CONVECTIVE MASS TRANSFER

In most practical applications, convective mass transfer is required in order to obtain higher rates of mass transfer. This needs bulk motion of the fluid preferably in turbulent flow regime.

For example, fluid is flowing inside a pipe/tube, where the internal wall is coated with a solid that dissolves in the fluid. If the flow is in the laminar regime, the transport of the solute will take place by the molecular diffusion. On the other hand, if the flow is in the turbulent flow regime, the solute mass transport will occur by the random motion of eddies (turbulent diffusion). The turbulent diffusion is much faster/higher than the molecular diffusion.

Videos of Turbulence

https://www.youtube.com/watch?v=nl75BGg9qdA

https://www.youtube.com/watch?v=LylMRupw4iE

Types of Mass-Transfer Coefficients

1. Definition of mass-transfer coefficients

For the mass transport involving the molecular and the turbulent diffusion,

$$J^{\star}_{Az} = -(D_{AB} + \varepsilon_M)\frac{d(c_A)}{dz}$$

Integrating using

$$J^{\star}_{A1} = -\frac{(D_{AB} + \overline{\varepsilon_M})}{(z_1 - z_2)}(c_{A1} - c_{A2})$$

$J^{\star}_{A1}$ is based on the surface area A_1. Also, $(z_1 - z_2)$ is not known. In a simple manner, one can write

$$J^{\star}_{A1} = \left[-\frac{(D_{AB} + \overline{\varepsilon_M})}{(z_1 - z_2)}\right](c_{A1} - c_{A2}) = k'_c(c_{A1} - c_{A2})$$

Here, $J^{\star}_{A1}$ is the flux of A from the surface A_1.

2. Mass-transfer coefficient for equi-molar counterdiffusion

$$N_A = -c(D_{AB} + \varepsilon_M)\frac{d(x_A)}{dz} + x_A(N_A + N_B)$$

For equimolar counterdiffusion, $N_A = -N_B$,

$$N_A = -c(D_{AB} + \varepsilon_M)\frac{d(x_A)}{dz}$$

Integrating the above equation gives,

$$N_A = \left[-\frac{(D_{AB} + \overline{\varepsilon_M})}{(z_1 - z_2)}\right](c_{A1} - c_{A2}) = k'_c(c_{A1} - c_{A2})$$

Gases:

$$N_A = k'_c(c_{A1} - c_{A2}) = k'_G(p_{A1} - p_{A2}) = k'_y(y_{A1} - y_{A2})$$

Liquids:

$$N_A = k'_c(c_{A1} - c_{A2}) = k'_L(c_{A1} - c_{A2}) = k'_x(x_{A1} - x_{A2})$$

Relationship between mass transfer coefficients

$$N_A = k'_y(y_{A1} - y_{A2}) = k'_y\left(\frac{c_{A1}}{c} - \frac{c_{A2}}{c}\right) = \frac{k'_y}{c}(c_{A1} - c_{A2}) = k'_c(c_{A1} - c_{A2})$$

3. Mass-transfer coefficient for A diffusing through stagnant, nondiffusing B

$$N_A = \left[-\frac{(D_{AB} + \overline{\varepsilon_M})}{(z_1 - z_2)}\right]\frac{c_{A1} - c_{A2}}{x_{BM}} = k'_c\frac{c_{A1} - c_{A2}}{x_{BM}} = \frac{\boldsymbol{k'_c}}{\boldsymbol{x_{BM}}}(c_{A1} - c_{A2}) = \boldsymbol{k_c}(c_{A1} - c_{A2})$$

Gases:

$$N_A = k_c(c_{A1} - c_{A2}) = k_G(p_{A1} - p_{A2}) = k_y(y_{A1} - y_{A2})$$

Liquids:

$$N_A = k_c(c_{A1} - c_{A2}) = k_L(c_{A1} - c_{A2}) = k_x(x_{A1} - x_{A2})$$

TABLE 7.2-1. *Flux Equations and Mass-Transfer Coefficients*

Flux equations for equimolar counterdiffusion

Gases: $N_A = k'_c(c_{A1} - c_{A2}) = k'_G(p_{A1} - p_{A2}) = k'_y(y_{A1} - y_{A2})$

Liquids: $N_A = k'_c(c_{A1} - c_{A2}) = k'_L(c_{A1} - c_{A2}) = k'_x(x_{A1} - x_{A2})$

Flux equations for A diffusing through stagnant, nondiffusing B

Gases: $N_A = k_c(c_{A1} - c_{A2}) = k_G(p_{A1} - p_{A2}) = k_y(y_{A1} - y_{A2})$

Liquids: $N_A = k_c(c_{A1} - c_{A2}) = k_L(c_{A1} - c_{A2}) = k_x(x_{A1} - x_{A2})$

Conversions between mass-transfer coefficients

Gases:

$$k'_c c = k'_c \frac{P}{RT} = k_c \frac{p_{BM}}{RT} = k'_G P = k_G p_{BM} = k_y y_{BM} = k'_y = k_c y_{BM} c = k_G y_{BM} P$$

Liquids:

$$k'_c c = k'_L c = k_L x_{BM} c = k'_L \rho/M = k'_x = k_x x_{BM}$$

(where ρ is density of liquid and M is molecular weight)

Units of mass-transfer coefficients

	SI Units	*Cgs Units*	*English Units*
k_c, k_L, k'_c, k'_L	m/s	cm/s	ft/h
k_x, k_y, k'_x, k'_y	$\frac{\text{kg mol}}{\text{s} \cdot \text{m}^2 \cdot \text{mol frac}}$	$\frac{\text{g mol}}{\text{s} \cdot \text{cm}^2 \cdot \text{mol frac}}$	$\frac{\text{lb mol}}{\text{h} \cdot \text{ft}^2 \cdot \text{mol frac}}$
k_G, k'_G	$\frac{\text{kg mol}}{\text{s} \cdot \text{m}^2 \cdot \text{Pa}}$ (preferred) $\quad \frac{\text{kg mol}}{\text{s} \cdot \text{m}^2 \cdot \text{atm}}$	$\frac{\text{g mol}}{\text{s} \cdot \text{cm}^2 \cdot \text{atm}}$	$\frac{\text{lb mol}}{\text{h} \cdot \text{ft}^2 \cdot \text{atm}}$

EXAMPLE 7.2-1. Vaporizing A and Convective Mass Transfer

A large volume of pure gas B at 2 atm pressure is flowing over a surface from which pure A is vaporizing. The liquid A completely wets the surface, which is a blotting paper. Hence, the partial pressure of A at the surface is the vapor pressure of A at 298 K, which is 0.20 atm. The k'_y has been estimated to be 6.78×10^{-5} kg mol/s · m² · mol frac. Calculate N_A, the vaporization rate, and also the value of k_y and k_G.

Solution: This is the case of A diffusing through B, where the flux of B normal to the surface is zero, since B is not soluble in liquid A. $p_{A1} = 0.20$ atm and $p_{A2} = 0$ in the pure gas B. Also, $y_{A1} = p_{A1}/P = 0.20/2.0 = 0.10$ and $y_{A2} = 0$. We can use Eq. (7.2-12) with mole fractions.

$$N_A = k_y(y_{A1} - y_{A2}) \qquad \textbf{(7.2-12)}$$

However, we have a value for k'_y which is related to k_y from Table 7.2-1 by

$$k_y\, y_{BM} = k'_y \qquad \textbf{(7.2-16)}$$

The term y_{BM} is similar to x_{BM} and is, from Eq. (7.2-11),

$$y_{BM} = \frac{y_{B2} - y_{B1}}{\ln (y_{B2}/y_{B1})} \qquad \textbf{(7.2-11)}$$

$$y_{B1} = 1 - y_{A1} = 1 - 0.10 = 0.90 \qquad y_{B2} = 1 - y_{A1} = 1 - 0 = 1.0$$

Substituting into Eq. (7.2-11),

$$y_{BM} = \frac{1.0 - 0.90}{\ln (1.0/0.90)} = 0.95$$

Then, from Eq. (7.2-16),

$$k_y = \frac{k'_y}{y_{BM}} = \frac{6.78 \times 10^{-5}}{0.95} = 7.138 \times 10^{-5} \text{ kg mol/s} \cdot \text{m}^2 \cdot \text{mol frac}$$

Also, from Table 7.2-1,

$$k_G\, y_{BM}\, P = k_y\, y_{BM} \qquad \textbf{(7.2-17)}$$

Hence, solving for k_G and substituting knowns,

$$k_G = \frac{k_y}{P} = \frac{7.138 \times 10^{-5}}{2 \times 1.01325 \times 10^5 \text{ Pa}} = 3.522 \times 10^{-10} \text{ kg mol/s} \cdot \text{m}^2 \cdot \text{Pa}$$

$$k_G = \frac{k_y}{P} = \frac{7.138 \times 10^{-5}}{2.0 \text{ atm}} = 3.569 \times 10^{-5} \text{ kg mol/s} \cdot \text{m}^2 \cdot \text{atm}$$

For the flux using Eq. (7.2-12),

$$N_A = k_y(y_{A1} - y_{A2}) = 7.138 \times 10^{-5}(0.10 - 0) = 7.138 \times 10^{-6} \text{ kg mol/s} \cdot \text{m}^2$$

Also,

$$p_{A1} = 0.20 \text{ atm} = 0.20(1.01325 \times 10^5) = 2.026 \times 10^4 \text{ Pa}$$

Using Eq. (7.2-12) again,

$$N_A = k_G(p_{A1} - p_{A2}) = 3.522 \times 10^{-10}(2.026 \times 10^4 - 0)$$
$$= 7.138 \times 10^{-6} \text{ kg mol/s} \cdot \text{m}^2$$
$$N_A = k_G(p_{A1} - p_{A2}) = 3.569 \times 10^{-5}(0.20 - 0)$$
$$= 7.138 \times 10^{-6} \text{ kg mol/s} \cdot \text{m}^2$$

Note that in this case, since the concentrations were dilute, y_{BM} is close to 1.0 and k_y and k'_y differ very little.

PREDICTION OF MASS TRANSFER COEFFICIENTS

Role of Dimensionless Numbers

$$N_{Re} = \frac{Lv\rho}{\mu} = \frac{Inertial\ forces}{Viscous\ forces}$$

$$N_{Sc} = \frac{(\mu/\rho)}{D_{AB}} = \frac{Momentum\ diffusivity}{Mass\ diffusivity}$$

$$N_{Sh} = k'_c \frac{L}{D_{AB}} = \frac{Convective\ mass\ trasport}{Molecular\ mass\ trasport};$$

$$N_{St} = \frac{k'_c}{v} = \frac{k'_y}{cv}$$

$$J_D = \frac{k'_c}{v}(N_{Sc})^{2/3} = \frac{N_{Sh}}{N_{Re}N_{Sc}^{1/3}}$$

$$J_H = \frac{h}{C_P G}(N_{Pr})^{2/3}$$

Reading/Discussion Assignment: Discuss the relevance various dimensionless number

Momentum, Heat and Mass Transfer Analogy

Reynolds/Prandtl/Von Karman/Chilton and Colburn J-factor analogy

$$\tau_{zx} = -\left(\frac{\mu}{\rho} + \varepsilon_t\right)\frac{d(\rho v_x)}{dz}$$

$$\frac{q_z}{A} = -(\alpha + \alpha_t)\frac{d(\rho C_P T)}{dz}$$

$$\left(\frac{\tau_{zx}}{\frac{q_z}{A}}\right) C_P dT = dv$$

In the turbulent flow, **assume** that the heat flux is analogous to the momentum flux, their ratio must be constant along the radial position. Integrating between interface condition $(v = 0, T = T_i)$ to bulk condition $(v = v_{av}, T = T)$

$$\left(\frac{\tau_s}{\frac{q}{A}}\right) C_P (T - T_i) = v_{av} - 0$$

$$\left(\frac{\frac{f}{2}\rho v_{av}^2}{h(T - T_i)}\right) C_P (T - T_i) = v_{av}$$

$$\frac{f}{2} = \frac{h}{C_P \rho v_{av}} = \frac{h}{C_P G}$$

Similarly, mass transfer and the momentum transfer can be assumed to be analogous, and using the expression

$$J_{A1}^{\star} = k_c'(c_{A1} - c_{A2})$$

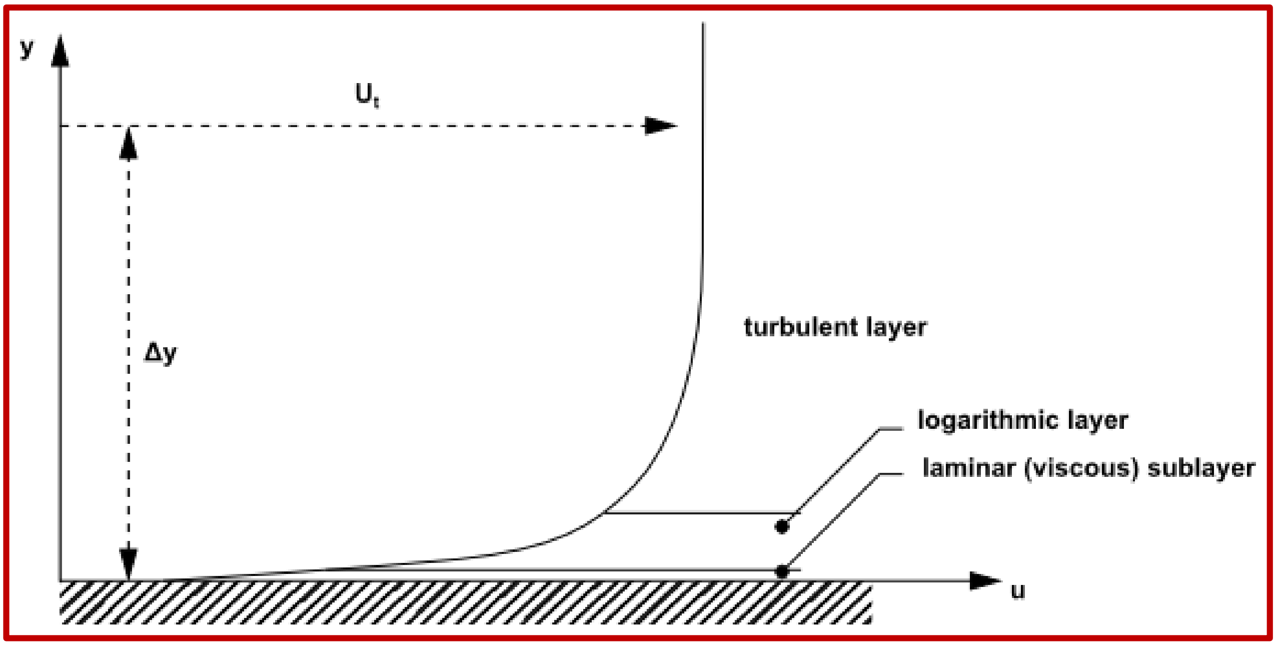
to give

$$\frac{f}{2} = \frac{h}{C_P G} = \frac{k'_c}{v_{av}}$$

3. Other analogies. The Reynolds analogy assumes that the turbulent diffusivities ε_t, α_t, and ε_M are all equal and that the molecular diffusivities μ/ρ, α, and D_{AB} are negligible compared to the turbulent diffusivities. When the Prandtl number $(\mu/\rho)/\alpha$ is 1.0, then $\mu/\rho = \alpha$; also, for $N_{Sc} = 1.0$, $\mu/\rho = D_{AB}$. Then, $(\mu/\rho + \varepsilon_t) = (\alpha + \alpha_t) = (D_{AB} + \varepsilon_M)$ and the Reynolds analogy can be obtained with the molecular terms present. However, the analogy breaks down when the viscous sublayer becomes important since the eddy diffusivities diminish to zero and the molecular diffusivities become important.

Prandtl modified the Reynolds analogy by writing the regular molecular diffusion equation for the viscous sublayer and a Reynolds-analogy equation for the turbulent core region. Then since these processes are in series, these equations were combined to produce an overall equation (G1). The results also are poor for fluids where the Prandtl and Schmidt numbers differ from 1.0.

Von Kármán further modified the Prandtl analogy by considering the buffer region in addition to the viscous sublayer and the turbulent core. These three regions are shown in the universal velocity profile in Fig. 3.10-4. Again, an equation is written for molecular diffusion in the viscous sublayer using only the molecular diffusivity and a Reynolds analogy equation for the turbulent core. Both the molecular and eddy diffusivity are used in an equation for the buffer layer, where the velocity in this layer is used to obtain an equation for the eddy diffusivity. These three equations are then combined to give the von Kármán analogy. Since then, numerous other analogies have appeared (P1, S4).



4. Chilton and Colburn J-factor analogy. The most successful and most widely used analogy is the Chilton and Colburn J-factor analogy (C2). This analogy is based on experimental data for gases and liquids in both the laminar and turbulent flow regions and is written as follows:

$$\frac{f}{2} = J_H = \frac{h}{c_p G}(N_{Pr})^{2/3} = J_D = \frac{k_c'}{v_{av}}(N_{Sc})^{2/3} \quad (7.3\text{-}13)$$

Although this is an equation based on experimental data for both laminar and turbulent flow, it can be shown to satisfy the exact solution derived from laminar flow over a flat plate in Sections 3.10 and 5.7.

Equation (7.3-13) has been shown to be quite useful in correlating momentum, heat, and mass transfer data. It permits the prediction of an unknown transfer coefficient when one of the other coefficients is known. In momentum transfer the friction factor is obtained for the total drag or friction loss, which includes form drag or momentum losses due to blunt objects and also skin friction. For flow past a flat plate or in a pipe where no form drag is present, $f/2 = J_H = J_D$. When form drag is present, such as in flow in packed beds or past other blunt objects, $f/2$ is greater than J_H or J_D and $J_H \cong J_D$.

Mass transfer Coefficients in Laminar Flow

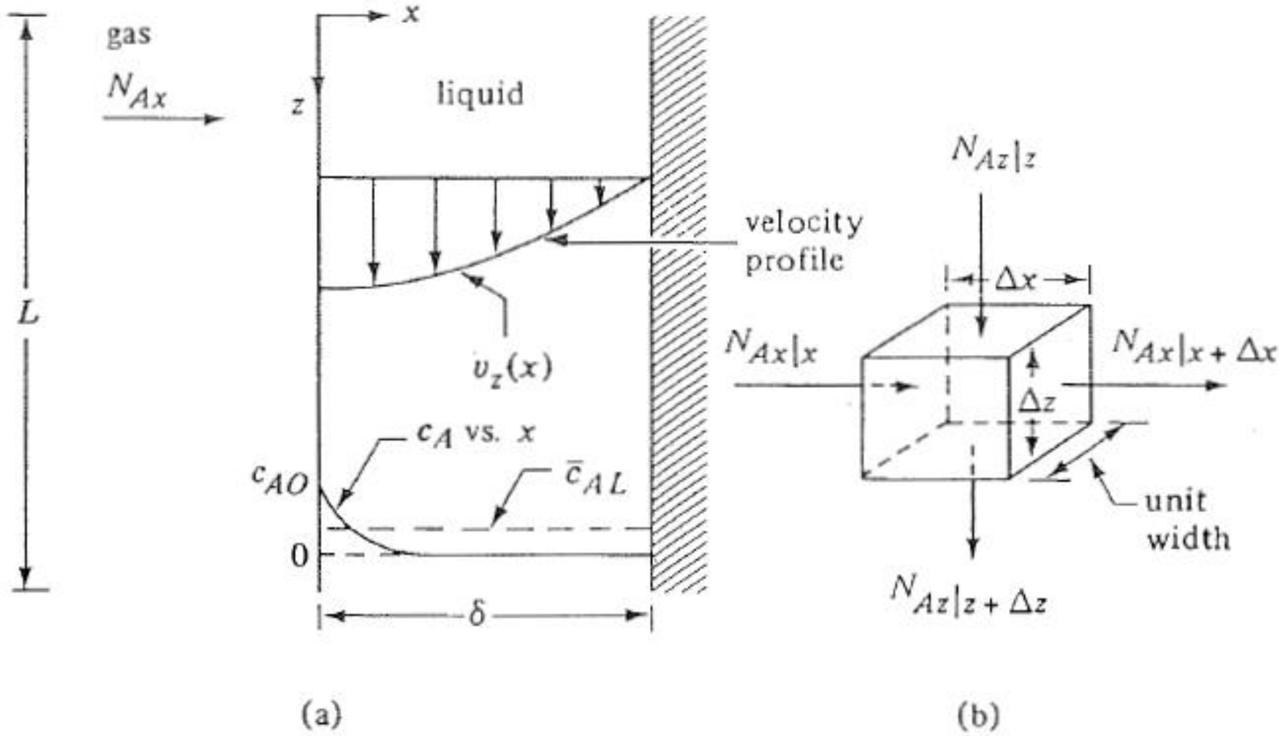


FIGURE 7.3-1. *Diffusion of solute A in a laminar falling film: (a) velocity profile and concentration profile, (b) small element for mass balance.*

$$\frac{C_A}{C_{A0}} = efrc\left(\frac{x}{\sqrt{4D_{AB}z/v_{max}}}\right)$$

$$[N_{Ax}(z)]_{x=0} = \left[-D_{AB}\frac{\partial c_A}{\partial x}\right]_{x=0} = c_{A0}\sqrt{\frac{D_{AB}v_{max}}{\pi z}}$$

Therefore, the total moles of A transferred per second over the entire length L and width W of the laminar falling film will be given by (after integration)

$$N_A(LW) = (LW)c_{A0}\sqrt{\frac{4D_{AB}v_{max}}{\pi L}}$$

Mass transfer coefficients for flow in pipes and tubes

- Laminar flow
- Turbulent flow

Mass transfer coefficients for flow outside solid surface

- Flow parallel to flat plates
- Flow past single spheres
- Packed beds of solids

Mass Transfer for Flow Inside Pipes

Mass transfer for laminar flow inside pipes (N_{Re} < 2,100)

Experimental data obtained for mass transfer from the walls for gases are shown in Fig. 7.3-2. In the figure, the y-axis is

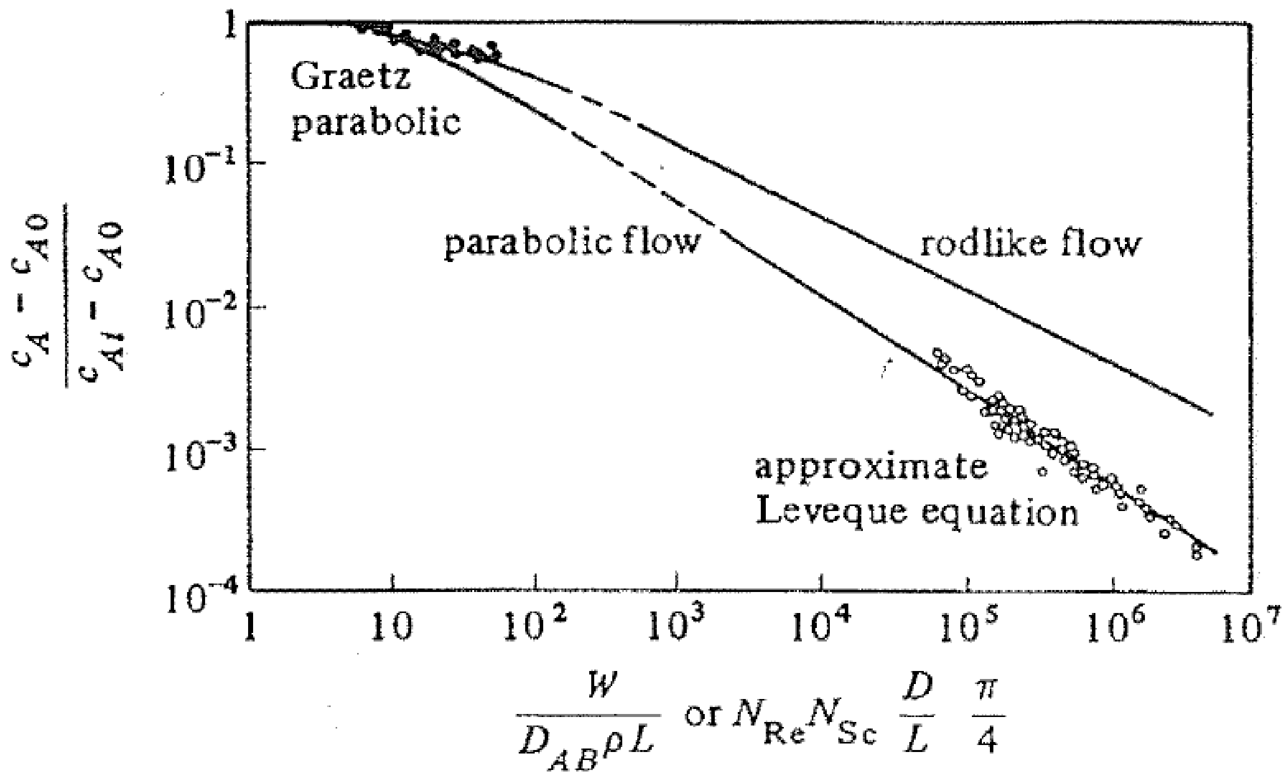
$$\frac{(C_A - C_{A0})}{(C_{Ai} - C_{A0})}$$

where C_A is the exit concentration, C_{A0} inlet concentration, and C_{Ai} is concentration at the interface between the wall and the gas. In the figure, x-axis is

$$\frac{W}{D_{AB}\rho L} = N_{Re} N_{Sc} \frac{D}{L}\frac{\pi}{4}$$

where W is flow in kg/s and L is length of mass-transfer section in m. For liquids that have small values of D_{AB}, data follow the parabolic flow line, which is as follows for $(W/D_{AB}\rho L) > 400$

$$\frac{(C_A - C_{A0})}{(C_{Ai} - C_{A0})} = 5.5\left(\frac{W}{D_{AB}\rho L}\right)^{-2/3}$$



Mass transfer for G/L turbulent flow inside pipes

$$N_{Sh} = k_c' \frac{D}{D_{AB}} = 0.023 \left(\frac{Dv\rho}{\mu}\right)^{0.83} \left(\frac{(\mu/\rho)}{D_{AB}}\right)^{0.33} = 0.023(N_{Re})^{0.83}(N_{Sc})^{0.33}$$

EXAMPLE 7.3-1:
A tube is coated on the inside with naphthalene and has an inside diameter of 20 mm and a length of 1.10 m. Air at **318 K** and an average pressure of 101.3 kPa flows through this pipe at a velocity of 0.80 m/s. Assuming that the absolute pressure remains essentially constant, calculate the concentration of naphthalene in the exit air (See Example 6.2-4 for physical properties).

Solution: From Example 6.2-4, $D_{AB} = 6.92 \times 10^{-6}$ m^2/s and the vapor pressure $p_{Ai} = 74.0$ Pa or $c_{Ai} = p_{Ai}/RT = 74.0/(8314.3 \times 318) = 2.799 \times 10^{-5}$ kg mol/m^3. For air from Appendix A.3, $\mu = 1.932 \times 10^{-5}$ Pa·s, $\rho = 1.114$ kg/m^3. The Schmidt number is

$$N_{Sc} = \frac{\mu}{\rho D_{AB}} = \frac{1.932 \times 10^{-5}}{1.114 \times 6.92 \times 10^{-6}} = 2.506$$

The Reynolds number is

$$N_{Re} = \frac{Dv\rho}{\mu} = \frac{0.020(0.80)(1.114)}{1.932 \times 10^{-5}} = 922.6$$

Hence, the flow is laminar. Then,

$$N_{Re}N_{Sc}\frac{D}{L}\frac{\pi}{4} = 922.6(2.506)\frac{0.020}{1.10}\frac{\pi}{4} = 33.02$$

Using Fig. 7.3-2 and the rodlike flow line, $(c_A - c_{A0})/(c_{Ai} - c_{A0}) = 0.55$. Also, c_{A0}(inlet) $= 0$. Then, $(c_A - 0)/(2.799 \times 10^{-5} - 0) = 0.55$. Solving, c_A(exit concentration) $= 1.539 \times 10^{-5}$ kg mol/m^3.

Question 3 (Test 2 Winter 2016 2017):

Air at 318 K and an average pressure of 101.3 kPa flows through a naphthalene tube that has an inside diameter of 20 mm and length L = 1.5 m, at a bulk velocity of 2.0 m/s. The D_{AB} of naphthalene in air at 318 K is $6.92 \times 10^{-6}\, m^2/s$ and the vapor pressure is 74 Pa. Note that the flow is **turbulent** in this case. Assuming that the absolute pressure remains essentially constant, determine

- N_{Sh} and the mass transfer coefficient (write appropriate units)
- Concentration of naphthalene concentration in the exiting gas stream in kgmol/m^3. (Since the **solution is dilute**, one can use arithmetic mean instead of log mean for simplicity of calculation)

Data: At T = 318 K, $\rho = 1.1\, kg/m^3$; $\mu = 1.93 \times 10^{-5} Pa \cdot s$

Solution Question 3 (Max. 30)

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$$N_{Re} = \left(\frac{Dv\rho}{\mu}\right) = \left(\frac{20 \times 10^{-3} \times 2 \times 1.1}{1.93 \times 10^{-5}}\right) = 2277.43$$

$$N_{Sc} = \frac{(\mu/\rho)}{D_{AB}} = \frac{(1.93 \times 10^{-5}/1.1)}{6.92 \times 10^{-6}} = 2.54$$

For turbulent flow:

$$N_{Sh} = 0.023(N_{Re})^{0.83}(N_{Sc})^{1/3} = 0.023(2274.43)^{0.83}(2.54)^{0.33} = 19.14$$

$$N_{Sh} = k_C' \frac{D_C}{D_{AB}}$$

$$k_C' = N_{Sh}\frac{D_{AB}}{D_P} = 19.14 \times \left(\frac{6.92 \times 10^{-6}}{20 \times 10^{-3}}\right) = 6.62 \times 10^{-3}\frac{m}{s}$$

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Mass transfer area of tube: $A = \pi D_c L = \pi \times 20 \times 10^{-3} \times 1.5 = 0.00942\, m^2$

Volumetric flow rate: $V = v \times \frac{\pi}{4} D_c^2 = 2 \times \frac{\pi}{4}(20 \times 10^{-3})^2 = 6.28 \times 10^{-4}\, \frac{m^3}{s}$

Mass transfer from the walls of the tube (Problem allows simplification using arithmetic mean):

$$N_A A = A k_C\left(C_{Ai} - \frac{C_{A1} + C_{A2}}{2}\right) = (0.00942)(6.62 \times 10^{-3})\left(\frac{74}{8314(273+45)} - \frac{0 + C_{A2}}{2}\right)$$

But, from material balance: $N_A A = V(C_{A2} - C_{A1}) = 6.28 \times 10^{-4}(C_{A2} - 0)$

Therefore, solution is: $C_{A2} = 1.86 \times 10^{-5}\frac{kg\,mol}{m^3}$

Using log means for driving force: $C_{A2} = 1.77 \times 10^{-5}\frac{kg\,mol}{m^3}$

Question 4 (35 pts, Test1 Fall 2019 2020:

Pure water at 26.l°C is flowing at a velocity of 0.10 m/s in a tube having an inside diameter of 6.35 mm. The tube length is 5 m having the walls coated with benzoic acid. Assuming that the velocity profile is fully developed, calculate the average concentration of benzoic acid at the outlet.

Assume that the D_{AB} of benzoic acid in water 26.l°C is 1.245×10^{-9} m^2/s. Use $\mu = 8.71\times10^{-4}$ Pa·s, $\rho = 996\ (kg/m^3)$. The solubility of benzoic acid in water is $0.02948\ kg\ mol/m^3$

- Determine the Reynolds number, N_{Re}, Schmidt number, N_{Sc}, and $N_{Re}N_{Sc}\frac{D}{L}\frac{\pi}{4}$
- Determine $\frac{(C_A - C_{A0})}{(C_{Ai} - C_{A0})}$ using appropriate correlation and exit concentration, C_A, in $kg\ mol/m^3$
- Determine $(C_{Ai} - C_A)_{LM}$
- Determine the amount of the benzoic acid removed from the tube in kg/s, if the molecular weight of the benzoic acid is 122.12 kg/kg mol
- Determine the mass transfer coefficient, $k'_C, m/s$

$$N_{Re} = \left(\frac{Dv\rho}{\mu}\right) = \left(\frac{6.35\times10^{-3}\times0.10\times996}{0.871\times10^{-3}}\right) = 726.3$$ **10**

$$N_{Sc} = \frac{(\mu/\rho)}{D_{AB}} = 697.37$$

$$N_{Re}N_{Sc}\frac{D}{L}\frac{\pi}{4} = \left(\frac{W}{D_{AB}\rho L}\right) = 505.1$$

$$\frac{(C_A - C_{A0})}{(C_{Ai} - C_{A0})} = 5.5\left(\frac{W}{D_{AB}\rho L}\right)^{-2/3} = 5.5(505.1)^{-2/3} = 0.0867$$ **10**

$$\frac{(C_A - 0)}{(0.02948 - 0)} = 0.0867$$

$$C_A = 2.556\times10^{-3}\ kg\ mol/m^3$$

$$(C_{Ai} - C_A)_{LM} = \frac{(C_{Ai} - C_{A0}) - (C_{Ai} - C_A)}{\ln[(C_{Ai} - C_{A0})/(C_{Ai} - C_A)]}$$ **5**

$$= \frac{(0.02948 - 0) - (0.02948 - 0.002556)}{\ln[(0.02948 - 0)/(0.02948 - 0.002556)]} = \mathbf{0.0282\ kg\ mol/m^3}$$

Cross-sectional area of the tube; $A_C = \frac{\pi}{4}(D_C)^2 = \frac{\pi}{4}(0.00635)^2 = 3.17\times10^{-5}\ m^2$ **5**

Volumetric flow rate in m^3/s; $V = v\times A_C = 0.1\times3.17\times10^{-5} = 3.17\times10^{-6}\ \frac{m^3}{s}$

Using the material balance,

$$V(C_A - C_{A0}) = 3.17\times10^{-6}(2.556\times10^{-3} - 0) = 8.1\times10^{-9}\frac{kg\ mol}{s}$$

$$V(C_A - C_{A0}) = 8.1\times10^{-9}\frac{kg\ mol}{s}\times122.12\ \frac{kg}{kg\ mol} = \mathbf{0.989\times10^{-6}\ kg\ /s}$$

Since, $N_A A = V(C_A - C_{A0})$. **5**

Therefore, $N_A A = Ak_C(C_{Ai} - C_A)_{LM} = 8.1\times10^{-9}\frac{kg\ mol}{s}$

Mass transfer area of tube: $A = \pi D_c L = \pi\times6.35\times10^{-3}\times5 = 0.09975\ m^2$

Equating, $k_C\times0.09975\times0.0282 = 8.1\times10^{-9}$

$$\mathbf{k_C = 2.87\times10^{-6}\ m/s}$$

Question:

A tube is coated on the inside with benzoic acid and has the inside diameter of 20 mm and length L = 5 m. Pure water at 26.1 °C flows through the tube at a velocity of 0.20 m/s. At 26.1 , the D_{AB} of benzoic acid is $1.254 \times 10^{-9}\, m^2/s$ and the solubility of benzoic acid in water is $2.948 \times 10^{-2}\, kg\ mol/m^3$. determine

- N_{Sh} and the mass transfer coefficient (write appropriate units)
- Concentration of benzoic acid concentration in the exiting gas stream in kgmol/m^3. (Since the **solution is dilute**, one can use arithmetic mean instead of log mean for simplicity of calculations)

(Data: At 26.1 °C for water, Density = $996.7\, kg/m^3$, Viscosity = $0.8718 \times 10^{-3} Pa.s$

MT Inside Tube Coated with Benzoic Acid	
Pressure	101300
T	**299.1**
Meu	8.72E-04
Rho	996.7
Dc	0.02
L	5
Ac	0.0003142
Tube Inside Area, A	3.142E-01
Uo	**0.2**
V=Uo*pi*(Dc^2)	6.283E-05
D_AB (given)	1.25E-09
N_Sc	702.56
N_Re	4,573
C_Ai	**2.948E-02**
C_A1	0
C_A2	1.94E-03
C_A_AM	2.851E-02
N_Sh	218.33
Kc'	**1.3591E-05**
A*Kc*C_A_AM	1.21734E-07
V*(C_A2-C_A1)	1.21731E-07
(n_Ax)-(n_Ay)	2.94041E-06

Mass Transfer for Flow Outside Solids Surfaces

Mass transfer in flow parallel to flat plates

Example:

- drying of inorganic and biological materials
- evaporation of solvents from paints
- plates in wind tunnels
- flow channels in chemical process equipment

When the fluid flows past a plate in a free stream in an open space the boundary layer is not fully developed. For gases or evaporation of liquids in the gas phase and for the laminar region of $N_{Re,L} = Lv\rho/\mu$ less than 15 000, the data can be represented within $\pm 25\%$ by the equation (S4)

$$J_D = 0.664 N_{Re,L}^{-0.5} \tag{7.3-26}$$

Writing Eq. (7.3-26) in terms of the Sherwood number N_{Sh},

$$\frac{k'_c L}{D_{AB}} = N_{Sh} = 0.664 N_{Re,L}^{0.5} N_{Sc}^{1/3} \tag{7.3-27}$$

where L is the length of plate in the direction of flow. Also, $J_D = J_H = f/2$ for this geometry. For gases and $N_{Re,L}$ of 15 000–300 000, the data are represented within $\pm 30\%$ by $J_D = J_H = f/2$ as

$$J_D = 0.036 N_{Re,L}^{-0.2} \tag{7.3-28}$$

Experimental data for liquids are correlated within about $\pm 40\%$ by the following for a $N_{Re,L}$ of 600–50 000 (L2):

$$J_D = 0.99 N_{Re,L}^{-0.5} \tag{7.3-29}$$

EXAMPLE 7.3-2: A large volume of pure water at 26.1°C is flowing parallel to a flat plate of solid benzoic acid, where L = 0.244 m in the direction of flow. The water velocity is 0.061 m/s. The solubility of benzoic acid in water is 0.02948 kg mol/m^3. The diffusivity of benzoic acid is 1.245 × 10^{-9} m^2/s. Calculate the mass-transfer coefficient k_L and the flux (N_A).

Solution: Since the solution is quite dilute, the physical properties of water at 26.1°C from Appendix A.2 can be used.

$$\mu = 8.71 \times 10^{-4}\ \text{Pa}\cdot\text{s}$$

$$\rho = 996\ \text{kg/m}^3$$

$$D_{AB} = 1.245 \times 10^{-9}\ \text{m}^2/\text{s}$$

The Schmidt number is

$$N_{Sc} = \frac{8.71 \times 10^{-4}}{996(1.245 \times 10^{-9})} = 702$$

The Reynolds number is

$$N_{Re,L} = \frac{Lv\rho}{\mu} = \frac{0.244(0.0610)(996)}{8.71 \times 10^{-4}} = 1.700 \times 10^4$$

Using Eq. (7.3-29),

$$J_D = 0.99 N_{Re,L}^{-0.5} = 0.99(1.700 \times 10^4)^{-0.5} = 0.00758$$

The definition of J_D from Eq. (7.3-5) is

$$J_D = \frac{k'_c}{v}(N_{Sc})^{2/3} \qquad (7.3\text{-}5)$$

Solving for k'_c, $k'_c = J_D v (N_{Sc})^{-2/3}$. Substituting known values and solving,

$$k'_c = 0.00758(0.0610)(702)^{-2/3} = 5.85 \times 10^{-6}\ \text{m/s}$$

In this case, diffusion is for A through nondiffusing B, so k_c in Eq. (7.2-10) should be used.

$$N_A = \frac{k'_c}{x_{BM}}(c_{A1} - c_{A2}) = k_c(c_{A1} - c_{A2}) \qquad (7.2\text{-}10)$$

Since the solution is very dilute, $x_{BM} \cong 1.0$ and $k'_c \cong k_c$. Also, $c_{A1} = 2.948 \times 10^{-2}$ kg mol/m^3 (solubility) and $c_{A2} = 0$ (large volume of fresh water). Substituting into Eq. (7.2-10),

$$N_A = (5.85 \times 10^{-6})(0.02948 - 0) = 1.726 \times 10^{-7}\ \text{kg mol/s}\cdot\text{m}^2$$

Mass transfer for flow past single spheres

For flow past single spheres and for very low Reynolds number, the Sherwood number $\left(N_{Sh} = k'_c\, D_p/D_{AB}\right)$ should approach a value of 2.0. Since

$$N_A = \frac{D_{AB}}{r_1}(c_{A1} - c_{A2}) = \frac{D_{AB}}{D_p/2}(c_{A1} - c_{A2}) = k_c(c_{A1} - c_{A2}) = k'_c(c_{A1} - c_{A2})\ (Why?)$$

Thus,

$$k'_c = \frac{D_{AB}}{D_p/2}$$

And

$$N_{Sh} = \left(k'_c\, D_p/D_{AB}\right) = 2$$

<u>For gases</u> for a Schmidt number range of 0.6–2.7 and a Reynolds number range of 1-48000,

$$N_{Sh} = k'_c \frac{D}{D_{AB}} = 2 + \ 0.552(N_{Re})^{0.53}(N_{Sc})^{0.33}$$

This equation also holds for heat transfer where the Prandtl number replaces the Schmidt number and the Nusselt number replaces the Sherwood number.

<u>For liquids</u> and a Reynolds number range of 2 to about 2000,

$$N_{Sh} = k'_c \frac{D}{D_{AB}} = 2 + \ 0.95(N_{Re})^{0.50}(N_{Sc})^{0.33}$$

<u>For liquids</u> and a Reynolds number of 2000–17000,

$$N_{Sh} = k'_c \frac{D}{D_{AB}} = 0.347(N_{Re})^{0.62}(N_{Sc})^{0.33}$$

EXAMPLE 7.3-3. Mass Transfer from a Sphere
Calculate the value of the mass-transfer coefficient and the flux for mass transfer from a sphere of napthalene to air at 45°C and 1 atm abs flowing at a velocity of 0.305 m/s. The diameter of the sphere is 25.4 mm. The diffusivity of naphthalene in air at 45°C is 6.92×10^{-6} m^2/s and the vapor pressure of solid naphthalene is 0.555 mm Hg. Use English and SI units.

Solution: In English units $D_{AB} = 6.92 \times 10^{-6}(3.875 \times 10^4) = 0.2682\ \text{ft}^2/\text{h}$. The diameter $D_p = 0.0254\ \text{m} = 0.0254(3.2808) = 0.0833\ \text{ft}$. From Appendix A.3 the physical properties of air will be used since the concentration of naphthalene is low.

$$\mu = 1.93 \times 10^{-5}\ \text{Pa}\cdot\text{s} = 1.93 \times 10^{-5}(2.4191 \times 10^3) = 0.0467\ \text{lb}_m/\text{ft}\cdot\text{h}$$

$$\rho = 1.113\ \text{kg/m}^3 = \frac{1.113}{16.0185} = 0.0695\ \text{lb}_m/\text{ft}^3$$

$$v = 0.305\ \text{m/s} = 0.305(3600 \times 3.2808) = 3600\ \text{ft/h}$$

The Schmidt number is

$$N_{Sc} = \frac{\mu}{\rho D_{AB}} = \frac{0.0467}{0.0695(0.2682)} = 2.505$$

$$N_{Sc} = \frac{1.93 \times 10^{-5}}{1.113(6.92 \times 10^{-6})} = 2.505$$

The Reynolds number is

$$N_{Re} = \frac{D_p v \rho}{\mu} = \frac{0.0833(3600)(0.0695)}{0.0467} = 446$$

$$N_{Re} = \frac{0.0254(0.3048)(1.113)}{1.93 \times 10^{-5}} = 446$$

Equation (7.3-33) for gases will be used.

$$N_{Sh} = 2 + 0.552(N_{Re})^{0.53}(N_{Sc})^{1/3} = 2 + 0.552(446)^{0.53}(2.505)^{1/3} = 21.0$$

From Eq. (7.3-3),

$$N_{Sh} = k'_c \frac{L}{D_{AB}} = k'_c \frac{D_p}{D_{AB}}$$

Substituting the knowns and solving,

$$21.0 = \frac{k'_c(0.0833)}{0.2682} \qquad k'_c = 67.6 \text{ ft/h}$$

$$21.0 = \frac{k'_c(0.0254)}{6.92 \times 10^{-6}} \qquad k'_c = 5.72 \times 10^{-3} \text{ m/s}$$

From Table 7.2-1,

$$k'_c c = k'_c \frac{P}{RT} = k'_G P$$

Hence, for $T = 45 + 273 = 318 \text{ K} = 318(1.8) = 574°\text{R}$,

$$k'_G = \frac{k'_c}{RT} = \frac{67.6}{(0.730)(573)} = 0.1616 \text{ lb mol/h} \cdot \text{ft}^2 \cdot \text{atm}$$

$$k'_G = \frac{5.72 \times 10^{-3}}{8314(318)} = 2.163 \times 10^{-9} \text{ kg mol/s} \cdot \text{m}^2 \cdot \text{Pa}$$

Since the gas is very dilute, $y_{BM} \cong 1.0$ and $k'_G \cong k_G$. Substituting into Eq. (7.2-12) for A diffusing through stagnant B and noting that $p_{A1} = 0.555/760 = 7.303 \times 10^{-4}$ atm $= 74.0$ Pa and $p_{A2} = 0$ (pure air),

$$\begin{aligned} N_A = k_G(p_{A1} - p_{A2}) &= 0.1616(7.303 \times 10^{-4} - 0) \\ &= 1.180 \times 10^{-4} \text{ lb mol/h} \cdot \text{ft}^2 \\ &= 2.163 \times 10^{-9}(74.0 - 0) = 1.599 \times 10^{-7} \text{ kg mol/s} \cdot \text{m}^2 \end{aligned}$$

The area of the sphere is

$$\begin{aligned} A = \pi D_p^2 &= \pi(0.0833)^2 = 2.18 \times 10^{-2} \text{ ft}^2 \\ &= (2.18 \times 10^{-2})\left(\frac{1}{3.2808}\right)^2 = 2.025 \times 10^{-3} \text{ m}^2 \end{aligned}$$

Total amount evaporated $= N_A A = (1.18 \times 10^{-4})(2.18 \times 10^{-2}) = 2.572 \times 10^{-6}$ lb mol/h $= (1.599 \times 10^{-7})(2.025 \times 10^{-3}) = 3.238 \times 10^{-10}$ kg mol/s

Question 2b (Test 2 Winter 2016 2017):

Mass transfer from a sphere of naphthalene that has a diameter of 30 mm to air at 45 ˚C and 1 atm. abs. flowing at a velocity of 0.50 m/s. The vapor pressure of solid naphthalene at 45 ˚C is 74 Pa. The mass transfer correlation for flow past a sphere is as follows:

$$N_{Sh} = 2 + 0.552(N_{Re})^{0.52}(N_{Sc})^{1/3}$$

You are given the diffusion coefficient of naphthalene at 27 ˚C as $D_{AB} = 6.25 \times 10^{-6}\, m^2/s$. **Find**

a) Compute the diffusion coefficient of the naphthalene at 45 ˚C
b) Predict the convective mass transfer coefficient using the above correlation
c) Determine k_G
d) Flux of naphthalene from the sphere.
e) At the given temperature, pressure and diameter, what is the minimum possible value of N_{Sh} and mass transfer coefficient

Data: At T = 45 ˚C, $\boldsymbol{\rho} = 1.1\, kg/m^3$; $\boldsymbol{\mu} = 1.93 \times 10^{-5} Pa \cdot s$; Gas Constant, $\boldsymbol{R} = 8314\, m^3 Pa/K \cdot kg\, mol$, $1\, atm = 101.3\ \times 10^3 Pa$;

	Solution Question 2b (Max. 27)
5	$(D_{AB})_{45\,C} = (D_{AB})_{27\,C}\left(\frac{273+45}{273+27}\right)^{1.75} = 6.25 \times 10^{-6} \times \left(\frac{273+45}{273+27}\right)^{1.75} = 6.92 \times 10^{-6}\frac{m^2}{s}$
10	$N_{Re} = \left(\frac{Dv\rho}{\mu}\right) = 855$ $N_{Sc} = \frac{(\mu/\rho)}{D_{AB}} = 2.5$ $N_{Sh} = 2 + 0.552(N_{Re})^{0.52}(N_{Sc})^{1/3}$ $N_{Sh} = 2 + 0.552(855)^{0.52}(2.5)^{1/3} = 27.2$ $N_{Sh} = k'_C\frac{D_P}{D_{AB}}$ $k'_C = N_{Sh}\frac{D_{AB}}{D_P} = 27.2 \times \left(\frac{6.92 \times 10^{-6}}{30 \times 10^{-3}}\right) = 6.27 \times 10^{-3}\frac{m}{s}$
3	$k'_G = \frac{k'_C}{RT} = \frac{6.27 \times 10^{-3}}{8314 \times (273+45)} = 2.37 \times 10^{-9}\frac{kgmol}{s \cdot m^2 \cdot Pa}$
4	For dilute case; $k_G \cong k'_G$ $N_A = k_G(P_{A1} - P_{A2}) = k_G(P_A^0 - 0) = 2.37 \times 10^{-9}(74 - 0) = 1.75 \times 10^{-7}\frac{kgmol}{s \cdot m^2}$
5	At the given temperature, pressure and diameter, when velocity is zero; $N_{Re} = 0$, Therefore, $N_{Sh} = 2 + 0.552(0)^{0.52}(2.5)^{1/3} = 2 + 0 = 2$ $k'_C = N_{Sh}\frac{D_{AB}}{D_P} = 2 \times \left(\frac{6.92 \times 10^{-6}}{30 \times 10^{-3}}\right) = 0.46 \times 10^{-3}\frac{m}{s}$