May 2013 MLC Solutions

1.   Key: D

Let k be the policy year, so that the mortality rate during that year is q_{30+k-1}. The objective is to determine the smallest value of k such that

$v^{k-1}\left({}_{k-1}p_{30}\right)(1000P_{30}) < v^k\left({}_{k-1}p_{30}\right)q_{30+k-1}(1000)$

$P_{30} < vq_{30+k-1}$

$\dfrac{0.10248}{15.8561} < \dfrac{q_{29+k}}{1.06}$

$q_{29+k} > 0.00685$

$29+k > 51 \Rightarrow k > 22$

Therefore, the smallest value that meets the condition is 23.

2.   Key: A

Expected Present Value of Benefits:

$5,000(10/1,000)/1.06 + 7,500(15/1,000)/1.06^2 + 10,000(18/1,000)/1.06^3$

$= 47.17 + 100.12 + 151.13 = 298.42.$

Expected Present Value of Premiums:

$[1 + (870/1,000)/1.06 + (701/1,000)/1.06^2]P = 2.4446P.$

The annual premium is $P = 298.42/2.4446 = 122$

3.   Key: C

There are three ways to approach this problem. In all cases, let π denote the benefit premium.,

The first approach is an intuitive result based on the solution to Sample Question 309. The key is that in addition to the pure endowment, there is a benefit equal in value to a temporary annuity due with annual payment π. However, if the insured survives the 20 years, the value of the annuity is not received.

$$\pi\ddot{a}_{40:\overline{20|}} = 100,000\ {}_{20}E_{40} + \pi\ddot{a}_{40:\overline{20|}} - {}_{20}p_{40}\ \ddot{a}_{\overline{20|}6\%}\pi.$$

Based on this equation,

$$\pi = \frac{100,000\ {}_{20}E_{40}}{{}_{20}p_{40}\ddot{a}_{\overline{20|}}} = \frac{100,000v^{20}}{\ddot{a}_{\overline{20|}}} = \frac{100,000}{\ddot{s}_{\overline{20|}}} = \frac{100,000}{38.993} = 2,565$$

The second approach uses random variables to derive the expected present value of the return of premium benefit. Let K be the curtate future lifetime of (40). The present value random variable is then

$$Y = \begin{cases} \pi\ddot{s}_{\overline{K+1|}}v^{K+1}, & K < 20 \\ 0, & K \geq 20 \end{cases}$$

$$= \begin{cases} \pi\ddot{a}_{\overline{K+1|}}, & K < 20 \\ 0, & K \geq 20 \end{cases}$$

$$= \begin{cases} \pi\ddot{a}_{\overline{K+1|}} - 0, & K < 20 \\ \pi\ddot{a}_{\overline{20|}} - \pi\ddot{a}_{\overline{20|}}, & K \geq 20 \end{cases}.$$

The first term is the random variable that corresponds to a 20-year temporary annuity. The second term is the random variable that corresponds to a payment with a present value of $\pi\ddot{a}_{\overline{20|}}$ contingent on surviving 20 years. The expected present value is then $\pi\ddot{a}_{40:\overline{20|}} - {}_{20}p_{40}\ \ddot{a}_{\overline{20|}}\pi$.

The third approach takes the most steps.

$$\pi\ddot{a}_{40:\overline{20|}} = 100,000\ {}_{20}E_{40} + \pi\sum_{k=0}^{19}v^{k+1}\ \ddot{s}_{\overline{k+1|}}\ {}_{k|}q_{40} = 100,000\ {}_{20}E_{40} + \pi\sum_{k=0}^{19}v^{k+1}\frac{(1+i)^{k+1}-1}{d}\ {}_{k|}q_{40}$$

$$= 100,000\ {}_{20}E_{40} + \frac{\pi}{d}\left(\sum_{k=0}^{19}{}_{k|}q_{40} - v^{k+1}\ {}_{k|}q_{40}\right) = 100,000\ {}_{20}E_{40} + \frac{\pi}{d}\left({}_{20}q_{40} - A^1_{40:\overline{20|}}\right)$$

$$= 100,000\ {}_{20}E_{40} + \frac{\pi}{d}\left({}_{20}q_{40} - 1 + d\ddot{a}_{40:\overline{20|}} + v^{20}\ {}_{20}p_{40}\right)$$

$$= 100,000\ {}_{20}E_{40} + \pi\ddot{a}_{40:\overline{20|}} - \pi\ {}_{20}p_{40}\frac{1-v^{20}}{d}$$

$$= 100,000\ {}_{20}E_{40} + \pi\ddot{a}_{40:\overline{20|}} - {}_{20}p_{40}\ \ddot{a}_{\overline{20|}6\%}\pi.$$

4.   **Key: C**

There are four career paths Joe could follow. Each is of the form:
$p_{35}(p_{36})(p_{37})$(transition probability 1)(transition probability 2)
where the survival probabilities depend on each year's employment type. For example, the first entry below corresponds to Joe being an actuary for all three years.

The probability that Joe is alive on January 1, 2016 is:

$$(0.9)(0.85)(0.8)(0.4)(0.4)$$
$$+(0.9)(0.85)(0.65)(0.4)(0.6)$$
$$+(0.9)(0.7)(0.8)(0.6)(0.8)$$
$$+(0.9)(0.7)(0.65)(0.6)(0.2) = 0.50832$$

The expected present value of the endowment is $(100,000)0.50832/(1.08)^3 = 40,352$

5.   **Key: A**

$$_{10}p_{65:60}^{02} = \int_0^{10} {}_t p_{65:60}^{00} \mu^{02}{}_{10-t} p_{65+t:60+t}^{22} \, dt$$

$$= \int_0^{10} e^{-0.01t}(0.005)e^{-0.008(10-t)} \, dt$$

$$= 0.005e^{-0.08} \int_0^{10} e^{-0.002t} \, dt$$

$$= 0.005e^{-0.08} \frac{1-e^{-0.02}}{0.002} = 0.0457$$

6.   **Key: B**

$$_t p_{50} = \left(1 - \frac{t}{50}\right), \, 0 \le t \le 50$$

$$_t p_{55} = e^{-0.04t}, t \ge 0$$

$$_t p_{50:55} = \Pr(T_{50:55} > t) = \begin{cases} \left(1 - \dfrac{t}{50}\right) e^{-0.04t}, & 0 \le t \le 50 \\ 0, & t > 50 \end{cases}$$

where $T_{50:55} = \min(T_{50}, T_{55})$ is the time until the first death.

$$L = 100 e^{-0.05 T_{50:55}} - 60 > 0 \Rightarrow e^{-0.05 T_{50:55}} > 0.6 \Rightarrow T_{50:55} < -20 \ln(0.6)$$

$$\begin{aligned}
\Pr(L > 0) &= \Pr\left[T_{50:55} < -20 \ln(0.60)\right] \\
&= 1 - \Pr\left[T_{50:55} \ge -20 \ln(0.60)\right] \\
&= 1 - \Pr\left[T_{50} \ge -20 \ln(0.60)\right] \Pr\left[T_{55} \ge -20 \ln(0.60)\right] \\
&= 1 - \left(1 - \frac{-20 \ln(0.60)}{50}\right) e^{-0.04[-20 \ln(0.60)]} \\
&= 0.4712
\end{aligned}$$

7.   **Key: D**

$$A_{60:\overline{3}|} = q_{60} v + (1 - q_{60}) q_{60+1} v^2 + (1 - q_{60})(1 - q_{60+1}) v^3 = 0.86545$$

$$q_{60+1} = \frac{A_{60:\overline{3}|} - q_{60} v - (1 - q_{60}) v^3}{(1 - q_{60}) v^2 - (1 - q_{60}) v^3} = \frac{0.86545 - \dfrac{0.01}{1.05} - \dfrac{0.99}{1.05^3}}{\dfrac{0.99}{1.05^2} - \dfrac{0.99}{1.05^3}} = 0.017 \text{ when } v = 1/1.05.$$

The primes indicate calculations at 4.5% interest.

$$\begin{aligned}
A'_{60:\overline{3}|} &= q_{60} v' + (1 - q_{60}) q_{60+1} v'^2 + (1 - q_{60})(1 - q_{60+1}) v'^3 \\
&= \frac{0.01}{1.045} + \frac{0.99(0.017)}{1.045^2} + \frac{0.99(0.983)}{1.045^3} \\
&= 0.87777
\end{aligned}$$

8. **Key: A**

$$Var(Z) = E(Z^2) - E(Z)^2$$

$$E(Z) = E\left[(1+0.2T)(1+0.2T)^{-2}\right] = E\left[(1+0.2T)^{-1}\right]$$

$$= \int_0^{40} \frac{1}{(1+0.2t)} f_T(t)dt = \frac{1}{40}\int_0^{40} \frac{1}{1+0.2t}dt$$

$$= \frac{1}{40}\frac{1}{0.2}\ln(1+0.2t)\Big|_0^{40} = \frac{1}{8}\ln(9) = 0.27465$$

$$E(Z^2) = E\{(1+0.2T)^2[(1+0.2T)^{-2}]^2\} = E[(1+0.2T)^{-2}]$$

$$= \int_0^{40} \frac{1}{(1+0.2t)^2} f_T(t) = \frac{1}{40}\frac{1}{0.2}\left[\frac{-1}{(1+0.2t)}\right]_0^{40}$$

$$= \frac{1}{8}\left(1-\frac{1}{9}\right) = \frac{1}{9} = 0.11111$$

$$Var(Z) = 0.11111 - (0.27465)^2 = 0.03568$$

9. **Key: E**

$$_{4.5}V = v^{0.5}{}_{0.5}p_{x+4.5}\,{}_5V + v^{0.5}{}_{0.5}q_{x+4.5}\,b, \text{ where } b = 10,000 \text{ is the death benefit during year 5}$$

$$_{0.5}q_{x+4.5} = \frac{_{0.5}q_{x+4}}{1-{}_{0.5}q_{x+4}} = \frac{0.5(0.04561)}{1-0.5(0.04561)} = 0.02334$$

$$_{0.5}p_{x+4.5} = 0.97666$$

$$_5V = \frac{(_4V + P)(1.03) - q_{x+4}b}{p_{x+4}}$$

$$_5V = \frac{(1,405.08 + 647.46)(1.03) - 0.04561(10,000)}{0.95439} = 1,737.25$$

$$_{4.5}V = (1.03)^{-0.5}(0.97666)(1,737.25) + (1.03)^{-0.5}(0.02334)(10,000)$$

$$= 1,671.81 + 229.98 = 1,902$$

$_{4.5}V$ can also be calculated retrospectively:

$$_{0.5}p_{x+4} = 0.5(0.04561) = 0.02281$$

$$_{4.5}V = \frac{(1,405.08 + 647.16)(1.03)^{0.5} - 0.02281(10,000)/(1.03)^{0.5}}{1-0.02281} = 1,902$$

The interest adjustment on the death benefit term is needed because the death benefit will not be paid for another one-half year.

10.   Key: E

This is an application of Thiele's differential equation for a multi-state model.

The components of $\frac{d}{dt}{}_tV^{(s)}$ are

Interest on the current reserve: $\delta\,{}_tV^{(s)}$

Rate of premiums received while in state s: 0
Rate of benefits paid while in state s: $-B$
Transition intensity for transition to state h, times the change in reserve upon transition (hold reserve for h and release reserve for s): $-\mu_{x+t}^{sh}({}_tV^{(h)}-{}_tV^{(s)})$

Similar to previous, noting that the reserve if dead is 0: $-\mu_{x+t}^{sd}(0-{}_tV^{(s)})$

Adding these terms yields the solution.

11.   Key: B

Outcome (z)	Prob (p)	$p\times z$	$p\times z^2$
$1000v=943.40$	0.04	37.74	35,603.92
Fv	0.20	$0.20Fv$	$0.20(Fv)^2$
0	0.76	0	0

$E(Z)=37.74+0.1887F$

$E(Z^2)=35,603.92+0.1780F^2$

$Var(Z)=35,603.92+0.1780F^2-(37.74+0.1887F)^2$

$\qquad = 34,174.61-14.243F+0.1424F^2$

Take derivative with respect to F.
Derivative $=0.2848F-14.243$
Set $=0$ and solve; get $F=14.243\,/\,0.2848=50$

It should be obvious that this is a minimum rather than a maximum; you could prove it by noting that the second derivative $=0.2848>0$.

Note: the result does not depend on v. If you carry v symbolically through all steps, all instances cancel.

12.   Key: B

Final average salary
$$\frac{50,000}{3}\left[(1.04)^{26}+(1.04)^{25}+(1.04)^{24}\right]=133,360.2$$

Annual retirement benefit
$$= 0.017(27)(\text{final average salary})(0.85)=52,030$$

Note that the factor of 0.85 is based on an interpretation of the 5% reduction as producing a factor of $1-3(0.05)=0.85$. An alternative approach is to use 0.95^3. Upon rounding, the same answer results.

13.   Key: A

$$Q=\begin{bmatrix}0.7 & 0.3 & 0.0\\ 0.2 & 0.4 & 0.4\\ 0.0 & 0.0 & 1.0\end{bmatrix}\qquad Q^2=\begin{bmatrix}0.55 & 0.33 & 0.12\\ 0.22 & 0.22 & 0.56\\ 0.00 & 0.00 & 1.00\end{bmatrix}$$

Let $p=0.75$ be the probability of renewing.

PV costs for Medium Risk:
$$=300v^{0.5}+\left[100(0.2)+300(0.4)+600(0.4)\right]pv^{1.5}+\left[100(0.22)+300(0.22)+600(0.56)\right]p^2v^{2.5}$$
$$=291.4+261.1+206.2=759$$

The present value of costs for the new portfolio is $(0.9)317+(0.1)759=361.2$. The increase is $(361.2/317)-1=0.14$, or 14%.

14.   Key: D

$$AV_{end} = \left[AV_{start} + P(1-f) - e - COI\right]\left(1+i^c\right)$$

where $COI = \left[(DB_{end} - AV_{end})/(1+i^q)\right] \times COI$ rate

For AV_{12} : $AV_{12} = \left[AV_{11} + 2000(1-0.035) - 50 - COI\right](1+0.005)$

$= \left[25,000 + 2000(1-0.035) - 50 - COI\right](1+0.005) = 27,014.40 - 1.005 \times COI$

$$COI = \frac{150,000 - (27,014.40 - 1.005COI)}{1.005} 0.001 = 122.37 + 0.001COI$$

$COI = 122.37 / 0.999 = 122.49$

$AV_{12} = 27,014.40 - 1.005COI = 27014.40 - 1.005(122.49) = 26,891$

15.   Key: B

Benefit Premium $= 10,000 A_{62} / \ddot{a}_{62} = 10,000(0.3967) / 10.6584 = 372.19$

$G = 372.19(1.02) = 379.63$

Let $_0L^*$ be the present value of future loss at issue for one policy.

$_0L^* = 10,000v^{K+1} - (G-5)\ddot{a}_{\overline{K+1}|} + 0.05G$

$\quad = 10,000v^{K+1} - (379.63 - 5)\dfrac{1-v^{K+1}}{d} + 0.05(379.63)$

$\quad = (10,000 + 6,618.46)v^{K+1} - 6,618.46 + 18.98$

$\quad = 16,618.46v^{K+1} - 6,599.48$

$E(_0L^*) = 16,618.46 A_{62} - 6,599.48 = 16,618.46(0.3967) - 6,599.48 = -6.94$

$Var(_0L^*) = 16,618.46^2 \left(^2A_{62} - A_{62}^2\right) = 16,618.46^2 \left(0.19941 - 0.3967^2\right) = 11,610,076$

Let $_0L$ be the aggregate loss for 600 such policies.

$E(_0L) = 600E(_0L^*) = 600(-6.94) = -4,164$

$Var(_0L) = 600Var(_0L^*) = 600(11,610,076) = 6,966,045,600$

$StdDev(_0L) = 6,966,045,600^{0.5} = 83,463$

$$\Pr\left(_0L < 60,000\right) = \Phi\left(\frac{60,000 + 4,164}{83,463}\right) = \Phi\left(0.77\right) = 0.7794$$

16.   Key: E

Gross premium = G

$$G\ddot{a}_{45} = 2000A_{45} + \underbrace{\left(1\left(\frac{2000}{1000}\right) + 20\right)}_{22} + \underbrace{\left(0.5\left(\frac{2000}{1000}\right) + 10\right)}_{11}\ddot{a}_{45} + 0.20G + 0.05G\ddot{a}_{45}$$

$$\left(0.95\ddot{a}_{45} - 0.20\right)G = 2000A_{45} + 22 + 11\ddot{a}_{45}$$

$$G = \frac{2000A_{45} + 22 + 11\ddot{a}_{45}}{0.95\ddot{a}_{45} - 0.20} = \frac{2000(0.20120) + 22 + 11(14.1121)}{0.95(14.1121) - 0.20} = 43.89$$

There are two ways to proceed. The first is to calculate the expense-augmented reserve and the benefit reserve and take the difference.

The benefit premium is $\dfrac{2000A_{45}}{\ddot{a}_{45}} = \dfrac{2000(0.2012)}{14.1121} = 28.52$.

The benefit reserve is $2000A_{55} - 28.52\ddot{a}_{55} = 2000(0.30514) - 28.52(12.2758) = 260.17$.

The expense-augmented reserve, which is the gross premium reserve, is
$$2000A_{55} + [0.05(43.89) + 0.5(2000/1000) + 10]\ddot{a}_{55} - 43.89\ddot{a}_{55}$$
$$= 2000(0.30514) + (13.19 - 43.89)(12.2758) = 233.41.$$
Expense reserve is $233.41 - 260.17 = -27$

The second is to calculate the expense reserve directly based on the pattern of expenses. The first step is to determine the expense premium.
The present value of expenses is
$$[0.05G + 0.5(2000/1000) + 10]\ddot{a}_{45} + 0.20G + 1.0(2000/1000) + 20$$
$$= 13.1945(14.1121) + 30.778 = 216.98.$$
The expense premium is $216.98/14.1121 = 15.38$
The expense reserve is the expected present value of future expenses less future expense premiums, that is,
$$[0.05G + 0.5(2000/1000) + 10]\ddot{a}_{55} - 15.38\ddot{a}_{55} = -2.1855(12.2758) = -27$$

There is a shortcut with the second approach based on recognizing that expenses that are level throughout create no expense reserve (the level expense premium equals the actual expenses). Therefore, the expense reserve in this case is created entirely from the extra first year expenses. They occur only at issue so the expected present value is 0.20(43.97)+1.0(2000/1000)+20 = 30.794. The expense premium for those expenses is then 30.794/14.1121 = 2.182 and the expense reserve is the present value of future non-level expenses (0) less the present value of those future expense premiums, which is 2.182(12.2758) = 27 for a reserve of –27.

17. **Key: E**

Benefit Premium for reserves $= 10(514.95)/8.5693 = 600.92$

$_1V_{70} = 10(530.26) - 8.2988(600.92) = 315.69$

$_2V_{70} = 10(545.60) - 8.0278(600.92) = 631.93$

Expected Profit $= \left[315.69 + 800(1-0.10)\right]1.07 - 10,000(0.03) - 631.93(1-0.03-0.04) = 220$

18. **Key: A**

$q_x^{NS} = q_{x+1}^{NS} = 1 - e^{-0.1} = 0.095$

Then the annual premium for the non-smoker policies is P^{NS}, where

$P^{NS}\left(1 + vp_x^{NS}\right) = 100,000vq_x^{NS} + 100,000v^2 p_x^{NS}q_{x+1}^{NS} + 30,000v^2 p_x^{NS}p_{x+1}^{NS}$

$$P^{NS} = \frac{100,000(0.926)(0.095) + 100,000(0.857)(0.905)(0.095) + 300,00(0.857)(0.905)^2}{1 + (0.926)(0.905)}$$

$P^{NS} = 20,251$

And then $P^S = 40,502.$

$q_x^S = q_{x+1}^S = 1.5(1 - e^{-0.1}) = 0.143$

$EPV(L^S) = 100,000vq_x^S + 100,000v^2 p_x^S q_{x+1}^S + 30,000v^2 p_x^S p_{x+1}^S - P^S - P^S vp_x^S$

$\quad = 100,000(0.926)(0.143) + 100,000(0.857)(0.857)(0.143)$

$\quad\quad + 30,000(0.857)(0.857)^2 - 40,502 - 40,502(0.926)(0.857)$

$\quad = -30,017$

19.   Key: D

$l_{65+1} = 1000 - 40 = 960$

$l_{66+1} = 955 - 45 = 910$

$\mathring{e}_{[65]} = \int_0^1 {}_t p_{[65]} dt + p_{[65]} \int_0^1 {}_t p_{66} \, dt + p_{[65]} p_{66} \mathring{e}_{67}$

$15.0 = \left[1 - \left(\frac{1}{2}\right)\left(\frac{40}{1000}\right)\right] + \frac{960}{1000}\left[1 - \left(\frac{1}{2}\right)\left(\frac{50}{960}\right)\right] + \left(\frac{960}{1000}\right)\left(\frac{910}{960}\right)\mathring{e}_{67}$

$\mathring{e}_{67} = \dfrac{15(1000) - (980 + 935)}{910} = 14.37912$

$\mathring{e}_{[66]} = \int_0^1 {}_t p_{[66]} dt + p_{[66]} \mathring{e}_{67} = \left[1 - \left(\frac{1}{2}\right)\left(\frac{45}{955}\right)\right] + \left(\frac{910}{955}\right)\mathring{e}_{67}$

$\mathring{e}_{[66]} = \left[1 - \left(\frac{1}{2}\right)\left(\frac{45}{955}\right)\right] + \left(\frac{910}{955}\right)(14.37912) = 14.678$

Note that because deaths are uniformly distributed over each year of age, $\int_0^1 {}_t p_x dt = 1 - 0.5 q_x$.

20.   Key: D

This is a mixed distribution for the population, since the vaccine will apply to all once available.

				S = # of survivors		
Available? (A)	$\Pr(A)$	$_2p\,|\,A$	$E(S|A)$	$Var(S|A)$	$E(S^2|A)$
Yes	0.2	0.9702	97,020	2,891	9,412,883,291
No	0.8	0.9604	96,040	3,803	9,223,685,403
			$E(S)$		$E(S^2)$
			96,236		9,261,524,981
			$Var(S)$	157,285	
			$SD(S)$	397	

As an example, the formulas for the "No" row are

$\Pr(\text{No}) = 1 - 0.2 = 0.8$

$_2p$ given No $= (0.98$ during year 1$)(0.98$ during year 2$) = 0.9604.$

$E(S\,|\,\text{No}), Var(S\,|\,\text{No})$ and $E(S^2\,|\,\text{No})$ are just binomial, $n = 100,000;$ p(success) $= 0.9604$

$E(S), E(S^2)$ are weighted averages,
$Var(S) = E(S^2) - E(S)^2$

Or, by the conditional variance formula:
$$Var(S) = Var[E(S\,|\,A)] + E[Var(S\,|\,A)]$$
$$= 0.2(0.8)(97,020 - 96,040)^2 + 0.2(2,891) + 0.8(3,803)$$
$$= 153,664 + 3,621 = 157,285$$
$StdDev(S) = 397$

21.   Key: A

$$E(Y) = \bar{a}_{\overline{10}|} + e^{-\delta(10)} e^{-\mu(10)} \bar{a}_{x+10}$$

$$= \frac{\left(1 - e^{-0.6}\right)}{0.06} + e^{-0.7} \frac{1}{0.07}$$

$$= 14.6139$$

$$Y > E(Y) \Rightarrow \left(\frac{1 - e^{-0.06T}}{0.06}\right) > 14.6139$$

$$\Rightarrow T > 34.90$$

$$\Pr[Y > E(Y)] = \Pr(T > 34.90) = e^{-34.90(0.01)} = 0.705$$

22.   Key: D

$$\ddot{a}_{x+10} = (1 - A_{x+10}) / d = (1 - 0.4) / (0.05 / 1.05) = 12.6$$

$$\ddot{a}_{x+10}^{(12)} \approx 12.6 - 11 / 24 = 12.142$$

$$_{10}V = 10,000 A_{x+10} + 100 \ddot{a}_{x+10} - 12 \ddot{a}_{x+10}^{(12)} (30)(1 - 0.05)$$

$$_{10}V = 10,000(0.4) + 100(12.6) - 12(12.142)(28.50)$$

$$_{10}V = 1107$$

23.   Key: D

k	q_{x+k-1}	$_{k-1|}q_{x+k-1}$	Benefit	v^k		
	(A)	(B)	(C)	(D)	$B \times C \times D$	$B \times C^2 \times D^2$
1	0.02	0.0200	2000	0.97	38.80	75,272
2	0.04	0.0392	4000	0.92	144.26	530,862

$$E(Z) = 38.80 + 144.26 = 183.06$$

$$E(Z^2) = 75,272 + 530,862 = 606,134$$

$$\text{Var}(Z) = 606,134 - 183^2 = 572,645$$

24.   Key: C

$$AS_1 = (0 + G - 0.10G - 5)(1.05) = (0.9G - 5)(1.05)$$
$$AS_2 = (AS_1 + G - 0.03G - 2)(1.05)$$
$$= (0.9G - 5)(1.05)^2 + (0.97G - 2)(1.05)$$
$$= \left[0.9(1.05)^2 + 0.97(1.05) \right] G - \left[5(1.05)^2 + 2(1.05) \right] = 64.11$$

Solving for G, $G = 35.67$

Note that both AS_1 and AS_2 have $(1 - q^{(d)} - q^{(w)})$ as a denominator term, but as there were no deaths or withdrawals, this term is 1 in both cases and so can be ignored.

25.   Key: B

$$\ddot{a}_{60:\overline{10|}} = \ddot{a}_{60} - {}_{10}E_{60}\ddot{a}_{70} = 7.2789$$
$$\ddot{a}_{60:\overline{20|}} = \ddot{a}_{60} - {}_{20}E_{60}\ddot{a}_{80} = 10.2652$$

$$\text{Annual level amount} = \frac{40 + 5\ddot{a}_{60:\overline{10|}} + 5\ddot{a}_{60:\overline{20|}}}{\ddot{a}_{60}} = \frac{127.72}{11.1454} = 11.46$$