Quantum Mechanics cheat sheet

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(Please, do not write on this paper)

1 Fundamental Constants

$\hbar =$	1.054572×10^{-34} Js	(09)
$m_e =$	9.109382×10^{-31} kg	(03)
$e =$	1.602176×10^{-19} C	(23)
$c =$	2.997924×10^{8} m/s	(28)
$\mu_B =$	9.274009×10^{-24} s/T	(08)
$N_A =$	6.022142×10^{23} g/mol	(24)
$\varepsilon_0 =$	8.8541×10^{-12}F/m	(32)
$a_0 =$	5.29177×10^{-11} m	$= \frac{\hbar}{\alpha m_e c}$
$\alpha =$	$\frac{1}{137}$	$= \frac{e^2}{4\pi\hbar}$

Note : You may use the built-in list of constants in your calculator. Press SHIFT, then 7. Enter the number of the constant, as listed here.

2 Mathematical Relations

The gamma function:

$$\Gamma(x) = \int_0^\infty t^{x-1} e^{-t} dt$$

Gaussian Integral :

$$\int_{-\infty}^{+\infty} dx e^{-x^2} = \sqrt{\pi}$$

Useful trigonometric identity

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$$

Taylor expansion:

$$f(x) \approx f(0) + \frac{df(x)}{dx}|_{x=0}\, x + \ldots$$

Fourier transform

$$\tilde{f}(k) = \frac{1}{\sqrt{2\pi\hbar}} \int dx f(x) e^{-ikx}$$

3 Commutators

$[A,[B,C]] + [B,[C,A]] + [C,[A,B]] = 0$	Jacobi Identity
$[A,BC] = [A,B]C + B[A,C]$	Product rule
$e^A B e^{-A} = B + [A,B] + \frac{1}{2!}[A,[A,B]] + \ldots$	Hadamard Lemma

4 Quantum Harmonic Oscillator

Operators:

$$\hat{a} = \frac{1}{\sqrt{2}}\left(\hat{X} + i\hat{P}\right)$$
$$\hat{a}^\dagger \quad \frac{1}{\sqrt{2}}\left(\hat{X} - i\hat{P}\right)$$
$$\hat{N} = \hat{a}^\dagger \hat{a}$$

Eigenfunctions:

$$\psi_0(x) = \langle x|0\rangle \quad , \psi_n = \langle x|n\rangle$$

$$\psi_0 = C_0 \exp\left(-\frac{m\omega x^2}{2\hbar}\right)$$

$$\psi_n = \frac{1}{\sqrt{n}}\left(\hat{a}^\dagger\right)^n \psi_0$$

5 Spherical Harmonics

For $\ell = 0$

$$Y_0^0(\theta,\varphi) = \frac{1}{2}\sqrt{\frac{1}{\pi}}$$

For $\ell = 1$

$$Y_1^{-1}(\theta,\varphi) = \frac{1}{2}\sqrt{\frac{3}{2\pi}} \cdot e^{-i\varphi} \cdot \sin\theta$$
$$Y_1^{0}(\theta,\varphi) = \frac{1}{2}\sqrt{\frac{3}{\pi}} \cdot \cos\theta$$
$$Y_1^{1}(\theta,\varphi) = -\frac{1}{2}\sqrt{\frac{3}{2\pi}} \cdot e^{i\varphi} \cdot \sin\theta$$

6 Radial Functions

$$R_{10} = 2\left(\frac{Z}{a_0}\right)^{3/2} e^{-Zr/a_0}$$
$$R_{20} = 2\left(\frac{Z}{a_0}\right)^{3/2}\left(1 - \frac{Zr}{2a_0}\right) e^{-Zr/2a_0}$$
$$R_{21} = \frac{1}{\sqrt{3}}\left(\frac{Z}{a_0}\right)^{3/2}\left(\frac{Zr}{a_0}\right) e^{-Zr/2a_0}$$

7 Angular Momentum and Spin

For (any) angular momentum operator :$\hat{J}_\pm = \hat{J}_x \pm i\hat{J}_y$, and their eigenvalues are:

$$J_+|j\, m\rangle = \hbar\sqrt{j(j+1) - m(m+1)}|j\, m+1\rangle$$
$$J_-|j\, m\rangle = \hbar\sqrt{j(j+1) - m(m-1)}|j\, m-1\rangle.$$

The Pauli spin matrices:

$$\sigma_1 = \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tag{1}$$
$$\sigma_2 = \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \tag{2}$$
$$\sigma_3 = \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{3}$$

The Clifford Algebra relation:

$$\{\sigma_1, \sigma_1\} = 2I$$

The spinor wavefunctions:

$$\psi_{z+} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \qquad \psi_{z-} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}. \tag{4}$$

8 Fine-Structure

Spin-orbit coupling term:

$$H_{so} = \frac{1}{2}\left(\frac{Ze^2}{4\pi\varepsilon_0}\right)\left(\frac{g_s}{2m_e^2 c^2}\right)\frac{\vec{L}\cdot\vec{S}}{r^3}$$

Total effect :

$$\Delta E = \frac{E_n (Z\alpha)^2}{n}\left(\frac{1}{j+\frac{1}{2}} - \frac{3}{4n}\right)$$

Where $E_n = -\dfrac{e^2}{8\pi\varepsilon_0 a_0 n^2}$