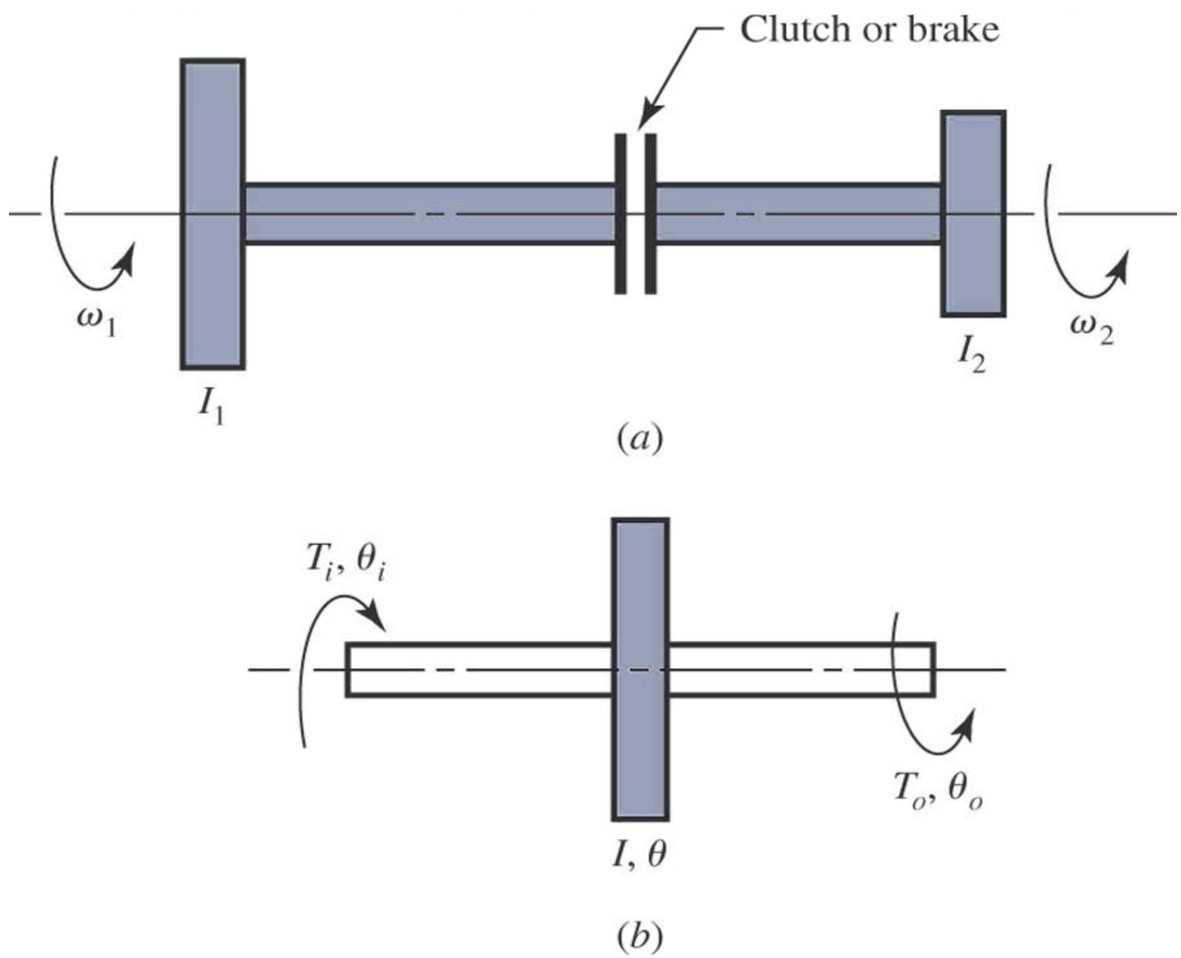
CHAPTER 16

Clutches, Brakes, Coupling, and Flywheels

- Clutches, brakes, couplings, and flywheels are a group of elements usually associated with rotation that have in common the function of storing and/or transferring rotating energy

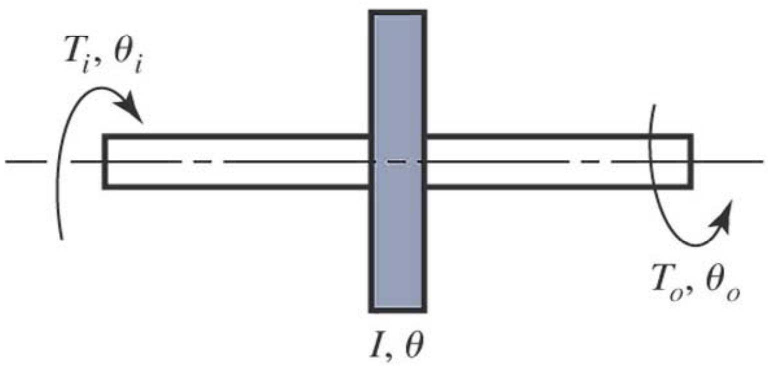
- In analyzing the performance of these devices we shall be interested in:
  - The actuating force
  - The torque transmitted
  - The energy loss
  - The temperature rise



- A simplified dynamic representation of a friction clutch or brake is shown in Fig. 16-la.

- Two inertias, I_1 and I_2, traveling at the respective angular velocities; $\omega_1$and ω_2, one of which may be zero in the case of brakes, are to be brought to the same speed by engaging the clutch or brake. Slippage occurs because the two elements are running at different speeds and energy is dissipated during actuation, resulting in a temperature rise.
- The varies types of devices to be studied may be classified as following:
  - Rim types with internal expanding shoes
  - Rim types with external contracting shoes
  - Band types
  - Disk or axial types
  - Cone types
  - Miscellaneous types

- A flywheel is an inertial energy-storage device.
- It absorbs mechanical energy by increasing its angular velocity and delivers energy by decreasing its velocity.
- Figure 16-lb is a mathematical representation of a flywheel.
- An input torque T_i, corresponding to a coordinate θ_i, will cause the flywheel speed to increase. And a load or output torque T_0, with coordinate θ_o, will absorb energy from the flywheel and cause it to slow down.



(b)

16.2 Internal Expanding Rim Clutches and Brakes

- The internal-shoe rim clutch shown in Fig. 16-3 consists essentially of three elements:
- The mating frictional surface.
- The means of transmitting the torque to and from the surfaces.
- The actuating mechanism.

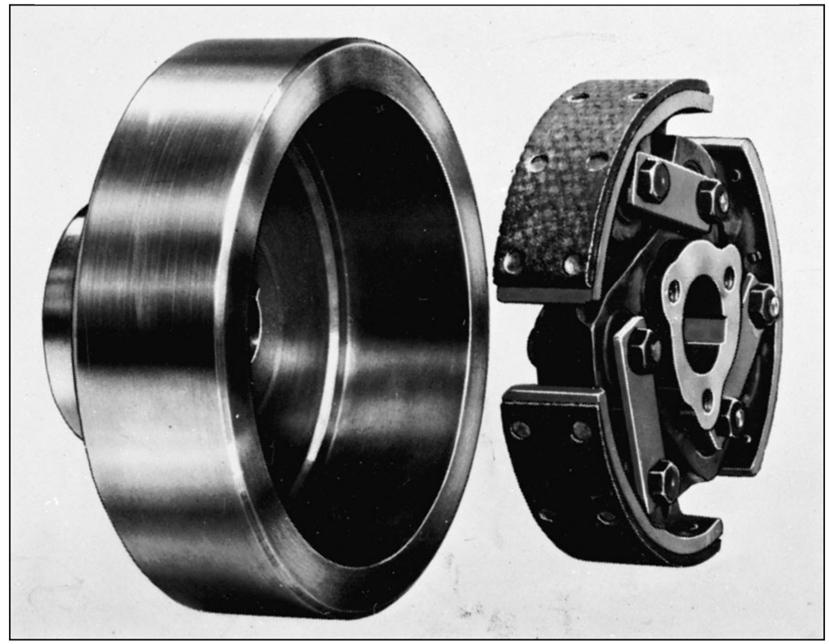


Figure 16-3: An internal expanding centrifugal-acting rim clutch

- Depending upon the operating mechanism, such clutches are further classified as *expanding-ring, centrifugal, magnetic, hydraulic,* and *pneumatic*.

- The expanding-ring clutch
  - used in textile machinery, excavators, and machine tools where the clutch may be located within the driving pulley.
  - Transmit high torque, even at low speeds;
  - require both positive engagement and ample release force.

- The centrifugal clutch
  - used mostly for automatic operation.
  - For no spring is used, the torque transmitted is proportional to the square of the speed.

- Magnetic clutches
  - used for automatic and remote-control systems.

- Hydraulic and pneumatic clutches
  - useful in drives having complex loading cycles and in automatic machinery, or in robots.

- In braking systems, the *internal-shoe* or *drum* brake
  - used mostly for automotive applications.

- To analyze an internal-shoe device, figure 16-4 shows a shoe pivoted at point A, with the actuating force acting at the other end of the shoe.
- Since the shoe is long, we cannot make the assumption that the distribution of normal forces is uniform.
- The mechanical arrangement permits no pressure to be applied at the heel, thus we will assume the pressure at this point to be zero.

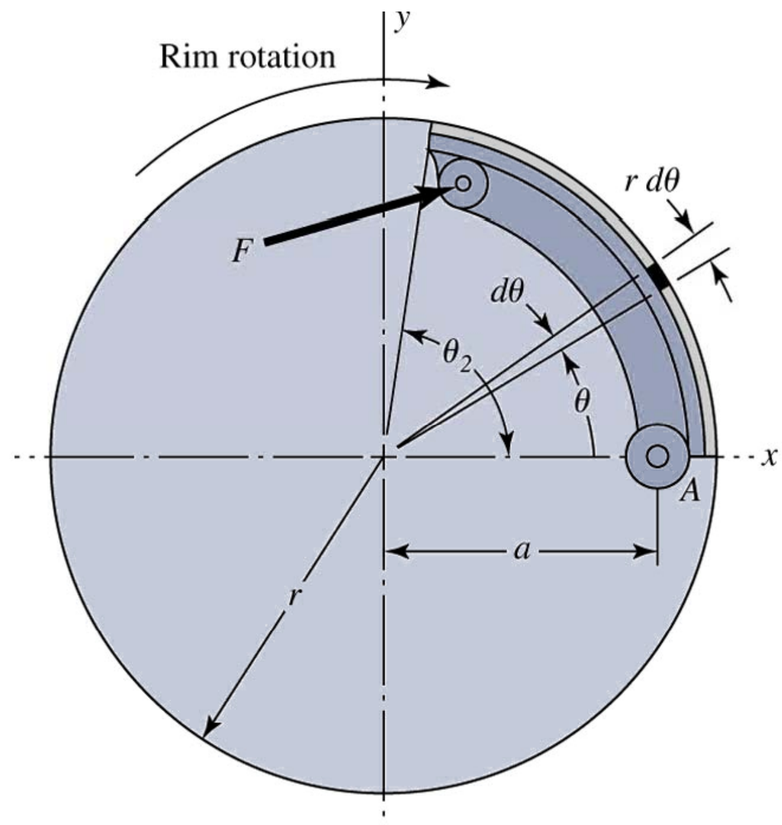


Figure 16-4: internal friction shoe geometry

- It is the usual practice to omit the friction material for a short distance away from point A. This eliminates interference, and the material would contribute little to the performance anyway, as will be shown.
- Let us consider the pressure p acting upon an element of area of the frictional material located at an angle θ from the hinge pin (Figure 16-4).
- We designate the maximum pressure p_a located at an angle θ_a from the hinge pin.

- To find the pressure distribution on the periphery of the internal shoe, consider point *B* on the shoe (Fig. 16-5).
- If the shoe deforms by an infinitesimal rotation $\Delta\Phi$ about the pivot point A, deformation perpendicular to *AB* is $h\ \Delta\Phi$. From the isosceles triangle *AOB*, $h = 2r\sin(\theta/2)$,so

$$h\ \Delta\Phi = 2\ r\ \Delta\Phi\ \sin(\theta/2)$$

- The deformation perpendicular to the rim is $h\ \Delta\Phi\ \cos(\theta/2)$, which is

$$h\ \Delta\Phi\ \cos(\theta/2)$$

$$= 2\ r\ \Delta\Phi\ \sin(\theta/2)\cos(\theta/2)$$

$$= r\ \Delta\Phi\ \sin\theta$$

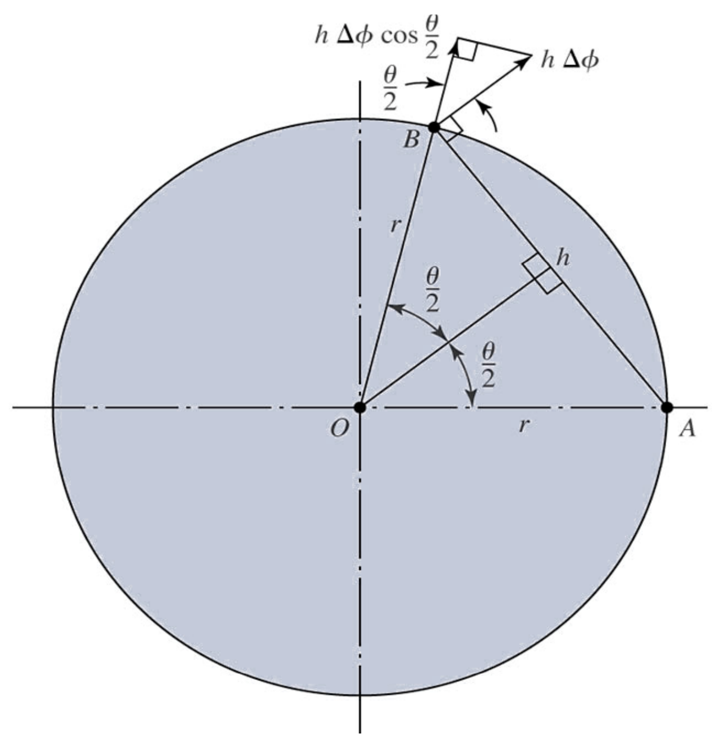


Figure 16-5: The geometry associated with an arbitrary point on the shoe

We now make the assumption that the pressure at any point is proportional to the vertical distance from the hinge pin. This vertical distance is proportional to sin θ

- Thus, the deformation, and consequently the pressure, is proportional to sin *θ*. In terms of the pressure at *B* and where the pressure is a maximum, this means

$$\frac{p}{\sin\theta} = \frac{p_a}{\sin\theta_a} \qquad \text{(a)}$$

- or

$$p = \frac{p_a}{\sin\theta_a}\sin\theta \qquad \text{(16-1)}$$

- p will be maximum when $\theta = 90^{o}$

- This pressure distribution has interesting and useful characteristics:
- The pressure distribution is sinusoidal with respect to the angle θ.
- If the shoe is short, as shown in Fig. 16-6*a*, the largest pressure *on the shoe* is p_a occurring at the end of the shoe, θ_2.
- If the shoe is long, as shown in Fig. 16-6*b,* the largest pressure on the shoe is p_a occurring at $\theta_a = 90°$.

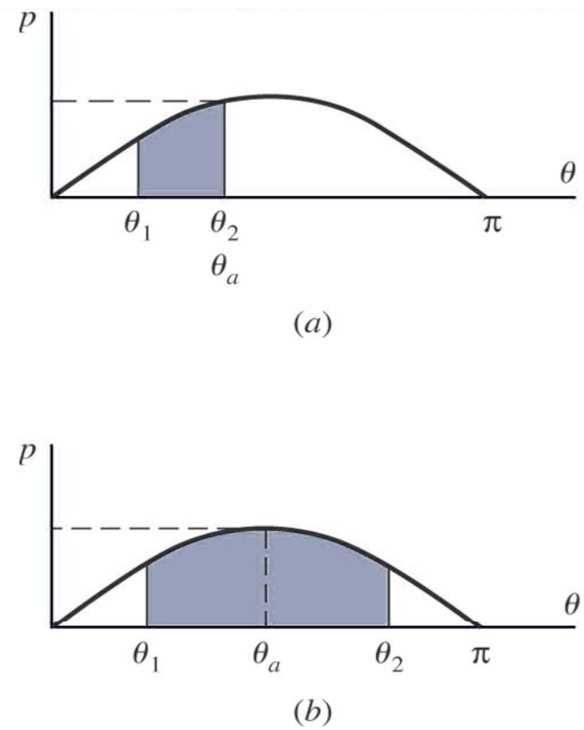


Figure 16-6: Defining the angle θ_a at which the maximum pressure p_a occurs when
(a) shoe exists in zone $\theta_1 \leq \theta_2 \leq \pi/2$
(b) shoe exists in zone $\theta_1 \leq \pi/2 \leq \theta_2$

- Since limitations on friction materials are expressed in terms of the largest allowable pressure on the lining, the designer wants to think in terms of p_a and not about the amplitude of the sinusoidal distribution that addresses locations off the shoe.
- When $\theta=0$, Eq. (16-1) shows that the pressure is zero.
- The frictional material located at the heel therefore contributes very little to the braking action and might as well be omitted.
- A good design would concentrate as much frictional material as possible in the neighborhood of the point of maximum pressure.
- Such a design is shown in Fig. 16-7.

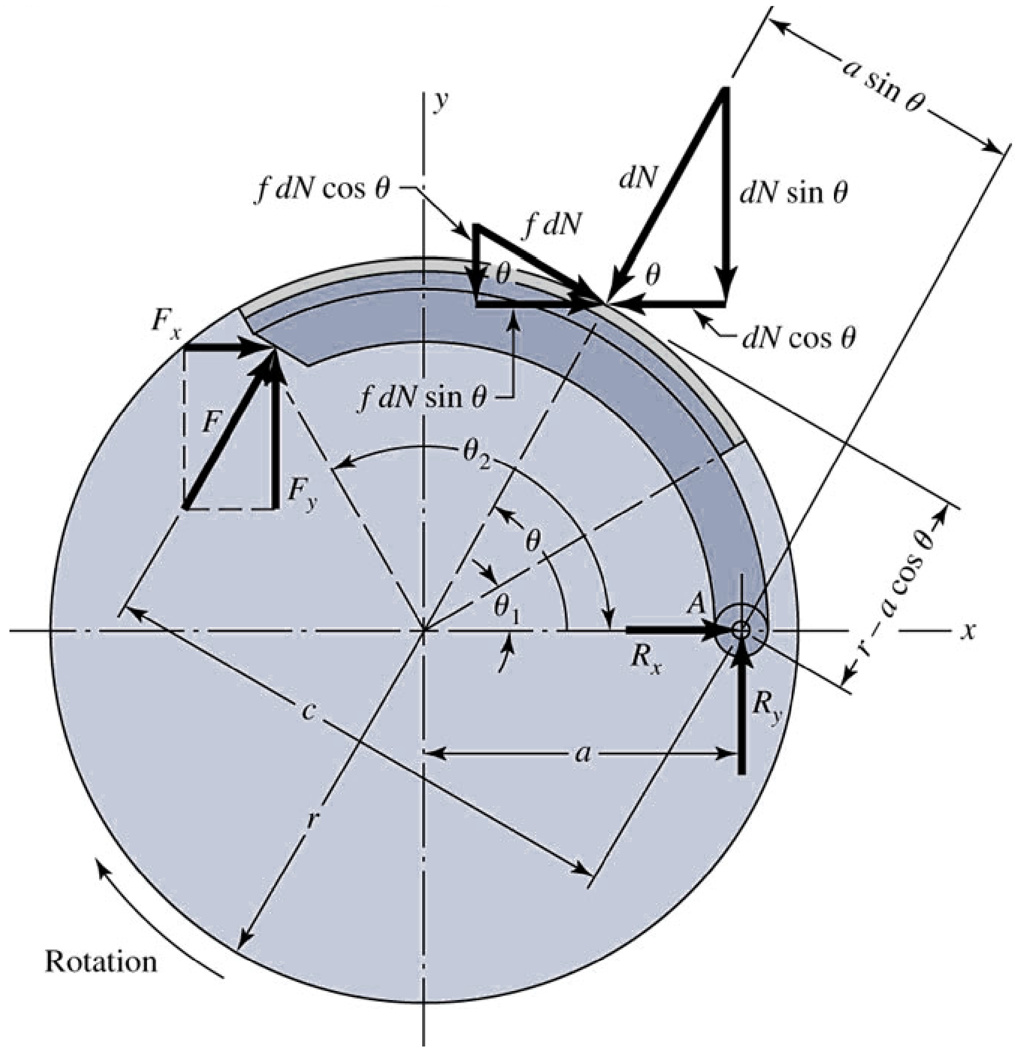
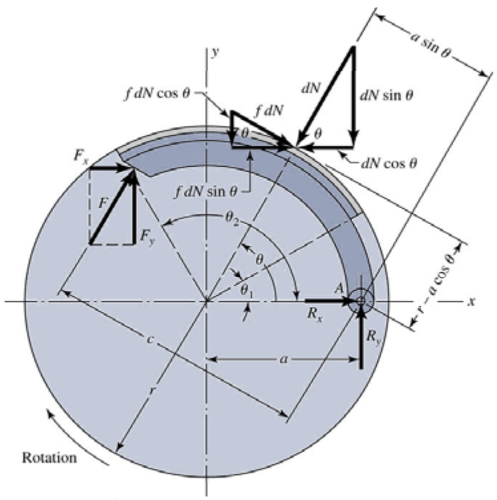


Figure 16-7: Forces on the shoe

- In this figure the frictional material begins at an angle θ_1, measured from the hinge pin A, and ends at an angle θ_2. Any arrangement such as this will give a good distribution of the frictional material.
- Proceeding now (Fig. 16-7), the hinge-pin reactions are R_x and R_y.
- The actuating force F has components F_x and F_y and operates at distance c from the hinge pin.

- At any angle θ from the hinge pin there acts a differential normal force dN whose magnitude is



$$dN = pbr\,d\theta \tag{b}$$

$$dN = \frac{p_a br\sin\theta\,d\theta}{\sin\theta_a} \tag{c}$$

$$M_f = \int f\,dN(r - a\cos\theta) = \frac{fp_a br}{\sin\theta_a}\int_{\theta_1}^{\theta_2}\sin\theta(r - a\cos\theta)\,d\theta \tag{16–2}$$

$$M_N = \int dN(a\sin\theta) = \frac{p_a bra}{\sin\theta_a}\int_{\theta_1}^{\theta_2}\sin^2\theta\,d\theta \tag{16–3}$$

- The actuating force F must balance these moments. Thus

$$F = \frac{M_N - M_f}{c} \tag{16-4}$$

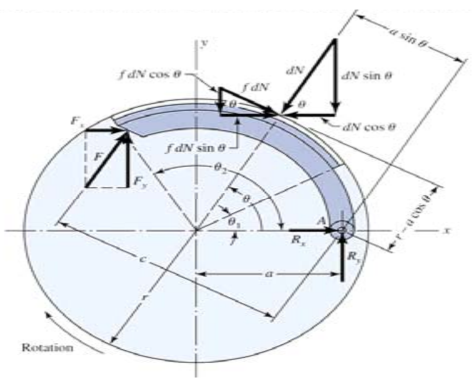
- If we make $M_N = M_f$, self-locking is obtained, and no actuating force is required. This furnishes us with a method for obtaining the dimensions for some self-energizing action. Thus the dimension of a must be such that

$$M_N > M_f \tag{16-5}$$

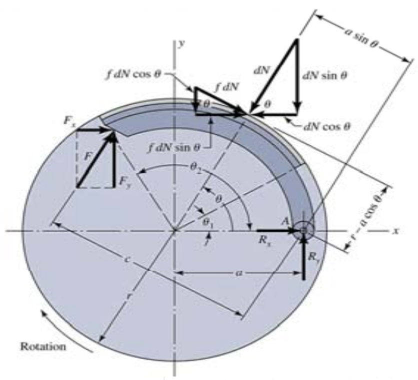
- The torque T applied to the drum by the brake shoe is the sum of the frictional forces $f\,dN$ times the radius of the drum:

$$T = \int f r\,dN = \frac{f p_a b r^2}{\sin\theta_a}\int_{\theta_1}^{\theta_2} \sin\theta\,d\theta$$

$$= \frac{f p_a b r^2(\cos\theta_1 - \cos\theta_2)}{\sin\theta_a} \qquad (16\text{–}6)$$



- The hinge-pin reactions are found by taking a summation of the horizontal and vertical forces. Thus, for R_x and R_y, we have



$$R_x = \int dN\cos\theta - \int f\,dN\sin\theta - F_x$$

$$= \frac{p_a b r}{\sin\theta_a}\left(\int_{\theta_1}^{\theta_2} \sin\theta\cos\theta\,d\theta - f\int_{\theta_1}^{\theta_2} \sin^2\theta\,d\theta\right) - F_x \quad \text{(d)}$$

$$R_y = \int dN\sin\theta + \int f\,dN\cos\theta - F_y$$

$$= \frac{p_a b r}{\sin\theta_a}\left(\int_{\theta_1}^{\theta_2} \sin^2\theta\,d\theta + f\int_{\theta_1}^{\theta_2} \sin\theta\cos\theta\,d\theta\right) - F_y \quad \text{(e)}$$

- The direction of the frictional forces is reversed if the rotation is reversed. Thus, for counterclockwise rotation the actuating force is:

$$F = \frac{M_N + M_f}{c} \qquad (16\text{–}7)$$

and since both moments have the same sense, the self-energizing effect is lost. Also, for counterclockwise rotation the signs of the frictional terms in the equations for the pin reactions change, and Eqs. (d) and (e) become

$$R_x = \frac{p_a b r}{\sin\theta_a}\left(\int_{\theta_1}^{\theta_2} \sin\theta\cos\theta\, d\theta + f\int_{\theta_1}^{\theta_2} \sin^2\theta\, d\theta\right) - F_x \qquad (f)$$

$$R_y = \frac{p_a b r}{\sin\theta_a}\left(\int_{\theta_1}^{\theta_2} \sin^2\theta\, d\theta - f\int_{\theta_1}^{\theta_2} \sin\theta\cos\theta\, d\theta\right) - F_y \qquad (g)$$

Equations (d), (e), (f), and (g) can be simplified to ease computations. Thus, let

$$A = \int_{\theta_1}^{\theta_2} \sin\theta \cos\theta \, d\theta = \left(\frac{1}{2}\sin^2\theta\right)_{\theta_1}^{\theta_2}$$

$$B = \int_{\theta_1}^{\theta_2} \sin^2\theta \, d\theta = \left(\frac{\theta}{2} - \frac{1}{4}\sin 2\theta\right)_{\theta_1}^{\theta_2} \tag{16–8}$$

Then, for clockwise rotation as shown in Fig. 16–7, the hinge-pin reactions are

$$R_x = \frac{p_a br}{\sin\theta_a}(A - fB) - F_x$$

$$R_y = \frac{p_a br}{\sin\theta_a}(B + fA) - F_y \qquad (16\text{–}9)$$

For counterclockwise rotation, Eqs. (f) and (g) become

$$R_x = \frac{p_a br}{\sin\theta_a}(A + fB) - F_x$$

$$R_y = \frac{p_a br}{\sin\theta_a}(B - fA) - F_y \qquad (16\text{–}10)$$

- In using these equations,
  - The reference system always has its origin at the center of the drum.
  - The positive x axis is taken through the hinge pin.
  - The positive y axis is always in the direction of the shoe, even if this should result in a left-handed system.

- The following assumptions are implied by the preceding analysis:
  - The pressure at any point on the shoe is assumed to be proportional to the distance from the hinge pin, being zero at the heel.
    - This should be considered from the standpoint that pressures specified by manufacturers are averages rather than maxima.
  - The effect of centrifugal force has been neglected.
    - In the case of brakes, the shoes are not rotating, and no centrifugal force exists.
    - In clutch design, the effect of this force must be considered in writing the equations of static equilibrium.

- The shoe is assumed to be rigid. Since this cannot be true, some deflection will occur, depending upon the load, pressure, and stiffness of the shoe.
  - The resulting pressure distribution may be different from that which has been assumed.
- The entire analysis has been based upon a coefficient of friction that does not vary with pressure.
  - Actually, the coefficient may vary with a number of conditions, including temperature, wear, and environment.

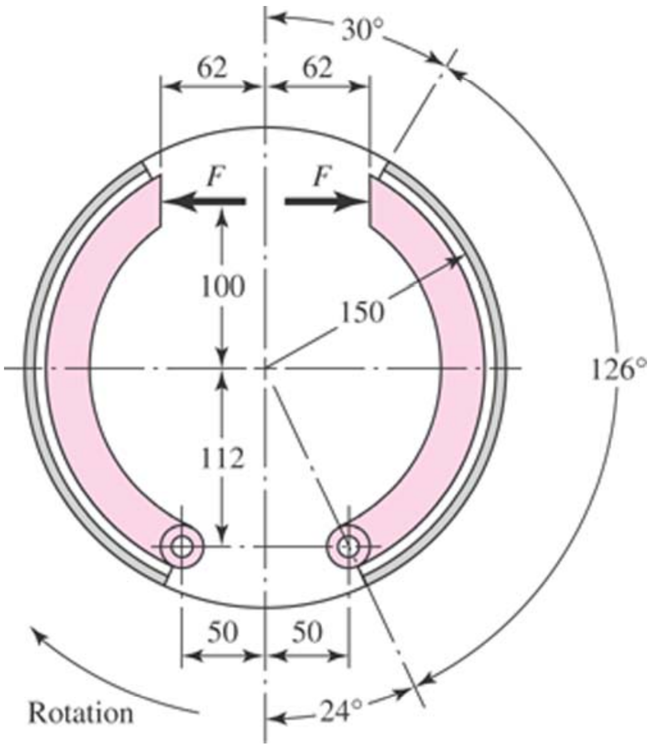
Example

The brake shown in Fig. 16–8 is 300 mm in diameter and is actuated by a mechanism that exerts the same force F on each shoe. The shoes are identical and have a face width of 32 mm. The lining is a molded asbestos having a coefficient of friction of 0.32 and a pressure limitation of 1000 kPa. Estimate the maximum

(*a*) Actuating force F.

(*b*) Braking capacity.

(*c*) Hinge-pin reactions.



(a) The right-hand shoe is self-energizing, and so the force F is found on the basis that the maximum pressure will occur on this shoe. Here $\theta_1 = 0°$, $\theta_2 = 126°$, $\theta_a = 90°$, and $\sin\theta_a = 1$. Also,

$$a = \sqrt{(112)^2 + (50)^2} = 122.7 \text{ mm}$$

$$M_f = \frac{f p_a b r}{\sin\theta_a}\left[\left(-r\cos\theta\right)_0^{\theta_2} - a\left(\frac{1}{2}\sin^2\theta\right)_0^{\theta_2}\right]$$

$$= \frac{f p_a b r}{\sin\theta_a}\left(r - r\cos\theta_2 - \frac{a}{2}\sin^2\theta_2\right)$$

$$M_f = (0.32)[1000(10)^3](0.032)(0.150)$$

$$\times\left[0.150 - 0.150\cos 126° - \left(\frac{0.1227}{2}\right)\sin^2 126°\right]$$

$$= 304 \text{ N}\cdot\text{m}$$

$$M_N = \frac{p_a b r a}{\sin\theta_a}\left(\frac{\theta}{2} - \frac{1}{4}\sin 2\theta\right)_0^{\theta_2}$$

$$= \frac{p_a b r a}{\sin\theta_a}\left(\frac{\theta_2}{2} - \frac{1}{4}\sin 2\theta_2\right)$$

$$= [1000(10)^3](0.032)(0.150)(0.1227)\left\{\frac{\pi}{2}\frac{126}{180} - \frac{1}{4}\sin[(2)(126°)]\right\}$$

$$= 788 \text{ N}\cdot\text{m}$$

$$F = \frac{M_N - M_f}{c} = \frac{788 - 304}{100 + 112} = 2.28 \text{ kN}$$

$$T_R = \frac{f p_a b r^2 (\cos\theta_1 - \cos\theta_2)}{\sin\theta_a}$$

$$= \frac{0.32[1000(10)^3](0.032)(0.150)^2(\cos 0^\circ - \cos 126^\circ)}{\sin 90^\circ} = 366 \text{ N} \cdot \text{m}$$

Right $F = \dfrac{M_N - M_f}{c}$ Left $F = \dfrac{M_N + M_f}{c}$

$$M_N = \int dN(a\sin\theta) = \frac{p_a b r a}{\sin\theta_a} \int_{\theta_1}^{\theta_2} \sin^2\theta \, d\theta$$

MN/788 = Pa/1000

$$M_N = \frac{788 p_a}{1000} \qquad M_f = \frac{304 p_a}{1000}$$

$$F = \frac{M_N + M_f}{c}$$

or

$$2.28 = \frac{(788/1000)\,p_a + (304/1000)\,p_a}{100 + 112}$$

Solving gives $p_a = 443$ kPa. Then, from Eq. (16–6), the torque on the left-hand shoe is

$$T_L = \frac{f p_a b r^2 (\cos\theta_1 - \cos\theta_2)}{\sin\theta_a}$$

Since $\sin\theta_a = \sin 90^\circ = 1$, we have

$$T_L = 0.32[443(10)^3](0.032)(0.150)^2(\cos 0^\circ - \cos 126^\circ) = 162 \text{ N}\cdot\text{m}$$

The braking capacity is the total torque:

$$T = T_R + T_L = 366 + 162 = 528 \text{ N}\cdot\text{m}$$

(c) In order to find the hinge-pin reactions, we note that $\sin\theta_a = 1$ and $\theta_1 = 0$. Then Eq. (16–8) gives

$$A = \frac{1}{2}\sin^2\theta_2 = \frac{1}{2}\sin^2 126^\circ = 0.3273$$

$$B = \frac{\theta_2}{2} - \frac{1}{4}\sin 2\theta_2 = \frac{\pi(126)}{2(180)} - \frac{1}{4}\sin[(2)(126^\circ)] = 1.3373$$

Also, let

$$D = \frac{p_a b r}{\sin\theta_a} = \frac{1000(0.032)(0.150)}{1} = 4.8 \text{ kN}$$

where $p_a = 1000$ kPa for the right-hand shoe. Then, using Eq. (16–9), we have

$$R_x = D(A - fB) - F_x = 4.8[0.3273 - 0.32(1.3373)] - 2.28\sin 24^\circ$$
$$= -1.410 \text{ kN}$$

$$R_y = D(B + fA) - F_y = 4.8[1.3373 + 0.32(0.3273)] - 2.28\cos 24^\circ$$
$$= 4.839 \text{ kN}$$

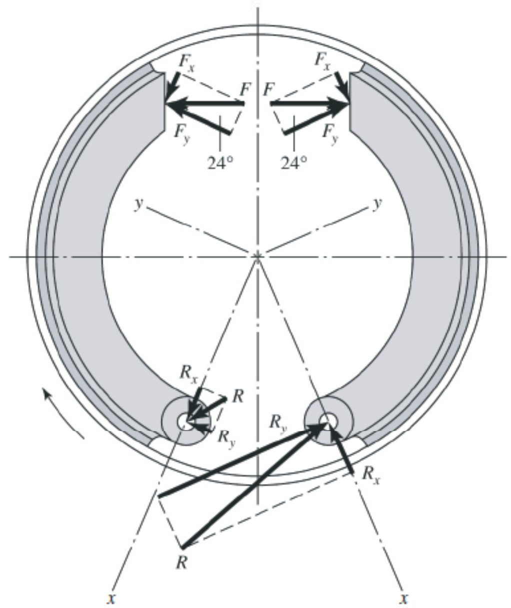
The resultant on this hinge pin is

$$R = \sqrt{(-1.410)^2 + (4.839)^2} = 5.04 \text{ kN} \qquad \textit{Ans.}$$

The reactions at the hinge pin of the left-hand shoe are found using Eqs. (16–10) for a pressure of 443 kPa. They are found to be $R_x = 0.678$ kN and $R_y = 0.538$ kN. The resultant is

$$R = \sqrt{(0.678)^2 + (0.538)^2} = 0.866 \text{ kN} \qquad \textit{Ans.}$$

The reactions for both hinge pins, together with their directions, are shown in the figure below.

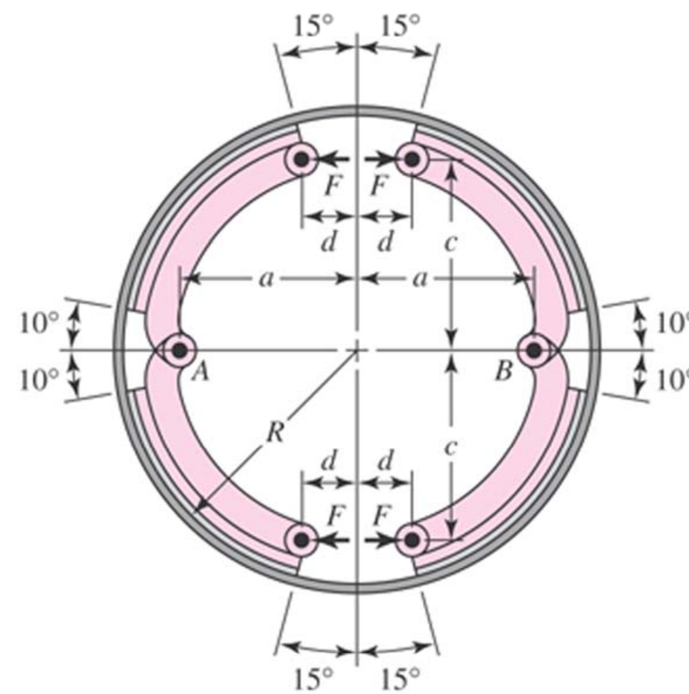


This example dramatically shows the benefit to be gained by arranging the shoes to be self-energizing. If the left-hand shoe were turned over so as to place the hinge pin at the top, it could apply the same torque as the right-hand shoe. This would make the capacity of the brake (2)(366) = 732 N · m instead of the present 528 N · m, a 30 percent improvement. In addition, some of the friction material at the heel could be eliminated without seriously affecting the capacity, because of the low pressure in this area. This change might actually improve the overall design because the additional rim exposure would improve the heat-dissipation capacity.

Example

The figure shows a 400-mm-diameter brake drum with four internally expanding shoes. Each of the hinge pins A and B supports a pair of shoes. The actuating mechanism is to be arranged to produce the same force F on each shoe. The face width of the shoes is 75 mm. The material used permits a coefficient of friction of 0.24 and a maximum pressure of 1000 kPa.

a) Determine the actuating force.
b) Estimate the brake capacity
c) Noting that rotation may be in either direction, estimate the hinge-pin reactions.



The dimensions in millimeters are $a = 150$, $c = 165$, $R = 200$, and $d = 50$.

EXAMPLE 16–2 Figure 16–8 shows a 400-mm-diameter brake drum with four internally expanding shoes. Each of the hinge pins A and B supports a pair of shoes. The actuating mechanism is to be arranged to produce the same force F on each shoe. The face width of the shoes is 75 mm. The material used permits a coefficient of friction of 0.24 and a maximum pressure of 1000 kPa.

(a) Determine the actuating force.

(b) Estimate the brake capacity.

(c) Noting that rotation may be in either direction, estimate the hinge-pin reactions.

Solution (a) Given: $\theta_1 = 10°$, $\theta_2 = 75°$, $\theta_a = 75°$, $p_a = 10^6$ Pa, $f = 0.24$, $b = 0.075$ m (shoe width), $a = 0.150$ m, $r = 0.200$ m, $d = 0.050$ m, $c = 0.165$ m. Some of the terms needed are evaluated as

$$A = \left[r\int_{\theta_1}^{\theta_2} \sin\theta\, d\theta - a\int_{\theta_1}^{\theta_2} \sin\theta\cos\theta\, d\theta \right] = r\big[-\cos\theta\big]_{\theta_1}^{\theta_2} - a\left[\frac{1}{2}\sin^2\theta\right]_{\theta_1}^{\theta_2}$$

$$= 200\big[-\cos\theta\big]_{10°}^{75°} - 150\left[\frac{1}{2}\sin^2\theta\right]_{10°}^{75°} = 77.5 \text{ mm}$$

Figure 16–8

The dimensions in millimeters are $a = 150$, $c = 165$, $R = 200$, and $d = 50$.

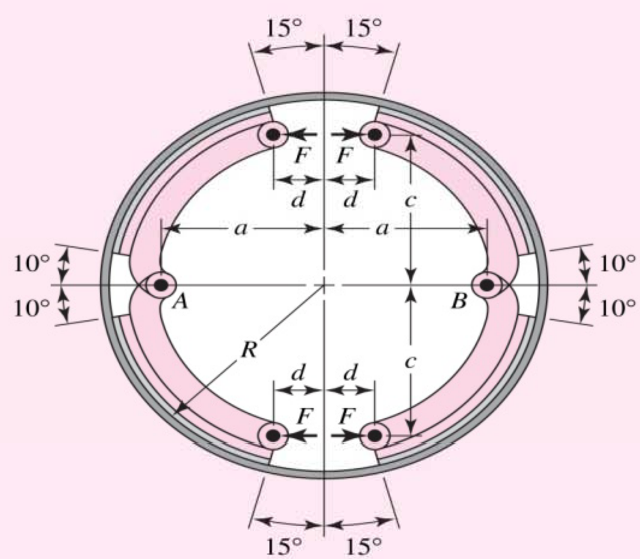
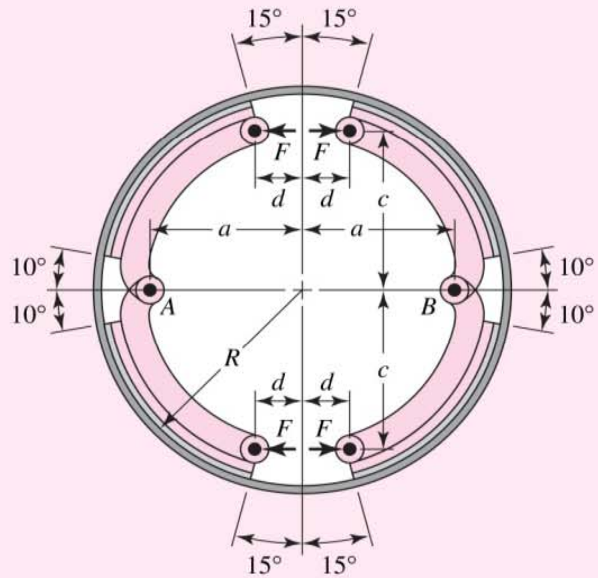


Figure 16–8

The dimensions in millimeters are $a = 150$, $c = 165$, $R = 200$, and $d = 50$.



$$B = \int_{\theta_1}^{\theta_2} \sin^2\theta \, d\theta = \left[\frac{\theta}{2} - \frac{1}{4}\sin 2\theta\right]_{10\pi/180 \text{ rad}}^{75\pi/180 \text{ rad}} = 0.528$$

$$C = \int_{\theta_1}^{\theta_2} \sin\theta \cos\theta \, d\theta = 0.4514$$

Now converting to pascals and meters, we have from Eq. (16–2),

$$M_f = \frac{f p_a b r}{\sin\theta_a} A = \frac{0.24[(10)^6](0.075)(0.200)}{\sin 75°}(0.0775) = 289 \text{ N} \cdot \text{m}$$

From Eq. (16–3),

$$M_N = \frac{p_a b r a}{\sin\theta_a} B = \frac{[(10)^6](0.075)(0.200)(0.150)}{\sin 75°}(0.528) = 1230 \text{ N} \cdot \text{m}$$

Finally, using Eq. (16–4), we have

Answer $$F = \frac{M_N - M_f}{c} = \frac{1230 - 289}{165} = 5.70 \text{ kN}$$

(*b*) Use Eq. (16–6) for the primary shoe.

$$T = \frac{f p_a b r^2(\cos\theta_1 - \cos\theta_2)}{\sin\theta_a}$$

$$= \frac{0.24[(10)^6](0.075)(0.200)^2(\cos 10^\circ - \cos 75^\circ)}{\sin 75^\circ} = 541\ \mathrm{N \cdot m}$$

For the secondary shoe, we must first find p_a.

Substituting

$$M_N = \frac{1230}{10^6} p_a \quad \text{and} \quad M_f = \frac{289}{10^6} p_a \quad \text{into Eq. (16–7),}$$

$$5.70 = \frac{(1230/10^6) p_a + (289/10^6) p_a}{165}, \quad \text{solving gives} \quad p_a = 619(10)^3\ \mathrm{Pa}$$

Then

$$T = \frac{0.24[0.619(10)^6](0.075)(0.200)^2(\cos 10^\circ - \cos 75^\circ)}{\sin 75^\circ} = 335\ \mathrm{N \cdot m}$$

Answer

so the braking capacity is $T_{total} = 2(541) + 2(335) = 1750\ \text{N} \cdot \text{m}$

(*c*) *Primary shoes*:

$$R_x = \frac{p_a br}{\sin\theta_a}(C - fB) - F_x$$

$$= \frac{(10^6)(0.075)(0.200)}{\sin 75^\circ}[0.4514 - 0.24(0.528)](10)^{-3} - 5.70 = -0.658\ \text{kN}$$

$$R_y = \frac{p_a br}{\sin\theta_a}(B + fC) - F_y$$

$$= \frac{(10^6)(0.075)(0.200)}{\sin 75^\circ}[0.528 + 0.24(0.4514)](10)^{-3} - 0 = 9.88\ \text{kN}$$

Secondary shoes:

$$R_x = \frac{p_a br}{\sin\theta_a}(C + fB) - F_x$$

$$= \frac{[0.619(10)^6](0.075)(0.200)}{\sin 75^\circ}[0.4514 + 0.24(0.528)](10)^{-3} - 5.70$$

$$= -0.143\ \text{kN}$$

$$R_y = \frac{p_a br}{\sin\theta_a}(B - fC) - F_y$$

$$= \frac{[0.619(10)^6](0.075)(0.200)}{\sin 75^\circ}[0.528 - 0.24(0.4514)](10)^{-3} - 0$$

$$= 4.03\ \text{kN}$$

Note from Figure 16–9 that $+y$ for secondary shoe is opposite to $+y$ for primary shoe.

Combining horizontal and vertical components,

$$R_H = -0.658 - 0.143 = -0.801\ \text{kN}$$

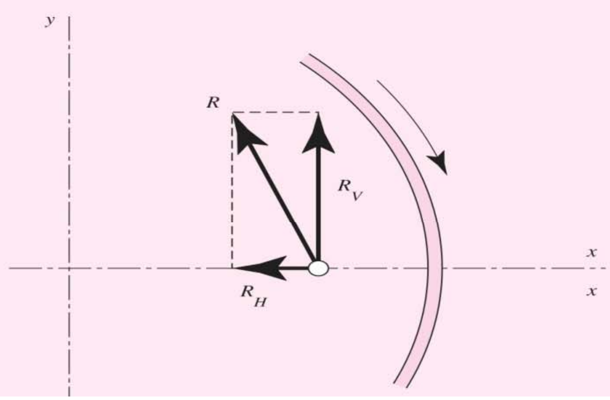
$$R_V = 9.88 - 4.03 = 5.85\ \text{kN}$$

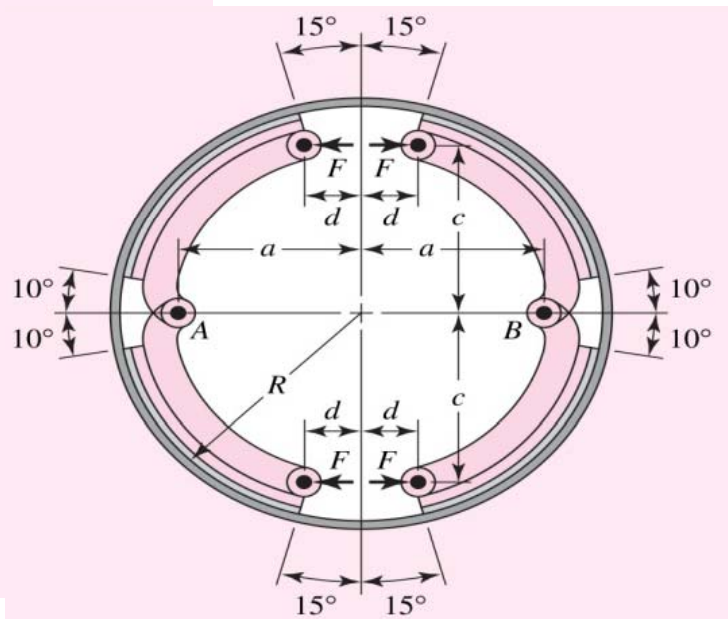
Answer

$$R = \sqrt{(0.801)^2 + (5.85)^2} = 5.90\ \text{kN}$$

Figure 16–9

Hinge-pin reactions.





16-3 External Contracting Rim Clutches and Brakes

- Operating mechanisms can be classified as:
  - Solenoids
  - Levers, linkages, or toggle devices
  - Linkages with spring loading
  - Hydraulic and pneumatic devices

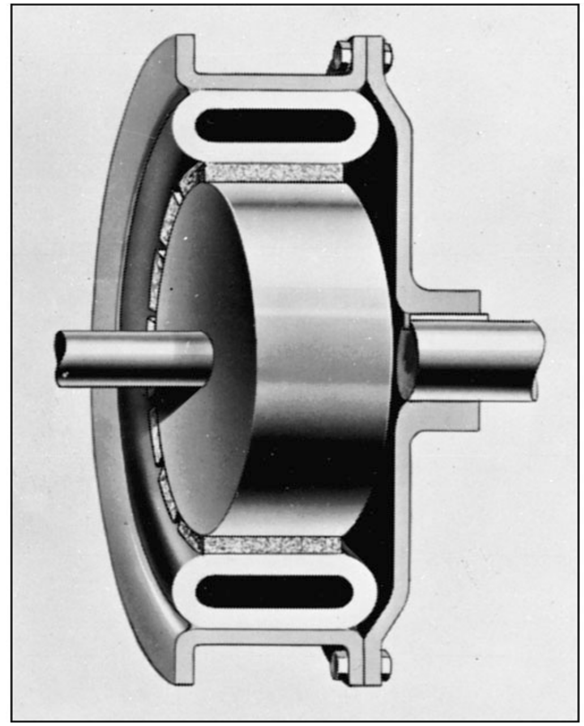


Figure 16-10: An external contracting clutch-brake that is engaged by expanding the flexible tube with compressed air.

- The notation for external contracting shoes is shown in Fig. 16-11.

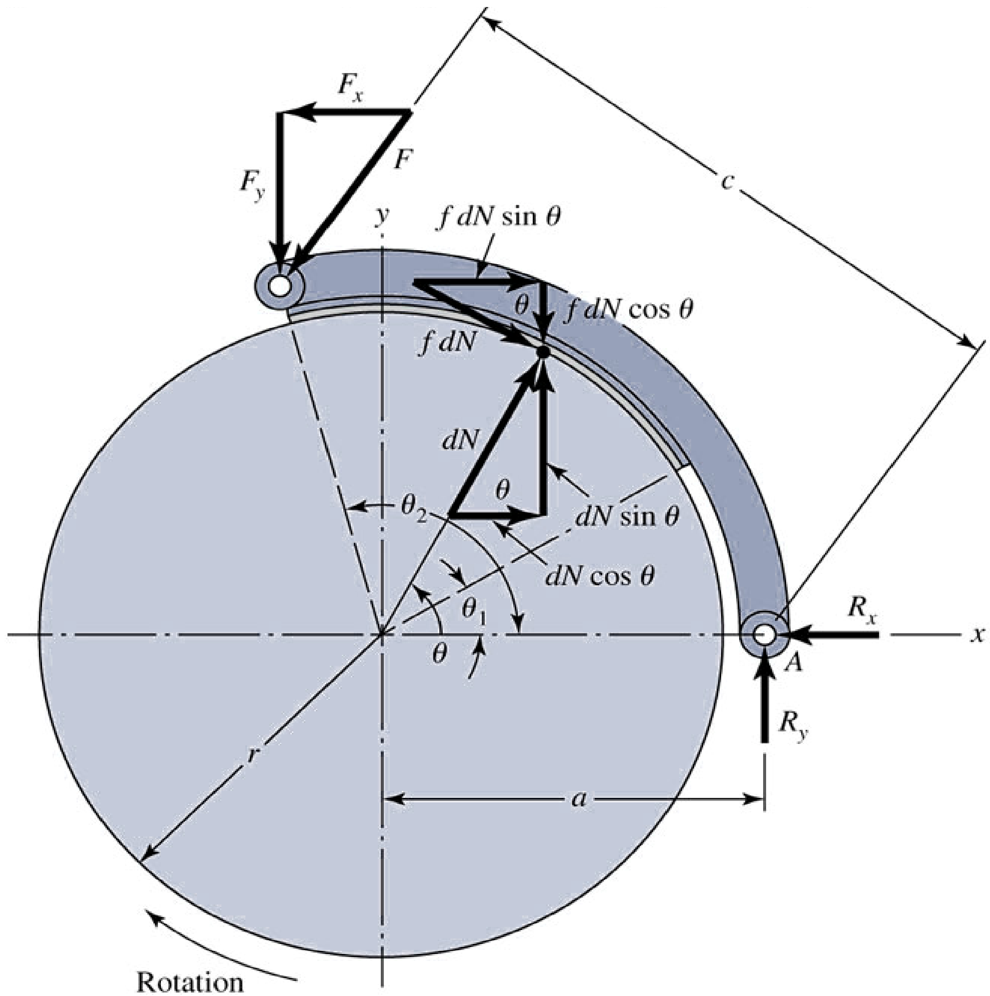
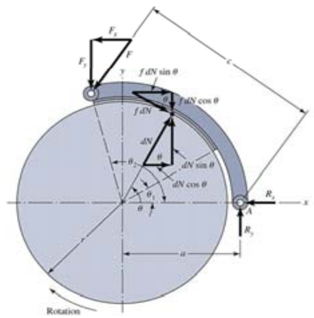


Figure 16-11: Notation of external contacting shoes

- The moments of the frictional and normal forces about the hinge pin are the same as for the internal expanding shoes. Equations (16-2) and (16-3) apply and are repeated here for convenience:



$$M_f = \frac{f p_a b r}{\sin\theta_a} \int_{\theta_1}^{\theta_2} \sin\theta (r - a\cos\theta)\, d\theta \tag{16–2}$$

$$M_N = \frac{p_a b r a}{\sin\theta_a} \int_{\theta_1}^{\theta_2} \sin^2\theta\, d\theta \tag{16–3}$$

Both these equations give positive values for clockwise moments (Fig. 16–11) when used for external contracting shoes. The actuating force must be large enough to balance both moments:

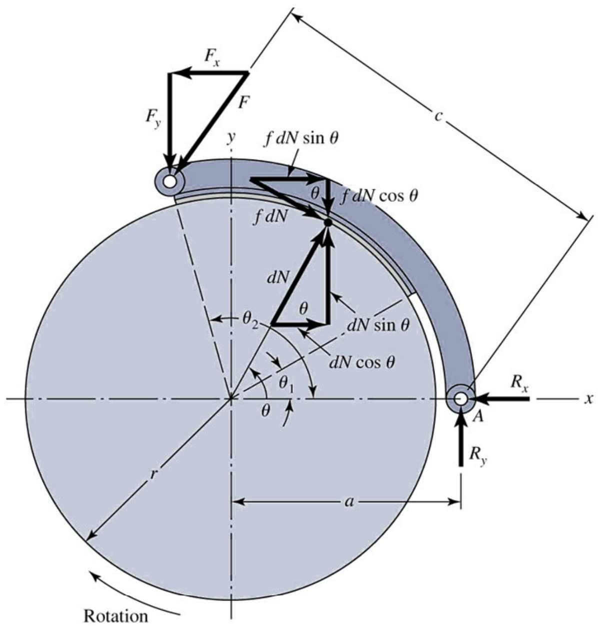
$$F = \frac{M_N + M_f}{c} \tag{16–11}$$

The horizontal and vertical reactions at the hinge pin are found in the same manner as for internal expanding shoes. They are

$$R_x = \int dN \cos\theta + \int f\, dN \sin\theta - F_x \tag{a}$$

$$R_y = \int f\, dN \cos\theta - \int dN \sin\theta + F_y \tag{b}$$

By using Eq. (16–8) and Eq. (c) from Sec. 16–2, we have



$$\begin{aligned} R_x &= \frac{p_a b r}{\sin\theta_a}(A + f B) - F_x \\ R_y &= \frac{p_a b r}{\sin\theta_a}(f A - B) + F_y \end{aligned} \tag{16–12}$$

If the rotation is counterclockwise, the sign of the frictional term in each equation is reversed. Thus Eq. (16–11) for the actuating force becomes

$$F = \frac{M_N - M_f}{c} \tag{16–13}$$

and self-energization exists for counterclockwise rotation. The horizontal and vertical reactions are found, in the same manner as before, to be

$$\begin{aligned} R_x &= \frac{p_a br}{\sin\theta_a}(A - fB) - F_x \\ R_y &= \frac{p_a br}{\sin\theta_a}(-fA - B) + F_y \end{aligned} \tag{16–14}$$

- Note:
  - when external contracting designs are used as clutches, the effect of centrifugal force is to decrease the normal force.
  - Thus, as the speed increases, a larger value of the actuating force *F* is required.

Symmetrically Located Pivot

- The pivot is symmetrically located
- The moment of the friction forces about the pivot is zero.

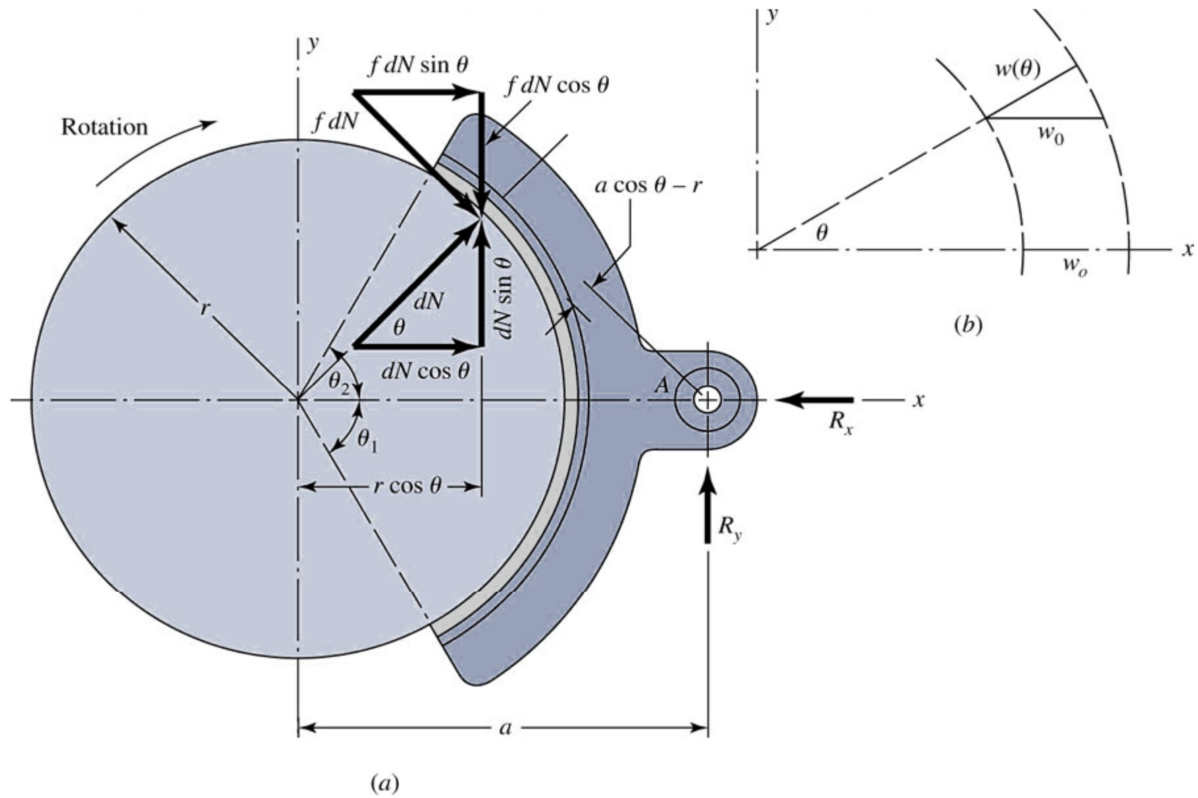


Figure 16-12: (a) Brake with symmertrical pivoted shoe; (b) wear of brake lining

- To get a pressure-distribution relation,
  - we note that lining wear is such as to retain the cylindrical shape, much as a milling machine cutter feeding in the x direction would do to the shoe held in a vise. See Fig. 16-12b.
  - This means the abscissa (x-axis) component of wear is w_o for all positions θ.
  - If wear in the radial direction is expressed as if $w(\theta)$, then

$$w(\theta)= w_o \cos \theta$$

Linear Sliding Wear

- Consider the sliding block depicted in figure 12-38 (page 652) moving along a plate with constant pressure P acting over area A, in the presence of a coefficient of sliding friction f_s .
- The work done by (force = $f_s PA$) during displacement S is: $f_s PAS$ or $f_s PAVt$, where V is the sliding velocity and t is time.

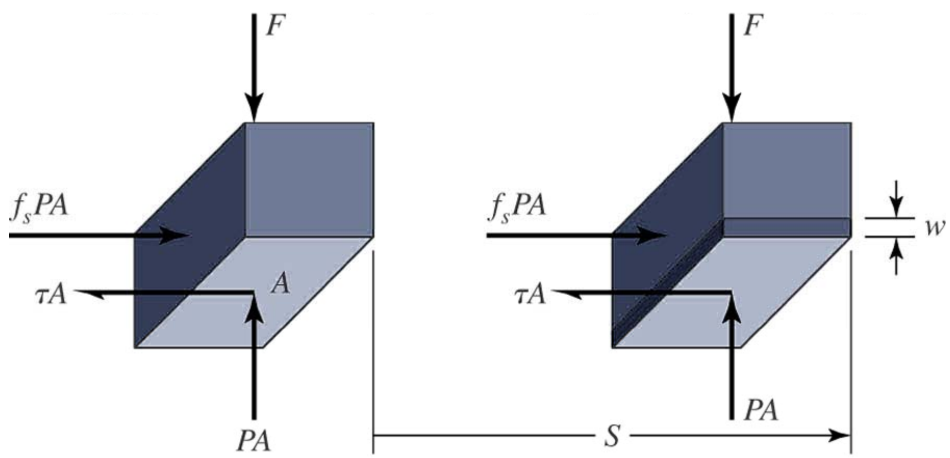


Figure 12-38: Sliding block subjected to wear

- The material volume removed due to wear is wA and is proportional to the work done, that is:

$$wA \propto f_s PAVt,$$

or

$$wA = K f_s PAVt$$

where K is the proportionality factor, which includes f_s, and is determined from laboratory testing.

- The linear wear is then expressed as

$$w = KPVt \qquad (12\text{-}26)$$

- Using Eq. (12-26) to express radial wear $w(\theta)$ as

$$w(\theta) = KPVt$$

  Where K is a material constant, P is pressure, V is rim velocity, and t is time.
- Then, denoting P as $p(\theta)$ above and solving for $p(\theta)$ gives:

$$p(\theta) = \frac{w(\theta)}{KVt} = \frac{w_0 \cos\theta}{KVt}$$

Since all elemental surface areas of the friction material see the same rubbing speed for the same duration, $w_0/(KVt)$ is a constant and

$$p(\theta) = (\text{constant}) \cos\theta = p_a \cos\theta \qquad \text{(c)}$$

where p_a is the maximum value of $p(\theta)$.

Proceeding to the force analysis, we observe from Fig. 16–12a that

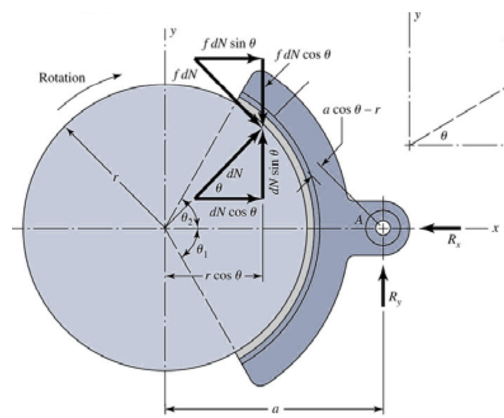
$$dN = pbr\, d\theta \tag{d}$$

or

$$dN = p_a br \cos\theta\, d\theta \tag{e}$$

The distance a to the pivot is chosen by finding where the moment of the frictional forces M_f is zero. First, this ensures that reaction R_y is at the correct location to establish symmetrical wear. Second, a cosinusoidal pressure distribution is sustained, preserving our predictive ability. Symmetry means $\theta_1 = \theta_2$, so

$$M_f = 2\int_0^{\theta_2} (f\,dN)(a\cos\theta - r) = 0$$



Substituting Eq. (e) gives

$$2fp_a br \int_0^{\theta_2} (a\cos^2\theta - r\cos\theta)\,d\theta = 0$$

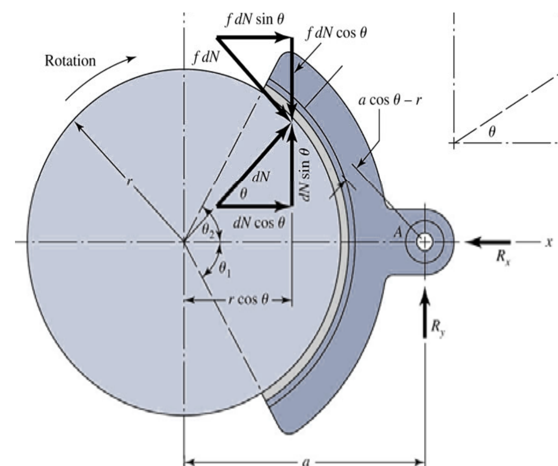
from which

$$a = \frac{4r\sin\theta_2}{2\theta_2 + \sin 2\theta_2} \qquad (16\text{–}15)$$

- The distance *a* depends on the pressure distribution.
- Mislocating the pivot makes (M_f=zero) about a different location, so the brake lining adjusts its local contact pressure (through wear) to compensate.
- The result is unsymmetrical wear, retiring the shoe lining, hence the shoe, sooner.

With the pivot located according to Eq. (16–15), the moment about the pin is zero, and the horizontal and vertical reactions are

$$R_x = 2\int_0^{\theta_2} dN\cos\theta = \frac{p_a b r}{2}(2\theta_2 + \sin 2\theta_2)$$



where, because of symmetry,

$$\int f\,dN\sin\theta = 0$$

Also,

$$R_y = 2\int_0^{\theta_2} f\,dN\cos\theta = \frac{p_a b r f}{2}(2\theta_2 + \sin 2\theta_2) \qquad (16\text{–}17)$$

where

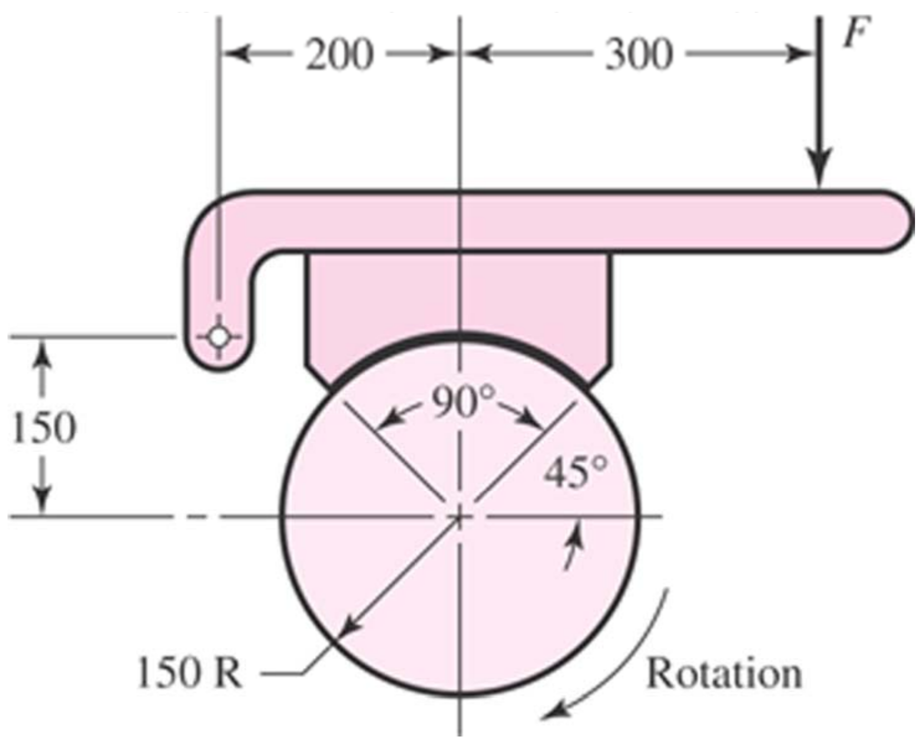
$$\int dN\sin\theta = 0$$

also because of symmetry. Note, too, that $R_x = -N$ and $R_y = -fN$, as might be expected for the particular choice of the dimension a. Therefore the torque is

$$T = afN \qquad (16\text{–}18)$$

Example

The block-type hand brake shown in the figure has a face width of 30 mm and a mean coefficient of friction of 0.25. For an estimated actuating force of 400 N, find the maximum pressure on the shoe and find the braking torque.



Given: Face width $b = 30$ mm, $F = 400$ N, $f = 0.25$.

Preliminaries: $\theta_1 = 45° - \tan^{-1}(150/200) = 8.13°$, $\theta_2 = 98.13°$, $\theta_a = 90°$,
$a = (150^2 + 200^2)^{1/2} = 250$ mm

Eq. (16-2):

$$M_f = \frac{f\,p_a br}{\sin\theta_a}\int_{\theta_1}^{\theta_2}\sin\theta(r - a\cos\theta)\,d\theta = \frac{0.25p_a(0.03)(0.15)}{1}\int_{8.13°}^{98.13°}\sin\theta(150 - 250\cos\theta)\,d\theta$$
$$= 5.59(10^{-5})p_a \text{ N}\cdot\text{m}$$

Eq. (16-3):

$$M_N = \frac{p_a bra}{\sin\theta_a}\int_{\theta_1}^{\theta_2}\sin^2\theta\,d\theta = \frac{p_a(0.03)(0.15)(0.25)}{1}\int_{8.13°}^{98.13°}\sin^2\theta\,d\theta$$
$$= 1.0406(10^{-3})p_a \text{ N}\cdot\text{m}$$

Eq. (16-4): Using $Fc = (M_N - M_f)/_c$, we obtain

$$400 = \frac{(104.06 - 5.59)p_a}{0.5(10^5)} \quad\Rightarrow\quad p_a = 203 \text{ kPa} \quad \textit{Ans.}$$

Eq. (16-6):

$$T = \frac{fp_a br^2\left(\cos\theta_1 - \cos\theta_2\right)}{\sin\theta_a} = \frac{0.25(203)(10^3)(0.03)(0.15^2)(\cos 8.13^\circ - \cos 98.13^\circ)}{1}$$

$$= 38.8\ \text{N}\cdot\text{m} \qquad \textit{Ans.}$$

16.4 Band-Type Clutches and Brakes

- Flexible clutch and brake bands are used in power excavators and in hoisting and other machinery. The analysis follows the notation of Fig. 16-13.

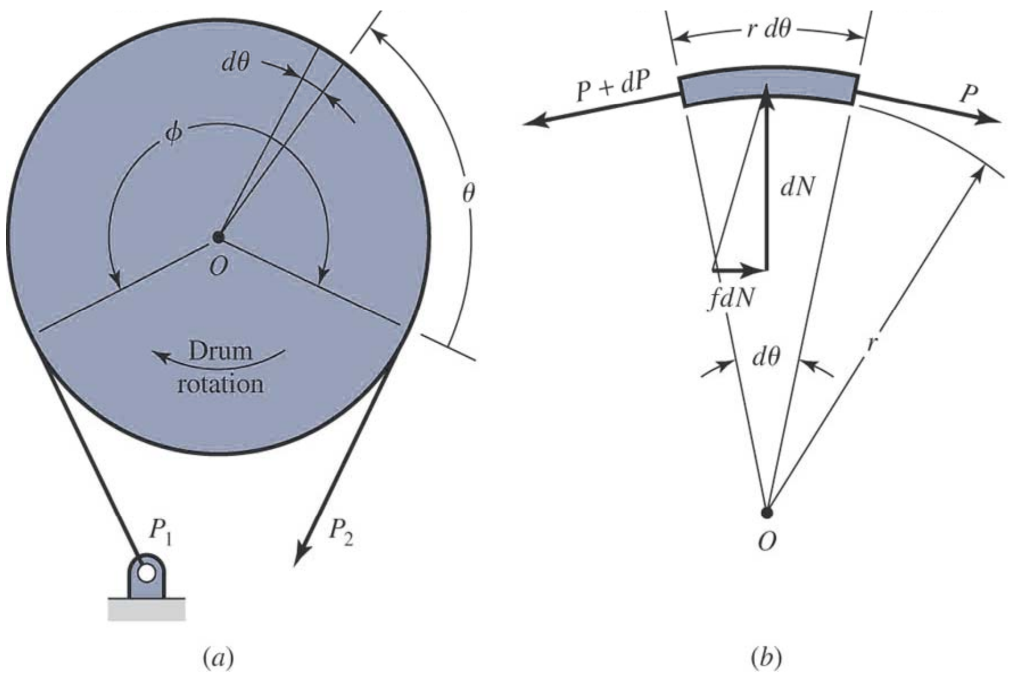


Figure 16-13: Forces on a brake band

- Because of friction and the rotation of the drum, the actuating force P_2 is less than the pin reaction P_1.
- Any element of the band, of angular length $d\theta$, will be in equilibrium under the action of the forces shown in the figure.

- Summing these forces in the vertical direction, we have

$$(P + dP)\sin\frac{d\theta}{2} + P\sin\frac{d\theta}{2} - dN = 0 \qquad (a)$$

$$dN = Pd\theta \qquad (b)$$

since for small angles $\sin d\theta/2 = d\theta/2$. Summing the forces in the horizontal direction gives

$$(P + dP)\cos\frac{d\theta}{2} - P\cos\frac{d\theta}{2} - f\,dN = 0 \qquad (c)$$

$$dP - f\,dN = 0 \qquad (d)$$

Since for small angles, $\cos(d\theta/2) \doteq 1$. Substituting the value of dN from Eq. (b) in (d) and integrating give

$$\int_{P_2}^{P_1} \frac{dP}{P} = f\int_0^{\phi} d\theta \qquad \text{or} \qquad \ln\frac{P_1}{P_2} = f\phi$$

and

$$\frac{P_1}{P_2} = e^{f\phi} \tag{16–19}$$

The torque may be obtained from the equation

$$T = (P_1 - P_2)\frac{D}{2} \tag{16–20}$$

The normal force dN acting on an element of area of width b and length $r\,d\theta$ is

$$dN = pbr\,d\theta \tag{e}$$

where p is the pressure. Substitution of the value of dN from Eq. (b) gives

$$P\,d\theta = pbr\,d\theta$$

Therefore

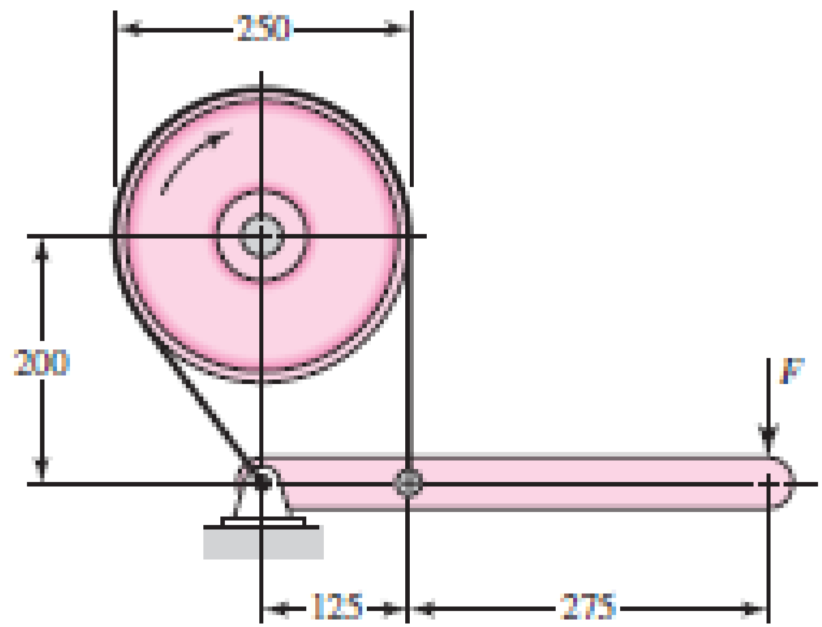
$$p = \frac{P}{br} = \frac{2P}{bD} \tag{16–21}$$

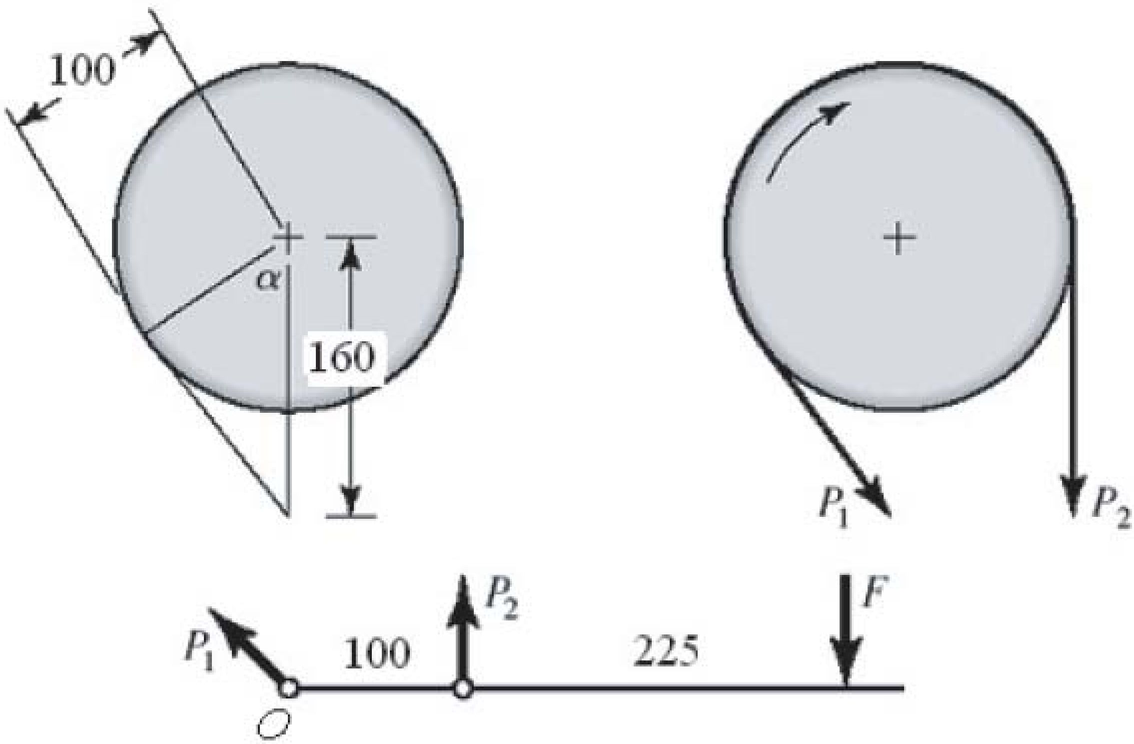
The pressure is therefore proportional to the tension in the band. The maximum pressure p_a will occur at the toe and has the value

$$p_a = \frac{2P_1}{bD} \tag{16–22}$$

Example

The brake shown in the figure has a coefficient of friction of 0.30 and is to operate using a maximum force F of 400 N. If the band width is 50 mm, find the band tensions and the braking torque.





$$\Sigma M_O = 0 = 100\,P_2 - 325\,F \quad \Rightarrow \quad P_2 = 325(300)/100 = 975\text{ N} \quad \textit{Ans.}$$

$$\alpha = \cos^{-1}\left(\frac{100}{160}\right) = 51.32^\circ$$

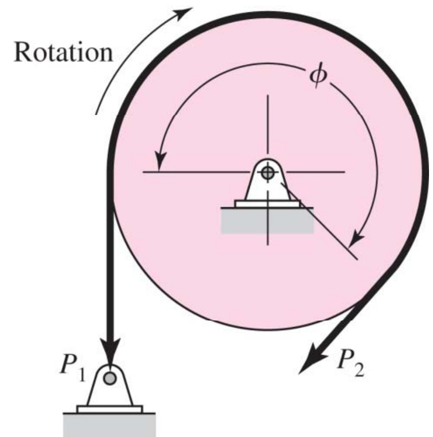
$$\phi = 270^\circ - 51.32^\circ = 218.7^\circ$$

$$f\phi = 0.30(218.7)\left(\pi / 180^\circ\right) = 1.145$$

$$P_1 = P_2 \exp(f\phi) = 975\exp(1.145) = 3064\text{ N} \qquad \textit{Ans.}$$

$$T = \left(P_1 - P_2\right)(D/2) = (3064 - 975)(200/2)$$

$$= 209\left(10^3\right)\text{ N}\cdot\text{mm} = 209\text{ N}\cdot\text{m} \quad \textit{Ans.}$$



Problem 16–11

16-11 Given: $D = 350$ mm, $b = 100$ mm, $p_a = 620$ kPa, $f = 0.30$, $\phi = 270°$.

Eq. (16-22):

$$P_1 = \frac{p_a bD}{2} = \frac{620(0.100)0.350}{2} = 10.85 \text{ kN} \qquad \textit{Ans.}$$

$$f\phi = 0.30(270°)(\pi / 180°) = 1.414$$

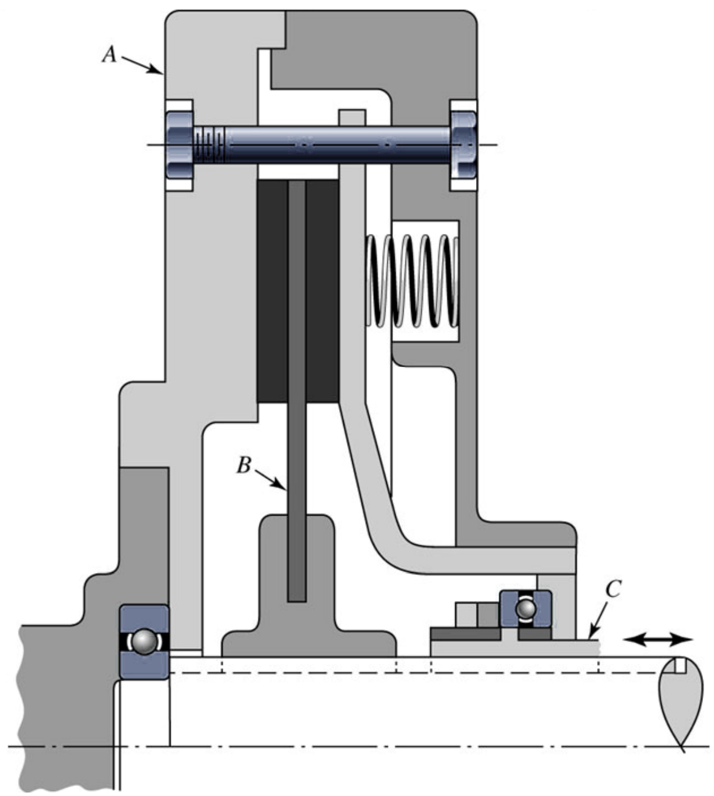
Eq. (16-19): $P_2 = P_1 \exp(-f\phi) = 10.85 \exp(-1.414) = 2.64$ kN *Ans.*

$$T = (P_1 - P_2)(D / 2) = (10.85 - 2.64)(0.350 / 2) = 1.437 \text{ kN} \cdot \text{m}$$

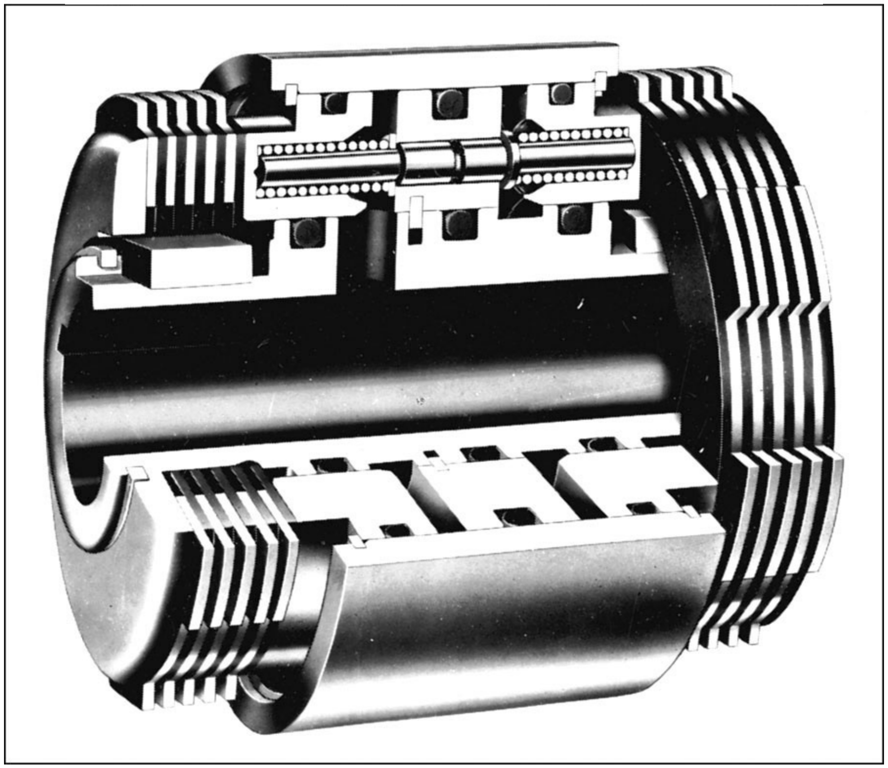
16.5 Frictional-Contact Axial Clutches

- An axial clutch is one in which the mating frictional members are moved in a direction parallel to the shaft.
- One of the earliest of these is the cone clutch, which is simple in construction and quite powerful.
- Advantages of the disk clutch include
  - the freedom from centrifugal effects,
  - the large frictional area that can be installed in a small space,
  - the more effective heat-dissipation surfaces,
  - the favorable pressure distribution.

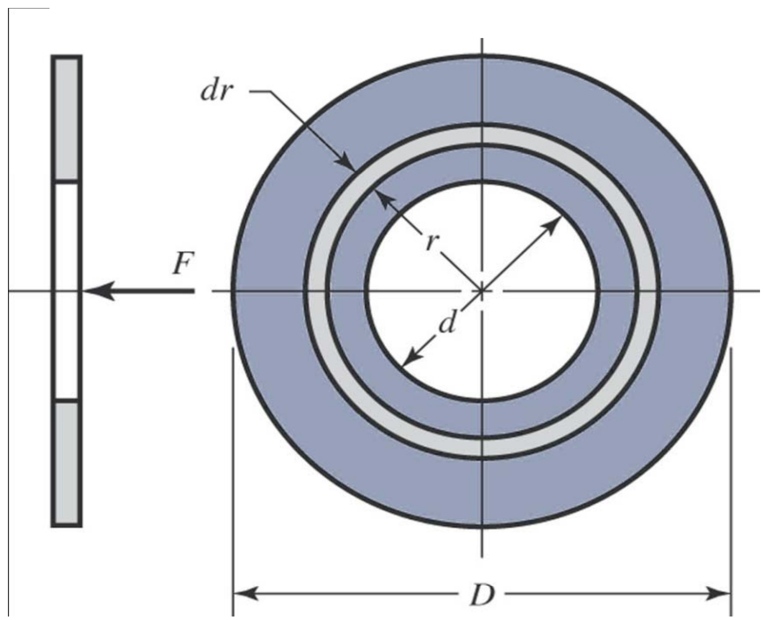
- Figure 16-14: Cross-sectional view of a single-plate clutch; A driver; B, driven plate (keyed to driven shaft); C actuator



- A multiple-disk clutch-brake is shown in Fig. 16-15.



- Let us now determine the capacity of such a clutch or brake in terms of the material and geometry.
- Figure 16-16 shows a friction disk having an outside diameter *D* and an inside di-ameter *d*.
- We are interested in obtaining the axial force *F* necessary to produce a certain torque *T* and pressure *p*.



- Two methods of solving the problem, depending upon the con-struction of the clutch, are in general use. If the disks are rigid, then the greatest amount of wear will at first occur in the outer areas, since the work of friction is greater in those areas.
- After a certain amount of wear has taken place, the pressure distribution will change so as to permit the wear to be uniform. This is the basis of the first method of solution.
- Another method of construction employs springs to obtain a uniform pressure over the area. It is this assumption of uniform pressure that is used in the second method of solution.

Uniform Wear

- After initial wear has taken place and the disks have worn down to a point where uniform wear is established, the axial wear can be expressed by Eq. (12-27) as

$$w = f_1 f_2 KPVt$$

in which only P and V vary from place to place in the rubbing surfaces. By definition uniform wear is constant from place to place; therefore,

$$PV = (\text{constant}) = C_1$$

$$pr\omega = C_2$$

$$pr = C_3 = p_{\max} r_i = p_a r_i = p_a \frac{d}{2} \qquad (a)$$

We can take an expression from Eq. (a), which is the condition for having the same amount of work done at radius r as is done at radius $d/2$. Referring to Fig. 16–16, we have an element of area of radius r and thickness dr. The area of this element is $2\pi r\,dr$, so that the normal force acting upon this element is $dF = 2\pi pr\,dr$. We can find the total normal force by letting r vary from $d/2$ to $D/2$ and integrating. Thus, with pr constant,

$$F = \int_{d/2}^{D/2} 2\pi pr\,dr = \pi p_a d \int_{d/2}^{D/2} dr = \frac{\pi p_a d}{2}(D - d) \qquad (16\text{–}23)$$

The torque is found by integrating the product of the frictional force and the radius:

$$T = \int_{d/2}^{D/2} 2\pi f pr^2\,dr = \pi f p_a d \int_{d/2}^{D/2} r\,dr = \frac{\pi f p_a d}{8}(D^2 - d^2) \qquad (16\text{–}24)$$

By substituting the value of F from Eq. (16–23) we may obtain a more convenient expression for the torque. Thus

$$T = \frac{Ff}{4}(D + d) \tag{16–25}$$

In use, Eq. (16–23) gives the actuating force for the selected maximum pressure p_a. This equation holds for any number of friction pairs or surfaces. Equation (16–25), however, gives the torque capacity for only a single friction surface.

Uniform Pressure

When uniform pressure can be assumed over the area of the disk, the actuating force F is simply the product of the pressure and the area. This gives

$$F = \frac{\pi p_a}{4}(D^2 - d^2) \tag{16–26}$$

As before, the torque is found by integrating the product of the frictional force and the radius:

$$T = 2\pi f p \int_{d/2}^{D/2} r^2\, dr = \frac{\pi f p}{12}(D^3 - d^3) \tag{16–27}$$

Since $p = p_a$, from Eq. (16–26) we can rewrite Eq. (16–27) as

$$T = \frac{Ff}{3}\,\frac{D^3 - d^3}{D^2 - d^2} \tag{16–28}$$

16.6 Disk Brakes

- There is no fundamental difference between a disk clutch and a disk brake.
- The analysis of the preceding section applies to disk brakes too.
- We have seen that rim or drum brakes can be designed for self-energization.
- While this feature is important in reducing the braking effort required, it also has a disadvantage.

- When drum brakes are used as vehicle brakes, only a slight change in the coefficient of friction will cause a large change in the pedal force required for braking.
- A not unusual 30 percent reduction in the coefficient of friction due to a temperature change or moisture, for example, can result in a 50 percent change in the pedal force required to obtain the same braking torque obtainable prior to the change.
- The disk brake has no self-energization, and hence is not so susceptible to changes in the coefficient of friction.

- Another type of disk brake is *the floating caliper brake,* shown in Fig. 16-18.
- The caliper supports a single floating piston actuated by hydraulic pressure.
- The action is much like that of a screw clamp, with the piston replacing the function of the screw.

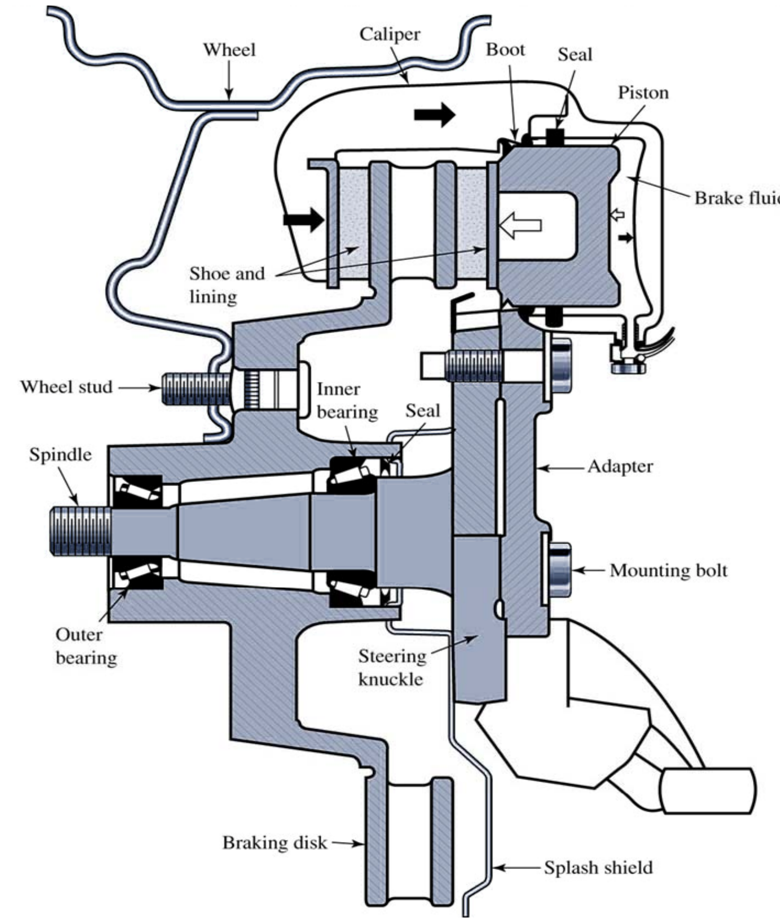
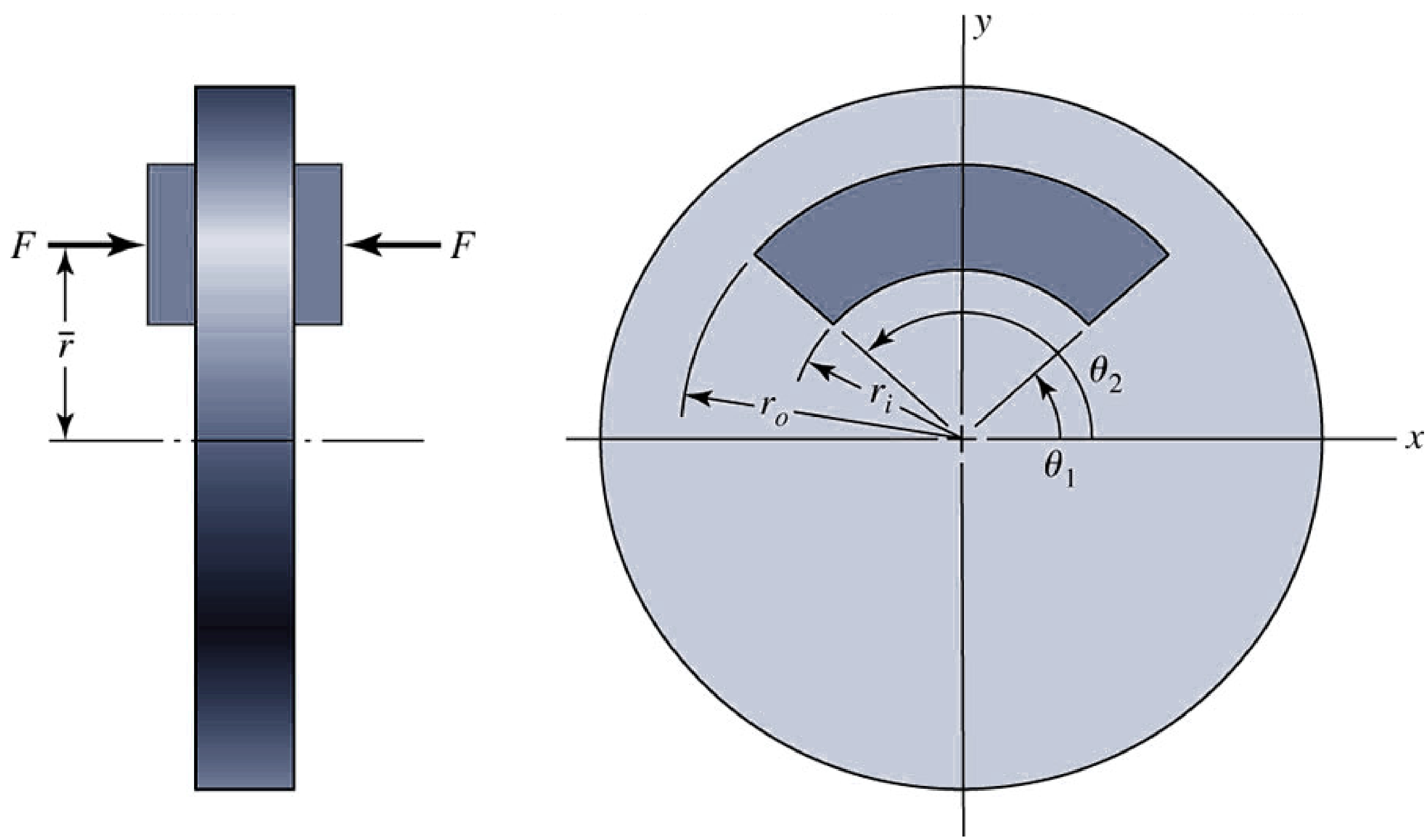


Figure 16-18: An automotive disk brake

- The floating action also compensates for wear and ensures a fairly constant pressure over the area of the friction pads.
- The seal and boot of Fig. 16-18 are designed to obtain clearance by backing off from the piston when the piston is released.
- Caliper brakes (named for the nature of the actuating linkage) and disk brakes (named for the shape of the unlined surface) press friction material against the face(s) of a rotating disk.
- Depicted in Fig. 16-19 is the geometry of an annular-pad brake contact area. The governing axial wear equation is Eq. (12-27):

$$w = f_1 f_2 KPVt$$

Figure 16-19: Geometry of contact area of an annular-pad segment of a caliper brake



The coordinate $\bar{r}$ locates the line of action of force F that intersects the y axis. Of interest also is the effective radius r_e, which is the radius of an equivalent shoe of infinitesimal radial thickness. If p is the local contact pressure, the actuating force F and the friction torque T are given by

$$F = \int_{\theta_1}^{\theta_2} \int_{r_i}^{r_o} pr\,dr\,d\theta = (\theta_2 - \theta_1) \int_{r_i}^{r_o} pr\,dr \tag{16–29}$$

$$T = \int_{\theta_1}^{\theta_2} \int_{r_i}^{r_o} fpr^2\,dr\,d\theta = (\theta_2 - \theta_1) f \int_{r_i}^{r_o} pr^2\,dr \tag{16–30}$$

The equivalent radius r_e can be found from $fFr_e = T$, or

$$r_e = \frac{T}{fF} = \frac{\int_{r_i}^{r_o} pr^2\,dr}{\int_{r_i}^{r_o} pr\,dr} \tag{16–31}$$

The locating coordinate $\bar{r}$ of the activating force is found by taking moments about the x axis:

$$M_x = F\bar{r} = \int_{\theta_1}^{\theta_2} \int_{r_i}^{r_o} pr(r \sin\theta)\, dr\, d\theta = (\cos\theta_1 - \cos\theta_2) \int_{r_i}^{r_o} pr^2\, dr$$

$$\bar{r} = \frac{M_x}{F} = \frac{(\cos\theta_1 - \cos\theta_2)}{\theta_2 - \theta_1} r_e \qquad (16\text{–}32)$$

Uniform Wear

- It is clear from Eq. (12-27) that for the axial wear to be the same everywhere, the prod-uct $P\ V$ must be a constant. From Eq. (a), Sec. 16-5, the pressure p can be expressed in terms of the largest allowable pressure p_a (which occurs at the inner radius r_i) as

$p = p_a r_i / r$. Equation (16–29) becomes

$$F = (\theta_2 - \theta_1) p_a r_i (r_o - r_i) \tag{16–33}$$

Equation (16–30) becomes

$$T = (\theta_2 - \theta_1) f p_a r_i \int_{r_i}^{r_o} r\, dr = \frac{1}{2}(\theta_2 - \theta_1) f p_a r_i \left(r_o^2 - r_i^2\right) \tag{16–34}$$

Equation (16–31) becomes

$$r_e = \frac{p_a r_i \displaystyle\int_{r_i}^{r_o} r\, dr}{p_a r_i \displaystyle\int_{r_i}^{r_o} dr} = \frac{r_o^2 - r_i^2}{2} \frac{1}{r_o - r_i} = \frac{r_o + r_i}{2} \tag{16–35}$$

Equation (16–32) becomes

$$\bar{r} = \frac{\cos\theta_1 - \cos\theta_2}{\theta_2 - \theta_1} \frac{r_o + r_i}{2} \tag{16–36}$$

Uniform Pressure

In this situation, approximated by a new brake, $p = p_a$. Equation (16–29) becomes

$$F = (\theta_2 - \theta_1) p_a \int_{r_i}^{r_o} r\,dr = \frac{1}{2}(\theta_2 - \theta_1) p_a \left(r_o^2 - r_i^2\right) \tag{16–37}$$

Equation (16–30) becomes

$$T = (\theta_2 - \theta_1) f p_a \int_{r_i}^{r_o} r^2\,dr = \frac{1}{3}(\theta_2 - \theta_1) f p_a \left(r_o^3 - r_i^3\right) \tag{16–38}$$

Equation (16–31) becomes

$$r_e = \frac{p_a \displaystyle\int_{r_i}^{r_o} r^2\,dr}{p_a \displaystyle\int_{r_i}^{r_o} r\,dr} = \frac{r_o^3 - r_i^3}{3} \frac{2}{r_o^2 - r_i^2} = \frac{2}{3} \frac{r_o^3 - r_i^3}{r_o^2 - r_i^3} \tag{16–39}$$

Equation (16–32) becomes

$$\bar{r} = \frac{\cos\theta_1 - \cos\theta_2}{\theta_2 - \theta_1} \frac{2}{3} \frac{r_o^3 - r_i^3}{r_o^2 - r_i^2} = \frac{2}{3} \frac{r_o^3 - r_i^3}{r_o^2 - r_i^2} \frac{\cos\theta_1 - \cos\theta_2}{\theta_2 - \theta_1} \tag{16–40}$$

Example

A plate clutch has a single pair of mating friction surfaces 300 mm OD by 225 mm ID. The mean value of the coefficient of friction is 0.25, and the actuating force is 5 kN.

a) Find the maximum pressure and the torque capacity using the uniform-wear model.
b) Find the maximum pressure and the torque capacity using the uniform-pressure model.

Example 2

A hydraulically operated multidisk plate clutch has an effective disk outer diameter of 150 mm and an inner diameter of 100 mm. The coefficient of friction is 0.24, and the limiting pressure is 800 kPa. There are six planes of sliding present.

a) Using the uniform wear model, estimate the axial force F and the torque T.
b) Let the inner diameter of the friction pairs d be a variable.

16-7 Cone Clutches and Brakes

- It consists of a cup keyed or splined to one of the shafts, a cone that must slide axially on splines or keys on the mating shaft, and a helical spring to hold the clutch in engagement.
- The clutch is disengaged by means of a fork that fits into the shifting groove on the friction cone.
- The cone angle α and the diameter and face width of the cone are the important geometric design parameters.

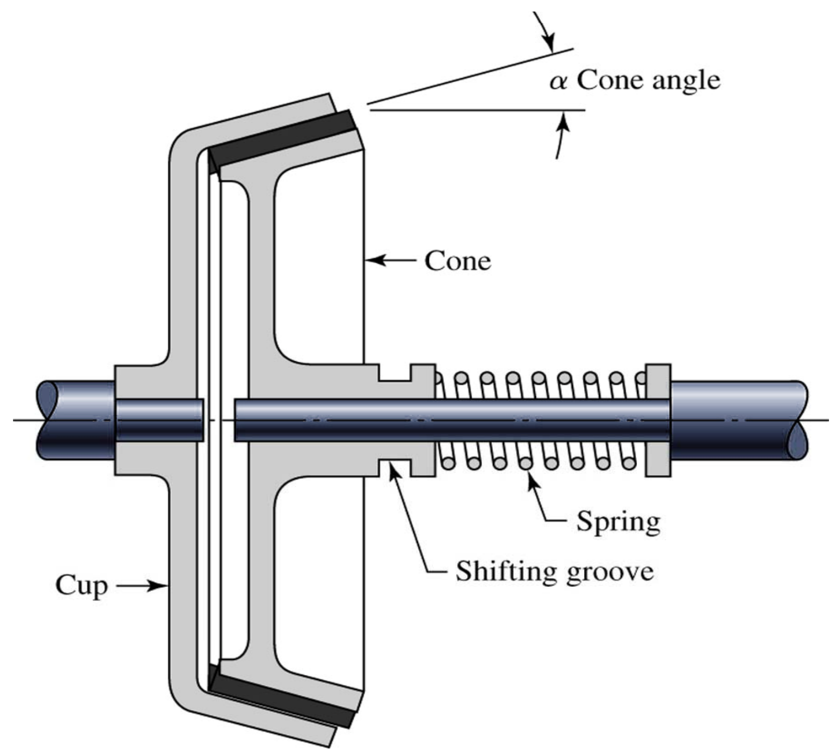


Figure 16-21: Cross section of a cone clutch

- If the cone angle (α) is too small, say, less than about 8°, then the force required to disengage the clutch may be quite large.
- And the wedging effect lessens rapidly when larger cone angles are used.
- Depending upon the characteristics of the friction materials, a good compromise can usually be found using cone angles between 10 and 15°.

- To find a relation between the operating force F and the torque transmitted, designate the dimensions of the friction cone as shown in Figure 16-22.
- As in the case of the axial clutch, we can obtain one set of relations for a uniform-wear and another set for a uniform-pressure assumption.

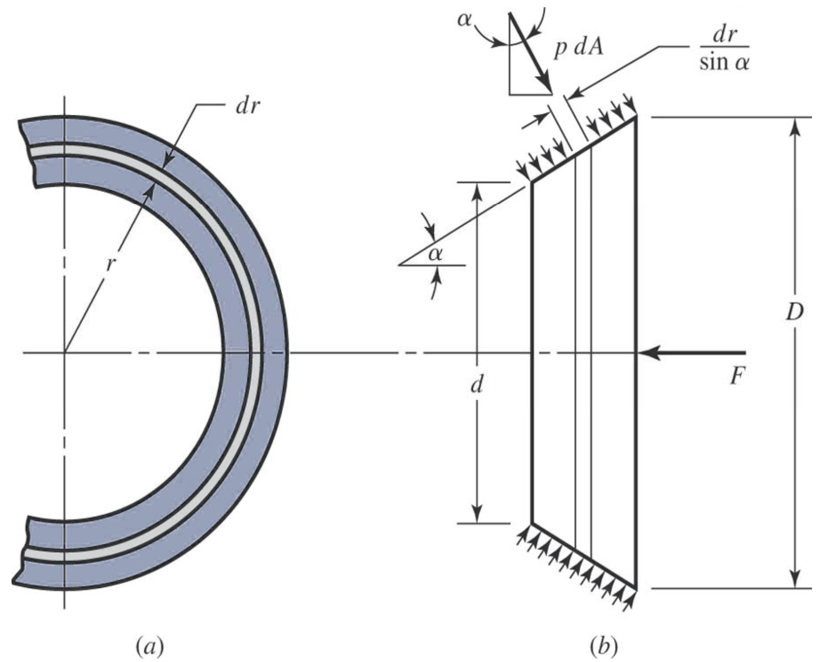


Figure 16-22: Contact area of a cone clutch

Uniform Wear

The pressure relation is the same as for the axial clutch:

$$p = p_a \frac{d}{2r} \tag{a}$$

Next, referring to Fig. 16–22, we see that we have an element of area dA of radius r and width $dr/\sin\alpha$. Thus $dA = (2\pi r\,dr)/\sin\alpha$. As shown in Fig. 16–22, the operating force will be the integral of the axial component of the differential force $p\,dA$. Thus

$$\begin{aligned} F &= \int p\,dA \sin\alpha = \int_{d/2}^{D/2} \left(p_a \frac{d}{2r}\right)\left(\frac{2\pi r\,dr}{\sin\alpha}\right)(\sin\alpha) \\ &= \pi p_a d \int_{d/2}^{D/2} dr = \frac{\pi p_a d}{2}(D-d) \end{aligned} \tag{16–44}$$

which is the same result as in Eq. (16–23).

The differential friction force is $f p \, dA$, and the torque is the integral of the product of this force with the radius. Thus

$$\begin{aligned} T &= \int r f p \, dA = \int_{d/2}^{D/2} (r f) \left(p_a \frac{d}{2r} \right) \left(\frac{2\pi r \, dr}{\sin \alpha} \right) \\ &= \frac{\pi f p_a d}{\sin \alpha} \int_{d/2}^{D/2} r \, dr = \frac{\pi f p_a d}{8 \sin \alpha} (D^2 - d^2) \end{aligned} \tag{16–45}$$

Note that Eq. (16–24) is a special case of Eq. (16–45), with $\alpha = 90°$. Using Eq. (16–44), we find that the torque can also be written

$$T = \frac{F f}{4 \sin \alpha} (D + d) \tag{16–46}$$

Uniform Pressure

Using $p = p_a$, the actuating force is found to be

$$F = \int p_a \, dA \sin\alpha = \int_{d/2}^{D/2} (p_a) \left(\frac{2\pi r \, dr}{\sin\alpha} \right) (\sin\alpha) = \frac{\pi p_a}{4} (D^2 - d^2) \qquad (16\text{–}47)$$

The torque is

$$T = \int r f p_a \, dA = \int_{d/2}^{D/2} (r f p_a) \left(\frac{2\pi r \, dr}{\sin\alpha} \right) = \frac{\pi f p_a}{12 \sin\alpha} (D^3 - d^3) \qquad (16\text{–}48)$$

Using Eq. (16–47) in Eq. (16–48) gives

$$T = \frac{Ff}{3\sin\alpha} \frac{D^3 - d^3}{D^2 - d^2} \qquad (16\text{–}49)$$

As in the case of the axial clutch, we can write Eq. (16–46) dimensionlessly as

$$\frac{T \sin \alpha}{f F d} = \frac{1 + d/D}{4} \tag{b}$$

and write Eq. (16–49) as

$$\frac{T \sin \alpha}{f F d} = \frac{1}{3} \frac{1 - (d/D)^3}{1 - (d/D)^2} \tag{c}$$

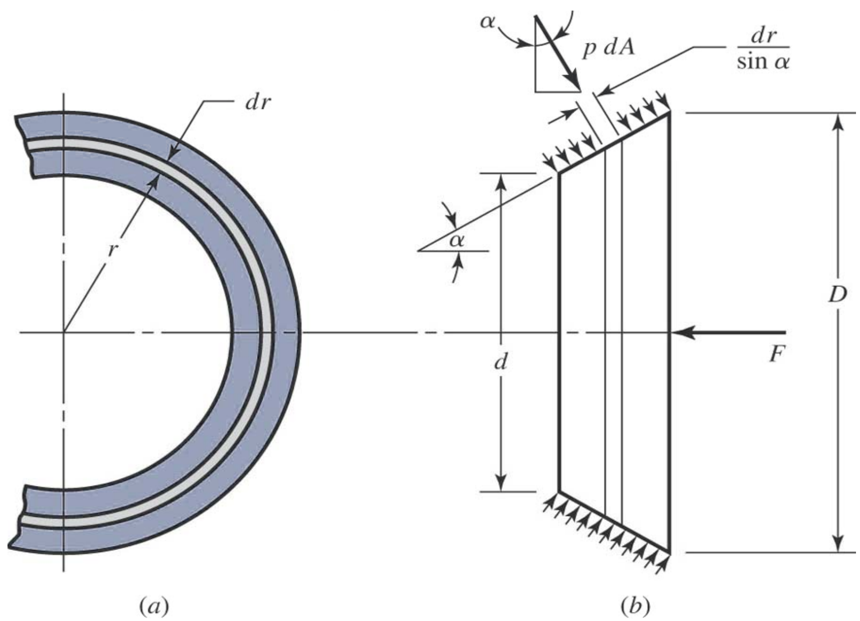
This time there are six (T, α, f, F, D, and d) parameters and four pi terms:

$$\pi_1 = \frac{T}{FD} \qquad \pi_2 = f \qquad \pi_3 = \sin \alpha \qquad \pi_4 = \frac{d}{D}$$

As in Fig. 16–17, we plot $T \sin \alpha/(f F D)$ as ordinate and d/D as abscissa. The plots and conclusions are the same. There is little reason for using equations other than Eqs. (16–44), (16–45), and (16–46).

Example

A cone clutch has *D = 330 mm, d = 306 mm, a cone length of 60 mm, and a coefficient of* friction of 0.26. A torque of 200 N · m is to be transmitted. For this requirement, estimate the actuating force and pressure by both models.



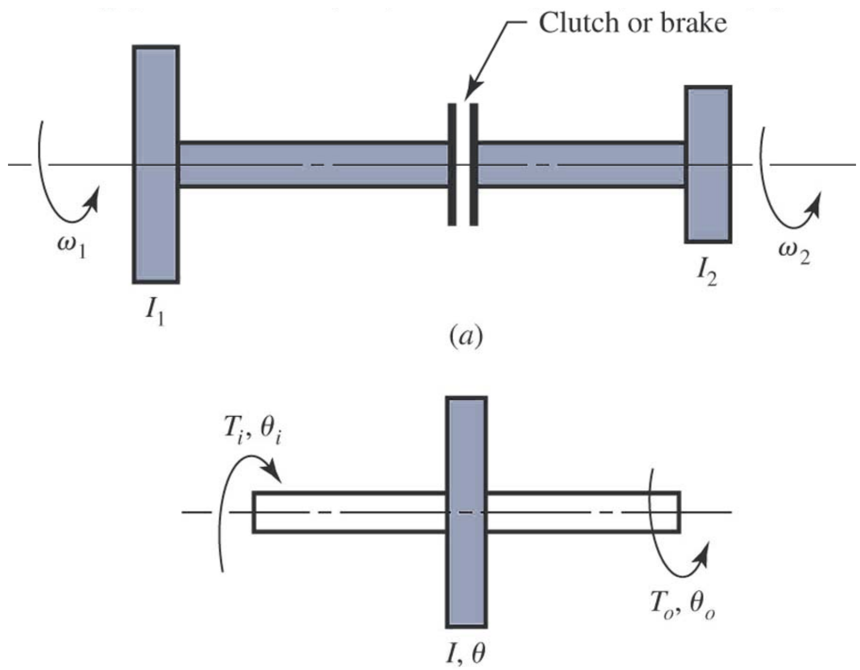
16-8 Energy Considerations

- When the rotating members of a machine are caused to stop by means of a brake, the kinetic energy of rotation must be absorbed by the brake.
- This energy appears in the brake in the form of heat.
- In the same way, when the members of a machine that are initially at rest are brought up to speed, slipping must occur in the clutch until the driven members have the same speed as the driver. Kinetic energy is absorbed during slippage of either a clutch or a brake, and this energy appears as heat.

- We have seen how the torque capacity of a clutch or brake depends upon the coefficient of friction of the material and upon a safe normal pressure.
- However, the character of the load may be such that, if this torque value is permitted, the clutch or brake may be destroyed by its own generated heat.
- The capacity of a clutch is therefore limited by two factors, the characteristics of the material and the ability of the clutch to dissipate heat.

- In this section, the amount of heat generated by a clutching or braking operation will be considered.
- If the heat is generated faster than it is dissipated, we have a temperature rise problem. (that is the subject of the next section and it will not cover in this class)

- To get a clear picture of what happens during a simple clutching or braking operation, refer to Fig. 16-la, which is a mathematical model of a two-inertia system connected by a clutch.
- As shown, inertias I_1 and I_2 have initial angular velocities of ω_1 and ω_2, respectively.
- During the clutch operation both angular velocities change and eventually become equal.
- We assume that the two shafts are rigid and that the clutch torque is constant.



(a)

(b)

Writing the equation of motion for inertia 1 gives

$$I_1\ddot{\theta}_1 = -T \tag{a}$$

where $\ddot{\theta}_1$ is the angular acceleration of I_1 and T is the clutch torque. A similar equation for I_2 is

$$I_2\ddot{\theta}_2 = T \tag{b}$$

We can determine the instantaneous angular velocities $\dot{\theta}_1$ and $\dot{\theta}_2$ of I_1 and I_2 after any period of time t has elapsed by integrating Eqs. (a) and (b). The results are

$$\dot{\theta}_1 = -\frac{T}{I_1}t + \omega_1 \tag{c}$$

$$\dot{\theta}_2 = \frac{T}{I_2}t + \omega_2 \tag{d}$$

where $\dot{\theta}_1 = \omega_1$ and $\dot{\theta}_2 = \omega_2$ at $t = 0$. The difference in the velocities, sometimes called the relative velocity, is

$$\begin{aligned}\dot{\theta} = \dot{\theta}_1 - \dot{\theta}_2 &= -\frac{T}{I_1}t + \omega_1 - \left(\frac{T}{I_2}t + \omega_2\right) \\ &= \omega_1 - \omega_2 - T\left(\frac{I_1 + I_2}{I_1 I_2}\right)t\end{aligned} \tag{16–50}$$

The clutching operation is completed at the instant in which the two angular velocities $\dot{\theta}_1$ and $\dot{\theta}_2$ become equal. Let the time required for the entire operation be t_1. Then $\dot{\theta} = 0$ when $\dot{\theta}_1 = \dot{\theta}_2$, and so Eq. (16–50) gives the time as

$$t_1 = \frac{I_1 I_2(\omega_1 - \omega_2)}{T(I_1 + I_2)} \tag{16–51}$$

This equation shows that the time required for the engagement operation is directly proportional to the velocity difference and inversely proportional to the torque.

We have assumed the clutch torque to be constant. Therefore, using Eq. (16–50), we find the rate of energy-dissipation during the clutching operation to be

$$u = T\dot{\theta} = T\left[\omega_1 - \omega_2 - T\left(\frac{I_1 + I_2}{I_1 I_2}\right)t\right] \tag{e}$$

This equation shows that the energy-dissipation rate is greatest at the start, when $t = 0$.

The total energy dissipated during the clutching operation or braking cycle is obtained by integrating Eq. (e) from $t = 0$ to $t = t_1$. The result is found to be

$$\begin{aligned} E &= \int_0^{t_1} u\,dt = T\int_0^{t_1}\left[\omega_1 - \omega_2 - T\left(\frac{I_1 + I_2}{I_1 I_2}\right)t\right]dt \\ &= \frac{I_1 I_2(\omega_1 - \omega_2)^2}{2(I_1 + I_2)} \end{aligned} \tag{16–52}$$

where Eq. (16–51) was employed. Note that the energy dissipated is proportional to the velocity difference squared and is independent of the clutch torque.

Note that E in Eq. (16–52) is the energy lost or dissipated; this is the energy that is absorbed by the clutch or brake. If the inertias are expressed in U.S. customary units ($\mathrm{lbf \cdot in \cdot s^2}$), then the energy absorbed by the clutch assembly is in $\mathrm{in \cdot lbf}$. Using these units, the heat generated in Btu is

$$H = \frac{E}{9336} \tag{16–53}$$

In SI, the inertias are expressed in kilogram-meter^2 units, and the energy dissipated is expressed in joules.

16-10 Friction Materials

- A brake or friction clutch should have the following lining material characteristics to a degree that is dependent on the severity of service:
  - High and reproducible coefficient of friction
  - Imperviousness to environmental conditions, such as moisture
  - The ability to withstand high temperatures, together with good thermal conductivity and diffusivity, as well as high specific heat capacity
  - Good resiliency
  - High resistance to wear, scoring, and galling
  - Compatible with the environment
  - Flexibility

- Table 16-2 gives area of friction surface required for several braking powers. Table 16-3 gives important characteristics of some friction materials for brakes and clutches.

Table 16-2: Area of Friction Material Required for a Given Average Brake Power

Duty Cycle	Typical Applications	Band and Drum Brakes	Plate Disk Brakes	Caliper Disk Brakes
		Ratio of Area to Average Braking Power, in^2/(Btu/s)		
Infrequent	Emergency brakes	0.85	2.8	0.28
Intermittent	Elevators, cranes, and winches	2.8	7.1	0.70
Heavy-duty	Excavators, presses	5.6–6.9	13.6	1.41

- The manufacture of friction materials is a highly specialized process, and it is advisable to consult manufacturers' catalogs and handbooks, as well as manufacturers directly, in selecting friction materials for specific applications.

- Selection involves a consideration of the many characteristics as well as the standard sizes available.

  - The *woven-cotton lining* is produced as a fabric belt that is impregnated with resins and polymerized. It is used mostly in heavy machinery and is usually supplied in rolls up to 50 ft in length. Thicknesses available range from 1/8 to 1 in, in widths up to about 12 in.

- A *woven-asbestos lining* is made in a similar manner to the cotton lining and may also contain metal particles. It is not quite as flexible as the cotton lining and comes in a smaller range of sizes. Along with the cotton lining, the asbestos lining was widely used as a brake material in heavy machinery.
- *Molded-asbestos linings* contain asbestos fiber and friction modifiers; a thermoset polymer is used, with heat, to form a rigid or semirigid molding. The principal use was in drum brakes.

- *Molded-asbestos pads* are similar to molded linings but have no flexibility; they were used for both clutches and brakes.
- *Sintered-metal pads* are made of a mixture of copper and/or iron particles with friction modifiers, molded under high pressure and then heated to a high temperature to fuse the material. These pads are used in both brakes and clutches for heavy-duty applications.
- *Cermet pads* are similar to the sintered-metal pads and have a substantial ceramic content.

Table 16-3 Characteristics of friction materials for brakes and Clutches

Material	Friction Coefficient f	Maximum Pressure p_{max}, psi	Maximum Temperature Instantaneous, °F	Maximum Temperature Continuous, °F	Maximum Velocity V_{max}, ft/min	Applications
Cermet	0.32	150	1500	750		Brakes and clutches
Sintered metal (dry)	0.29–0.33	300–400	930–1020	570–660	3600	Clutches and caliper disk brakes
Sintered metal (wet)	0.06–0.08	500	930	570	3600	Clutches
Rigid molded asbestos (dry)	0.35–0.41	100	660–750	350	3600	Drum brakes and clutches
Rigid molded asbestos (wet)	0.06	300	660	350	3600	Industrial clutches
Rigid molded asbestos pads	0.31–0.49	750	930–1380	440–660	4800	Disk brakes
Rigid molded nonasbestos	0.33–0.63	100–150		500–750	4800–7500	Clutches and brakes
Semirigid molded asbestos	0.37–0.41	100	660	300	3600	Clutches and brakes
Flexible molded asbestos	0.39–0.45	100	660–750	300–350	3600	Clutches and brakes
Wound asbestos yarn and wire	0.38	100	660	300	3600	Vehicle clutches
Woven asbestos yarn and wire	0.38	100	500	260	3600	Industrial clutches and brakes
Woven cotton	0.47	100	230	170	3600	Industrial clutches and brakes
Resilient paper (wet)	0.09–0.15	400	300		$PV < 500\,000$ psi · ft/min	Clutches and transmission bands

Table 16-4: Some Properties of Brake Linings

	Woven Lining	Molded Lining	Rigid Block
Compressive strength, kpsi	10–15	10–18	10–15
Compressive strength, MPa	70–100	70–125	70–100
Tensile strength, kpsi	2.5–3	4–5	3–4
Tensile strength, MPa	17–21	27–35	21–27
Max. temperature, °F	400–500	500	750
Max. temperature, °C	200–260	260	400
Max. speed, ft/min	7500	5000	7500
Max. speed, m/s	38	25	38
Max. pressure, psi	50–100	100	150
Max. pressure, kPa	340–690	690	1000
Frictional coefficient, mean	0.45	0.47	0.40–45

- Table 16-4 lists properties of typical brake linings. The linings may consist of a mixture of fibers to provide strength and ability to withstand high temperatures, various friction particles to obtain a degree of wear resistance as well as a higher coefficient of friction, and bonding materials.

Table 16-5: Friction Materials for Clutches

Material	Friction Coefficient Wet	Friction Coefficient Dry	Max. Temperature °F	Max. Temperature °C	Max. Pressure psi	Max. Pressure kPa
Cast iron on cast iron	0.05	0.15–0.20	600	320	150–250	1000–1750
Powdered metal* on cast iron	0.05–0.1	0.1–0.4	1000	540	150	1000
Powdered metal* on hard steel	0.05–0.1	0.1–0.3	1000	540	300	2100
Wood on steel or cast iron	0.16	0.2–0.35	300	150	60–90	400–620
Leather on steel or cast iron	0.12	0.3–0.5	200	100	10–40	70–280
Cork on steel or cast iron	0.15–0.25	0.3–0.5	200	100	8–14	50–100
Felt on steel or cast iron	0.18	0.22	280	140	5–10	35–70
Woven asbestos* on steel or cast iron	0.1–0.2	0.3–0.6	350–500	175–260	50–100	350–700
Molded asbestos* on steel or cast iron	0.08–0.12	0.2–0.5	500	260	50–150	350–1000
Impregnated asbestos* on steel or cast iron	0.12	0.32	500–750	260–400	150	1000
Carbon graphite on steel	0.05–0.1	0.25	700–1000	370–540	300	2100

*The friction coefficient can be maintained with ±5 percent for specific materials in this group.

- Table 16-5 includes a wider variety of clutch friction materials, together with some of their properties. Some of these materials may be run wet by allowing them to dip in oil or to be sprayed by oil. This reduces the coefficient of friction somewhat but carries away more heat and permits higher pressures to be used.

- A *woven-asbestos lining* is made in a similar manner to the cotton lining and may also contain metal particles. It is not quite as flexible as the cotton lining and comes in a smaller range of sizes. Along with the cotton lining, the asbestos lining was widely used as a brake material in heavy machinery.