

# Mathematics of Finance II: Derivative securities

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Second term 2015–2016

**Chapter 3 : Combined options**

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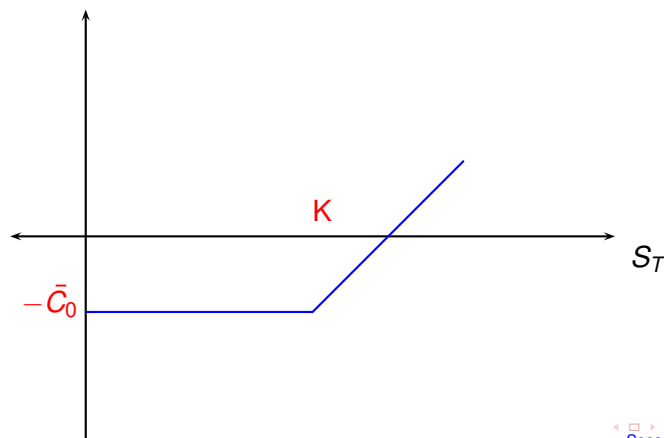
**Chapter 3 : Combined options**



# long a call

The buyer of call speculates on the increase of the price of the underlying

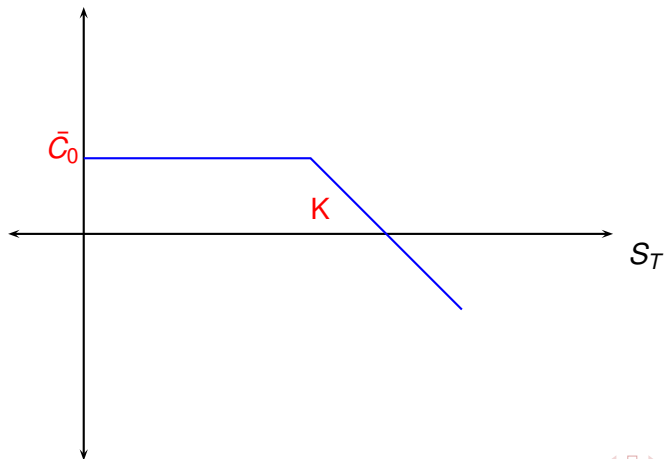
$$\text{Profit} = -\text{Premium} + \text{payoff} = -\bar{C}_0 + (S_T - K)^+$$



## Sell a call: SP on a call

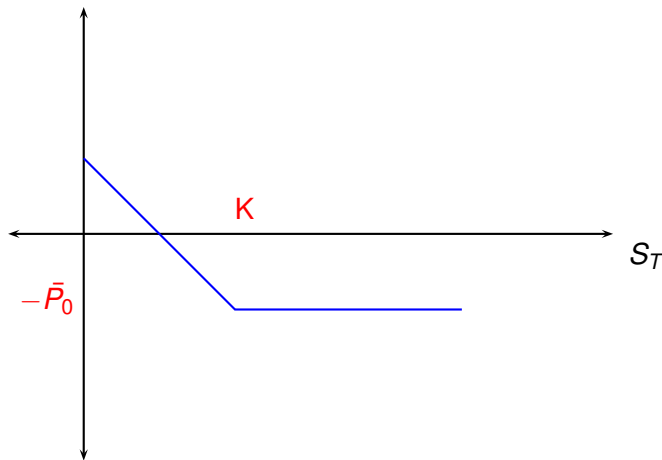
The following figure shows the profit of SP on a call.

$$\text{Profit} = \text{FV(Premium)} - \text{payoff} = \bar{C}_0 - (S_T - K)^+$$



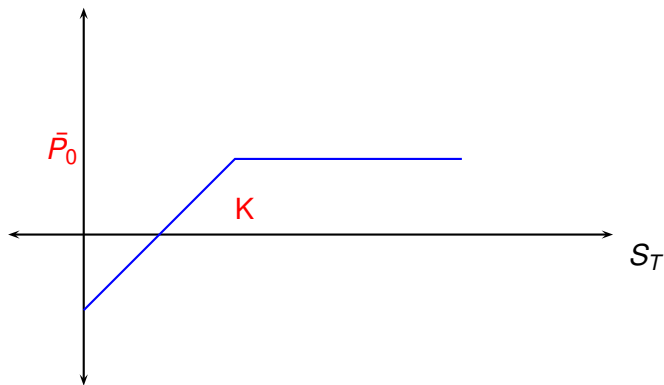
# Long a put

$$\text{Profit} = -\text{Premium} + \text{payoff} = -\bar{P}_0 + (K - S_T)^+$$



## short a put or written put

$$\text{Profit} = \text{Premium} - \text{payoff} = \bar{P}_0 - (K - S_T)^+$$



# Spreads

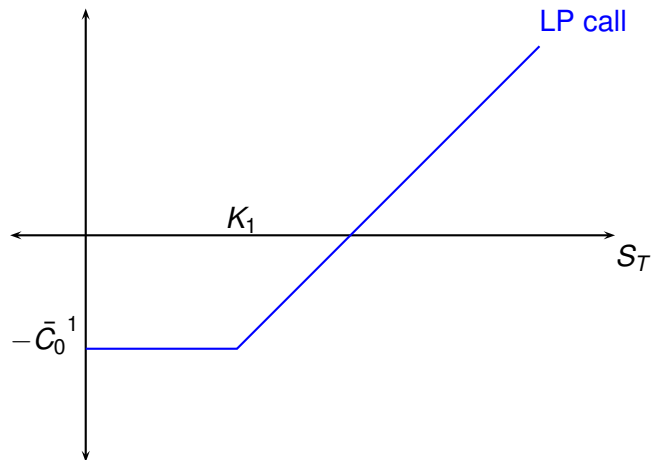
## bull spreads

**Bull call spread** : Long (buy) a call with  $K_1$  and short (sell) a call with strike  $K_2 > K_1$ , the underlying and maturity are the same



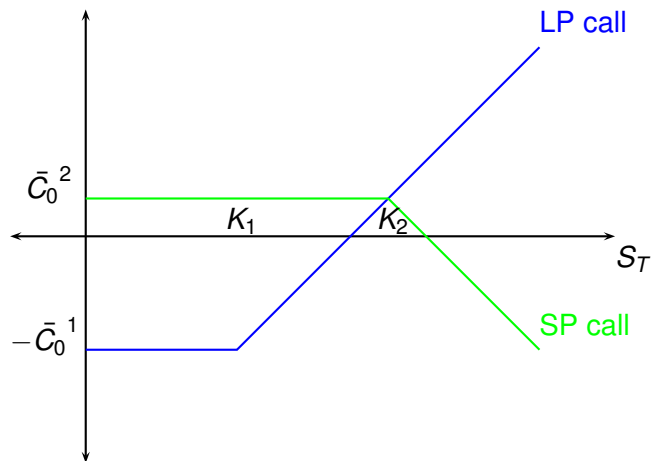
# Bull call spread

Profit of the purchased call  $= (S_T - K_1)^+ - \bar{C}_0$



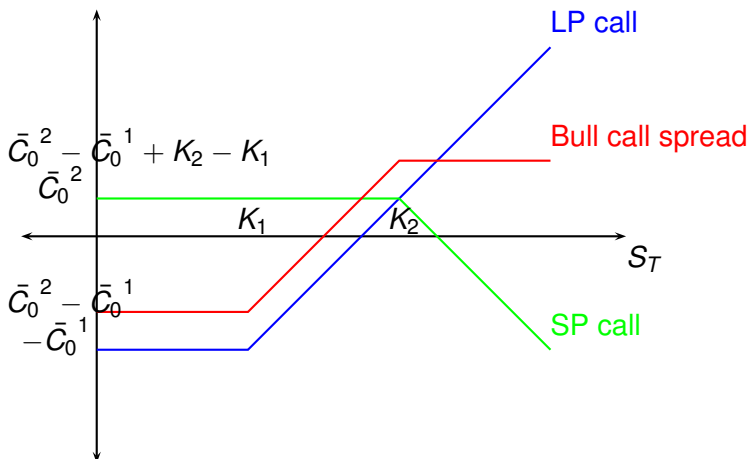
# Bull call spread

Profit of the written call =  $\bar{C}_0^2 - (S_T - K_2)^+$



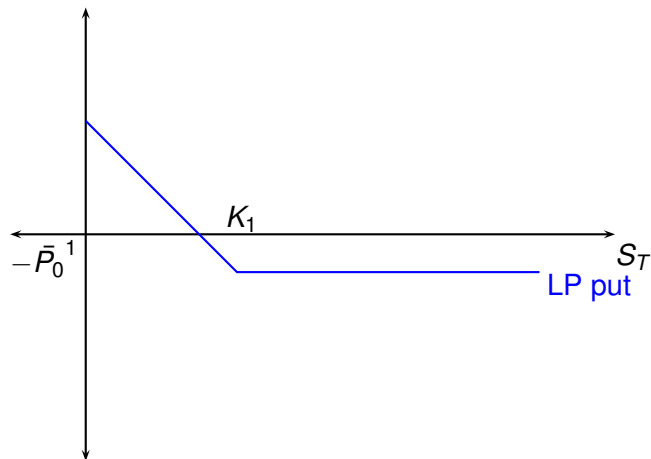
# Bull call spread

$$\text{Profit} = (S_T - K_1)^+ - \bar{C}_0^1 + \bar{C}_0^2 - (S_T - K_2)^+$$



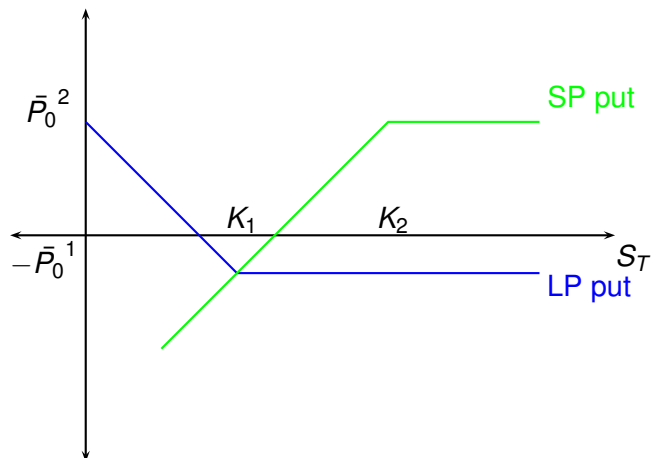
# Bull put spread

Profit of the purchased put  $(K_1 - S_T)^+ - \bar{P}_0$



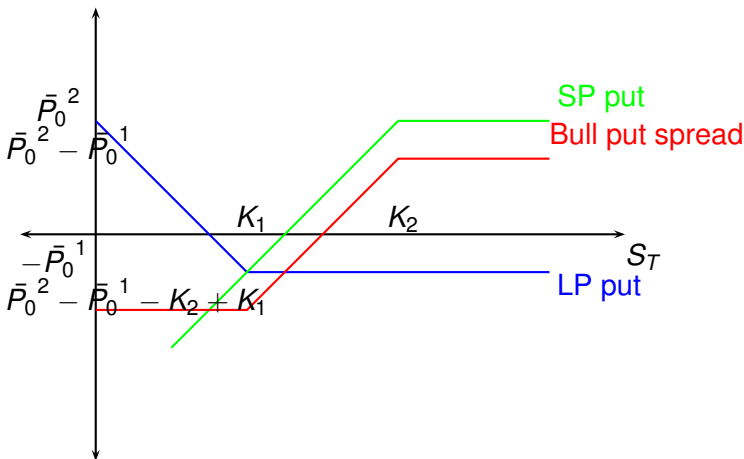
# Bull put spread

Profit of the written put =  $\bar{P}_0^2 - (K_2 - S_T)^+$



# Bull put spread

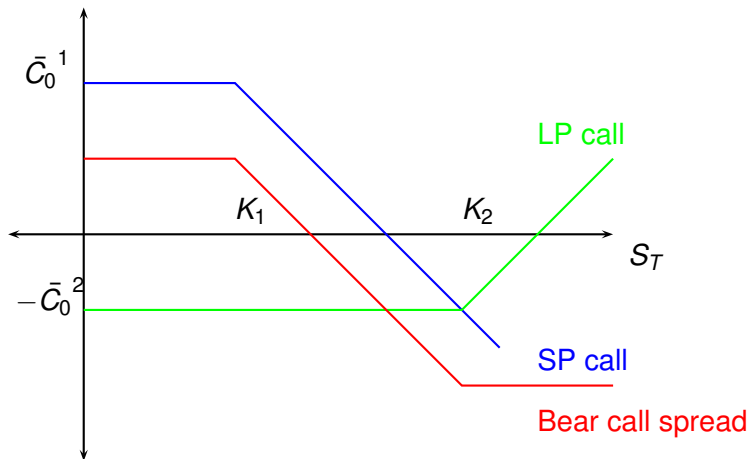
$$\text{Profit} = \bar{P}_0^2 - (K_2 - S_T)^+ + (K_1 - S_T)^+ - \bar{P}_0^1$$



# Bear call spreads

# Bear call spreads

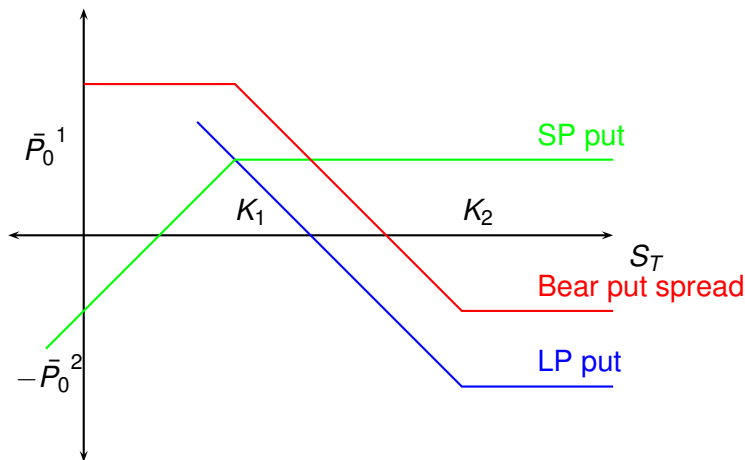
$$\text{Profit of bear spread call} = \bar{C}_0^1 - \bar{C}_0^2 - (S_T - K_1)^+ + (S_T - K_2)^+$$





# Bear put spreads

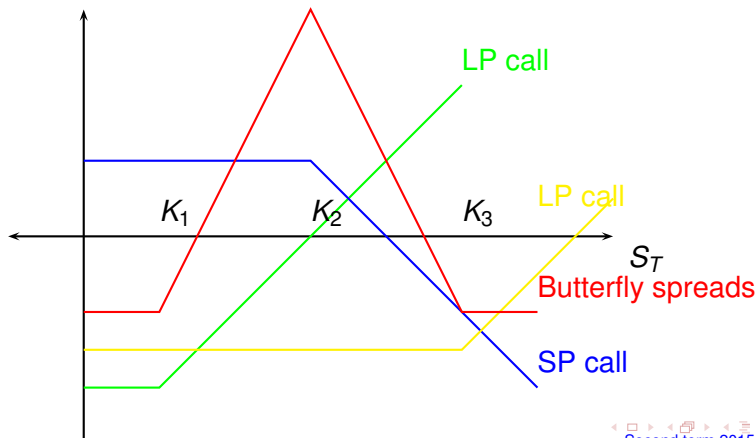
$$\text{Profit of bear spread put} = \bar{P}_0^1 - \bar{P}_0^2 - (K_1 - S_T)^+ + (K_2 - S_T)^+$$



# Butterfly spreads

Long butterfly spreads can be constructed by buying one lower striking call, writing two calls and buying another higher striking call.

$$\text{Profit} = 2\bar{C}_3 - 2(S_T - K_3)^+ + (S_T - K_1)^+ - \bar{C}_1 + (S_T - K_2)^+ - \bar{C}_2$$

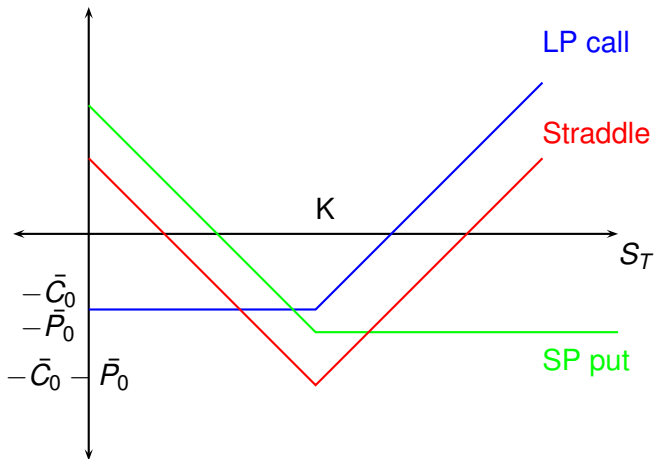


# Other combinaisons : Strategies for volatility

## Straddles

# Buy a Straddle

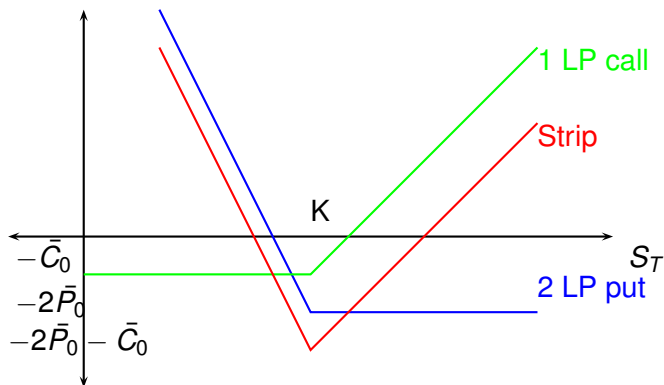
$$\text{Profit} = -\bar{C}_0 - \bar{P}_0 + (S_T - K)^+ + (K - S_T)^+$$



# Strips and Straps

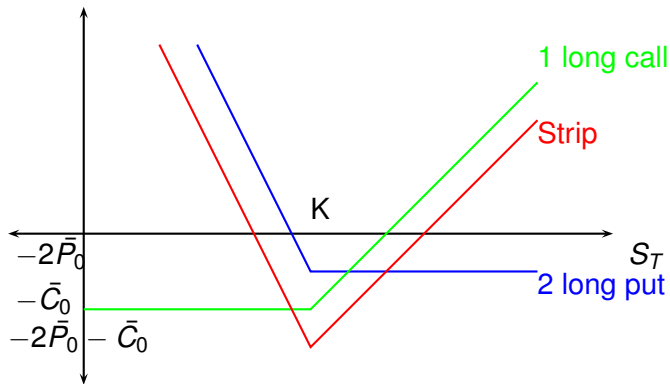
**Profit of the gain of a strip  $\bar{C}_0 \leq 2\bar{P}_0$**

$$\text{Profit} = 2(K - S_T)^+ - 2\bar{P}_0 + (S_T - K)^+ - \bar{C}_0$$



## Profil of the gain of strip $\bar{C}_0 \geq 2\bar{P}_0$

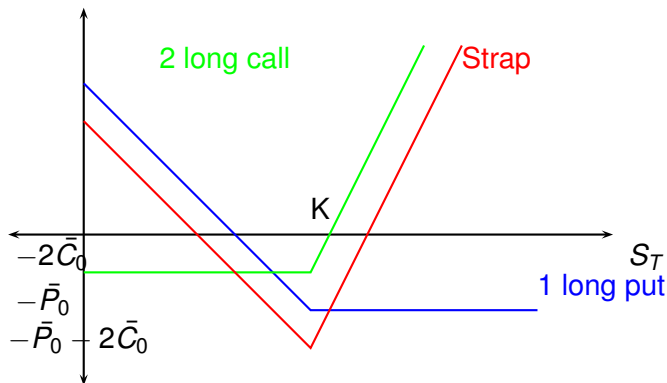
$$\text{Profit} = 2(K - S_T)^+ - 2\bar{P}_0 + (S_T - K)^+ - \bar{C}_0$$



# Strap

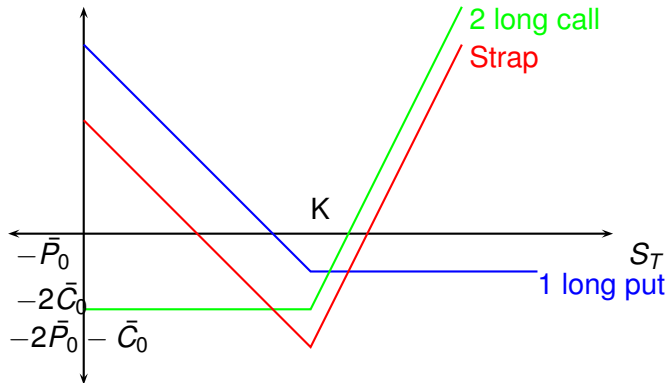
**Profil of the gain of a strap  $2\bar{C}_0 \leq \bar{P}_0$**

$$\text{Profit} = 2(S_T - K)^+ - 2\bar{C}_0 + (K - S_T)^+ - \bar{P}_0$$



## Profil du gain d'un strap $2\bar{C}_0 \geq \bar{P}_0$

$$\text{Profit} = 2(S_T - K)^+ - 2\bar{C}_0 + (K - S_T)^+ - \bar{P}_0$$

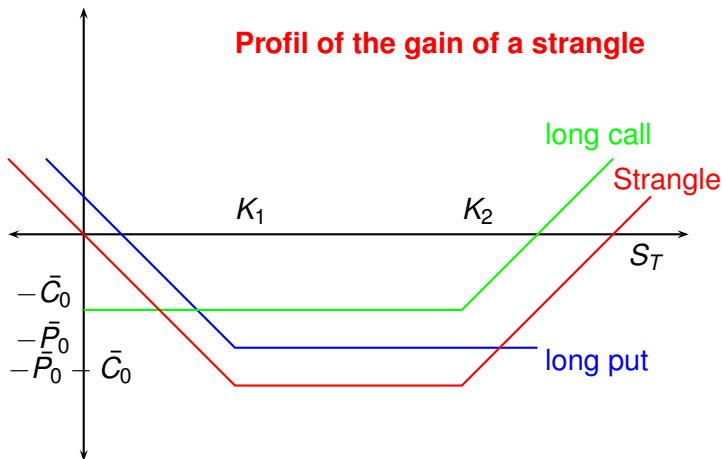




# Strategy on the volatility: strangle

$$\text{Profit} = (S_T - K_2)^+ - \bar{C}_0 + (K_1 - S_T)^+ - \bar{P}_0$$

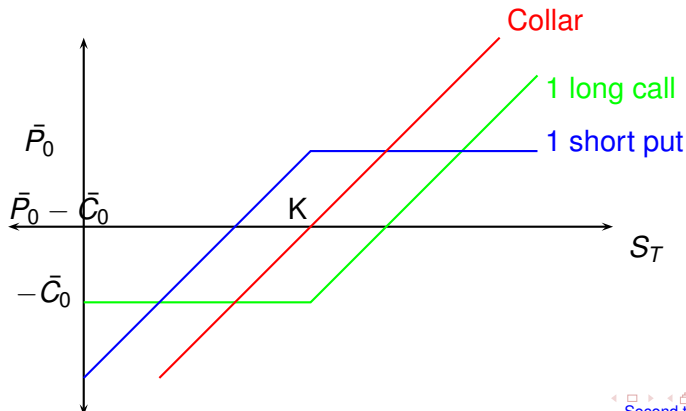
## Profil of the gain of a strangle



# Collar

The collar corresponds to long an option (call or put) associated to short an option in opposite side (put or call). This type of strategy is insensitive to the variation of the volatility.

$$\text{Profit} = (S_T - K)^+ - \bar{C}_0 + \bar{P}_0 - (K - S_T)^+$$

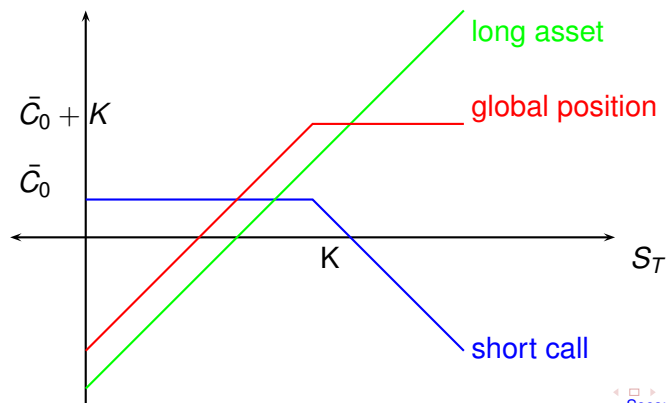


# Strategies based on options and stocks

## Sell the covered call

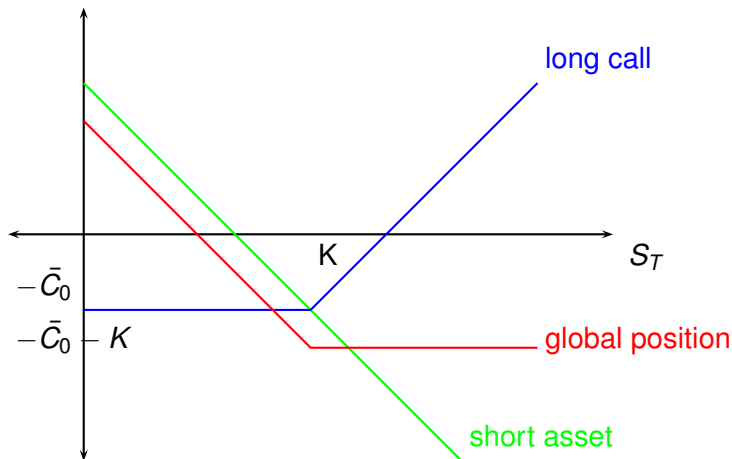
The position consists to long asset and short call. Indeed, the long position on the asset **hedges** the short call.

$$\text{Profit} = \bar{C}_0 - (S_T - K)^+ + S_T$$



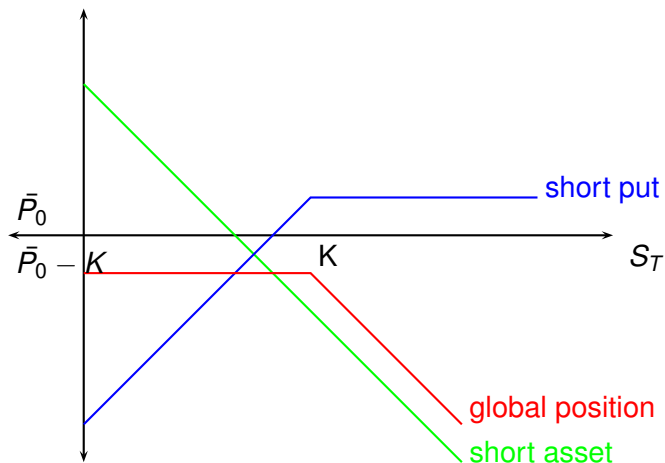
# short asset and long call

$$\text{Profit} = (S_T - K)^+ - \bar{C}_0 - S_T$$



# short asset and short put

$$\text{Profit} = \bar{P}_0 - (K - S_T)^+ - S_T$$



# longue the asset and long the put

$$\text{Profit} = -\bar{P}_0 + (K - S_T)^+ + S_T$$

