MLC Written Answer Model Solutions
Spring 2014

1. **Learning Outcomes:** (2a) (3a) (3b) (3d)

Sources:
Textbook references: 4.4, 5.6, 5.11, 6.5, 9.4

(a) Show that the expected present value of the death benefit is 2020 to the nearest 10. You should calculate the value to the nearest 1.

Solution:

EPV Death Benefit:
$$20000\ {}_{20}E_{40}\ A_{60} = 2023.9$$

Commentary:
Almost all candidates earned full marks for this part of the question.

(b) Calculate the annualized net premium for the policy.

Solution:

EPV Premium of P per year:
$$P\ddot{a}_{40}^{(12)} \approx P\left(\ddot{a}_{40} - \frac{11}{24}\right) \approx P\left(14.8166 - 0.45833\right) \approx 14.3583P$$

which gives the annual premium
$$P = \frac{2023.9}{14.3583} = 140.96$$

Commentary:
This part was also done well by most candidates. Some candidates used the 3-term Woolhouse formula rather than the 2-term. No points were deducted for doing this correctly, but this is a substantially more time-consuming calculation, and many candidates lost points through errors in the formula or calculation. Other candidates used the $\alpha(m)$ and $\beta(m)$ formula based on an assumption of uniform distribution of deaths to calculate the annuity value. This resulted in a small deduction..

(c) State two reasons why the annualized net premium rate will change if premiums are payable continuously, giving the direction of change for each reason.

Solution:
Reason 1: On average, premiums paid continuously are paid later than premiums paid monthly, leading to a loss of interest income on the premium payments.

This will lead to an increase in the annualized net premium.

Reason 2: In the year of death, on average the total premium received if premiums are continuous will be less than the total for monthly premiums.

This will lead to an increase in the annualized net premium.

Commentary:
Stronger candidates gave good answers to this part. Brief explanations were often better than longer ones. No credit was given for irrelevant comments.
A number of candidates stated that the annualized premium would increase because (i) $\bar{a}_x < \ddot{a}_x^{(12)}$ and (ii) the expected present value of benefits does not change. While this is true, it does not address the reasons for the change, which was what the question asked for. This answer received only partial credit.
Some candidates proposed that continuous premiums meant that the interest and/or mortality rates would change. This is incorrect and received no credit.
A few candidates wrote that continuous premium meant that the death benefits would now be paid at the moment of death. This is incorrect and received no credit.

(d) Calculate the increase in the annualized net premium due to the premium waiver.

Solution:

Let P^* denote the revised premium. Premiums are paid while both (40) and (50) survive. This can be valued as a joint life annuity, so the EPV of premiums is now

$$P^*\left(\ddot{a}_{40:50}^{(12)}\right) \approx P^*\left(\ddot{a}_{40:50} - \frac{11}{24}\right) = 12.0201P^*$$

The EPV of benefits is 2023.9 as above, so the revised premium is

$$P^* = \frac{2023.9}{12.0201} = 168.38$$

which gives an increase in the premium of

$$P^* - P = 27.42$$

1. Continued

Alternative solution:

$$P^*\ddot{a}_{40}^{(12)} = 2023.9 + P^*\ddot{a}_{50|40}^{(12)}$$

$$= 2023.9 + P^*\left(\ddot{a}_{40}^{(12)} - \ddot{a}_{40:50}^{(12)}\right)$$

$$\Rightarrow P^* = \frac{2023.90}{\ddot{a}_{40:50}^{(12)}} \quad \text{as above}$$

Commentary:

A good proportion of candidates answered this part correctly, but most did not. Most candidates who used reversionary annuities did so incorrectly.
A candidate who made the same mistake twice did not have points deducted twice. For example, a candidate who used the UDD annuity formula in part (b) and in part (d) would have lost some credit in part (b), but would have received full credit in part (d) provided the rest of the calculation was correct.

2. **Learning Outcomes:** (1a) (1b) (1d) (1e) (2a) (3c) (3d)

Sources:
Textbook References: 8.4, 8.5, 8.6

(a) Write down the Kolmogorov forward differential equations with associated boundary conditions (initial conditions) for ${}_tp_0^{00}$, ${}_tp_0^{01}$, and ${}_tp_0^{02}$ under this model.

Solution:

$$\frac{d}{dt}\,{}_tp_0^{00} = p_0^{01}\mu_t^{10} - {}_tp_0^{00}\mu_t^{01}$$

$$\frac{d}{dt}\,{}_tp_0^{01} = {}_tp_0^{00}\mu_t^{01} - {}_tp_0^{01}\left(\mu_t^{10} + \mu_t^{12}\right)$$

$$\frac{d}{dt}\,{}_tp_0^{02} = {}_tp_0^{01}\mu_t^{12}$$

Boundary Conditions:

$${}_0p_0^{00} = 1 \quad {}_0p_0^{01} = 0 \quad p_0^{02} = 0$$

Commentary:
The question asks for the Kolmogorov forward differential equations as well as boundary conditions. Most candidates were able to give the differential equations, but quite a few did not provide boundary conditions.
For full credit, candidates were expected to use the specific model given in the question, which meant that subscripts and superscripts needed to correspond with the notation of the question for full credit.

(b) Using the student's approach:

(a) Calculate ${}_{0.5}p_0^{01}$.

(b) Show that the probability that a television will become Broken Beyond Repair within a year of purchase is 0.18 to the nearest 0.01. You should calculate the probability to the nearest 0.001.

Solution:

(i) The Euler equation for ${}_{t+h}p_0^{01}$ is:

$${}_{t+h}p_0^{01} = {}_tp_0^{01} + h\left({}_tp_0^{00}\mu_t^{01} - {}_tp_0^{01}(\mu_t^{10} + \mu_t^{12})\right)$$

so, using $t = 0, h = .5$ gives:

$$\Rightarrow {}_{0.5}p_0^{01} = 0.5({}_0p_0^{00}\mu_0^{01}) \quad \text{as } {}_0p_0^{01} = 0$$

$$\Rightarrow {}_{0.5}p_0^{01} = 0.5 \times 0.5 = 0.25$$

2. **Continued**

(ii)

The Euler equation for ${}_{t+h}p_0^{02}$ is

$${}_{t+h}p_0^{02} = {}_t p_0^{02} + h\left({}_t p_0^{01}\mu_t^{12}\right)$$

so, using $t = 0.5,$ and $t = 1,$ and using $h = .5$ gives:

$${}_{0.5}p_0^{02} = {}_0 p_0^{02} + 0.5({}_0 p_0^{01}\mu^{12}) = 0$$

$$\Rightarrow {}_1 p_0^{02} = 0 + 0.5\left(0.25 \times 2^{0.5}\right) = 0.177$$

Commentary:
Candidates were required to use Euler's method for full credit. Many candidates' solutions lacked clarity in this part, which made it difficult to award partial credit for incomplete answers.

(c) Using the probabilities from the student's approach and $i^{(2)} = 8\%$:

(i) Calculate the actuarial present value at issue of the replacement cost payments for this policy.

(ii) Calculate the semi-annual net premium for this policy.

Solution:

(i)

$$APV = 1000({}_{0.5}p_0^{02}\,v + {}_1 p_0^{02}\,v^2) \quad at \;\; 4\%$$
$$= 163.64$$

(ii)

$$P(1 + {}_{0.5}p_0^{00}v) = APV$$
$$\Rightarrow P(1.72115) = APV$$
$$\Rightarrow P = 95.08$$

Commentary:
This part was done well by most candidates who attempted it. Note that all the probability values required were given in the question, so it was not necessary to answer (b) to answer (c) correctly. Full credit was given for answers using the rounded probability values given in the question.

2. **Continued**

(d) Suggest a change to the student's approach that would improve the accuracy of the probability calculations.

Solution:
Use smaller h for greater accuracy.

Commentary:
Virtually all candidates who did not omit the question entirely answered this correctly.

3. **Learning Outcomes:** (1a) (1c)

Sources:
Textbook References: 3.2, 11.4

(a) Calculate the mean and variance of the number of deaths in the control group.

Solution:
Let D^c denote deaths in the control group. Then:
$D^c \sim Bin(1000, 0.20)$
$\Rightarrow E[D^c] = 1000 \times 0.20 = 200 \quad$ and $\quad V[D^c] = 1000 \times 0.20 \times 0.80 = 160$

Commentary:
Performance on part (a) was very good, with most candidates receiving full credit.

(b) Calculate the mean and variance of the number of deaths in Cohort A.

Solution:
Let D^A denote deaths in cohort A. Then

$$D^A \mid q \sim Bin(1000, q)$$
$$\Rightarrow \quad E[D^A \mid q] = 1000q \qquad \text{and} \quad V[D^A \mid q] = 1000q(1-q)$$
$$where \;\; q = \begin{cases} 0.2 & \text{w.prob. } 0.8 \\ 0.05 & \text{w.prob. } 0.2 \end{cases}$$

So

$$E[D^A] = E[E[D^A \mid q]] = 0.8 \times 200 + 0.2 \times 50 = 170$$
$$V[D^A] = E[V[D^A \mid q]] + V[E[D^A \mid q]] = E[1000q(1-q)] + V[1000q]$$
$$= 1000(0.17 - 0.0325) + 1000^2(0.0036) = 3737.5$$

(c) Calculate the mean and variance of the number of deaths in Cohort B.

Solution:
Let D_i^B be a Bernoulli indicator function for the death of individual i and D^B denote the deaths in cohort B. Then $D_i^B \mid q$ are independent Bernoulli RVs with parameter q, and $D^B = \sum D_i^B$ so that

3. Continued

$$E[D_i^B] = E[E[D_i^B \mid q]] = E[q] = 0.17$$

$$\Rightarrow E[D^B] = E\left[\sum_{i=1}^{1000} D_i^B\right] = \sum_{i=1}^{1000} E[D_i^B] = (1000)(0.17) = 170.$$

$$V[D_i^B] = E[V[D_i^B \mid q]] + V[E[D_i^B \mid q]] = E[q(1-q)] + V[q]$$
$$= E[q] - E[q^2] + V[q] = 0.17 - 0.0325 + 0.0036 = 0.1411$$

$$V[D^B] = V\left[\sum_{i=1}^{1000} D_i^B\right] = \sum_{i=1}^{1000} V[D_i^B] = (1000)(0.1411) = 141.1$$

Commentary on (b) and (c):
Parts (b) and (c) tested candidates' understanding of the difference between an uncertain mortality rate applying to all subjects, and uncertainty in the mortality rate applying to each individual subject. In Cohort A, if you know the mortality of one life, then you know the mortality of all lives in the cohort. In Cohort B, knowing the mortality rate for one life gives no information about the other lives.

The main point was to understand how these cases are different, and to be able to calculate the mean and variance under each form of uncertainty. While the better candidates did very well on these parts, many candidates failed to distinguish the cases correctly. Some candidates calculated the two parts identically, while some switched the calculations for these two parts.

(d) For each of the 3 groups, state whether the mortality risk is diversifiable or not diversifiable, and justify your answer. You do not need to include formulas or calculations in your justification.

Solution:

Control Group

- Mortality is diversifiable since the lives are independent.

Cohort A

- For cohort A the variance is in two parts. The first part is $E[Nq(1-q)]$ and this increases in proportion to N and is therefore diversifiable. The second part is $V[Nq]$ which increases in N^2 which means that the risk is not diversifiable.

Cohort B

- For Cohort B the lives are independent. Hence, the risk is diversifiable.

3. Continued

Commentary:

Relatively few candidates answered this part well. Statements with no justification received little or no credit. Similarly, comments that were unrelated to diversifiability received no credit.

Some candidates gave mathematical arguments based on the formula given; these were given full credit if correct.

4. **Learning Outcomes:** (3a) (4a) (4c)

Sources:
Textbook References: 6.6, 7.3

Commentary on Question:
All parts of this question were answered well by most candidates.

(a) Show that the gross premium for each policy is 1700 to the nearest 10. You should calculate the premium to the nearest 1.

Solution:

$$EPV\ \ Benefits: \quad 100000A_{45} = 20120$$
$$EPV\ \ Premiums: \quad P\ddot{a}_{45} = 14.1121P$$
$$EPV\ \ Expenses: \quad 20\ddot{a}_{45} + 80 + 200A_{45} + 0.1P\ddot{a}_{45} + 0.65P$$
$$= 402.48 + 2.0612P$$

Equate values for
$$P = \frac{20120 + 402.48}{14.1121 - 2.0612} = 1702.98$$

Commentary:
It is easiest to deal with recurring expenses starting in year 1 (for all years) and add the extra first year expense. Candidates who tried to incorporate expenses year by year were more likely to make calculation errors. A common error was to omit the settlement expenses (200), or fail to discount them. The answer was given to nearest 10. Candidates who found a different answer to (a) were expected to answer the remaining parts with the given answer.

(b) Calculate the gross premium reserve for a policy in force at the end of policy year 1.

Solution:

$${}_1V = 100200A_{46} - 0.9P\ddot{a}_{46} + 20\ddot{a}_{46}$$
$$= 100200(0.21012) - 13.9546(1702.98(0.9) - 20) = -54.85$$

Alternative Solution:
$${}_1V = \left((P - E)(1.06) - q_{45}100200\right) / \ p_{45}$$
$$= \frac{((1702.98)(0.25) - 100)(1.06) - .004(100200)}{0.996} = -55.73$$

4. **Continued**

Commentary:
Candidates could answer using the prospective approach or the recursion/retrospective approach, which gives a slightly different answer due to rounding. A few candidates forgot expenses completely, others subtracted expenses when they should have added them (prospective) or vice versa (recursion). Another common mistake was to use the wrong expenses, for example, use the renewal expenses instead of first year in the recursive formula, or to omit settlement expenses. Some candidates used the actual interest rate (1.07) instead of the pricing rate (1.06), which led to partial credit if the rest of the solution was correct.

(c) For each of interest, expense and mortality, in that order, calculate the gain or loss by source in policy year 1 on this block of policies.

Solution:
Interest Gain:

$$10,000\ (0.07-0.06)\left(P-(0.75P+100)\right)=32,575$$

Expenses Gain:

$$10000\left((0.75P+100)-(0.75P+105)(1.07)\right)=-50,000(1.07)=-53,500$$

Mortality Gain: 0

Commentary:
Candidates were expected to indicate clearly whether the amount calculated is a gain or a loss. A few candidates calculated only the total gain/loss; this received small partial credit. One relatively common mistake was to use a factor of .07 instead of 1.07 when calculating the expense gain and loss.

(d) Explain the sources and direction of any gains or losses in policy year 2. Exact values are not necessary.

Solution:
Interest: No gain or loss, as experience matches the assumption.

Expenses: Loss, as settlement expenses exceed the assumption.

Mortality: Expected deaths $=10000\,p_{45}\,q_{46}=43$. There will be a loss, as actual deaths exceed expected deaths.

Commentary:
A few candidates compared the experience of year 2 with year 1. For example, "interest in year 2 was lower than in year 1 so there was a loss due to interest". This did not directly answer the question and did not receive full credit. A few candidates proposed that there would be a gain due to mortality because there were more deaths than expected.

5. **Learning Outcomes:** (3a) (4a) (4c) (4g) (4h)

Sources:
Textbook References: 6.4, 7.3, 7.4

(a) Calculate ${}_0L^{\mathrm{Q}}$ when $T_{35}=0.30$.

Solution:
Given $T_{35}=0.3$ then the benefit is paid at $t=1$ and two premiums are paid, at $t=0$ and $t=0.25$. Allowing, also, for the expenses at $t=0$ and $t=0.25$ we have:

$${}_0L^Q \mid (T_{35}=0.3) \; = 100{,}000v + 100 + 16v^{0.25} - 240(1+v^{0.25}) = 93{,}979$$

Commentary:
Many candidates answered this correctly, but most did not, with common errors including (i) assuming the benefit is paid at the moment of death; (ii) omitting the initial expenses (iii) omitting the renewal expenses and (iv) only allowing for a single premium payment.

(b) Calculate $E\left[{}_0L^{\mathrm{Q}}\right]$.

Solution:

$E[{}_0L^Q] =$ EPV Death benefit +EPV expenses -EPV premiums at issue:

EPV Death Benefit $= 100000A_{35} = 12872$

EPV Expenses $= 84 + (4\times 16)\ddot{a}^{(4)}_{35:\overline{10}|}$

$$\text{where } \ddot{a}^{(4)}_{35:\overline{10}|} = \ddot{a}_{35:\overline{10}|} - \frac{3}{8}\left(1 - {}_{10}E_{35}\right) = 7.7272 - \frac{3}{8}\left(1-0.54318\right) = 7.5559$$

$\Rightarrow$ EPV Expenses $= 567.6$

EPV Premiums $= (4\times 240)\ddot{a}^{(4)}_{35}$

where $\ddot{a}^{(4)}_{35} = \ddot{a}_{35} - \frac{3}{8} = 15.3926 - \frac{3}{8} = 15.0176$

$\Rightarrow$ EPV Premiums $= 14417$

$\Rightarrow E[{}_0L^Q] = 12872 + 568 - 14417 = -977$

Commentary:
As for part (a), there were relatively few fully correct answers to this question. Common errors included omitting the factor of 4 for the quarterly premiums and expenses, and omitting the 10-year term on the commissions. Candidates who made errors on intermediate calculations, such as the annual term annuity-due, could receive partial credit for that part if they showed their working.

5. **Continued**

(c) Show that ${}_5V^Q$ is 2545, to the nearest 5. You should calculate the value to the nearest 1.

Solution:

$${}_5V^Q = 100000A_{40} + (4\times 16)\ddot{a}^{(4)}_{40:\overline{5}|} - (4\times 240)\ddot{a}^{(4)}_{40}$$

where $A_{40} = 0.16132$ and $\ddot{a}^{(4)}_{40:\overline{5}|} = 4.3408$ (given in question)

and $\ddot{a}^{(4)}_{40} = 14.8166 - \dfrac{3}{8}$

$${}_5V^Q = 16132 + 278 - 13864 = 2546$$

Alternative Solution:

$$\left({}_4V^Q + 4((240)-16)\ddot{a}^{(4)}_{39:\overline{1}|}\right)(1.06) = q_{39}(100000) + p_{39}\,{}_5V^Q$$

$$\Rightarrow \left(1765 + 896(0.97785)\right)(1.06) = 260.0 + 0.9974\,{}_5V^Q$$

as $\ddot{a}^{(4)}_{39:\overline{1}|} = 1 - \dfrac{3}{8}(1 - {}_1E_{39}) = 0.97785$

$$\Rightarrow {}_5V^Q = \frac{2539.6}{0.9974} = 2546$$

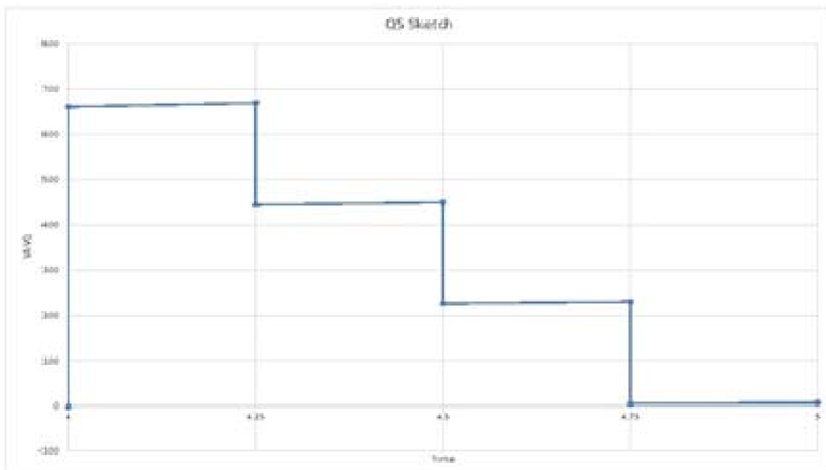
Commentary:

This part was done better than (b), though many of the comments for (b) apply also to (c). Errors that were penalized in (b) were not penalized again in (c).

5. **Continued**

(d) Sketch the graph of ${}_tV^{\mathrm{A}} - {}_tV^{\mathrm{Q}}$ for $4 \le t \le 5$. An exact calculation of the difference at intermediate times is not expected. Label the scales on the *x* (time) and *y* (reserve difference) axes.

Solution:



Commentary:
Key points for credit were:

- *The differences are -2 at time 4 and +7 at time 5.*
- *There is a jump up immediately after time 4 as the annual premium (net of expenses) is much greater than the quarterly premium (net of expenses).*
- *The curve steps down each 1/4 year; the steps are approximately equal to 224, which is the quarterly premium, net of expenses.*

This part was answered correctly by a relatively small set of the very best candidates. Many candidates sketched a smooth curve, and many others omitted this part.

6. **Learning Outcomes:** (1b) (3a) (3c) (4e) (4g)

Sources:
Textbook References: 8.9, 12.4, 12.5, 13.4

Commentary on Question:
Many candidates omitted this question. Those who attempted it generally did well on parts (a) and (b). Part (c) is more challenging, and only a small number of strong candidates answered the question fully.

(a) Show that the account value at the end of year 1, for a policy in force, is 1300 to the nearest 100. You should calculate the account value to the nearest 1.

Solution:
The recursion equation for the Account Value, AV, is

$$\left(AV_1 + 0.9P - 50 - 0.00747(100000)v_{5\%}\right)(1.05) = AV_2$$

$$AV_2 = 1500 \Rightarrow AV_1 = 1290.0$$

Alternative solution:

$$AV_1 = \left(0.4(5000) - 200 - 0.006(100000)v_{5\%}\right)(1.05) = 1290.0$$

Commentary:
This part was generally answered well. The most common minor error was failing to discount the cost of insurance. More significantly, a number of candidates incorrectly used information from the `Assumptions' table in this part. It is important in UL to understand the different roles of the policy terms and conditions (used to project account values) and the valuation assumptions (used to determine profitability).

(b) Calculate the total expected present value at issue of profits for the first three years per policy issued.

Solution:

$$EPV = -1206v_{12\%} + 374\,p_{60}^{(\tau)}\,v_{12\%}^{2} + 400\,{}_2p_{60}^{(\tau)}\,v_{12\%}^{3}$$

$$p_{60}^{(\tau)} = 1 - 0.004 - 0.1 = 0.8960; \quad {}_2p_{60}^{(\tau)} = 0.8960(0.9450) = 0.8467$$

$$\Rightarrow EPV = -568.6$$

Commentary:
This part was generally answered well. Candidates losing marks generally applied discount or probability factors that were one year too short or long.

(c) Your company's pricing actuary asks you to redo the profit test using the end of year cash surrender values as reserves, with no other changes.

(i) Calculate the revised expected profits for policy year 2, per policy in force at the start of year 2.

(ii) Explain why the expected profit in policy year 2 decreases due to this change.

(iii) State with reasons whether the total present value of profits in all years increases, decreases, or stays the same due to this change.

Solution:

(i) Revised profit test table for year $t = 2$:

${}_{t-1}V$	P	E	I	EDB	ESV	$E\,{}_tV$	$\Pr_t$
690	1,000	60	97.8	507.5	65	1,228.5	−73.2

Details:

${}_1V = 1290 - 600 = 690$

$Premium,\ P\ = 1000$

$Expenses,\ E\ =\ 0.05P + 10 = 60$

$Interest,\ I\ =\ 0.06\left(690 + 1000 - 60\right) = 97.80$

Expected cost of death benefit, EDB

$q_{61}^{(d)} \times (100000 + AV_2) = 507.5$

Expected cost of surrenders, ESV

$q_{61}^{(w)} \times (1500 - 200) = 65$

Expected cost of year end reserve, E_tV

$p_{61}^{(\tau)} \times (1500 - 200) = 1228.5$

(ii) In year 2 the beginning of year reserve, brought forward with interest during year 2 is much lower using the cash value. The end year reserve is also reduced, but the change is smaller as the surrender charge is lower. Hence, the surplus emerging in year 2 decreases.

(iii) Overall the total present value of profits will increase, because the profits are released earlier when reserves are smaller. This increases the NPV whenever the risk discount rate is greater than the assumed earned rate.

6. Continued

Commentary:
A lot of candidates omitted these parts of the question. Of the candidates who attempted it, many answered (c)(i) correctly or with a minor error (such as omitting AV_2 *from the expected cost of death benefit).*
Parts (c)(ii) and (iii) proved to be challenging conceptual questions. A few strong candidates correctly answered (c)(ii), and a handful correctly (and impressively) answered (c)(iii). Some candidates offered one-word answers (`increase' or `decrease'), which received no credit.

7. **Learning Outcomes:** (2a) (5a) (5c)
Sources:
Textbook References: 10.2, 10.3, 10.6

Commentary on Question:
This was a relatively straightforward question that many students omitted. Those who attempted this question generally did well.

(a) List three reasons why employers sponsor pensions for their employees.

Solution:

Reasons include:

- To compete for new employees
- To retain employees in productive years
- To facilitate turnover of employees at older ages
- To offer tax efficient remuneration
- As a tool in negotiations with unions (or other employee collective bargaining units)
- To fulfill responsibility to provide for long-serving employees.
- To improve morale of employees

Commentary on Question:
Most candidates offered at least one valid reason, and many offered the three requested.

7. Continued

(b) Calculate the actuarial present value on January 1, 2014 of Chris' death benefit.

Solution:

Let S_x denote the salary earned in year of age x to x + 1. We have
$S_x = 50000(1.03)^{(x-38)}$.
Also let $v(t)$ denote the t-year discount factor at the valuation date, based on the spot rates given. The EPV of the death benefit is then
$(2S_{62})q_{62}\,v(1) + (2S_{63})\,{}_{1|}q_{62}\,v(2) + (2S_{64})\,{}_{2|}q_{62}\,v(3)$
Where
$v(1) = (1.035)^{-1}; \quad v(2) = (1.04)^{-2}; \quad v(3) = (1.045)^{-3} \quad q_{62} = 0.08$
${}_{1|}q_{62} = .92(0.09) = 0.0828; \; {}_{2|}q_{62} = (0.92)(0.91)(0.1) = 0.08372$

So the EPV of the Death Benefit is
$2(50000)(1.03)^{24}\left(0.08v_{3.5\%} + (1.03)(0.0828)v_{4\%}^{2} + (1.03)^{2}(0.08372)v_{4.5\%}^{3}\right) = 47562$

(c) Calculate the actuarial present value on January 1, 2014 of Chris' retirement benefit.

Solution:

The EPV of the retirement benefit is
$0.03 \times 27 \times (FAS) \times {}_{3}p_{62} \times v(3) \times \ddot{a}_{65}$
Where FAS is the final average salary, i.e.

$$FAS = 50000\left(\frac{(1.03)^{24} + (1.03)^{25} + (1.03)^{26}}{3}\right) = 104,719$$

So the EPV is
$(0.03)(27)(104,719)(0.75348)(0.87630)(4.7491) = 265,978$

Commentary on Question:
Parts (b) and (c) were answered well. The most common minor error was counting years of service incorrectly, which resulted in a small penalty for part (b), and none for (c) if the answer was consistent with (b). A few candidates justified their use of 26 years of service by proposing that Chris was 1 day short of 27 years. Although this interpretation is incorrect, candidates were not penalized if they gave this justification.
Some candidates were confused by the spot rates.