

Continuous Probability Distributions

Chapter 7

Learning Objectives

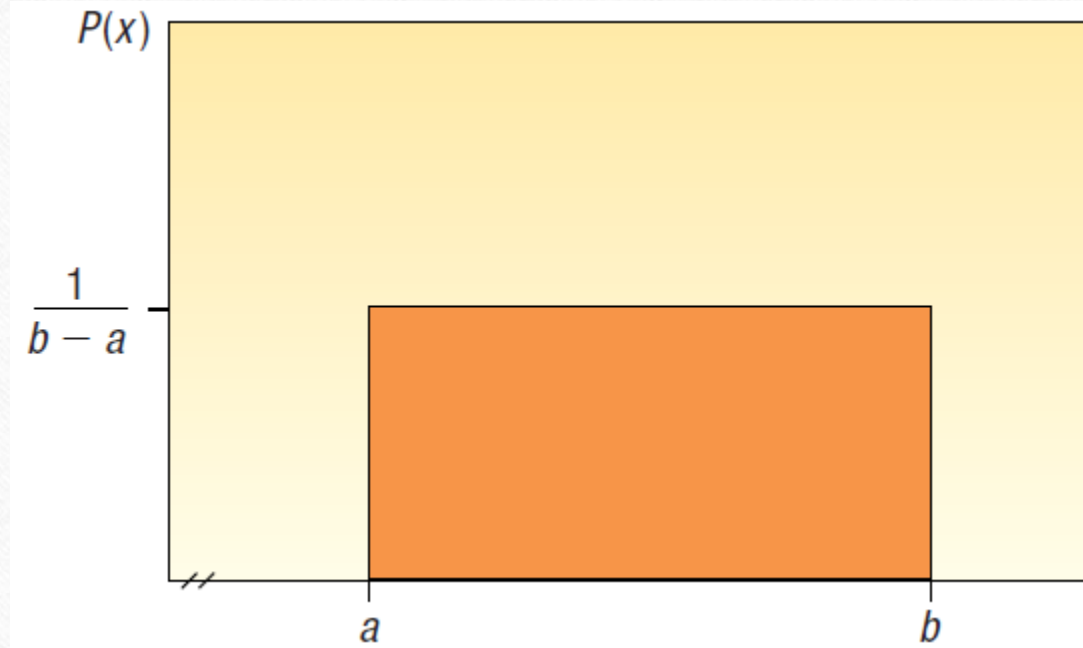
- List the characteristics of the uniform distribution.
- Compute probabilities using the uniform distribution
- List the characteristics of the normal probability distribution.
- Define and calculate z values.
- Determine the probability an observation is between two points on a normal probability distribution.
- Determine the probability an observation is above (or below) a point on normal probability distribution.

Uniform Probability Distribution

1. The distribution shape is rectangular.
2. Has a maximum value (b) and a minimum value (a).
3. the height of the distribution is constant or uniform for all values between a and b
4. The area of the rectangular is 1
5. The equation for the uniform probability distribution is:

$$P(x) = \frac{1}{b - a} \quad \text{if } a \leq x \leq b \text{ and } 0 \text{ elsewhere}$$

Uniform Probability Distribution



Uniform Probability Distribution

MEAN OF THE UNIFORM DISTRIBUTION

$$\mu = \frac{a + b}{2}$$

[7-1]

STANDARD DEVIATION
OF THE UNIFORM DISTRIBUTION

$$\sigma = \sqrt{\frac{(b - a)^2}{12}}$$

[7-2]

Uniform Probability Distribution

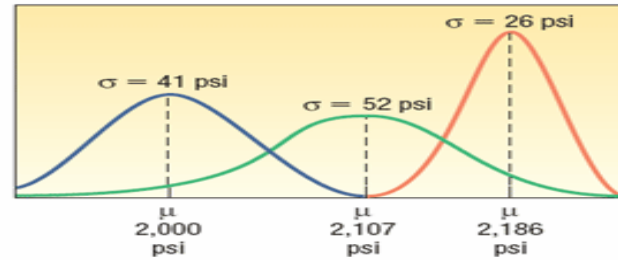
Southwest Arizona State University provides bus service to students while they are on campus. A bus arrives at the North Main Street and College Drive stop every 30 minutes between 6 A.M. and 11 P.M. during weekdays. Students arrive at the bus stop at random times. The time that a student waits is uniformly distributed from 0 to 30 minutes.

1. Draw a graph of this distribution.
2. Show that the area of this uniform distribution is 1.00.
3. How long will a student “typically” have to wait for a bus? In other words, what is the mean waiting time? What is the standard deviation of the waiting times?
4. What is the probability a student will wait more than 25 minutes?
5. What is the probability a student will wait between 10 and 20 minutes?

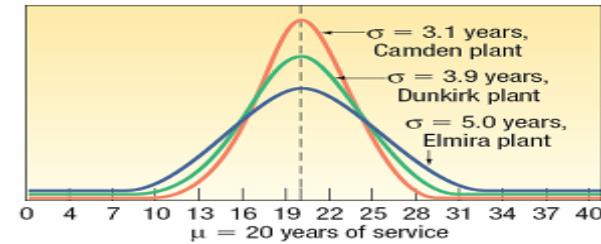
Normal Probability distribution

1. It is **bell-shaped** and has a single peak at the center of the distribution.
2. It is **symmetrical** about the mean
3. It is **asymptotic**: The curve gets closer and closer to the X -axis but never actually touches it.
4. The location of a normal distribution is determined by the mean, μ , the dispersion or spread of the distribution is determined by the standard deviation, σ .
5. The arithmetic **mean, median, and mode are equal**
6. The total **area under the curve is 1.00**; half the area under the normal curve is to the right of this center point and the other half to the left of it

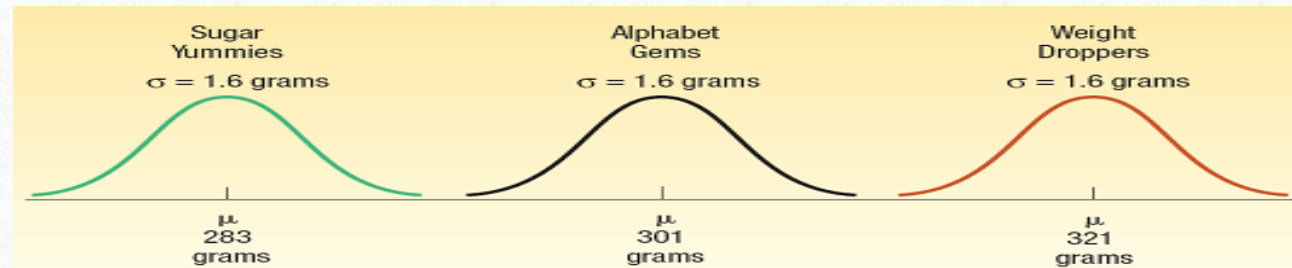
Normal Probability distribution



Different Means and Standard Deviations

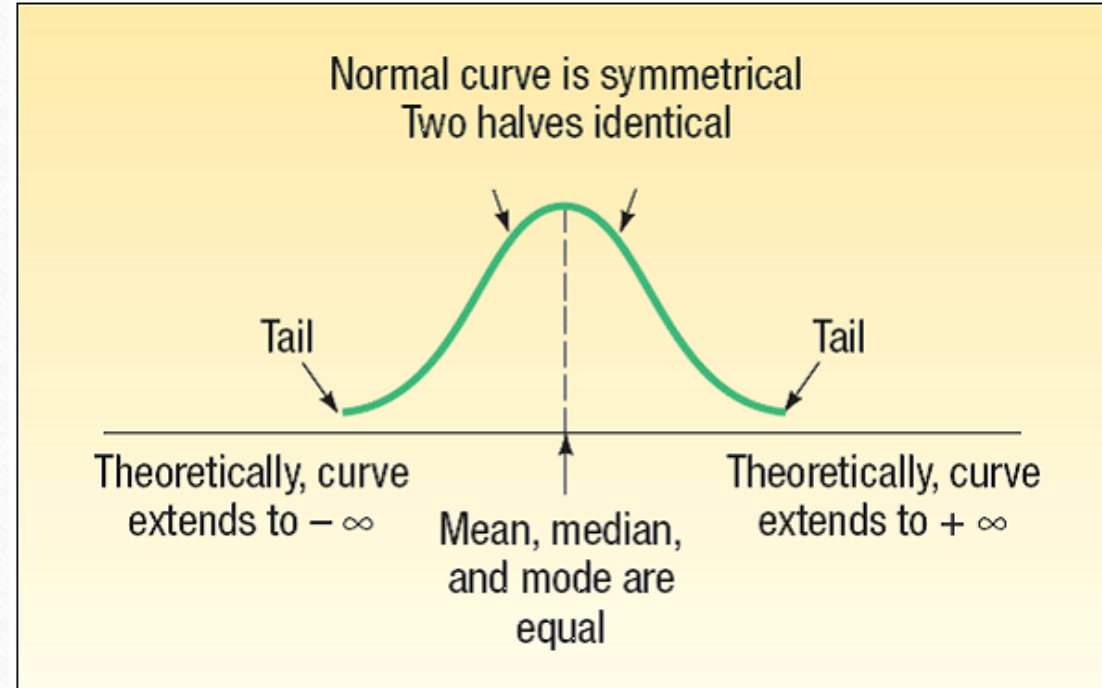


Equal Means and Different Standard Deviations



Different Means and Equal Standard Deviations

Normal Probability Distribution



The Empirical Rule

As part of its quality assurance program, the Autolite Battery Company conducts tests on battery life. For a particular D-cell alkaline battery, the mean life is 19 hours. The useful life of the battery follows a normal distribution with a standard deviation of 1.2 hours.

Answer the following questions.

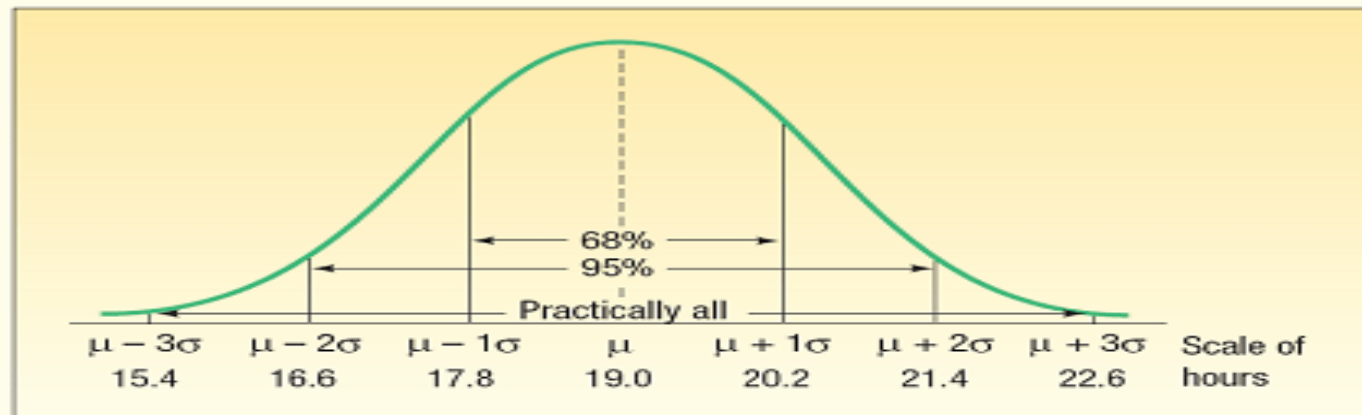
1. About 68 percent of the batteries failed between what two values?
2. About 95 percent of the batteries failed between what two values?
3. Virtually all of the batteries failed between what two values?

The Empirical Rule

We can use the results of the Empirical Rule to answer these questions.

1. About 68 percent of the batteries will fail between 17.8 and 20.2 hours by $19.0 \pm 1(1.2)$ hours.
2. About 95 percent of the batteries will fail between 16.6 and 21.4 hours by $19.0 \pm 2(1.2)$ hours.
3. Virtually all failed between 15.4 and 22.6 hours, found by $19.0 \pm 3(1.2)$

This information is summarized on the following chart.



The Standard Normal Probability Distribution

- The standard normal distribution is a normal distribution with a mean of 0 and a standard deviation of 1.
- It is also called the z distribution.
- A **z-value** is the signed distance between a selected value, designated X , and the population mean μ , divided by the population standard deviation, σ .
- The formula is:

$$Z = \frac{X - \mu}{\sigma}$$

Z-value

The weekly incomes of shift foremen in the glass industry follow the normal probability distribution with a mean of \$1,000 and a standard deviation of \$100.

What is the z-value for the income, let's call it X , of a foreman who earns \$1,100 per week? For a foreman who earns \$900 per week?

Example 1

- The mean weekly income of a shift foreman in the glass industry is normally distributed with a mean of \$1,000 and a standard deviation of \$100. What is the likelihood of selecting a foreman whose weekly income is between \$1,000 and \$1,100?
- We want $P(1000 < X < 1100)$
- First we have to get the z values.
- $P(0 < Z < 1) = ??$

Example 2

- The mean weekly income of a shift foreman in the glass industry is normally distributed with a mean of \$1,000 and a standard deviation of \$100. What is the likelihood of selecting a foreman whose weekly income is between \$773 and \$1,000?
- We want $P(773 < X < 1000)$
- First we have to get the z values.
- $P(-2.27 < Z < 0) = ??$

Example 3

- The mean weekly income of a shift foreman in the glass industry is normally distributed with a mean of \$1,000 and a standard deviation of \$100. What is the likelihood of selecting a foreman whose weekly income is between \$1,100 and \$1,200?
- We want $P(1100 < X < 1200)$
- First we have to get the z values.
- $P(1 < Z < 2) = ??$

Example 4

- The mean weekly income of a shift foreman in the glass industry is normally distributed with a mean of \$1,000 and a standard deviation of \$100. What is the likelihood of selecting a foreman whose weekly income is between \$800 and \$950?
- We want $P(800 < X < 1200)$
- First we have to get the z values.
- $P(-2 < Z < -0.5) = ??$

Example 5

- The mean weekly income of a shift foreman in the glass industry is normally distributed with a mean of \$1,000 and a standard deviation of \$100. What is the likelihood of selecting a foreman whose weekly income is between \$865 and \$1,157?
- We want $P(865 < X < 1157)$
- First we have to get the z values.
- $P(-1.35 < Z < 1.57) = ??$

Example 6

- The mean weekly income of a shift foreman in the glass industry is normally distributed with a mean of \$1,000 and a standard deviation of \$100. What is the likelihood of selecting a foreman whose weekly income is greater than \$1220?
- We want $P(X > 1220)$
- First we have to get the z values.
- $P(Z > 2.2) = ??$

Example 7

- The mean weekly income of a shift foreman in the glass industry is normally distributed with a mean of \$1,000 and a standard deviation of \$100. What is the likelihood of selecting a foreman whose weekly income is less than \$1050?
- We want $P(X < 1050)$
- First we have to get the z values.
- $P(Z < 0.5) = ??$

Example 8

- The mean weekly income of a shift foreman in the glass industry is normally distributed with a mean of \$1,000 and a standard deviation of \$100. What is the likelihood of selecting a foreman whose weekly income is greater than \$788?
- We want $P(X > 788)$
- First we have to get the z values.
- $P(Z > -2.12) = ??$

Example 9

- The mean weekly income of a shift foreman in the glass industry is normally distributed with a mean of \$1,000 and a standard deviation of \$100. What is the likelihood of selecting a foreman whose weekly income is less than \$940?
- We want $P(X < 940)$
- First we have to get the z values.
- $P(Z < -0.6) = ??$

Example 10

- The mean weekly income of a shift foreman in the glass industry is normally distributed with a mean of \$1,000 and a standard deviation of \$100. What is the minimum weekly income for the top 2.5%?
- We have $P(X > x) = 0.025$ and we want to know the value of x .
- First we have to get the z values.
- $P(Z > 1.96) = 0.025$
- Now, we got the z value, we know μ and σ . How can we get x ??

Example 11

- The mean weekly income of a shift foreman in the glass industry is normally distributed with a mean of \$1,000 and a standard deviation of \$100. What is the likelihood of selecting a foreman whose weekly income is between \$865 and \$1,157?
- We want $P(865 < X < 1157)$
- First we have to get the z values.
- $P(-1.35 < Z < 1.57) = ??$