

**\*\*BEGINNING OF EXAMINATION\*\***

**1.** For a 2-year select and ultimate mortality model, you are given:

(i)  $q_{[x]+1} = 0.95 q_{x+1}$

(ii)  $l_{76} = 98,153$

(iii)  $l_{77} = 96,124$

Calculate  $l_{[75]+1}$ .

(A) 96,150

(B) 96,780

(C) 97,420

(D) 98,050

(E) 98,690

2. You are given:

(i)  $p_x = 0.97$

(ii)  $p_{x+1} = 0.95$

(iii)  $e_{x+1.75} = 18.5$

(iv) Deaths are uniformly distributed between ages  $x$  and  $x+1$ .

(v) The force of mortality is constant between ages  $x+1$  and  $x+2$ .

Calculate  $e_{x+0.75}$ .

(A) 18.6

(B) 18.8

(C) 19.0

(D) 19.2

(E) 19.4

**3.** For a fully discrete 3-year term insurance of 10,000 on (40), you are given:

- (i)  $\mu_{40+t}$ ,  $t \geq 0$ , is a force of mortality consistent with the Illustrative Life Table.
- (ii)  $\mu_{40+t} + 0.02$ ,  $t = 0, 1, 2$ , is the force of mortality for this insured.
- (iii)  $i = 0.06$

Calculate the annual benefit premium for this insurance.

- (A) 196
- (B) 214
- (C) 232
- (D) 250
- (E) 268

4. For a 3-year term insurance of 1000 on  $[75]$ , you are given:

- (i) Death benefits are payable at the end of the year of death.
- (ii) Level premiums are payable at the beginning of each quarter.
- (iii) Mortality follows a select and ultimate life table with a two-year select period:

$x$	$l_{[x]}$	$l_{[x]+1}$	$l_{x+2}$	$x+2$
75	15,930	15,668	15,286	77
76	15,508	15,224	14,816	78
77	15,050	14,744	14,310	79

- (iv) Deaths are uniformly distributed over each year of age.
- (v)  $i = 0.06$

Calculate the amount of each quarterly benefit premium.

- (A) 5.3
- (B) 5.5
- (C) 5.7
- (D) 5.9
- (E) 6.1

**5.** For a whole life insurance on (80):

- (i) Level premiums of 900 are payable at the start of each year.
- (ii) Commission expenses are 50% of premium in the first year and 10% of premium in subsequent years, payable at the start of the year.
- (iii) Maintenance expenses are 3 per month, payable at the start of each month.
- (iv)  $\delta = 0.0488$
- (v)  $\ddot{a}_{80} = 6.000$
- (vi)  $\mu_{80} = 0.033$

Using the first 3 terms of Woolhouse's formula, calculate the expected value of the present-value-of-expenses random variable at age 80.

- (A) 920
- (B) 940
- (C) 1010
- (D) 1100
- (E) 1120

6. Two whales, Hannibal and Jack, occupy the Ocean World aquarium.

Both are currently age 6 with independent future lifetimes. Hannibal arrived at Ocean World at age 4 and Jack at age 6.

The standard mortality for whales is as follows:

$x$	$q_x$
6	0.06
7	0.07
8	0.08
9	0.09

Whales arriving at Ocean World have mortality according to the following:

- 150% of standard mortality in year 1
- 130% of standard mortality in year 2
- 110% of standard mortality in year 3
- 100% of standard mortality in years 4 and later.

As whales can get lonely on their own, a fund is set up today to provide 100,000 to replace a whale at the end of 3 years if exactly one survives.

$$i = 0.06$$

Calculate the initial value of the fund using the equivalence principle.

- (A) 24,500
- (B) 29,200
- (C) 29,300
- (D) 30,900
- (E) 31,900

7. For a single premium product replacement insurance on a computer, you are given:

- (i) The insurance pays a percentage of the cost of a new computer at the end of the year of failure within the first 5 years.
- (ii) The insurance will pay only once.
- (iii) The insurance is bought at the time of the purchase of the computer.
- (iv) The cost of a new computer increases by 2% each year, compounded annually. The insurance pays based on the increased cost.
- (v) The insurance benefit and probabilities of failure are as follows:

Year of failure $k$	Percentage of cost of new computer covered	Probability of computer failure ${}_{k-1 }q_0$
1	100%	0.01
2	80%	0.01
3	60%	0.02
4	40%	0.02
5	20%	0.02

- (vi) The commission is  $c\%$  of the gross premium.
- (vii) Other expenses are 5% of the gross premium.
- (viii) The gross premium, calculated using the equivalence principle, is 10% of the original cost of the computer.
- (ix)  $i = 0.04$

Calculate  $c\%$ .

- (A) 40%
- (B) 45%
- (C) 50%
- (D) 55%
- (E) 60%

**8.** For a fully discrete whole life insurance of 1000 on (80):

(i)  $i = 0.06$

(ii)  $\ddot{a}_{80} = 5.89$

(iii)  $\ddot{a}_{90} = 3.65$

(iv)  $q_{80} = 0.077$

Calculate  ${}_{10}V^{\text{FPT}}$ , the full preliminary term reserve for this policy at the end of year 10.

(A) 340

(B) 350

(C) 360

(D) 370

(E) 380



9. A special fully discrete 3-year endowment insurance on  $(x)$  pays death benefits as follows:

<u>Year of Death</u>	<u>Death Benefit</u>
1	30,000
2	40,000
3	50,000

You are given:

- (i) The maturity benefit is 50,000.
- (ii) Annual benefit premiums increase at 10% per year, compounded annually.
- (iii)  $i = 0.05$
- (iv)  $q_x = 0.09$      $q_{x+1} = 0.12$      $q_{x+2} = 0.15$

Calculate  ${}_2V$ , the benefit reserve at the end of year 2.

- (A) 28,000
- (B) 29,000
- (C) 30,000
- (D) 31,000
- (E) 32,000

**10.** For a universal life insurance policy on Julie, you are given:

- (i) At  $t = 0$ , Julie will pay a premium of 100.
- (ii) At  $t = 1$ , Julie will pay a premium of 50 with a probability of 0.8 or will pay nothing otherwise.
- (iii) At  $t = 2$ , Julie will pay a premium of 50 with probability of 0.9 if she paid the premium at  $t = 1$  or will pay nothing otherwise.
- (iv) During the first three years, Julie will pay no other premiums.
- (v)  $i = 0.04$
- (vi) Assume no deaths or surrenders in the first three years.

Calculate the standard deviation of the present value at issue of premiums paid during the first three years.

- (A) 30
- (B) 32
- (C) 34
- (D) 36
- (E) 38

**11.** For a universal life insurance policy with death benefit of 10,000 plus account value, you are given:

(i)

<u>Policy Year</u>	<u>Monthly Premium</u>	<u>Percent of Premium Charge</u>	<u>Cost of Insurance Rate Per Month</u>	<u>Monthly Expense Charge</u>	<u>Surrender Charge</u>
1	100	30%	0.001	5	300
2	100	10%	0.002	5	100

(ii) The credited interest rate is  $i^{(12)} = 0.048$ .

(iii) The actual cash surrender value at the end of month 11 is 1000.

(iv) The policy remains in force for months 12 and 13, with the monthly premiums of 100 being paid at the start of each month.

Calculate the cash surrender value at the end of month 13.

- (A) 1130
- (B) 1230
- (C) 1330
- (D) 1400
- (E) 1460

**12.** Employees in Company ABC can be in:

- State 0: Non-executive employee
- State 1: Executive employee
- State 2: Terminated from employment

John joins Company ABC as a non-executive employee at age 30.

You are given:

- (i)  $\mu^{01} = 0.01$  for all years of service
- (ii)  $\mu^{02} = 0.006$  for all years of service
- (iii)  $\mu^{12} = 0.002$  for all years of service
- (iv) Executive employees never return to the non-executive employee state.
- (v) Employees terminated from employment never get rehired.
- (vi) The probability that John lives to age 65 is 0.9, regardless of state.

Calculate the probability that John will be an executive employee of Company ABC at age 65.

- (A) 0.232
- (B) 0.245
- (C) 0.258
- (D) 0.271
- (E) 0.284

**13.** Lorie's Lorries rents lavender limousines.

On January 1 of each year they purchase 30 limousines for their existing fleet; of these, 20 are new and 10 are one-year old.

Vehicles are retired according to the following 2-year select-and-ultimate table, where selection is age at purchase:

Limousine age ( $x$ )	$q_{[x]}$	$q_{[x]+1}$	$q_{x+2}$	$x+2$
0	0.100	0.167	0.333	2
1	0.100	0.333	0.500	3
2	0.150	0.400	1.000	4
3	0.250	0.750	1.000	5
4	0.500	1.000	1.000	6
5	1.000	1.000	1.000	7

Lorie's Lorries has rented lavender limousines for the past ten years and has always purchased its limousines on the above schedule.

Calculate the expected number of limousines in the Lorie's Lorries fleet immediately after the purchase of this year's limousines.

- (A) 93
- (B) 94
- (C) 95
- (D) 96
- (E) 97

14. XYZ Insurance Company sells a one-year term insurance product with a gross premium equal to 125% of the benefit premium.

You are given:

- (i) The death benefit is payable at the end of the year of death, and the amount depends on whether the cause of death is accidental or not.
- (ii) Death benefit amounts, together with the associated one-year probabilities of death, are as follows:

Cause of Death	Death Benefit	Probabilities of Death
Non-accidental	5	0.10
Accidental	10	0.01

- (iii)  $i = 0\%$

Using the normal approximation without continuity correction, calculate the smallest number of policies that XYZ must sell so that the amount of gross premium equals or exceeds the 95<sup>th</sup> percentile of the distribution of the total present value of death benefits.

- (A) 378
- (B) 431
- (C) 484
- (D) 537
- (E) 590

**15.** For a 20-year temporary life annuity-due of 100 per year on (65), you are given:

(i)  $\mu_x = 0.001x, \quad x \geq 65$

(ii)  $i = 0.05$

(iii)  $Y$  is the present-value random variable for this annuity.

Calculate the probability that  $Y$  is less than 1000.

(A) 0.54

(B) 0.57

(C) 0.61

(D) 0.64

(E) 0.67

**16.** For a special continuous joint life annuity on  $(x)$  and  $(y)$ , you are given:

- (i) The annuity payments are 25,000 per year while both are alive and 15,000 per year when only one is alive.
- (ii) The annuity also pays a death benefit of 30,000 upon the first death.
- (iii)  $i = 0.06$
- (iv)  $\bar{a}_{xy} = 8$
- (v)  $\bar{a}_{\overline{xy}} = 10$

Calculate the actuarial present value of this special annuity.

- (A) 239,000
- (B) 246,000
- (C) 287,000
- (D) 354,000
- (E) 366,000



**17.** Your company issues fully discrete whole life policies to a group of lives age 40. For each policy, you are given:

- (i) The death benefit is 50,000.
- (ii) Assumed mortality and interest are the Illustrative Life Table at 6%.
- (iii) Annual gross premium equals 125% of benefit premium.
- (iv) Assumed expenses are 5% of gross premium, payable at the beginning of each year, and 300 to process each death claim, payable at the end of the year of death.
- (v) Profits are based on gross premium reserves.

During year 11, actual experience is as follows:

- (a) There are 1000 lives inforce at the beginning of the year.
- (b) There are five deaths.
- (c) Interest earned equals 6%.
- (d) Expenses equal 6% of gross premium and 100 to process each death claim.

For year 11, you calculate the gain due to mortality and then the gain due to expenses.

Calculate the gain due to expenses during year 11.

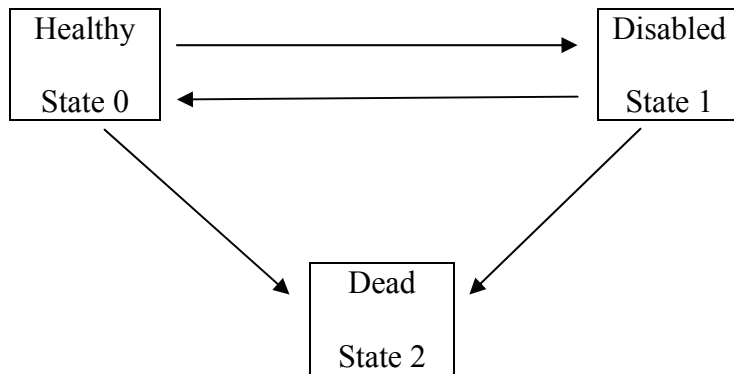
- (A) -5900
- (B) -6200
- (C) -6400
- (D) -6700
- (E) -7000

- 18.** For a fully discrete whole life policy on (50) with death benefit 100,000, you are given:
- (i) Reserves equal benefit reserves calculated using the Illustrative Life Table at 6%.
  - (ii) The gross premium equals 120% of the benefit premium calculated using the Illustrative Life Table at 6%.
  - (iii) Expected expenses equal 40 plus 5% of gross premium, payable at the beginning of each year.
  - (iv) Expected mortality equals 70% of the Illustrative Life Table.
  - (v) The expected interest rate is 7%.

Calculate the expected profit in the eleventh policy year, for a policy in force at the beginning of that year.

- (A) 683
- (B) 719
- (C) 756
- (D) 792
- (E) 829

19. You are pricing disability insurance using the following model:



You assume the following constant forces of transition:

(i)  $\mu^{01} = 0.06$

(ii)  $\mu^{10} = 0.03$

(iii)  $\mu^{02} = 0.01$

(iv)  $\mu^{12} = 0.04$

Calculate the probability that a disabled life on July 1, 2012 will become healthy at some time before July 1, 2017 but will not then remain continuously healthy until July 1, 2017.

- (A) 0.012
- (B) 0.015
- (C) 0.018
- (D) 0.021
- (E) 0.024

20. Jenny joins XYZ Corporation today as an actuary at age 60. Her starting annual salary is 225,000 and will increase by 4% each year on her birthday. Assume that retirement takes place on a birthday immediately following the salary increase.

XYZ offers a plan to its employees with the following benefits:

- A single sum retirement benefit equal to 20% of the final salary at time of retirement for each year of service. Retirement is compulsory at age 65; however, early retirement is permitted at ages 63 and 64, but with the retirement benefit reduced by 40% and 20%, respectively. The retirement benefit is paid on the date of retirement.
- A death benefit, payable at the end of the year of death, equal to a single sum of 100% of the annual salary rate at the time of death, provided death occurs while the employee is still employed.

You are given that  $\delta = 5\%$  and the following multiple decrement table ( $w =$  withdrawal;  $r =$  retirement; and  $d =$  death):

Age $x$	$l_x^{(\tau)}$	$d_x^{(w)}$	$d_x^{(r)}$	$d_x^{(d)}$
60	100	21	0	1
61	78	13	0	1
62	64	7	0	1
63	56	0	6	1
64	49	0	5	1
65	43	0	43	0

Calculate the expected present value of Jenny's total benefits.

- (A) 85,000  
 (B) 92,250  
 (C) 99,500  
 (D) 106,750  
 (E) 113,750

**21.** For a fully continuous whole life insurance of 1000 on  $(x)$ , you are given:

- (i) Benefit premiums are 10 per year.
- (ii)  $\delta = 0.05$
- (iii)  $\mu_{x+20.2} = 0.026$
- (iv)  ${}_tV$  denotes the benefit reserve at time  $t$  for this insurance.
- (v)  $\frac{d}{dt}({}_tV)$  at  $t = 20.2$  is equal to 20.5.

Calculate  ${}_{20.2}V$ .

- (A) 480
- (B) 540
- (C) 610
- (D) 670
- (E) 730

**22.** For a fully discrete 10-year deferred whole life insurance of 100,000 on (30), you are given:

- (i) Mortality follows the Illustrative Life Table.
- (ii)  $i = 6\%$
- (iii) The benefit premium is payable for 10 years.
- (iv) The gross premium is 120% of the benefit premium.
- (v) Expenses, all incurred at the beginning of the year, are as follows:

Policy Year	Percent of gross premium	Per policy
1	10%	100
2-10	3%	40
11 and later	0%	40

- (vi)  $L_0$  is the present value of future losses at issue random value.

Calculate  $E[L_0]$ .

- (A) -334
- (B) -496
- (C) -658
- (D) -820
- (E) -982

**23.** On January 1 an insurer issues 10 one-year term life insurance policies to lives age  $x$  with independent future lifetimes. You are given:

- (i) Each policy pays a benefit of 1000 at the end of the year if that policyholder dies during the year.
- (ii) Each policyholder pays a single premium of 90.
- (iii)  $q_x$  is the same for every policyholder. With probability 0.30,  $q_x = 0.0$  for every policyholder. With probability 0.70,  $q_x = 0.2$  for every policyholder.
- (iv)  $i = 0.04$

Calculate the variance of the present value of future losses at issue random variable for the entire portfolio.

- (A) 800,000
- (B) 900,000
- (C) 1,000,000
- (D) 1,400,000
- (E) 1,800,000

24. For a three-year term insurance of 10,000 on (65), payable at the end of the year of death, you are given:

(i)

$x$	$q_x$
65	0.00355
66	0.00397
67	0.00444

- (ii) Forward interest rates at the date of issue of the contract, expressed as annual rates, are as follows:

Start time	End time	Annual forward rate
0	3	0.050
1	3	0.070
2	3	0.091

Calculate the expected present value of this insurance.

- (A) 105
- (B) 110
- (C) 113
- (D) 115
- (E) 120



- 25.** An insurer issues fully discrete whole life insurance policies of 100,000 to insureds age 40 with independent future lifetimes.

You are given:

- (i) Issue expenses are incurred upon policy issuance and are 20% of the first year premium.
- (ii) Renewal expenses are 6% of each premium, incurred on the premium due date, starting in the second year.
- (iii) The annual premium is calculated using the percentile premium principle and the normal approximation, such that the probability of a loss on the portfolio is 5%.
- (iv)  $A_{40} = 0.161$
- (v)  ${}^2A_{40} = 0.048$
- (vi)  $\ddot{a}_{40} = 14.822$
- (vii)  $i = 0.06$
- (viii) The annual premium per policy for a portfolio of 2000 policies is 1215.

Calculate the difference in annual premium per policy for a portfolio of 2000 policies and for a portfolio of 40,000 policies.

- (A) 37
- (B) 42
- (C) 48
- (D) 52
- (E) 56

**26.** For a special fully discrete whole life insurance on (35) you are given:

- (i) Mortality follows the Illustrative Life Table.
- (ii)  $i = 0.06$
- (iii) Initial annual premiums are level for the first 30 years; thereafter, annual premiums are one-third of the initial annual premium.
- (iv) The death benefit is 60,000 during the first 30 years and 15,000 thereafter.
- (v) Expenses are 40% of the first year's premium and 5% of all subsequent premiums.
- (vi) Expenses are payable at the beginning of the year.

Calculate the initial annual premium using the equivalence principle.

- (A) 290
- (B) 310
- (C) 330
- (D) 350
- (E) 370

- 27.** For a universal life insurance policy with death benefit of 100,000 on (40), you are given:
- (i) The account value at the end of year 4 is 2029.
  - (ii) A premium of 200 is paid at the start of year 5.
  - (iii) Expense charges in renewal years are 40 per year plus 10% of premium.
  - (iv) The cost of insurance charge for year 5 is 400.
  - (v) Expense and cost of insurance charges are payable at the start of the year.
  - (vi) Under a no lapse guarantee, after the premium at the start of year 5 is paid, the insurance is guaranteed to continue until the insured reaches age 49.
  - (vii) If the expected present value of the guaranteed insurance coverage is greater than the account value, the company holds a reserve for the no lapse guarantee equal to the difference. The expected present value is based on the Illustrative Life Table at 6% interest and no expenses.

Calculate the reserve for the no lapse guarantee, immediately after the premium and charges have been accounted for at the start of year 5.

- (A) 0
- (B) 10
- (C) 20
- (D) 30
- (E) 40

**28.** You are using Euler's method to calculate estimates of probabilities for a multiple state model with states  $\{0, 1, 2\}$ . You are given:

(i) The only possible transitions between states are:

0 to 1  
1 to 0  
1 to 2

(ii) For all  $x$ ,  $\mu_x^{01} = 0.3$ ;  $\mu_x^{10} = 0.1$ ;  $\mu_x^{12} = 0.1$ .

(iii) Your step size is 0.1.

(iv) You have calculated that

(a)  ${}_{0.6}p_x^{00} = 0.8370$

(b)  ${}_{0.6}p_x^{01} = 0.1588$

(c)  ${}_{0.6}p_x^{02} = 0.0042$

Calculate the estimate of  ${}_{0.8}p_x^{01}$  using the specified procedure.

(A) 0.20

(B) 0.21

(C) 0.22

(D) 0.23

(E) 0.24

29. Your actuarial student has constructed a multiple decrement table using independent mortality and lapse tables. The multiple decrement table values, where decrement  $d$  is death and decrement  $w$  is lapse, are as follows:

$l_{60}^{(\tau)}$	$d_{60}^{(d)}$	$d_{60}^{(w)}$	$l_{61}^{(\tau)}$
950,000	2,580	94,742	852,678

You discover that an incorrect value of  $q_{60}^{(w)}$  was taken from the independent lapse table. The correct value is 0.05.

Decrements are uniformly distributed over each year of age in the multiple decrement table.

You correct the multiple decrement table, keeping  $l_{60}^{(\tau)} = 950,000$ .

Calculate the correct value of  $d_{60}^{(w)}$ .

- (A) 47,310
- (B) 47,340
- (C) 47,370
- (D) 47,400
- (E) 47,430

**30.** For a special 20-year temporary life annuity-due payable monthly on (50), you are given:

- (i) Mortality follows the Illustrative Life Table.
- (ii) Deaths are uniformly distributed over each year of age.
- (iii) 100 is payable at the beginning of each month from age 50 for 10 years.
- (iv) 400 is payable at the beginning of each month from age 60 for 10 years.
- (v)  $i = 0.06$

Calculate the expected present value of these annuity payments.

- (A) 24,000
- (B) 26,000
- (C) 28,000
- (D) 30,000
- (E) 32,000

**\*\*END OF EXAMINATION\*\***