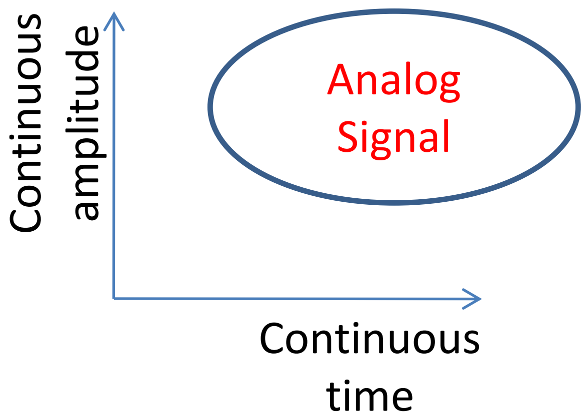
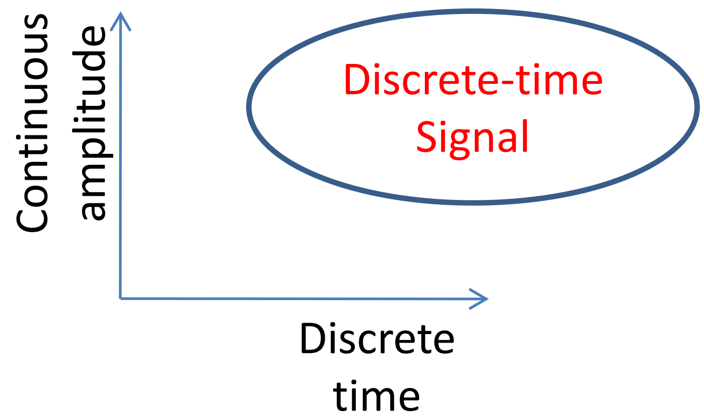
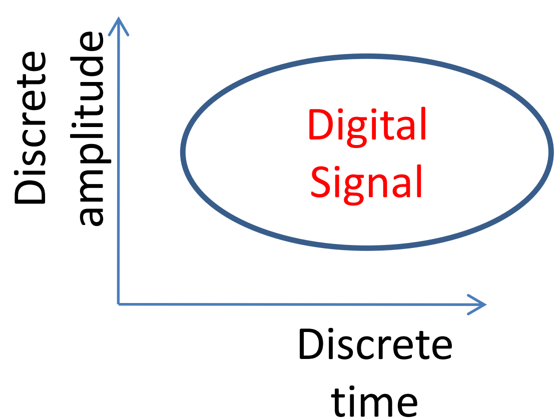
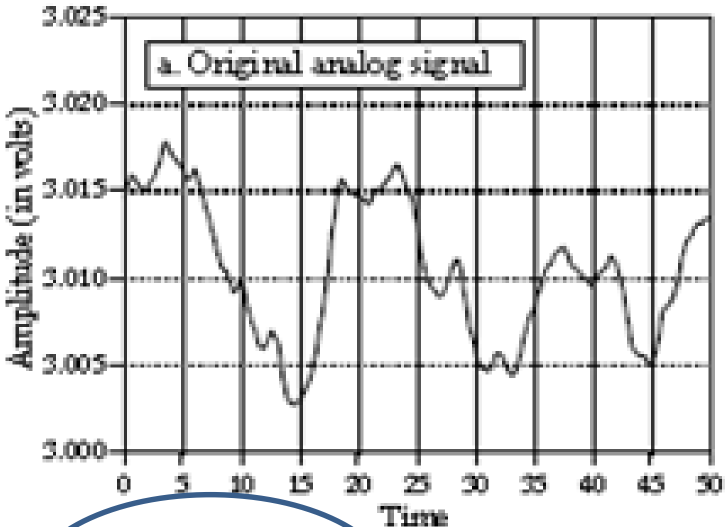
Digital Signal



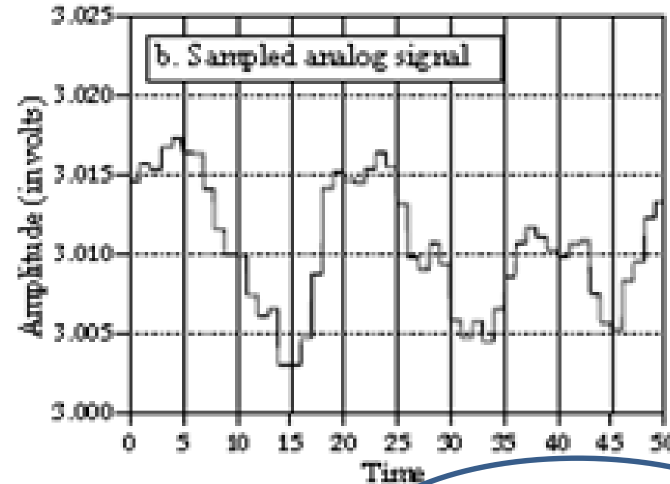




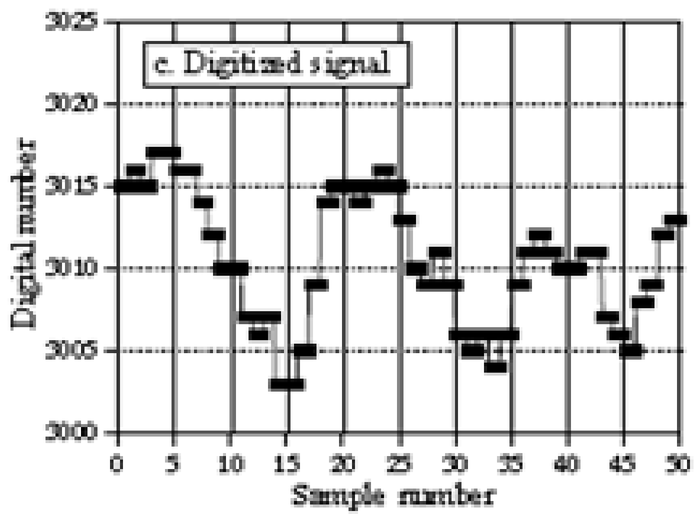
Digital Signal – contd.



Analog Signal



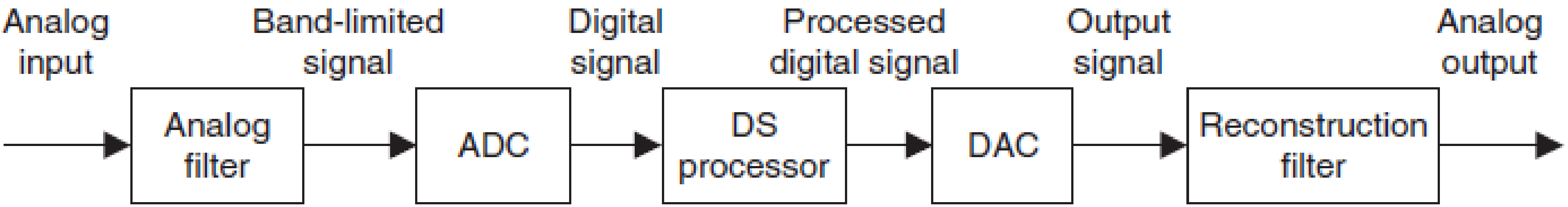
Discrete-time Signal



Digital Signal

DSP (Digital Signal Processing)

A digital signal processing scheme



To avoid aliasing for sampling

Analog to Digital Converter

Digital to Analog Converter

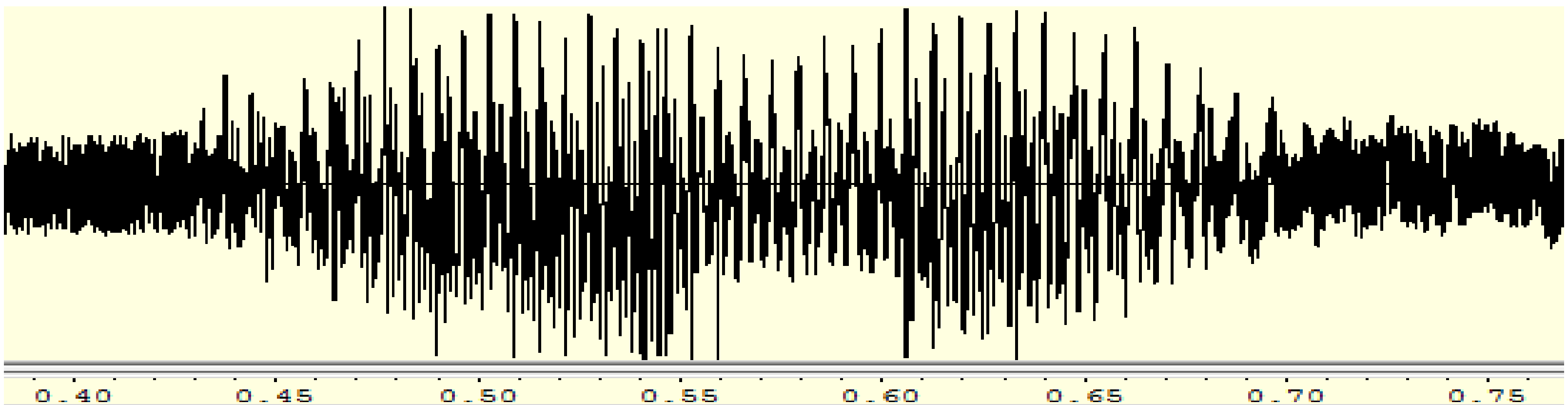
To avoid aliasing for sampling

Computer / microprocessor / micro controller/ etc.

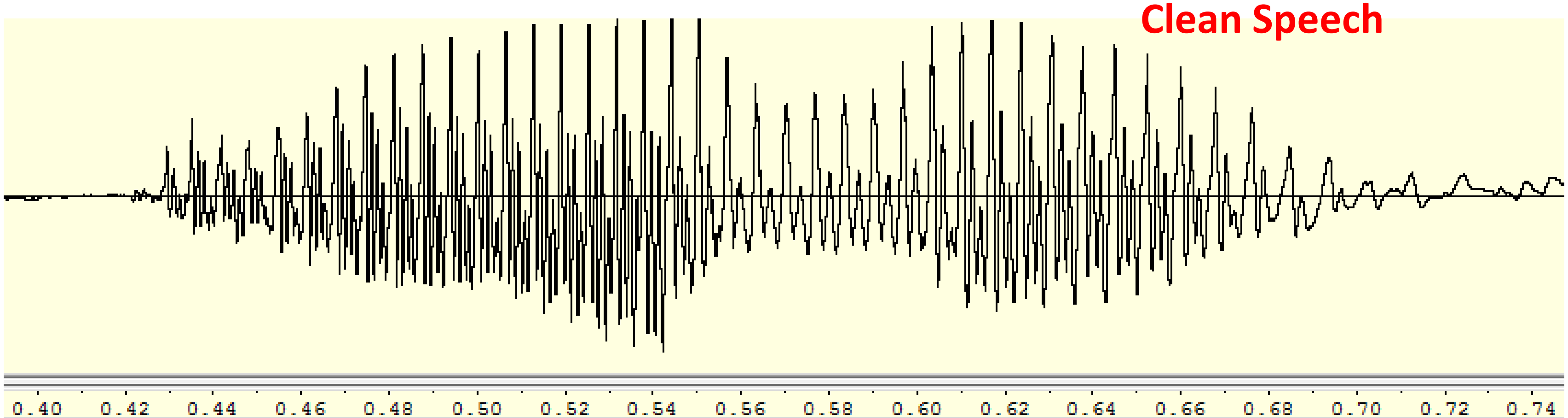
Some Applications of DSP

- Noise removal from speech.

Noisy Speech



Clean Speech



Some Applications of DSP

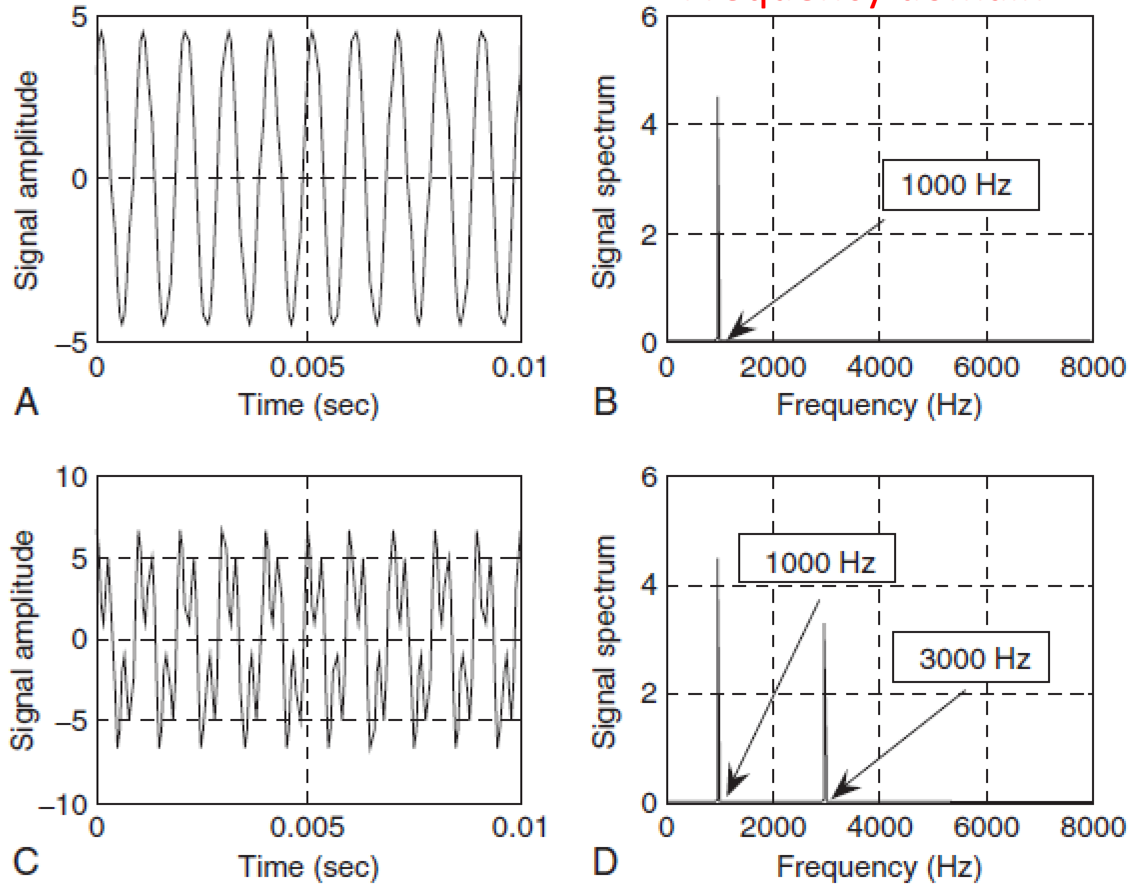
- **Signal spectral analysis.**

Time domain

Frequency domain

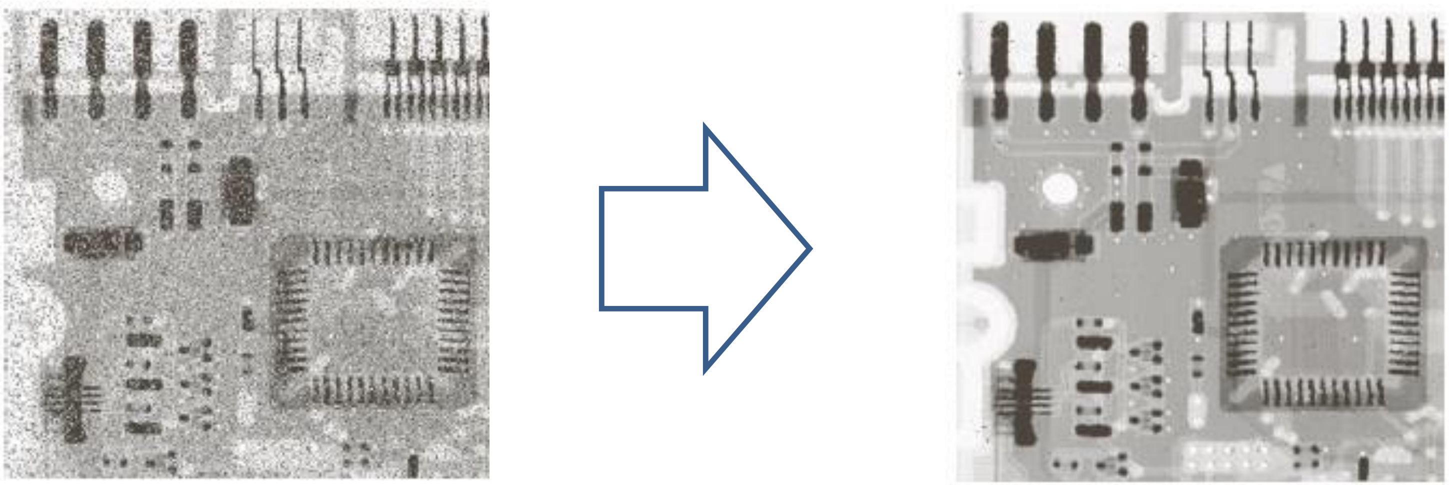
Single tone: 1000 Hz

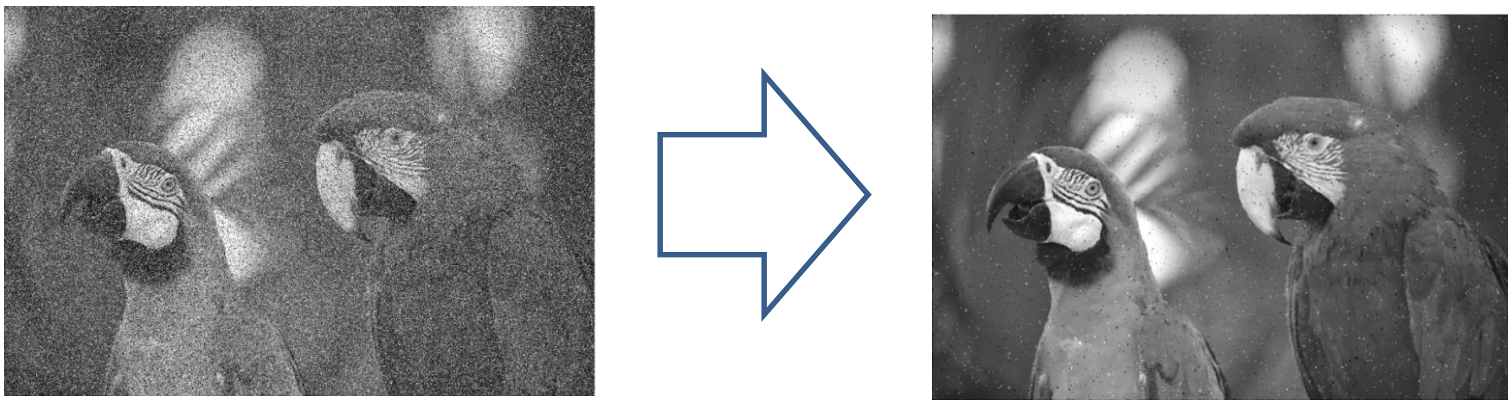
Double tone: 1000 Hz and 3000 Hz



Some Applications of DSP

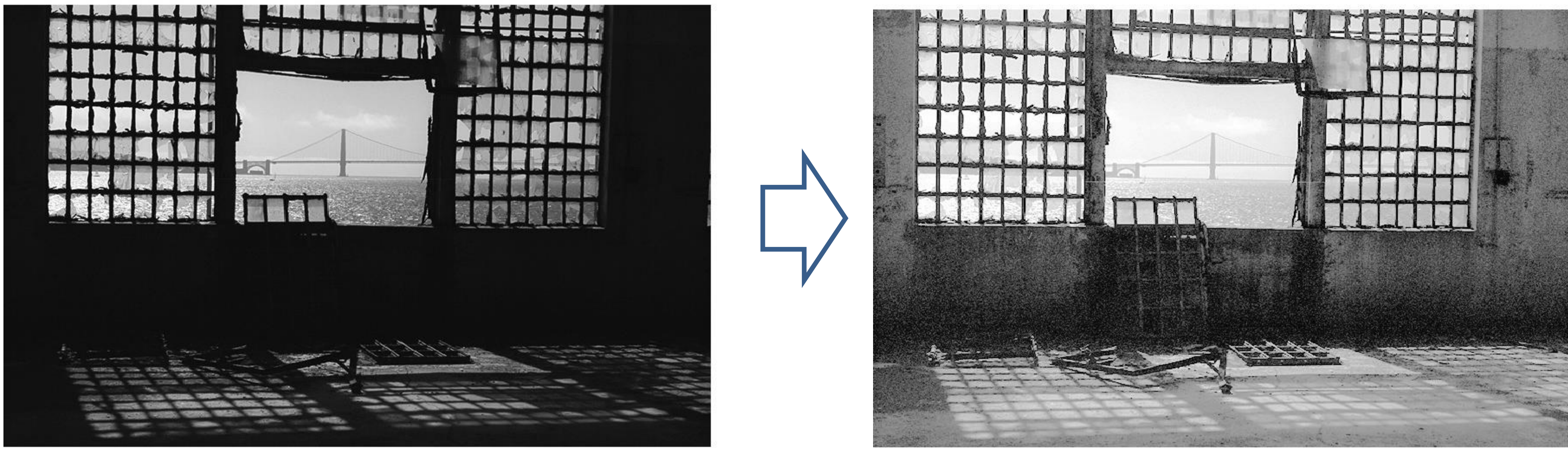
- Noise removal from image.





Some Applications of DSP

- **Image enhancement.**



Summary Applications of DSP

Digital speech and audio:

- Speech recognition
- Speaker recognition
- Speech synthesis
- Speech enhancement
- Speech coding

Digital Image Processing:

- Image enhancement
- Image recognition
- Medical imaging
- Image forensics
- Image coding

Multimedia:

- Internet audio, video, phones
- Image / video compression
- Text-to-voice & voice-to-text
- Movie indexing

……..

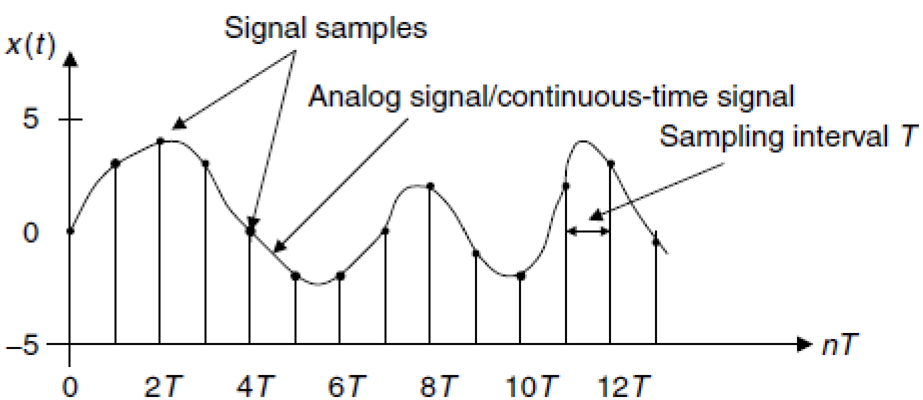
Sampling

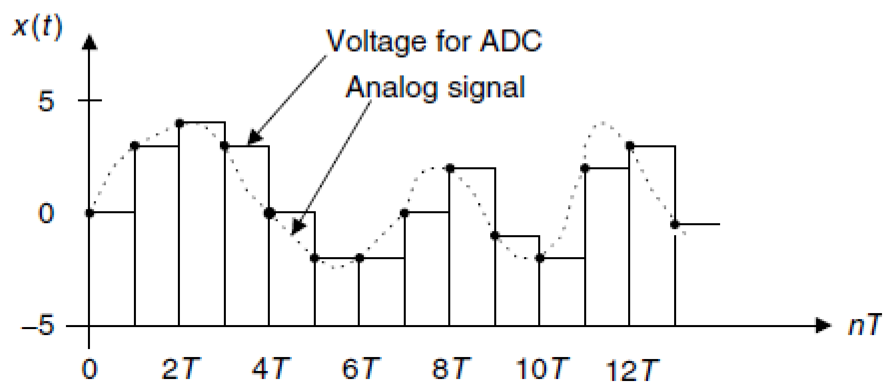
For a given sampling interval T, which is defined as the time span between two sample points, the sampling rate is given by

$$f_s = \frac{1}{T}$$

samples per second (Hz).

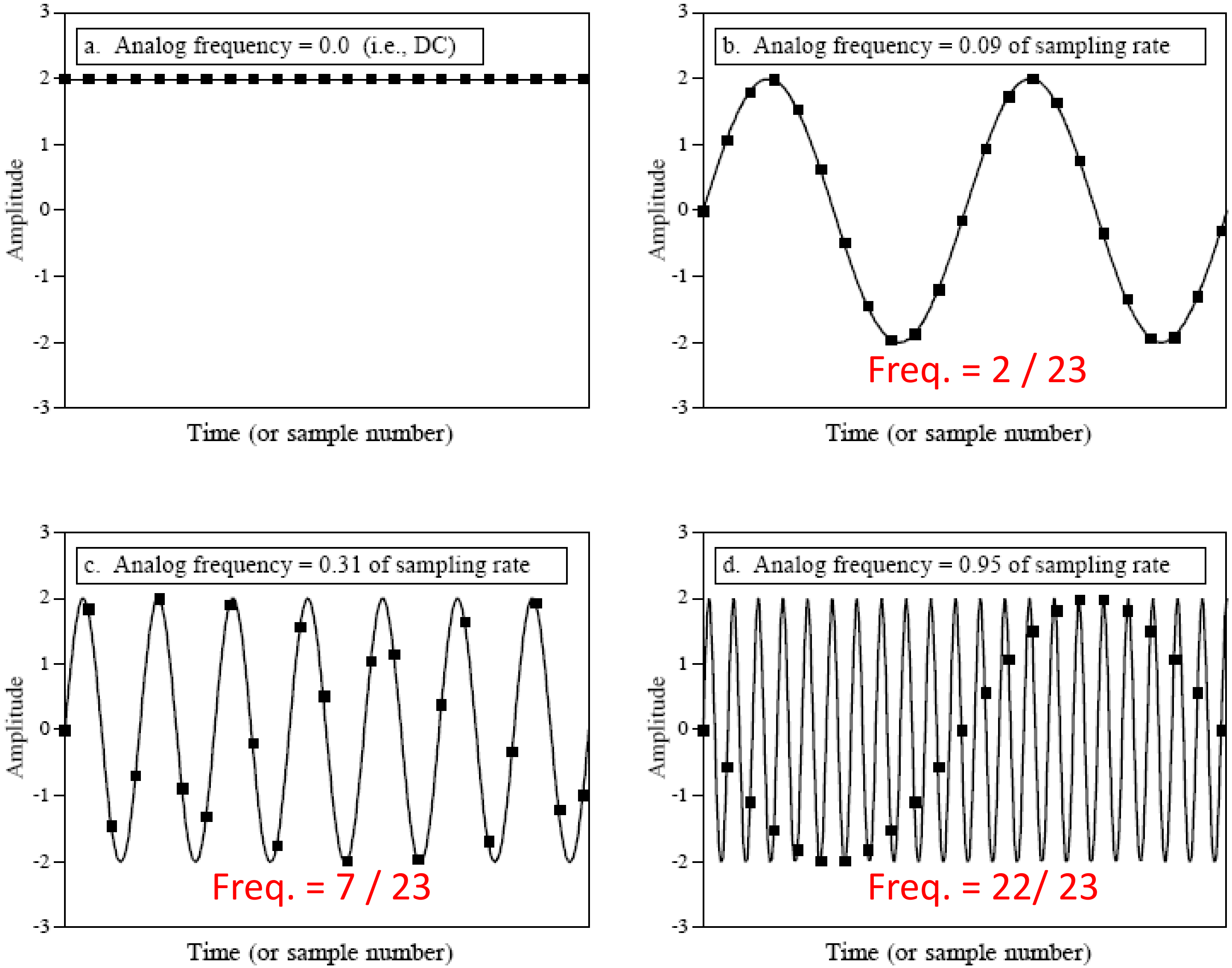
For example, if a sampling period is T = 125 microseconds, the sampling rate is determined as fs =1/125 μs or 8,000 samples per second (Hz).





Sample and Hold

Sampling - Theorem



Sampling - Theorem

The sampling theorem guarantees that an analog signal can be in theory perfectly recovered as long as the sampling rate is at least twice as large as the highest-frequency component of the analog signal to be sampled.

The condition is:

$$f_s \geq 2 f_{\max},$$

where $f_{\max}$ is the maximum-frequency component of the analog signal to be sampled.

For example, to sample a speech signal containing frequencies up to 4 kHz, the minimum sampling rate is chosen to be at least 8 kHz, or 8,000 samples per second.

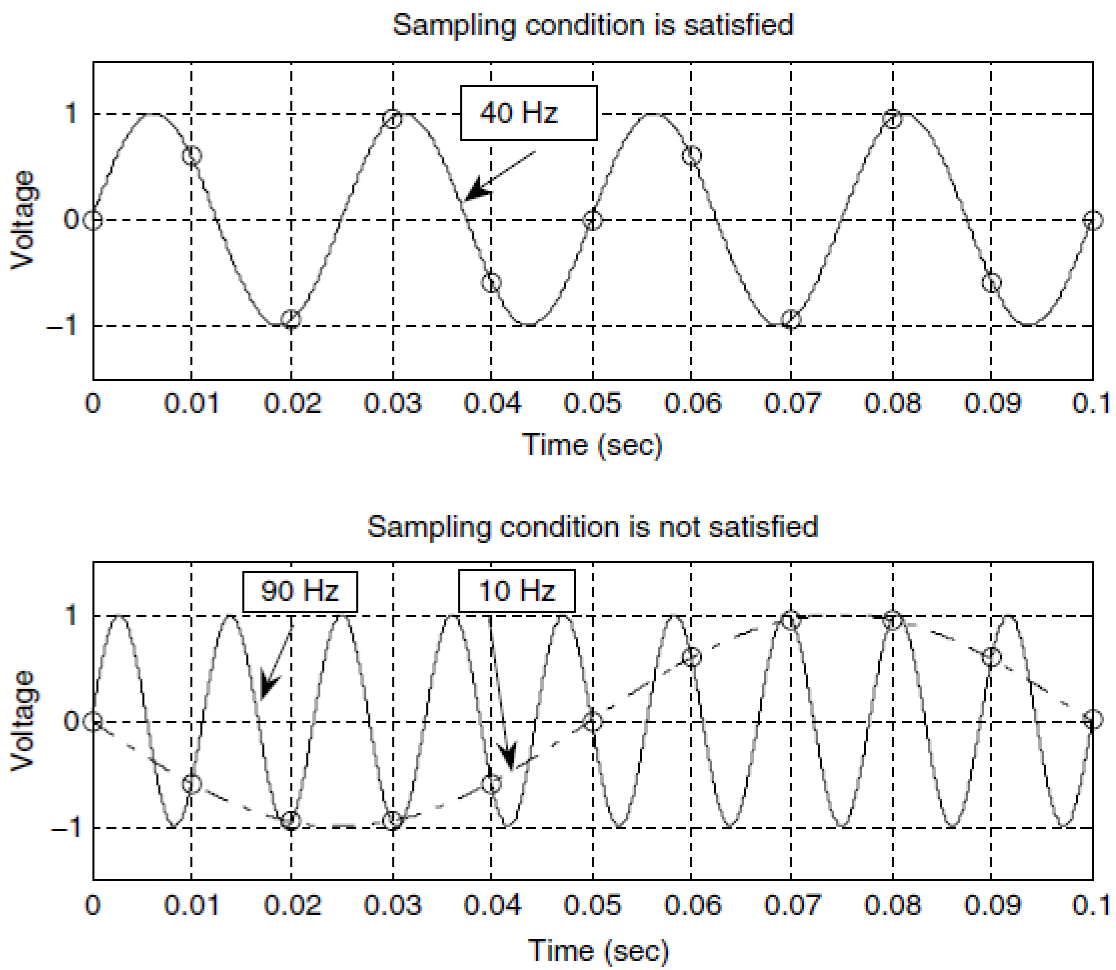
Sampling - Theorem

Sampling interval T= 0.01 s
Sampling rate f_s= 100 Hz
Sinusoid freq. = 4 cycles / 0.1
= 40 Hz

$$2f_{\max} = 80 \text{ Hz} < f_s.$$

Sampling condition is satisfied, so reconstruction from digital to analog is possible.

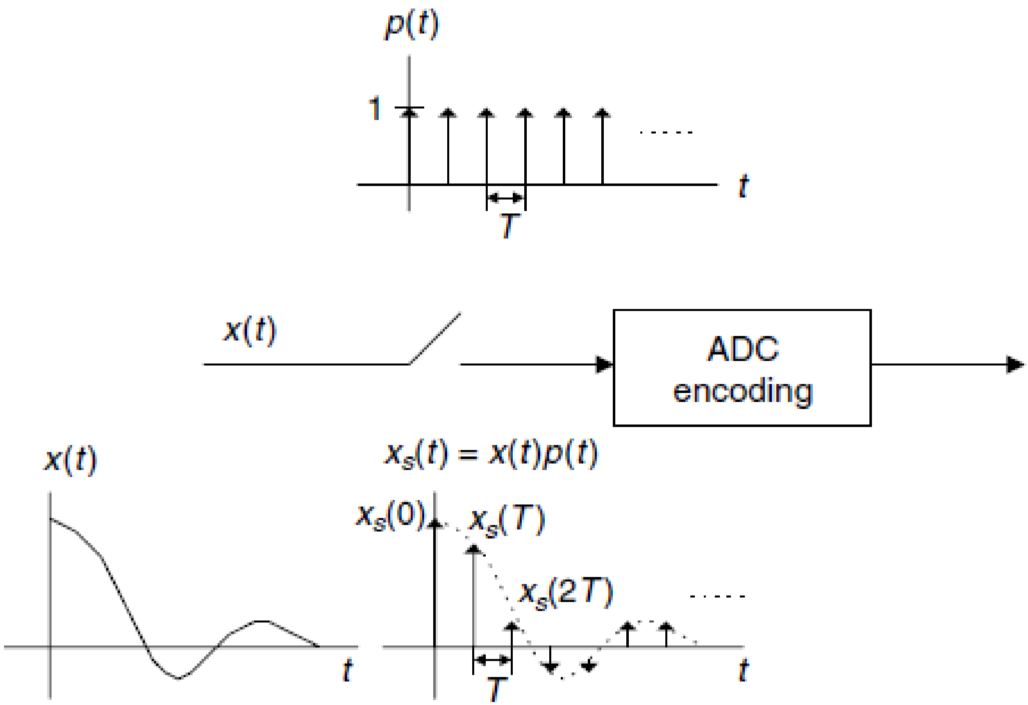
Do this by yourself! →



Sampling Process

$$x_s(t) = x(t)p(t)$$

x(t): Input analog signal
p(t): Pulse train



$$T = \frac{1}{f_s}$$

Sampling Process

In frequency domain:

$$X_s(f) = \frac{1}{T}\sum_{n=-\infty}^{\infty} X(f - nf_s)$$

X_s(f): Sampled spectrum
X(f): Original spectrum
X(f±nf_s): Replica spectrum

$$\Rightarrow \quad X_s(f) = \cdots + \frac{1}{T}X(f+f_s) + \frac{1}{T}X(f) + \frac{1}{T}X(f-f_s) + \cdots$$

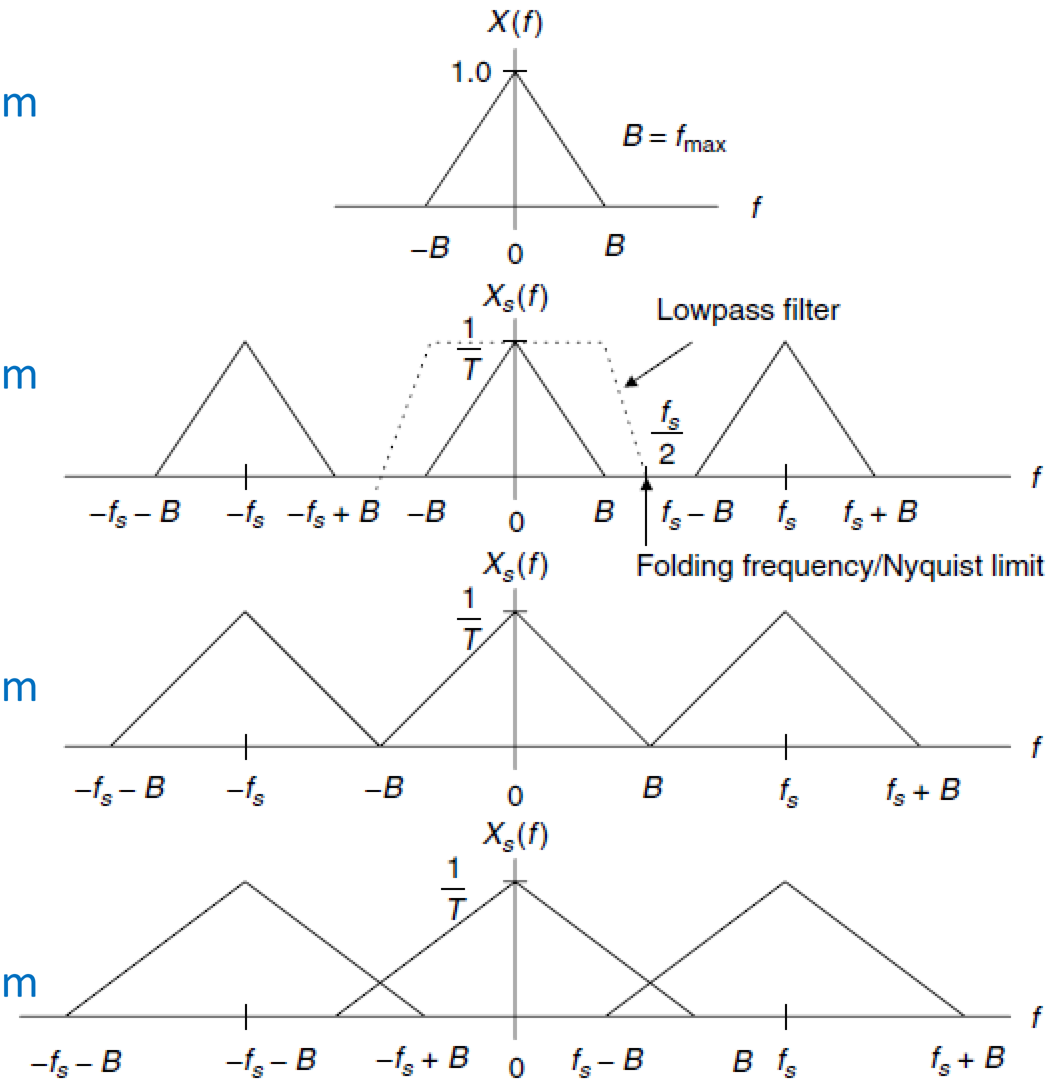
Sampling Process

Original spectrum

Original spectrum plus its replicas

Original spectrum plus its replicas

Original spectrum plus its replicas



Minimum requirement for Reconstruction

Reconstruction not possible

Shannon Sampling Theorem

For a uniformly sampled DSP system, an analog signal can be perfectly recovered as long as the sampling rate is at least twice as large as the highest-frequency component of the analog signal to be sampled.

$$f_s - f_{\max} \geq f_{\max} \quad \Rightarrow \quad f_s \geq 2f_{\max}$$

Half of the sampling frequency $f_s/2$ is usually called the Nyquist frequency (Nyquist limit), or folding frequency.

Example 1

Problem:

Suppose that an analog signal is given as

$$x(t) = 5\cos(2\pi \cdot 1000t), \text{ for } t \geq 0$$

and is sampled at the rate of 8,000 Hz.

a. Sketch the spectrum for the original signal.

b. Sketch the spectrum for the sampled signal from 0 to 20 kHz.

Solution:

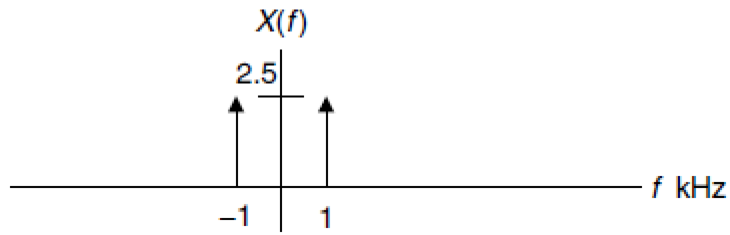
Using Euler's identity,

$$5\cos(2\pi \times 1000t) = 5 \cdot \left(\frac{e^{j2\pi \times 1000t} + e^{-j2\pi \times 1000t}}{2} \right) = 2.5e^{j2\pi \times 1000t} + 2.5e^{-j2\pi \times 1000t}$$

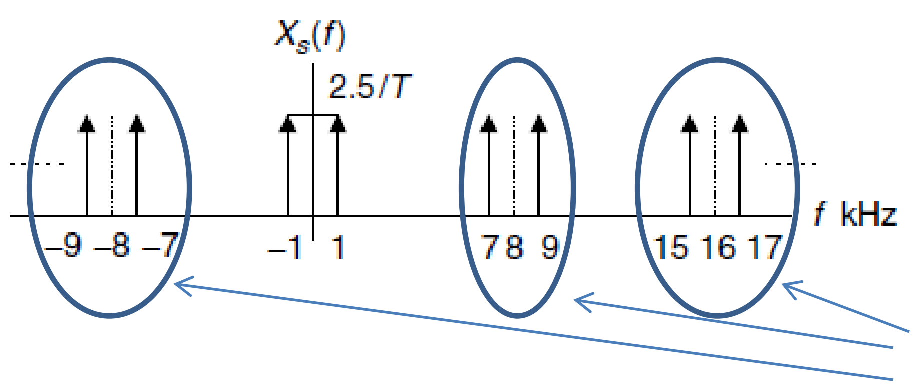
Hence, the Fourier series coefficients are: $c_1 = 2.5$, and $c_{-1} = 2.5$.

Example 1 – contd.

a.



b. After the analog signal is sampled at the rate of 8,000 Hz, the sampled signal spectrum and its replicas centered at the frequencies $\pm nf_s$, each with the scaled amplitude being 2.5/T

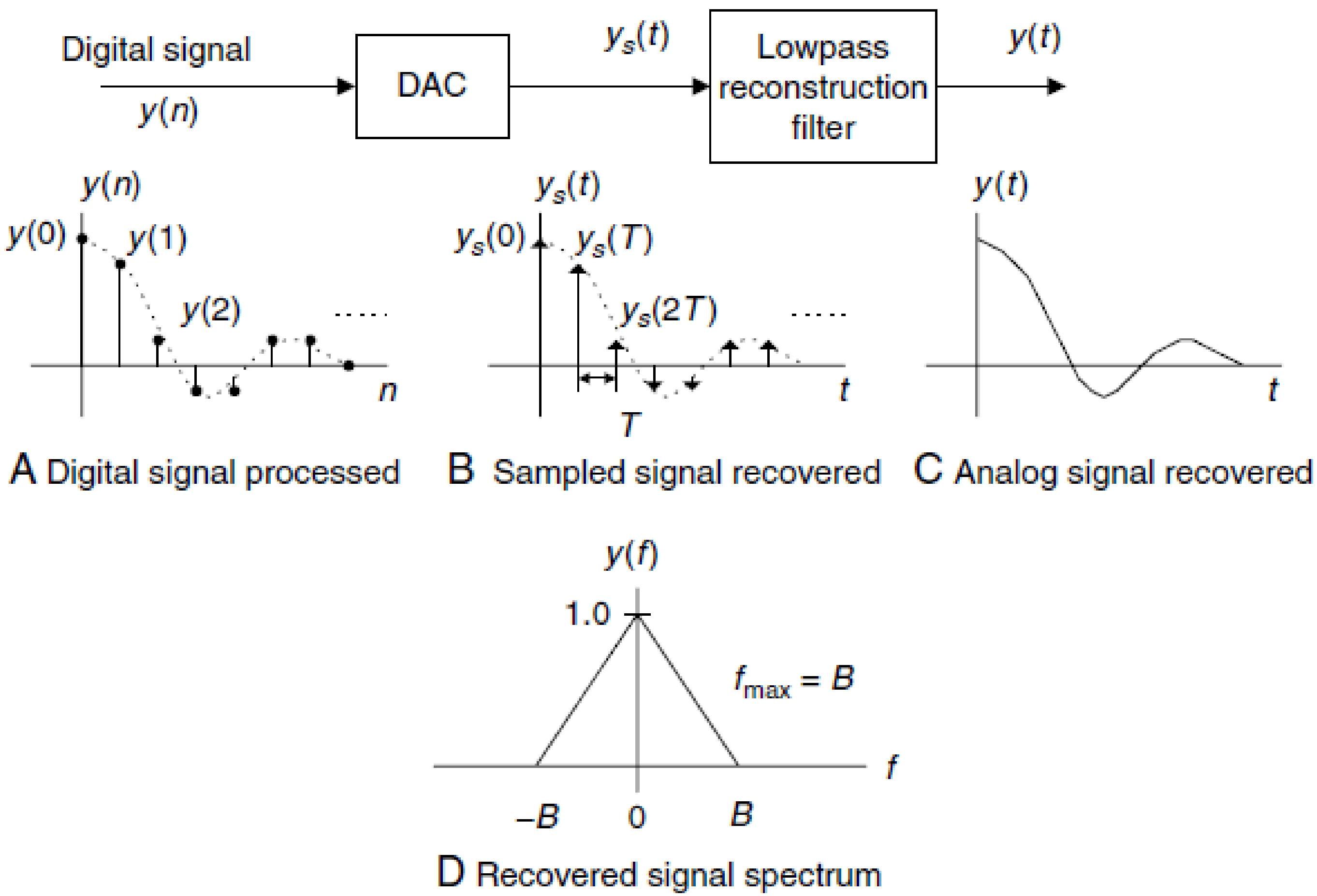


Replicas, no additional information.

Signal Reconstruction

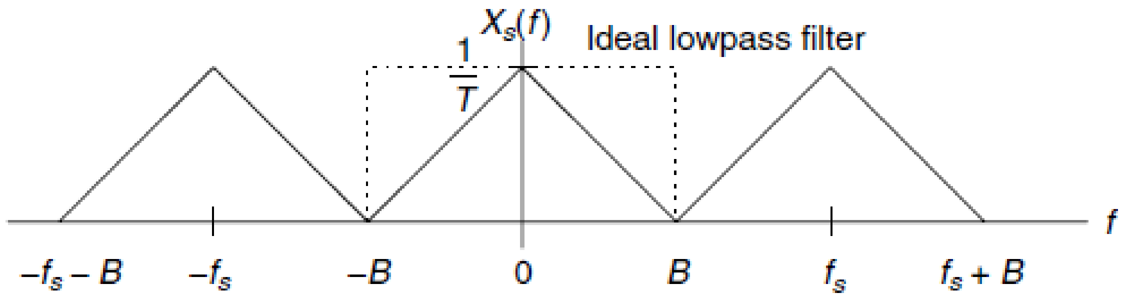
First, the digitally processed data y(n) are converted to the ideal impulse train $y_s(t)$, in which each impulse has its amplitude proportional to digital output y(n), and two consecutive impulses are separated by a sampling period of T;

second, the analog reconstruction filter is applied to the ideally recovered sampled signal $y_s(t)$ to obtain the recovered analog signal.

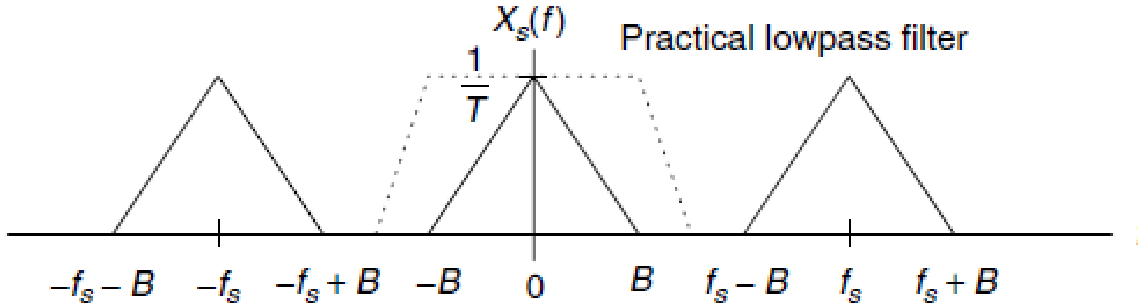


Signal Reconstruction

Case 1: $f_s = 2f_{\max}$

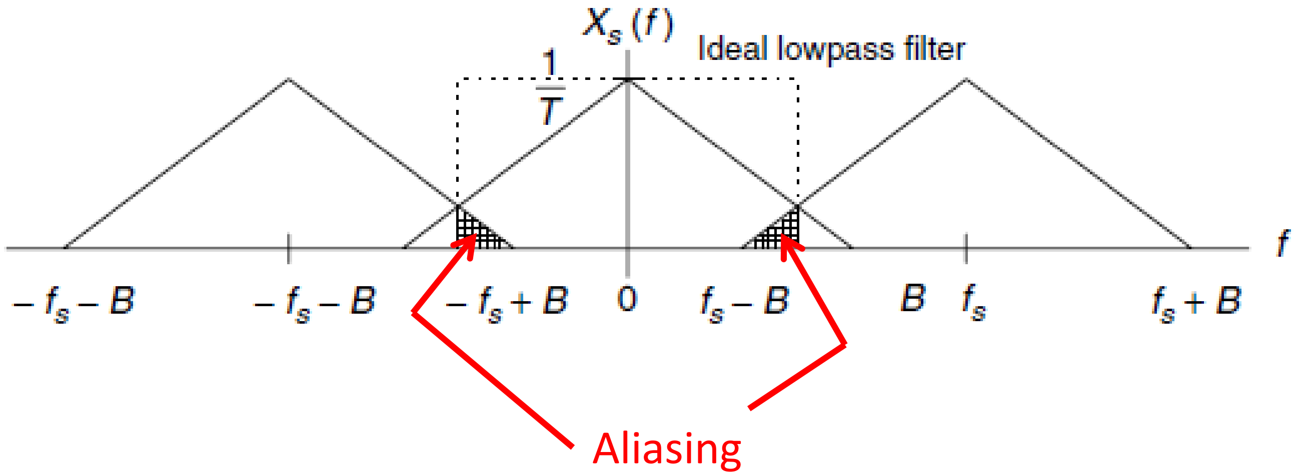


Case 2: $f_s > 2f_{\max}$



Signal Reconstruction

Case 3: $f_s < 2f_{max}$



Perfect reconstruction is not possible, even if we use ideal low pass filter.

Example 2

Problem:

Assuming that an analog signal is given by

$$x(t) = 5\cos(2\pi \cdot 2000t) + 3\cos(2\pi \cdot 3000t), \text{ for } t \geq 0$$

and it is sampled at the rate of 8,000 Hz,

a. Sketch the spectrum of the sampled signal up to 20 kHz.

b. Sketch the recovered analog signal spectrum if an ideal lowpass filter with a cutoff frequency of 4 kHz is used to filter the sampled signal ($y(n) = x(n)$ in this case) to recover the original signal.

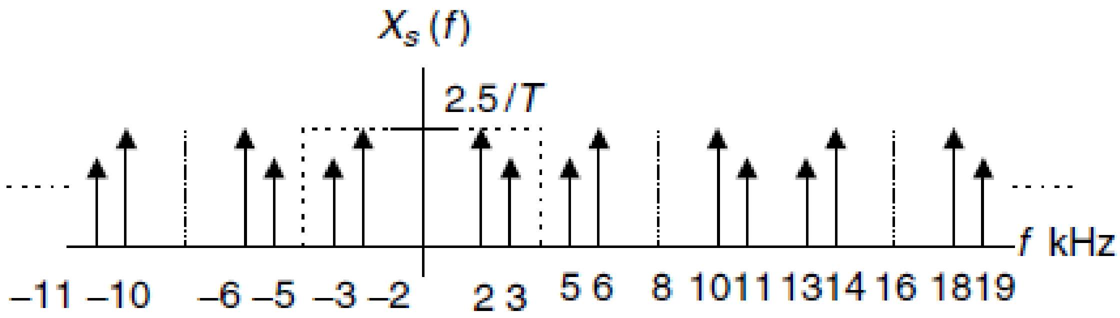
Solution:

Using the Euler's identity:

$$x(t) = \frac{3}{2}e^{-j2\pi \cdot 3000t} + \frac{5}{2}e^{-j2\pi \cdot 2000t} + \frac{5}{2}e^{j2\pi \cdot 2000t} + \frac{3}{2}e^{j2\pi \cdot 3000t}$$

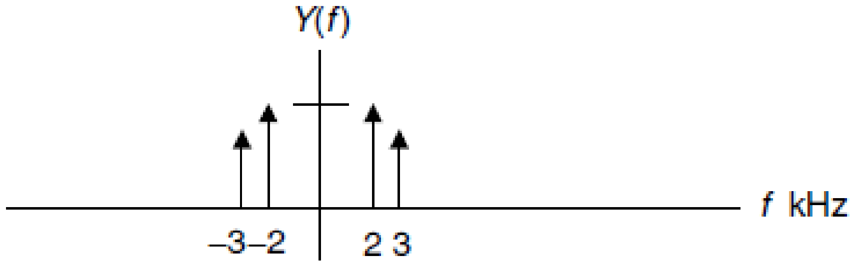
Example 2 - contd.

a.



b.

The Shannon sampling theory condition is satisfied.



Example 3

Problem:

Given an analog signal

$$x(t) = 5\cos(2\pi \times 2000t) + 1\cos(2\pi \times 5000t), \text{ for } t \geq 0,$$

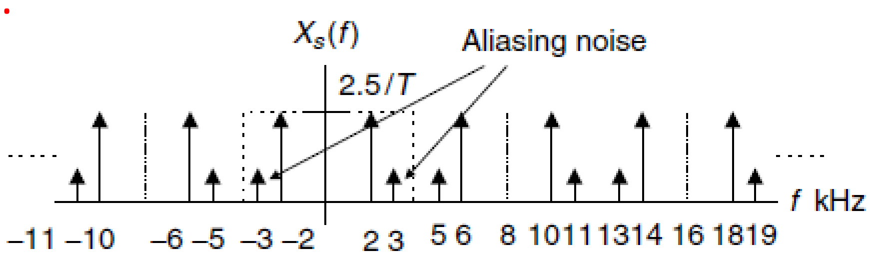
which is sampled at a rate of 8,000 Hz,

a. Sketch the spectrum of the sampled signal up to 20 kHz.

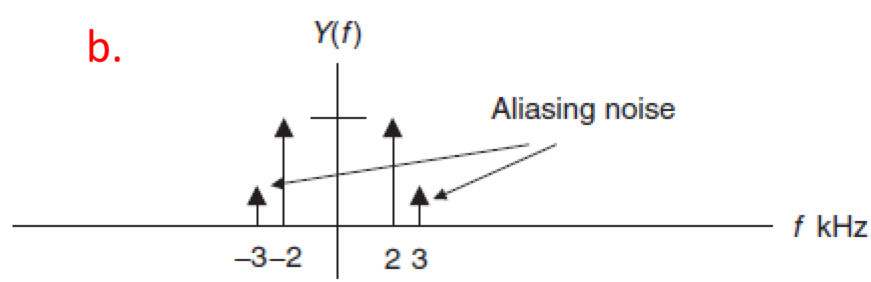
b. Sketch the recovered analog signal spectrum if an ideal lowpass filter with a cutoff frequency of 4 kHz is used to recover the original signal ($y(n) = x(n)$ in this case).

Solution:

a.



b.



Quantization

$$\Delta = \frac{(x_{\max} - x_{\min})}{L}$$

$$L = 2^m$$

$$i = round\left(\frac{x - x_{\min}}{\Delta}\right)$$

$$x_q = x_{\min} + i\Delta, \text{ for } i = 0,\ 1, \ldots, L - 1,$$

L: No. of quantization level

m: Number of bits in ADC

Δ: Step size of quantizer

i: Index corresponding to binary code

x_q: Quantization level

x_{max}: Max value of analog signal

x_{min}: Min value of analog signal

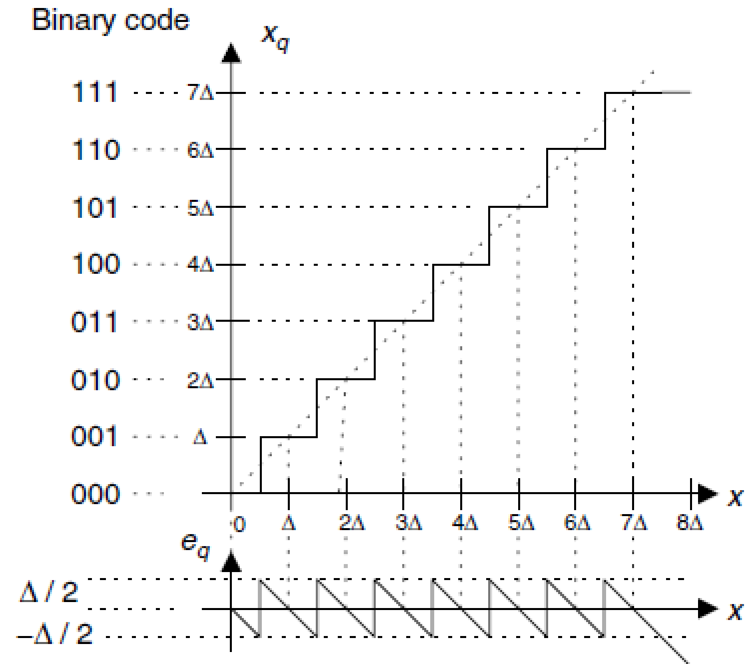
Example:

$$x_{\min} = 0,\ x_{\max} = 8\Delta, \text{ and } m = 3$$

$$x_q = 0 + i\Delta, i = 0, 1, \ldots,\ L - 1,$$

$$L = 2^3 = 8$$

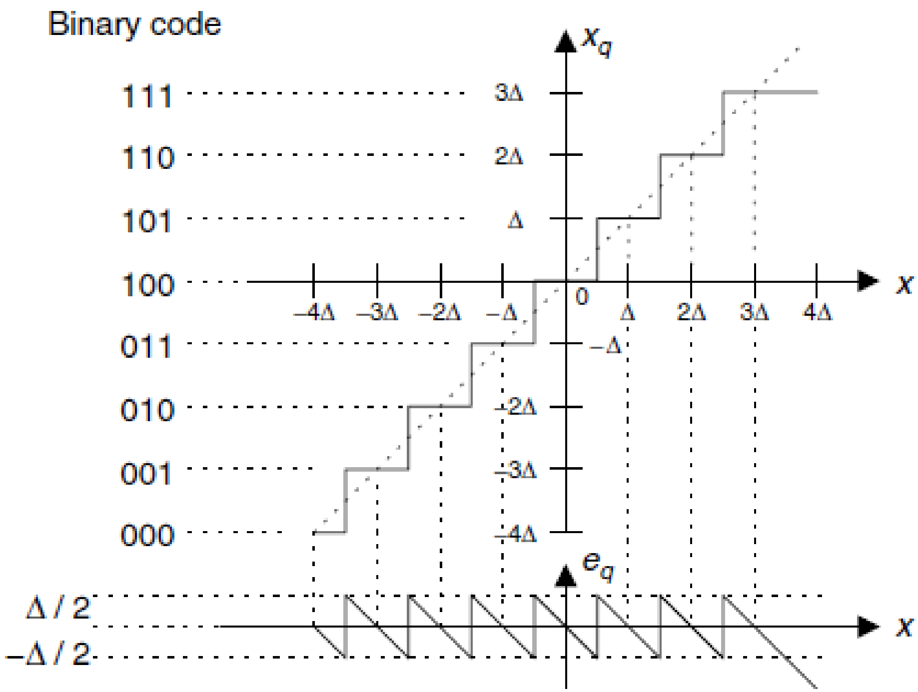
Unipolar



Quantization – contd.

$x_{\min} = -4\Delta,\ x_{\max} = 4\Delta,\ \text{and}\ m = 3.$

Bipolar



Example 4

Problem:

Assuming that a 3-bit ADC channel accepts analog input ranging from 0 to 5 volts, determine the following:

a. number of quantization levels

b. step size of the quantizer or resolution

c. quantization level when the analog voltage is 3.2 volts

d. binary code produced by the ADC

Solution:

$$x_{\min} = 0 \text{ volt}, \ x_{\max} = 5 \text{ volts, and } m = 3 \text{ bits}$$

a. $L = 2^m = 2^3 = 8.$

b. $\Delta = \frac{5-0}{8} = 0.625 \text{ volt}$

c. $x = 3.2 \frac{\Delta}{0.625} = 5.12\Delta$

$$i = round\left(\frac{x - x_{\min}}{\Delta}\right) = round(5.12) = 5.$$

$$x_q = 0 + 5\Delta = 5 \times 0.625 = 3.125 \text{ volts.}$$

d. 101

Quantization error: $e_q = x_q - x = 3.125 - 3.2 = -0.075 \text{ volt.}$