

Discrete Probability Distributions

Chapter 6

Learning Objectives

- Define terms random variable and probability distribution.
- Distinguish between discrete and continuous probability distributions.
- Calculate the mean, variance, and standard deviation of a discrete probability distribution.
- Describe the characteristics of binomial distribution and compute probabilities using binomial distribution.
- Describe the characteristics of hypergeometric distribution and compute probabilities using hypergeometric distribution.
- Describe the characteristic of Poisson distribution and compute probability using Poisson distribution.

Random Variable

- **Random variable** is a quantity resulting from an experiment that, by chance, can assume different values.
- Experiment: Tossing a coin three times
- Random variable: the number of heads.
- Experiment: Rolling a dice.
- Random variable: the number appears face up.

Types of Random Variables

- **Discrete Random Variable** can assume only certain clearly separated values. It is usually the result of counting something
- **Continuous Random Variable** can assume infinite number of values within a given range. It is usually the result of some type of measurement

Discrete Random Variables - Examples

- The number of students in a class.
- The number of children in a family.
- The number of cars entering a carwash in an hour.

Continuous Random Variables - Examples

- The distance students travel to class.
- The time it takes an executive to drive to work.
- The length of an afternoon nap.
- The length of time of a particular phone call.

Probability Distribution

PROBABILITY DISTRIBUTION A listing of all the outcomes of an experiment and the probability associated with each outcome.

Experiment: Toss a coin three times. Observe the number of heads. The possible results are: zero heads, one head, two heads, and three heads.

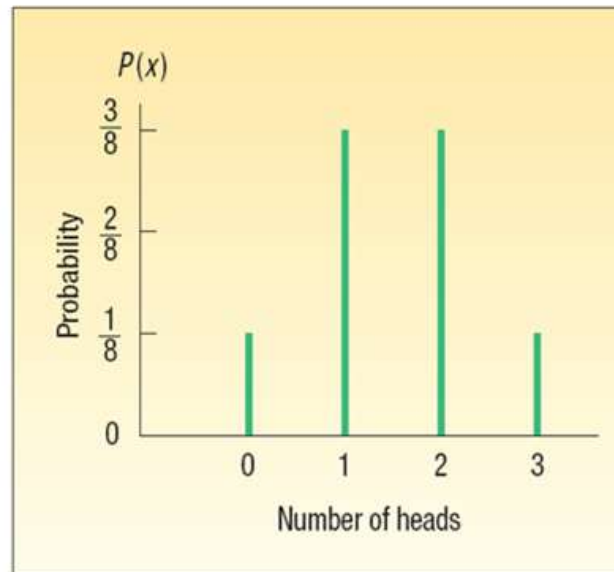
What is the probability distribution for the number of heads?

Probability Distribution

Possible Result	Coin Toss			Number of Heads
	First	Second	Third	
1	T	T	T	0
2	T	T	H	1
3	T	H	T	1
4	T	H	H	2
5	H	T	T	1
6	H	T	H	2
7	H	H	T	2
8	H	H	H	3

Probability Distribution

Number of Heads, x	Probability of Outcome, $P(x)$
0	$\frac{1}{8} = .125$
1	$\frac{3}{8} = .375$
2	$\frac{3}{8} = .375$
3	$\frac{1}{8} = .125$
Total	$\frac{8}{8} = 1.000$



Probability Distribution

CHARACTERISTICS OF A PROBABILITY DISTRIBUTION

1. The probability of a particular outcome is between 0 and 1 inclusive.
2. The outcomes are mutually exclusive events.
3. The list is exhaustive. So the sum of the probabilities of the various events is equal to 1.

Discrete Distribution

The main features of a discrete probability distribution are:

- The sum of the probabilities of the various outcomes is 1.00.
- The probability of a particular outcome is between 0 and 1.00.
- The outcomes are mutually exclusive.

The Mean

- The mean is a typical value used to represent the central location of a probability distribution.
- The mean of a probability distribution is also referred to as its **expected value**.

MEAN OF A PROBABILITY DISTRIBUTION

$$\mu = \sum[xP(x)]$$

[6-1]

The Variance and Standard Deviation

- Measures the amount of spread in a distribution
- The computational steps are:
 1. Subtract the mean from each value, and square this difference.
 2. Multiply each squared difference by its probability.
 3. Sum the resulting products to arrive at the variance.
- The standard deviation is found by taking the positive square root of the variance.

VARIANCE OF A PROBABILITY DISTRIBUTION

$$\sigma^2 = \sum [(x - \mu)^2 P(x)]$$

[6-2]

Example



John Ragsdale sells new cars for Pelican Ford. John usually sells the largest number of cars on Saturday. He has developed the following probability distribution for the number of cars he expects to sell on a particular Saturday.

1. What type of distribution is this?
2. On a typical Saturday, how many cars does John expect to sell?
3. What is the variance and the standard deviation of the distribution?

Number of Cars Sold, x	Probability, $P(x)$
0	.10
1	.20
2	.30
3	.30
4	.10
Total	1.00

Example

1. This is a discrete probability distribution.
2. To get how many cars does John expect to sell on a typical Saturday? We need to calculate the mean (the expected value).

$$\mu = \sum [XP(X)]$$

$$\mu = 0(0.1) + 1(0.2) + 2(0.3) + 3(0.3) + 4(0.1) = 2.1 \text{ cars}$$

Example

- To calculate the variance:

$$\sigma^2 = \sum[(X - \mu)^2 P(X)]$$

$$\sigma^2 = [(0 - 2.1)^2(0.1)] + [(1 - 2.1)^2(0.2)] + [(2 - 2.1)^2(0.3)] + [(3 - 2.1)^2(0.3)] + [(4 - 2.1)^2(0.1)]$$

$$\sigma^2 = 1.29$$

- To calculate the standard deviation:

$$\sigma = \sqrt{1.29} = 1.136 \text{ cars}$$

Example 2

- A quiz has 3 multiple choice questions. Each question has 5 choices, and only one of them is correct. What is the probability that a student will guess all the correct choices?

For each question we have 0.2 chance of guessing the correct answer and 0.8 chance of guessing the wrong answer.

Therefore, the probability that a student will guess all the correct choices is:

$$0.2 * 0.2 * 0.2 = (0.2)^3 = 0.008$$

Example 2

Now, if we define a random variable X to be the number of guessing the correct answers.

X can take values 0, 1, 2, and 3.

We answered $P(X = 3)$ or $P(3)$ and it is 0.008

What is $P(0)$?

We can see that $P(0) = 0.8 * 0.8 * 0.8 = (0.8)^3 = 0.512$

Example 2

What is $P(1)$?

Recall the probability of guessing a correct answer is 0.2 and the probability of guessing a wrong answer is 0.8.

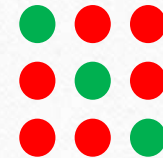
So, the probability of guessing one correct answer is

$$0.2 * 0.8 * 0.8 = (0.2)^1 * (0.8)^2 = 0.128$$

How many ways can we get one correct answer?

There are the three possible ways of getting one correct answer.

Therefore, $P(1) = 3 * 0.128 = 0.384$



Example 2

What is $P(2)$?

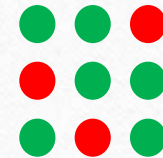
Recall the probability of guessing a correct answer is 0.2 and the probability of guessing a wrong answer is 0.8.

So, the probability of guessing two correct answers is

$$0.2 * 0.2 * 0.8 = (0.2)^2 * (0.8)^1 = 0.032$$

How many ways can we get two correct answers?

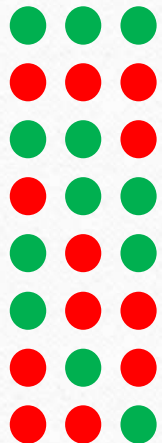
There are the three possible ways of getting two correct answers.



Therefore, $P(2) = 3 * 0.032 = 0.096$

Example 2

- Therefore, the probability distribution of X is



X	$P(X)$
0	0.512
1	0.384
2	0.096
3	0.008

$$\mu = ?, \sigma^2 = ?$$

Binomial Distribution

Characteristics of a Binomial Probability Experiment

1. An outcome on each trial of an experiment is classified into one of two mutually exclusive categories- a success or a failure.
2. The random variable counts the number of successes in a fixed number of trials.
3. The probability of success and failure stay the same for each trial.
4. The trials are independent, meaning that the outcome of one trial does not affect the outcome of any other trial.

Binomial Distribution

BINOMIAL PROBABILITY FORMULA

$$P(x) = {}_nC_x \pi^x (1 - \pi)^{n-x}$$

[6-3]

where:

C denotes a combination.

n is the number of trials.

x is the random variable defined as the number of successes.

π is the probability of a success on each trial.

Binomial Dist. – Mean and Variance

MEAN OF A BINOMIAL DISTRIBUTION

$$\mu = n\pi$$

[6-4]

VARIANCE OF A BINOMIAL DISTRIBUTION

$$\sigma^2 = n\pi(1 - \pi)$$

[6-5]

Binomial Distribution

Example: There are five flights daily from Pittsburgh via US Airways into the Bradford, Pennsylvania, Regional Airport. Suppose the probability that any flight arrives late is .20.

1. What is the probability that none of the flights are late today?
2. What is the average number of late flights?
3. What is the variance of the number of late flights?

Hypergeometric Distribution

Characteristics of a Hypergeometric Probability Experiment:

1. An outcome on each trial of an experiment is classified into one of two mutually exclusive categories—a success or a failure.
2. The random variable is the number of successes in a fixed number of trials.
3. The trials are *not independent*.
4. We assume that we sample from a finite population without replacement and $n/N > 0.05$. So, the probability of a success *changes* for each trial.

Hypergeometric Distribution

HYPERGEOMETRIC DISTRIBUTION

$$P(x) = \frac{{}_s C_x {}_{N-s} C_{n-x}}{{}_N C_n}$$

[6-6]

where:

N is the size of the population.

S is the number of successes in the population.

x is the number of successes in the sample. It may be 0, 1, 2, 3,

n is the size of the sample or the number of trials.

C is the symbol for a combination.

Hypergeometric Distribution

Example: Horwege Discount Brokers plans to hire 5 new financial analysts this year. There is a pool of 12 approved applicants, and George Horwege, the owner, decides to randomly select those who will be hired. There are 8 men and 4 women among the approved applicants. What is the probability that 3 of the 5 hired are men?

Hypergeometric Distribution

Example: PlayTime Toys Inc. employs 50 people in the Assembly Department. Forty of the employees belong to a union and ten do not. Five employees are selected at random to form a committee to meet with management regarding shift starting times. What is the probability that four of the five selected for the committee belong to a union?

Binomial vs. Hypergeometric

In order for you to compare the two probability distributions, Table 6–5 shows the hypergeometric and binomial probabilities for the PlayTime Toys Inc. example. Because 40 of the 50 Assembly Department employees belong to the union, we let $\pi = 0.80$ for the binomial distribution. The binomial probabilities for Table 6–5 come from the binomial distribution with $n = 5$ and $\pi = 0.80$.

Binomial vs. Hypergeometric

Number of Union Members on Committee	Hypergeometric Probability, $P(x)$	Binomial Probability ($n = 5$ and $\pi = .80$)
0	.000	.000
1	.004	.006
2	.044	.051
3	.210	.205
4	.431	.410
5	.311	.328
	<u>1.000</u>	<u>1.000</u>

Binomial vs. Hypergeometric

When the binomial requirement of a constant probability of success cannot be met, the hypergeometric distribution should be used. However, as Table 6–5 shows, under certain conditions the results of the binomial distribution can be used to approximate the hypergeometric. This leads to a rule of thumb:

If selected items are not returned to the population, the binomial distribution can be used to closely approximate the hypergeometric distribution when $n < 0.05N$. In words, the binomial will suffice if the sample is less than 5 percent of the population.

Poisson Distribution

- The **Poisson probability distribution** describes the number of times some event occurs during a specified interval. The interval may be time, distance, area, or volume.
- Assumptions of the Poisson Distribution
 - (1) The probability is proportional to the length of the interval.
 - (2) The intervals are independent.

Poisson Distribution

- The characteristics of Poisson distribution:
 1. The random variable is the number of times some event occurs during a define interval.
 2. The probability of the event is proportional to the size of the interval.
 3. The intervals do not overlap and are independent.

Poisson Distribution

- The Poisson distribution can be described mathematically using the formula:

POISSON DISTRIBUTION

$$P(x) = \frac{\mu^x e^{-\mu}}{x!}$$

[6-7]

where:

μ (μ) is the mean number of occurrences (successes) in a particular interval.

e is the constant 2.71828 (base of the Napierian logarithmic system).

x is the number of occurrences (successes).

$P(x)$ is the probability for a specified value of x .

Poisson Dist. – Mean and Variance

- The Mean .(the expected value) of the Poisson Distribution is μ
- The variance is also μ .

Example 1

- Assume baggage is rarely lost by Northwest Airlines. Suppose a random sample of 1,000 flights shows a total of 300 bags were lost. Thus, the arithmetic mean number of lost bags per flight is 0.3 ($300/1,000$). If the number of lost bags per flight follows a Poisson distribution with $\mu = 0.3$, find the probability of not losing any bags.

Example 2

- In the past, schools in Los Angeles County have closed an average of three days each year for weather emergencies. What is the probability that schools in Los Angeles will close for four-days next year?