\mu_D = \mu_1 - \mu_2$			
$\begin{array}{c} 100(1-\alpha)\%\\ \text{Confidence}\\ \text{Interval for }\mu_D \end{array}$	$\overline{D} \pm t_{1-rac{lpha}{2}}rac{S_D}{\sqrt{n}}$, $df = v = n-1$			
Test Hypotheses for $\mu_D = \mu_1 - \mu_2$				
Hypotheses	$H_0: \mu_1 = \mu_2 \ vs$ $H_A: \mu_1 \neq \mu_2$ or $H_0: \mu_D = 0 \ vs$ $H_A: \mu_D \neq 0$	$H_0: \mu_1 \le \mu_2 vs$ $H_A: \mu_1 > \mu_2$ or $H_0: \mu_D \le 0 vs$ $H_4: \mu_D \ge 0$	$H_{0}: \mu_{1} \ge \mu_{2} vs$ $H_{A}: \mu_{1} < \mu_{2}$ or $H_{0}: \mu_{D} \ge 0 vs$ $H_{A}: \mu_{D} < 0$	
Test Statistic (T.S.)	$T = \frac{\overline{D}}{S_D / \sqrt{n}} \sim t(n-1), df = v = n-1$			
Rejection Region (R.R.) & Acceptance Region (A.R.) of H_0	$\begin{array}{c c} & 1-\alpha \\ \alpha/2 \\ R.R. \\ \text{of } H_0 - t_{1-\frac{\alpha}{2}} \\ \end{array} \begin{array}{c} \alpha/2 \\ R.R. \\ t_{1-\frac{\alpha}{2}} \\ \end{array} \begin{array}{c} \alpha/2 \\ R.R. \\ \text{of } H_0 \end{array}$	$ \begin{array}{c} 1 - \alpha \\ A.R. \text{ of } H_0 \\ t_{1-\alpha} \\ \end{array} R.R. \\ \text{of } H_0 \end{array} $	$\begin{array}{c} & & & \\ \alpha & & \\ R.R. \\ of H_0 & -t_{1-\alpha} \end{array}$	
Critical Value	$-t_{1-\frac{\alpha}{2}}$ and $t_{1-\frac{\alpha}{2}}$	t_{1-lpha}	$-t_{1-\alpha}$	
	We reject H_0 (and accept H_A) at the significance level α if:			
Decision	$T < -t_{1-\frac{\alpha}{2}} \text{ or} $ $T > t_{1-\frac{\alpha}{2}}$	$T > t_{1-\alpha}$	$T < -t_{1-\alpha}$	

4. <u>Test Hypotheses for the Population Proportion (*p*):</u>

Test Procedure:

Hypotheses	$H_0: p = p_0 \ vs$ $H_A: p \neq p_0$	$ \begin{array}{l} H_0: p \leq p_0 \ vs \\ H_A: p > p \end{array} $	$ \begin{array}{l} H_0: p \geq p_0 \ vs \\ H_A: p < p_0 \end{array} $
Test Statistic (T.S.)	$Z = \frac{1}{\sqrt{p}}$	$\frac{\hat{p} - p_0}{\frac{p_0(1 - p_0)}{n}} \sim N(0, 1) ,$	$\hat{p} = \frac{X}{n}$
Rejection Region (R.R.) & Acceptance Region (A.R.) of H_0	$\begin{array}{c c} & 1-\alpha \\ \alpha/2 \\ R.R. \\ of H_0 - Z_{1-\frac{\alpha}{2}} \\ \end{array} \begin{array}{c} \alpha/2 \\ R.R. \\ A.R. of H_0 \\ Z_{1-\frac{\alpha}{2}} \\ \end{array} \begin{array}{c} \alpha/2 \\ R.R. \\ of H_0 \\ \end{array}$	$ \begin{array}{c} 1 - \alpha \\ A.R. \text{ of } H_0 \\ \hline Z_{1-\alpha} \\ \end{array} R.R. $	$\begin{array}{c} \alpha \\ A.R. \text{ of } H_0 \\ -Z_{1-\alpha} \end{array}$
Critical Value	$-Z_{1-\frac{\alpha}{2}}$ and $Z_{1-\frac{\alpha}{2}}$	$Z_{1-\alpha}$	$-Z_{1-\alpha}$
	We reject H_0 (and accept H_A) at the significance level α if:		
Decision	$Z < -Z_{1-\frac{\alpha}{2}} \text{ or}$ $Z > Z_{1-\frac{\alpha}{2}}$	$Z > Z_{1-\alpha}$	$Z < -Z_{1-\alpha}$

5. <u>Test Hypotheses for the Difference Between Two Population</u> <u>Proportions $(p_1 - p_2)$:</u>

Test Procedure:

Hypotheses	$ \begin{array}{l} H_0: p_1 = p_2 \ vs \\ H_A: p_1 \neq p_2 \end{array} $	$H_0: p_1 \le p_2 \ vs \ H_A: p_1 > p_2$	$\begin{array}{l} H_0: p_1 \geq p_2 \ vs \\ H_A: p_1 < p_2 \end{array}$
Test Statistic (T.S.)	$Z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\frac{\bar{p}(1 - \bar{p})}{n_1} + \frac{\bar{p}(1 - \bar{p})}{n_2}}} \sim N(0, 1)$ $\hat{p}_1 = \frac{X_1}{n_1}, \ \hat{p}_2 = \frac{X_2}{n_2}, \ \bar{p} = \frac{X_1}{n_1} + \frac{X_2}{n_2}$		
Rejection Region (R.R.) & Acceptance Region (A.R.) of H_0	$\begin{array}{c c} & 1-\alpha \\ \alpha/2 \\ R.R. \\ of H_0 - Z_{1-\frac{\alpha}{2}} \\ \end{array} \begin{array}{c} \alpha/2 \\ R.R. \\ Z_{1-\frac{\alpha}{2}} \\ \end{array} \begin{array}{c} \alpha/2 \\ R.R. \\ \sigma f H_0 \\ \end{array}$	$1 - \alpha$ A.R. of H_0 $Z_{1-\alpha}$ R.R. of H_0	$\begin{array}{c} & & & \\ \alpha & & \\ \alpha & & \\ R.R. \\ of H_0 & -Z_{1-\alpha} \end{array}$
Critical Value	$-Z_{1-\frac{\alpha}{2}}$ and $Z_{1-\frac{\alpha}{2}}$	$Z_{1-\alpha}$	$-Z_{1-\alpha}$
	We reject H_0 (and accept H_A) at the significance level α if:		
Decision	$Z < -Z_{1-\frac{\alpha}{2}} \text{or} \\ Z > Z_{1-\frac{\alpha}{2}}$	$Z > Z_{1-\alpha}$	$Z < -Z_{1-\alpha}$