

October 2014 MLC Multiple Choice Solutions

1. Key: C

$$\begin{aligned}
 1000 {}_5|q_{\overline{60:70}} &= 1000 \left[{}_5p_{60} {}_5p_{70} q_{65} q_{75} + {}_5q_{60} {}_5p_{70} q_{75} + {}_5q_{70} {}_5p_{60} q_{65} \right] \\
 1000 &[(0.92)(0.88)(0.02132)(0.05169) + (1-0.92)(0.88)(0.05169) \\
 &\quad + (1-0.88)(0.92)(0.02132)] \\
 &= [0.0008922 + 0.00363898 + 0.00235373]1000 = 6.8849
 \end{aligned}$$

2. Key: A

By U.D.D.:

$$\begin{aligned}
 0.2 &= q_x^{(2)} = q_x^{(2)} \left(1 - \frac{1}{2} q_x^{(1)} \right) \\
 &= q_x^{(2)} \left(1 - \frac{1}{2} (0.1) \right) = 0.95 q_x^{(2)} \\
 \Rightarrow q_x^{(2)} &= 0.2105 \\
 p_x^{(\tau)} &= p_x^{(1)} p_x^{(2)} = (0.90)(1 - 0.2105) = 0.71055 \\
 q_x^{(1)} &= q_x^{(\tau)} - q_x^{(2)} = 1 - p_x^{(\tau)} - q_x^{(2)} \\
 &= 1 - 0.71055 - 0.2 = 0.08945
 \end{aligned}$$

3. Key: C

<u>Path</u>	<u>Probability</u>
S → S → S → S	0.216
S → S → C → S	0.012
S → C → S → S	0.012
S → C → C → S	0.010
Total across all paths:	0.250

4. Key: D

$$\begin{aligned}A_{35} &= A_{35:\overline{15}|}^1 + A_{35:\overline{15}|}^{\frac{1}{2}} A_{50} \\0.32 &= 0.25 + 0.14 A_{50} \\A_{50} &= \frac{0.07}{0.14} = 0.50\end{aligned}$$

5. Key: D

$$\begin{aligned}P &= 30,000 {}_{20|}\ddot{a}_{40} + PA_{40:\overline{20}|}^1 \\ \Rightarrow P &= 30,000 {}_{20|}\ddot{a}_{40} / (1 - A_{40:\overline{20}|}^1) \\ &= 30,000(3.0554) / (1 - 0.0601267) = 97,526\end{aligned}$$

6. Key: E

$$\begin{aligned}1,000,000 &= \text{APV}(\text{benefits}) = 100,000 A_{65} + R\ddot{a}_{65:65} + 0.60R\ddot{a}_{65|65} + 0.70R\ddot{a}_{65|65} \\ &\quad \left(\text{where } \ddot{a}_{65|65} = \ddot{a}_{65} - \ddot{a}_{65:65}\right) \\ &= 100,000 A_{65} + R(1.3\ddot{a}_{65} - 0.3\ddot{a}_{65:65})\end{aligned}$$

$$\begin{aligned}R &= \frac{1,000,000 - 100,000 A_{65}}{1.3\ddot{a}_{65} - 0.3\ddot{a}_{65:65}} & A_{65} &= 0.43980 \\ & & \ddot{a}_{65} &= 9.8969 \\ & & \ddot{a}_{65:65} &= 7.8552\end{aligned}$$

$$R = \frac{956,020}{10.50941} = 90,968$$

7. Key: C

Let π be the annual premium, so that $\pi \ddot{a}_{50} = A_{50} + 0.01 \ddot{a}_{50} + 0.19$

$$\Rightarrow \pi = \frac{A_{50} + 0.19}{\ddot{a}_{50}} + 0.01 = \frac{0.24905 + 0.19}{13.2668} + 0.01 = 0.04309389$$

Loss at issue: $L_0 = v^{k+1} - (\pi - 0.01) \ddot{a}_{\overline{k+1}|} (1 - v^{k+1}) / d + 0.19$

$$\begin{aligned} \Rightarrow \text{Var}[L_0] &= \left(1 + \frac{(\pi - 0.01)}{d}\right)^2 ({}^2A_{50} - A_{50}^2) \\ &= (2.511143)(0.09476 - 0.24905^2) \\ &= (2.511143)(0.032734) \\ &= 0.082 \end{aligned}$$

8. Key: B

EPV(premiums) = EPV(benefits)

$$P(1 + v p_x + v^2 {}_2p_x) = P(v q_x + 2v^2 p_x q_{x+1}) + 10000(v^3 {}_2p_x q_{x+2})$$

$$P\left(1 + \frac{0.9}{1.04} + \frac{0.9 \times 0.88}{1.04^2}\right) = P\left(\frac{0.1}{1.04} + \frac{2 \times 0.9 \times 0.12}{1.04^2}\right) + 10000\left(\frac{0.9 \times 0.88 \times 0.15}{1.04^3}\right)$$

$$2.5976P = 0.29588P + 1056.13$$

$$P = 459$$

9. Key: C

$$P \times \ddot{a}_{75:\overline{15}|} = 1000\left(A_{75:\overline{15}|}^1 + 15 \times P \times A_{75:\overline{15}|}^{\frac{1}{2}}\right) \rightarrow P = \frac{1000 A_{75:\overline{15}|}^1}{\ddot{a}_{75:\overline{15}|} - 15 \times A_{75:\overline{15}|}^{\frac{1}{2}}}$$

$$A_{75:\overline{15}|}^1 = A_{75:\overline{15}|} - A_{75:\overline{15}|}^{\frac{1}{2}} = 0.7 - 0.11 = 0.59$$

$$\ddot{a}_{75:\overline{15}|} = \frac{1 - A_{75:\overline{15}|}^{\frac{1}{2}}}{d} = (1 - 0.7) / 0.04 = 7.5$$

$$\text{So } P = \frac{590}{7.5 - 15(0.11)} = 100.85$$

10. Key: C

Let the monthly net premium = π

$$12\pi = \frac{100,000 \bar{A}_{45}}{\ddot{a}_{45:\overline{20}|}^{(12)}}$$

$$\alpha(12) = 1.00028$$

$$\beta(12) = 0.46812$$

$$\frac{i}{\delta} = 1.02971$$

$$100,000 \bar{A}_{45} = 100,000 \frac{i}{\delta} A_{45} = (1.02971)20,120 = 20,717.7652$$

$$\ddot{a}_{45:\overline{20}|}^{(12)} = \alpha(12)\ddot{a}_{45:\overline{20}|} - \beta(12)(1 - {}_{20}E_{45})$$

$$\left(\ddot{a}_{45} - {}_{20}E_{45} \ddot{a}_{65} \right)$$

$$= 1.00028[14.1121 - 0.25634(9.8969)] - 0.46812(1 - 0.25634)$$

$$= 11.230$$

$$12\pi = \frac{20717.7652}{11.230}$$

$$12\pi = 1844.858878$$

$$\pi = 153.7382$$

11. Key: D

$$G\ddot{a}_{x:\overline{30}|} = \text{APV}[\text{gross premium}] = \text{APV}[\text{Benefits} + \text{expenses}]$$

$$= FA_x + (30 + 30\ddot{a}_x) + G(0.6 + 0.10\ddot{a}_{x:\overline{30}|} + 0.10\ddot{a}_{x:\overline{15}|})$$

$$G = \frac{FA_x + 30 + 30\ddot{a}_x}{\ddot{a}_{x:\overline{30}|} - 0.6 - 0.1\ddot{a}_{x:\overline{30}|} - 0.1\ddot{a}_{x:\overline{15}|}}$$

$$= \frac{FA_x + 30 + 30(15.3926)}{14.0145 - 0.6 - 0.1(14.0145) - 0.1(10.1329)}$$

$$= \frac{FA_x + 491.78}{10.9998}$$

$$= \frac{FA_x}{10.9998} + \frac{491.78}{10.9998} = \frac{FA_x}{10.9998} + 44.71$$

$$\Rightarrow h = 44.71$$

12. Key: D

Intuitively:

- (A) A lower interest rate increases premium, but a higher recovery rate decreases premium, because there are lower projected benefits and more policyholders paying premiums.
- (B) A lower death rate of healthy lives \rightarrow more pay premium \rightarrow lower premium
- (C) A higher death rate of sick lives \rightarrow fewer benefits \rightarrow lower premium
- (D) A lower recovery rate \rightarrow higher sickness benefits \rightarrow higher premium
A lower death rate of sick lives \rightarrow higher sickness benefits \rightarrow higher premium
- (E) A higher rate of interest \rightarrow lower premium
A lower mortality rate for healthy lives may result in lower premium because more healthy lives are paying premium.

13. Key: A

$$\begin{aligned}({}_5V + 0.96G - 50)(1.05) &= q_{50}(100, 200) + p_{50} {}_6V \\(5500 + 0.96G - 50)(1.05) &= (0.009)(100, 200) + (1 - 0.009)(7100) \\(1.05)(0.96)G + 5722.5 &= 7937.9 \\(1.05)(0.96)G &= 2215.4 \\G &= 2197.8\end{aligned}$$

14. Key: E

$$\begin{aligned}{}_{15.6}V(1+i)^{0.4} &= {}_{0.4}P_{x+15.6} {}_{16}V + {}_{0.4}q_{x+15.6} 100 \\{}_{15.6}V(1.05)^{0.4} &= 0.957447(49.78) + 0.042553(100) \\{}_{15.6}V &= 50.91\end{aligned}$$

15. Key: D

APV future benefits

$$= 1000 \left[0.04v + 0.05 \times 0.96v^2 + 0.96 \times 0.95 \times (0.06 + 0.94 \times 0.683)v^3 \right] = 630.25$$

$$\text{APV future premiums} = 130(1 + 0.96v) = 248.56$$

$$E[{}_3L] = 630.25 - 248.56 = 381.69$$

16. Key: D

$$\frac{V[{}_tL]}{V[{}_{t+1}L]} = \frac{\left(1 + \frac{p}{d}\right)^2 ({}^2A_{x+t} - A_{x+t}^2)}{\left(1 + \frac{p}{d}\right)^2 ({}^2A_{x+t+1} - A_{x+t+1}^2)}$$

$$A_{x+t} = vq_{x+t} + vp_{x+t} A_{x+t+1}$$

$$= (0.90703)^{1/2}(0.02067) + (0.90703)^{1/2}(1 - 0.02067)(0.52536) = 0.50969$$

$${}^2A_{x+t} = v^2q_{x+t} + v^2p_{x+t} {}^2A_{x+t+1}$$

$$= (0.90703)(0.02067) + (0.90703)(1 - 0.02067)(0.30783) = 0.29219$$

$$\Rightarrow \frac{\text{Var}({}_kL)}{\text{Var}({}_{k+1}L)} = \frac{(0.29219) - (0.50969)^2}{(0.30783) - (0.52536)^2} = \frac{0.03241}{0.03183} = 1.018$$

17. Key: B

$$\text{Pr}_2 = {}_1V + P - E + I - \text{EDB} - E_2V$$

$$= (400 + 1500 - 100)1.072 - 1200 - 0.988 \times 700$$

$$= 38.00$$

18. Key: A

x	t	Premium	Expense charges	COI rate	Amount at Risk	COI	Credited Interest	End of Year Account Value
40	0	2500	75.00	0.0028	100,000	266.67	97.12	2255.45
41	1	3000	80.00	0.0030	95,000	271.43	255.00	5159.03

The COI calculation reflects discounting the amount at risk at 5%, (e.g., in year 1, $(100,000 / 1.05) \times 0.0028 = 266.67$)
Credited interest is 4.5% in year 1 and 5.2% in year 2.

19. Key: E

Annual Retirement Benefit

$$(0.0175) \left[525,000 + \sum_{k=0}^9 (50,000)(1.03)^k \right] = 19,218.39$$

APV at age 65

$$(19,218.39) {}_{10|}\ddot{a}_{55}^{(12)} = 19,218.39(1.04)^{-10} ({}_{10}p_{55}) \ddot{a}_{65}^{(12)}$$

$$= 19,218.39(0.6756)(0.925)(12.60) = 151,328$$

20. Key: B

$$\begin{aligned} {}_{2.5}q_{[50]+0.4} &= 1 - {}_{2.5}p_{[50]+0.4} = 1 - {}_{2.9}p_{[50]} / (p_{[50]})^{0.4} \\ &= 1 - \left\{ p_{[50]} p_{[50]+1} (p_{52})^{0.9} \right\} / (1 - q_{[50]})^{0.4} \\ &= 1 - \left\{ (1 - q_{[50]}) (1 - q_{[50]+1}) (1 - q_{52})^{0.9} \right\} / (1 - q_{[50]})^{0.4} \\ &= 1 - \left\{ (1 - 0.0050) (1 - 0.0063) (1 - 0.0080)^{0.9} \right\} / (1 - 0.0050)^{0.4} \\ &= 0.01642 \end{aligned}$$

$$1000 {}_{2.5}q_{[50]+0.4} = 16.42$$