CONTINUOUS UNIFORM DISTRIBUTION:

Q1. If the random variable X has a uniform distribution on the interval (0,10), then

1. P(X<6) equals to

(A) 0.4 (B) 0.6 (C) 0.8 (D) 0.2 (E) 0.1

2. The mean of X is

(A) 5 (B) 10 (C) 2 (D) 8 (E) 6

3. The variance X is

(A) 33.33 (B) 28.33 (C) 8.33 (D) 25 (E) None

1) $\int_0^6 \frac{1}{10} dx = 0.6$

2) $\int_0^{10} \frac{x}{10} dx = 5$

3) $\left(\int_0^{10} \frac{x^2}{10} dx - \left(\int_0^{10} \frac{x}{10} dx \right)^2 \right) = 8.33$

Q2. Suppose that the random variable X has the following uniform distribution:

$$f(x) = \begin{cases} 3 & , \frac{2}{3} < x < 1 \\ 0 & , other\ wise \end{cases}$$

1. $P(0.33 < \text{X} < 0.5) =$

(A) 0.49 (B) 0.51 (C) 0 (D) 3

2. $P(\text{X} > 125) =$

(A) 0 (B) 1 (C) 0.5 (D) 0.33

3. The variance of X is

(A) 0.00926 (B) 0.333 (C) 9 (D) 0.6944

3) $\left(\int_{2/3}^{1} 3 * x^2\, dx - \left(\int_{2/3}^{1} 3x\, dx \right)^2 \right) = 0.00926$

Q3. Suppose that the continuous random variable X has the following probability density function (pdf): f(x)=0.2 for 0<x<5. Then

1. P(X>1) equals to

(A) 0.4 (B) 0.2 (C) 0.1 (D) 0.8

2. P(X≥1) equals to

(A) 0.05 (B) 0.8 (C) 0.15 (D) 0.4

3. The mean μ=E(X) equals to

(A) 2.0 (B) 2.5 (C) 3.0 (D) 3.5

4. E(X^2) equals to

(A)8.3333 (B)7.3333 (C) 9.3333 (D) 6.3333

(5) Var(X) equals to

(A) 8.3333(B) 69.444 (C) 5.8333 (D) 2.0833

(6) If F(x) is the cumulative distribution function (CDF) of X, then F(1) equals to

(A) 0.75 (B) 0.25 (C) 0.8 (D) 0.2

1 & 2) $\int_1^5 0.2\, dx = 0.8$

3) $\int_0^5 0.2 * x\, dx = 2.5$

4) $\int_0^5 0.2 * x^2\, dx = 8.33$

5) $\int_0^5 0.2 * x^2\, dx - \left(\int_0^5 0.2 * x\, dx\right)^2 = 2.0833$

6) F(a)=0.2 a, F(1)=0.2

9. EXPONENTIAL DISTRIBUTION

Q1. If the random variable X has an exponential distribution with the mean 4, then:

1. P(X<8) equals to

(A) 0.2647 (B) 0.4647 (C) 0.8647 (D) 0.6647 (E) 0.0647

2. The variance of X is

(A) 4 (B) 16 (C) 2 (D) 1/4 (E) 1/2

$f(x) = \frac{1}{4} e^{-\frac{x}{4}}; \; x > 0$

1) $\int_0^8 \frac{1}{4} \text{Exp}[-\frac{1}{4}x]\, dx = 0.8647$

2) V(X)=16

Q2. Suppose that the failure time (in hours) of a certain electrical device is distributed with a probability density function given by:

$$f(x) = \frac{1}{70} e^{-x/70}, \quad x > 0,$$

1. The probability that a randomly selected device will fail within the first 50 hours is:

(A) 0.4995 (B) 0.7001 (C) 0.5105 (D) 0.2999

2. The probability that a randomly selected device will last more than 150 hours is:

(A) 0.8827 (B) 0.2788 (C) 0.1173 (D) 0.8827

3. The average failure time of the electrical device is:

(A) 1/70 (B) 70 (C) 140 (D) 35

4. The variance of the failure time of the electrical device is:

(A) 4900 (B) 1/49000 (C) 70 (D) 1225

[Hint: $\int e^{-ax}\, dx = -\frac{1}{a} e^{-ax} + c$]

1) $\int_{0}^{50} \frac{1}{70} \text{Exp}[-\frac{1}{70}x]\, dx = 0.5105$

2) $\int_{150}^{\infty} \frac{1}{70} \text{Exp}[-\frac{1}{70}x]\, dx = 0.1173$

3) $\mu = 70$

4) V(X)=70^2=4900

Q5. Assume the length (in minutes) of a particular type of a telephone conversation is a random variable with a probability density function of the form:

$$f(x) = \begin{cases} 0.2\ e^{-0.2x} & ;\ x \geq 0 \\ 0 & ;\ elsewhere \end{cases}$$

1. $P(3 < \text{X} < 10) =$ is:
   (a) 0.587 (b) -0.413 (c) 0.413 (d) 0.758
2. For this random variable, $P(\ \mu\ -2\sigma \leq X \leq \mu\ +2\sigma\ )$ will have an exact value equals:
   (a) 0.250 (b) 0.750 (c) 0.950 (d) 0.3175

1) $\int_{3}^{10} 0.2 * \text{Exp}[-0.2 * x]\, dx = 0.413$

Q6. The length of time for one customer to be served at a bank is a random variable X that follows the exponential distribution with a mean of 4 minutes.

1. The probability that a customer will be served in less than 2 minutes is:

   (A) 0.9534 (B) 0.2123 (C) 0.6065 (D) 0.3935

2. The probability that a customer will be served in more than 4 minutes is:

   (A) 0.6321 (B) 0.3679 (C) 0.4905 (D) 0.0012

3. The probability that a customer will be served in more than 2 but less than 5 minutes is:

   (A) 0.6799 (B) 0.32 (C) 0.4018 (D) 0.5523

4. The variance of service time at this bank is

   (A) 2 (B) 4 (C) 8 (D) 16

$f(x) = \frac{1}{4} e^{-\frac{x}{4}};\ \ x > 0$

1) $\int_0^2 \frac{1}{4} \text{Exp}[-\frac{1}{4} x]\, dx = 0.3935$

2) $\int_4^\infty \frac{1}{4} \text{Exp}[-\frac{1}{4} x]\, dx = 0.3679$

3) $P(2 \leq X \leq 5) = F(5) - F(2) = \left(1 - e^{-\frac{5}{4}}\right) - \left(1 - e^{-\frac{2}{4}}\right) = e^{-\frac{2}{4}} - e^{-\frac{5}{4}} = 0.32$

H.W: Q3, Q4.

8. NORMAL DISTRIBUTION:

Q1. (A) Suppose that Z is distributed according to the standard normal distribution.

1. The area under the curve to the left of $z = 1.43$ is:

(A) 0.0764 (B) 0.9236 (C) 0 (D) 0.8133

2. The area under the curve to the left of z = 1.39 is:

(A) 0.7268 (B) 0.9177 (C) .2732 (D) 0.0832

3. The area under the curve to the right of $z = -0.89$ is:

(A) 0. 7815 (B) 0.8133 (C) 0.1867 (D) 0.0154

4. The area under the curve between $z = -2.16$ and $z = -0.65$ is:

(A) 0.7576 (B) 0.8665 (C) 0.0154 (D) 0.2424

5. The value of k such that $P(0.93 < Z < k) = 0.0427$ is:

(A) 0.8665 (B) −1.11 (C) 1.11 (D) 1.00

(B) Suppose that Z is distributed according to the standard normal distribution. Find:

1. P(Z< –3.9)=0

2. P(Z> 4.5)=0

1) P(Z< 3.7)=1

2) P(Z> –4.1)=1

Q2. The finished inside diameter of a piston ring is normally distributed with a mean of 12 centimetres and a standard deviation of 0.03 centimeter. Then,

1. The proportion of rings that will have inside diameter less than 12.05 centimetres is:

(A) 0.0475 (B) 0.9525 (C) 0.7257 (D) 0.8413

2. The proportion of rings that will have inside diameter exceeding 11.97 centimetres is:

(A) 0.0475 (B) 0.8413 (C) 0.1587 (D) 0.4514

3. The probability that a piston ring will have an inside diameter between 11.95 and 12.05 centimetres is:

(A) 0.905 (B) –0.905 (C) 0.4514 (D) 0.7257

$$X \sim N(12,0.03)$$

1) $P(X < 12.05) = P\left(\frac{X-\mu}{\sigma} < \frac{12.05-12}{0.03}\right) = P(Z < 1.666) = P(Z < 1.67) = 0.9525$

2) $P(X > 11.97) = P\left(\frac{X-\mu}{\sigma} > \frac{11.97-12}{0.03}\right) = P(Z > -1) = 1 - P(Z < -1) = 0.8413$

3) $P(11.95 < X < 12.05) = P(-1.67 < Z < 1.67) = P(Z < 1.67) - P(Z < -1.67) = 0.905$

Q3. The average life of a certain type of small motor is 10 years with a standard deviation of 2 years. Assume the live of the motor is normally distributed. The manufacturer replaces free all motors that fail while under guarantee. If he is willing to replace only 1.5% of the motors that fail, then he should give a guarantee of :

(A) 10.03 years (B) 8 years (C) 5.66 years (D) 3 years

Q4. A machine makes bolts (that are used in the construction of an electric transformer). It produces bolts with diameters (X) following a normal distribution with a mean of 0.060 inches and a standard deviation of 0.001 inches. Any bolt with diameter less than 0.058 inches or greater than 0.062 inches must be scrapped. Then

1. The proportion of bolts that must be scrapped is equal to
(A) 0.0456 (B) 0.0228 (C) 0.9772 (D) 0.3333 (E) 0.1667

2. If P(X>a)= 0.1949, then a equals to:
(A) 0.0629 (B) 0.0659 (C) 0.0649 (D) 0.0669 (E) 0.0609

Q5. The diameters of ball bearings manufactured by an industrial process are normally distributed with a mean μ = 3.0 cm and a standard deviation σ = 0.005 cm. All ball bearings with diameters not within the specifications $\mu \pm d$ cm (d > 0) will be scrapped.

1. Determine the value of d such that 90% of ball bearings manufactured by this process will not be scrapped.

2. If d = 0.005, what is the percentage of manufactured ball bearings that will be scraped?

Q6. The weight of a large number of fat persons is nicely modeled with a normal distribution with mean of 128 kg and a standard deviation of 9 kg.

1. The percentage of fat persons with weights at most 110 kg is $P(X<110)=P(Z<-2)$

   (A) 0.09 % (B) 90.3 % (C) 99.82 % (D) 2.28 %

2. The percentage of fat persons with weights more than 149 kg is $P(X>149)=P(Z>2.33)$

   (A) 0.09 % (B) 0.99 % (C) 9.7 % (D) 99.82 %

3. The weight x above which 86% of those persons will be

   (A) 118.28 (B) 128.28 (C) 154.82 (D) 81.28

4. The weight x below which 50% of those persons will be

   (A) 101.18 (B) 128 (C) 154.82 (D) 81

Q7. The random variable X, representing the lifespan of a certain electronic device, is normally distributed with a mean of 40 months and a standard deviation of 2 months. Find

1. $P(X<38)$. (0.1587)
2. $P(38<X<40)$. (0.3413)
3. $P(X=38)$. (0.0000)
4. The value of x such that $P(X<x)=0.7324$. (41.24)

Q8. If the random variable X has a normal distribution with the mean μ and the variance σ^2, then $P(X<\mu+2\sigma)$ equals to

(A) 0.8772 (B) 0.4772 (C) 0.5772 (D) 0.7772 (E) 0.9772

Q9. If the random variable X has a normal distribution with the mean μ and the variance 1, and if $P(X<3)=0.877$, then μ equals to

(A) 3.84 (B) 2.84 (C) 1.84 (D) 4.84 (E) 8.84

Q10. Suppose that the marks of the students in a certain course are distributed according to a normal distribution with the mean 70 and the variance 25. If it is known that 33% of the student failed the exam, then the passing mark x is

(A) 67.8 (B) 60.8 (C) 57.8 (D) 50.8 (E) 70.8

Q11.H.W If the random variable X has a normal distribution with the mean 10 and the variance 36, then

1. The value of X above which an area of 0.2296 lie is

(A) 14.44 (B) 16.44 (C) 10.44 (D) 18.44 (E) 11.44

2. The probability that the value of X is greater than 16 is

(A) 0.9587 (B) 0.1587 (C) 0.7587 (D) 0.0587 (E) 0.5587

Q12. Suppose that the marks of the students in a certain course are distributed according to a normal distribution with the mean 65 and the variance 16. A student fails the exam if he obtains a mark less than 60. Then the percentage of students who fail the exam is $P(X<60)=P(Z<-1.25)=0.1056$

(A) 20.56% (B) 90.56% (C) 50.56% (D) 10.56% (E)40.56%

Q13. The average rainfall in a certain city for the month of March is 9.22 centimetres. Assuming a normal distribution with a standard deviation of 2.83 centimetres, then the probability that next March, this city will receive:

1. Less than 11.84 centimetres of rain is:

(A) 0.8238 (B) 0.1762 (C) 0.5 (D) 0.2018

2. More than 5 centimetres but less than 7 centimetres of rain is:

(A) 0.8504 (B) 0.1496 (C) 0.6502 (D) 0.34221

3. More than 13.8 centimetres of rain is:

(A) 0.0526   (B) 0.9474   (C) 0.3101   (D) 0.4053

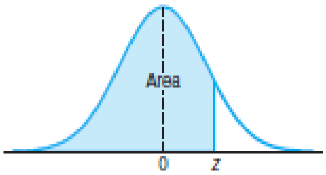

Table A.3 Areas under the Normal Curve

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
−3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
−3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
−3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
−3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
−2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
−2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
−2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
−2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
−2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
−2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
−2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
−2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
−2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
−2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
−1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
−1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
−1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
−1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
−1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
−1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
−1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
−1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
−1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
−1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
−0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
−0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
−0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
−0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
−0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
−0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
−0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
−0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
−0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
−0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

Table A.3 (continued) Areas under the Normal Curve

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998