

MLC Fall 2014 Written Answer Questions

Model Solutions

MLC Fall 2014 Question 1 Model Solution

Learning Objectives: 2(a), 3(a), 4(f)

Textbook References: 5.4, 6.6, 13.5

(a) The EPV of the commission payments is

$$\begin{aligned} E[C] &= 0.1 G \ddot{a}_{35:\overline{10}|} + 0.9G \\ \ddot{a}_{35:\overline{10}|} &= 15.3926 - 0.54318(14.1121) = 7.7272 \\ \text{So } E[C] &= 1.6727 G \end{aligned}$$

(b) The EPV is

$$8 \ddot{a}_{35} + 92 = 215.14$$

(c) The Equation of Value is

$$\begin{aligned} G \ddot{a}_{35} &= 1.1 (100\,000 A_{35} + 1.6727 G + 215.14) \\ \Rightarrow G &= \frac{14395.86}{15.3926 - 1.1(1.6727)} = 1062.22 \end{aligned}$$

Comments:

Parts (a), (b) and (c) were all done well, with a large majority of candidates scoring full marks. A few candidates did not allow for the 10-year term on commissions.

(d) Consider the random variable C :

$$\begin{aligned} C &= 0.1 G \ddot{a}_{\overline{\min(K_{35}+1, 10)}|} + 0.9G \\ &= 0.1 G \frac{1 - v^{\min(K_{35}+1, 10)}}{d} + 0.9G \end{aligned}$$

$$\text{So } \text{Var}[C] = \left(\frac{0.1 G}{d} \right)^2 \left({}^2 A_{35:\overline{10}|} - A_{35:\overline{10}|}^2 \right)$$

Now

$$\begin{aligned} {}^2 A_{35:\overline{10}|} &= {}^2 A_{35} - ({}_{10} E_{35} v^{10}) {}^2 A_{45} + {}_{10} E_{35} v^{10} \\ &= 0.03488 - 0.54318 v^{10} 0.06802 + 0.54318 v^{10} = 0.31756 \end{aligned}$$

$$A_{35:\overline{10}|} = A_{35} - {}_{10} E_{35} A_{45} + {}_{10} E_{35} = 0.56261$$

$$\text{so } \text{Var}[C] = \left(\frac{0.1 * 1062.22}{0.05660} \right)^2 (0.001030) = 3627.6$$

Comments:

This part proved more challenging to candidates, with relatively few gaining full credit. Most candidates earned partial credit, although a good number omitted this part entirely.

It was not necessary to write down the random variable C for full credit, but candidates who included this step were more likely to get to the correct answer than those who tried to write the variance down directly.

A number of candidates tried to calculate the annuity variance directly, and found that there was no feasible way to get the correct answer. It is worth remembering that it is always easier to calculate the variance of a death (or endowment) benefit than to calculate, directly, the variance of a life annuity.

Some candidates calculated a negative variance. Candidates who noted that this answer is impossible could receive partial credit for their working.

(e)

$$E[C] = 1.6727G = 1776.77$$

Let c denote the 1st year commission rate. Then

$$E[C^*] = 0.1 * 12 * 95 * \ddot{a}_{35:\overline{10}|}^{(12)} + (c - 0.1) * 12 * 95 * \ddot{a}_{35:\overline{1}|}^{(12)}$$

$$\ddot{a}_{35:\overline{10}|}^{(12)} = \ddot{a}_{35:\overline{10}|} - \frac{11}{24} (1 - {}_{10}E_{35}) = 7.5178$$

$$\ddot{a}_{35:\overline{1}|}^{(12)} = 1 - \frac{11}{24} (1 - {}_1E_{35}) = 0.97319$$

$$\text{So } E[C^*] = 857.03 + 1109.43(c - 0.1)$$

$$\Rightarrow c = 0.9290$$

Comments:

This part also proved quite challenging. A relatively small number of candidates earned full credit. Another group earned nearly full credit, but did not make the adjustment for the 10% renewal commission. Many candidates omitted this part.

- (f) Universal Life policies have flexible premiums. Premiums may be increased, decreased or even suspended for a period. Hence, the commissions would be more variable than a whole life policy, in general.

Comments:

Most candidates omitted this part. A small number answered with comments about Universal Life that were not relevant to the question (for example, that the crediting rate in UL can vary) but most candidates who attempted this part earned full credit.

MLC Fall 2014 Question 2 Model Solution

Learning Outcomes: 5(a), 5(c)

Textbook References: 10.1, 10.2, 10.3, 10.4, 10.6

- (a) Let S_x denote salary in year of age x to $x + 1$. The projected salaries in the final three years are:

$$S_{62} = 80\,000 (1.04)^{17} = 155\,832$$

$$S_{63} = 80\,000 (1.04)^{18} = 162\,065$$

$$S_{64} = 80\,000 (1.04)^{19} = 168\,548$$

$$\text{Average} = 162\,148$$

So the projected benefit per month is

$$25 (0.02 * 100\,000 + 0.03 * 62\,148) / 12 = 8050.92$$

Comment: This part was done well, with most candidates earning full credit. The most common error was miscounting years of service.

- (b) The Replacement Ratio (RR) is

$$\frac{12 * 8051}{168\,548} = 57.3\%$$

Comment: Again, this part was done well. The most common error was using the final average salary instead of the final year's salary.

- (c) The EPV is

$$EPV = 8051 * 12 * \ddot{a}_{65}^{(12)}$$

$$\ddot{a}_{65}^{(12)} = \frac{1 - A_{65}^{(12)}}{d^{(12)}}$$

$$d^{(12)} = 12(1 - v^{1/12}) = 0.04869$$

$$\Rightarrow \ddot{a}_{65}^{(12)} = 10.885$$

$$\Rightarrow EPV = 1\,051\,620$$

Comment: Many candidates gained full credit on this part. The most common errors were (i) errors in calculation of $d^{(12)}$ and (ii) forgetting to multiply the monthly benefit by 12.

(d) We require $(0.8-0.573)=0.227$ replacement ratio from the DC plan.

The cost of adding 22.7% RR is

$$0.227 * 168\,548 * \ddot{a}_{65}^{(12)} = 416\,464$$

The accumulated contributions to age 65, where c is the contribution rate, are:

$$\begin{aligned} & c S_{45} \left((1.07)^{20} + (1.04)(1.07)^{19} + \dots + (1.04)^{19}(1.07) \right) \\ &= c S_{45} \frac{(1.07)^{20} - (1.04)^{20}}{1 - (1.04)/(1.07)} \\ &= 4\,789\,500 c \end{aligned}$$

Equating contributions and benefit value gives

$$c = 8.70\%$$

Comment: This part was less well done, although a number of strong candidates scored full marks. Quite a few candidates omitted this part entirely. The most common errors, which earned partial credit, were miscalculating the geometric series, or omitting entirely either the 4% or 7% terms in the series.

MLC Fall 2014 Question 3 Model Solution

Learning Objectives: 1(a), 1(b), 1(f)

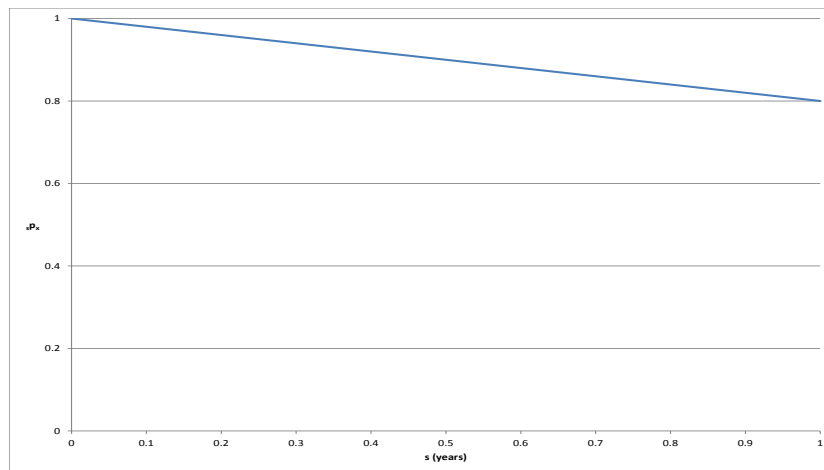
Textbook References: 9.3, 9.4, 3.3

- (a) The symbol q_{xy} is the probability that at least one of two lives, currently age x and y , dies within 1 year.

Comments:

Part (a) was well done by almost all candidates. Some candidates mistakenly described the last survivor status. For full credit, candidates were required to specify the 1-year time period for the mortality probability.

- (b) The function is



Comments:

Most candidates did well on this part. Some candidates lost marks for failing to mark key numerical values on the axes. Candidates who sketched a non-linear function for ${}_i p_x$, or who sketched a different probability, or who sketched a line from 1 to 0, received no credit for this part.

(c)

$$\begin{aligned} {}_s q_{xy} &= 1 - {}_s p_{xy} = 1 - {}_s p_x {}_s p_y \quad (\text{independence}) \\ &= 1 - (1 - {}_s q_x)(1 - {}_s q_y) = 1 - (1 - {}_s q_x)(1 - {}_s q_y) \quad (\text{UDD}) \\ &= s(q_x + q_y) - s^2 q_x q_y \end{aligned}$$

Now $q_x + q_y = q_{xy} + q_{\overline{xy}}$ and $q_x q_y = q_{\overline{xy}}$

$$\Rightarrow {}_s q_{xy} = s(q_{xy} + q_{\overline{xy}}) - s^2 q_{\overline{xy}}$$

$${}_s q_{xy} = s q_{xy} + (s - s^2)q_{\overline{xy}}$$

$$\Rightarrow g(s) = (s - s^2)$$

Comments:

Performance on this part was mixed. Many candidates received full credit. A number of candidates made a reasonable start but could not derive the final result. Partial credit was awarded in these cases. The most common error was assuming that UDD applied to the joint life status.

MLC Fall 2014 Question 4 Model Solution

Learning Outcomes: 1(a), 1(b), 1(d), 1(e), 2(a)

Textbook References: 8.4.1, 8.5, 8.6

(a)

$$\begin{aligned}\frac{d}{dt} {}_t p_x^{10} &= {}_t p_x^{11} \mu_{x+t}^{10} - {}_t p_x^{10} (\mu_{x+t}^{01} + \mu_{x+t}^{03}) \\ \frac{d}{dt} {}_t p_x^{11} &= {}_t p_x^{10} \mu_{x+t}^{01} - {}_t p_x^{11} (\mu_{x+t}^{10} + \mu_{x+t}^{12} + \mu_{x+t}^{13}) \\ \text{Boundary Conditions: } &{}_0 p_x^{10} = 0 \quad {}_0 p_x^{11} = 1\end{aligned}$$

Comments:

Most candidates achieved full credit for this part. Common errors included omitting the boundary conditions, or omitting terms on the right hand side. A few candidates forgot the $\frac{d}{dt}$ on the left hand side.

(b) From the Kolmogorov equation for $\frac{d}{dt} {}_t p_{80}^{10}$, we have

$$\lim_{h \rightarrow 0} \frac{{}_{t+h} p_{80}^{10} - {}_t p_{80}^{10}}{h} = {}_t p_{80}^{11} \mu_{80+t}^{10} - {}_t p_{80}^{10} (\mu_{80+t}^{01} + \mu_{80+t}^{03})$$

So, for small h ,

$$\begin{aligned}\frac{{}_{t+h} p_{80}^{10} - {}_t p_{80}^{10}}{h} &\approx {}_t p_{80}^{11} \mu_{80+t}^{10} - {}_t p_{80}^{10} (\mu_{80+t}^{01} + \mu_{80+t}^{03}) \\ \Rightarrow {}_{t+h} p_{80}^{10} &\approx {}_t p_{80}^{10} + h ({}_t p_{80}^{11} \mu_{80+t}^{10} - {}_t p_{80}^{10} (\mu_{80+t}^{01} + \mu_{80+t}^{03})) \\ \Rightarrow \frac{1}{3} p_{80}^{10} &\approx 0 + \frac{1}{3} (0.08) = 0.02667 \\ \frac{2}{3} p_{80}^{10} &\approx \frac{1}{3} p_{80}^{10} + \frac{1}{3} (0.90346 * 0.08 - 0.02667 * 0.13082) = 0.04960 \\ {}_1 p_{80}^{10} &\approx \frac{2}{3} p_{80}^{10} + \frac{1}{3} (0.81652 * 0.08 - 0.04960 * 0.13186) = 0.06919\end{aligned}$$

ALTERNATIVE SOLUTION

$${}_1 p_{80}^{10} = \left(\frac{1}{3} p_{80}^{11} \frac{1}{3} p_{80 \frac{1}{3}}^{10} \frac{1}{3} p_{80 \frac{2}{3}}^{00} \right) + \left(\frac{2}{3} p_{80}^{11} \frac{1}{3} p_{80 \frac{2}{3}}^{10} \right) + \left(\frac{1}{3} p_{80}^{10} \frac{1}{3} p_{80 \frac{1}{3}}^{00} \frac{1}{3} p_{80 \frac{2}{3}}^{00} \right)$$

We approximate the probabilities as follows

$${}_{\frac{1}{3}}p_{80+t}^{10} \approx \frac{1}{3}\mu_{80+t}^{10} = 0.02667$$

$${}_{\frac{1}{3}}p_{80+t}^{01} \approx \frac{1}{3}\mu_{80+t}^{01} = 0.03333$$

$${}_{\frac{1}{3}}p_{80\frac{1}{3}}^{00} \approx 1 - \frac{1}{3}(\mu_{80\frac{1}{3}}^{01} + \mu_{80\frac{1}{3}}^{03}) = 0.95606$$

$${}_{\frac{1}{3}}p_{80\frac{2}{3}}^{00} \approx 1 - \frac{1}{3}(\mu_{80\frac{2}{3}}^{01} + \mu_{80\frac{2}{3}}^{03}) = 0.95605$$

so

$$\begin{aligned} {}_1p_{80}^{10} &\approx 0.90346 \times 0.02667 \times 0.95605 + 0.81652 \times 0.02667 \\ &\quad + 0.02667 \times 0.95606 \times 0.95605 \\ &\approx 0.02304 + 0.02178 + 0.02438 = 0.06920 \end{aligned}$$

Comments:

- *This part was quite well done, with many candidates receiving full credit.*
- *Those who followed the alternative method were more prone to numerical errors.*
- *Some candidates lost marks by using the wrong values from the tables. Deductions for this were fairly small if the rest of the solution was correct.*
- *Many candidates calculated values for ${}_n p_{80}^{11}$, not realizing these had been given to them in the question.*

(c(i)) The expected present value of the service fees is

$$\begin{aligned} EPV &= 8000(\bar{a}_{80}^{00} + \bar{a}_{80}^{01}) \\ &= 8000(5.5793 + 1.3813) = 55\,685 \end{aligned}$$

(c(ii)) The expected present values of the level 2 care costs is

$$\begin{aligned} EPV &= 30\,000\bar{a}_{80}^{02} + 10\,000 {}_5|\bar{a}_{80}^{02} \\ &= 30\,000\bar{a}_{80}^{02} + 10\,000 ({}_5p_{80}^{00}v^5\bar{a}_{85}^{02} + {}_5p_{80}^{01}v^5\bar{a}_{85}^{12} + {}_5p_{80}^{02}v^5\bar{a}_{85}^{22}) \\ &= 23\,005 \end{aligned}$$

Alternative answers for (c)(ii)

$$\begin{aligned}EPV &= 30\,000\bar{a}_{80:\overline{5}|}^{02} + 40\,000\ {}_5|\bar{a}_{80}^{02} \\{}_5|\bar{a}_{80}^{02} &= v^5 \left({}_5p_{80}^{00}\bar{a}_{85}^{02} + {}_5p_{80}^{01}\bar{a}_{85}^{12} + {}_5p_{80}^{02}\bar{a}_{85}^{22} \right) = 0.4678 \\ \bar{a}_{80:\overline{5}|}^{02} &= \bar{a}_{80}^{02} - {}_5|\bar{a}_{80}^{02} = 0.14308 \\ \Rightarrow EPV &= 23\,005\end{aligned}$$

$$\begin{aligned}EPV &= 40\,000\bar{a}_{80}^{02} - 10\,000\bar{a}_{80:\overline{5}|}^{02} \\ \bar{a}_{80:\overline{5}|}^{02} &= \bar{a}_{80}^{02} - v^5 \left({}_5p_{80}^{00}\bar{a}_{85}^{02} + {}_5p_{80}^{01}\bar{a}_{85}^{12} + {}_5p_{80}^{02}\bar{a}_{85}^{22} \right) \\ \Rightarrow \bar{a}_{80:\overline{5}|}^{02} &= 0.14308 \\ \Rightarrow EPV &= 23\,005\end{aligned}$$

Comments:

- *This part is designed to test understanding of the multiple state model annuity factor notation, and ability to manipulate the probabilities and annuities to create term annuity factors. It was one of the more challenging parts of the exam overall.*
- *Many candidates achieved full credit for (c)(i). Those who did not tended to combine, for example, \bar{a}_{80}^{00} and \bar{a}_{80}^{11} , which would require Ada to be in state 0 and state 1 simultaneously at age 80.*
- *The best candidates correctly noted for (c)(ii) that there are three cases to allow for in creating the five-year deferred annuity, corresponding to Ada being in state 0, state 1 or state 2 in five years. However, most candidates did not take all three cases into consideration.*
- *Some answers allowed appropriately for one or two cases, and these received partial credit.*
- *Some candidates combined, for example, ${}_5p_{80}^{02}$ with \bar{a}_{85}^{02} , which is a more serious error, as it indicates a lack of understanding of the functions involved.*

MLC Fall 2014 Question 5 Model Solution

Learning Objectives: 4(c), 4(g)

Textbook References: 12.3, 12.4, 12.5

(a)

$$\begin{aligned} Pr_2 &= ({}_1V + P - E)(1 + i_2) - q_{51}S - p_{51}({}_2V) \\ &= (400 + 1100 - 55)(1.02) - 0.00642 * 100\,000 - 0.99358 * 800 = 37.04 \end{aligned}$$

Comments:

Most candidates earned full credit for this part. Those who did not tended to struggle with the rest of the question.

(b) Profit test table:

| t | ${}_{t-1}V$ | P | E_t | I_t | EDB_t | EMB | E_tV | Pr_t |
|-----|-------------|------|-------|-------|---------|---------|--------|---------|
| 0* | | | 155 | | | | | -155.00 |
| 1 | 0 | 1100 | 55 | 10.45 | 592 | | 397.63 | 65.82 |
| 2 | 400 | 1100 | 55 | 28.90 | 642 | | 794.86 | 37.04 |
| 3 | 800 | 1100 | 55 | 55.35 | 697 | 1092.33 | 0.00 | 111.02 |

The profit vector is the final column.

P is the premium

E_t denotes expenses;

I_t denotes interest on funds in year t

$EDB_t = 100,000 q_{x+t-1}$

EMB in year 3 is $1100 p_{52}$

$E_tV = p_{50+t-1} {}_tV$.

Comments:

Many candidates received full credit for this part, and a larger number received partial credit. The most common errors included (i) ignoring the maturity benefit and (ii) incorrect use of probabilities in E_tV . Some candidates used a profit table, others calculated each term in the profit vector individually. Either approach was acceptable. It is not necessary for candidates to define all terms, but it can be helpful when graders are considering partial credit.

(c)

The profit signature at t is Π_t where

$$\Pi_0 = Pr_0 \quad \text{and} \quad \Pi_t = {}_{t-1}p_{50} Pr_t$$

$$\Rightarrow (\Pi_0, \Pi_1, \Pi_2, \Pi_3) = (-155.00, 65.82, 36.82, 109.65)$$

The NPV is

$$NPV = \sum_{k=0}^3 \Pi_k v_{14\%}^k = 5.08$$

Comments:

Most candidates did this part well. Some candidates used incorrect probabilities (${}_t p_{50}$ instead of ${}_{t-1} p_{50}$ in the profit signature), and others used incorrect rates for discounting the profits for the NPV.

(d) The IRR of B is j where

$$155 = 210v_j^3 \Rightarrow j = 10.65\%.$$

and the IRR of A is greater than 14%, because the NPV at 14% is positive. Hence IRR of B is less than IRR of A.

Product C has lower reserves in year 1, which will allow an earlier release of surplus compared with Product A, which gives a higher NPV than A at the 14% hurdle rate, but does not necessarily mean that C has a higher IRR.

The lower reserve in year 1 results in the following profit signature for C:

$$(-155.00, 165.23, -64.58, 109.65)$$

We note that the NPV of A is a little larger than 14%, because the NPV at 14% is close to zero. Calculating the NPV of A and C at 16% gives -0.65 for A and 9.7 for C. Hence, the IRR for C is greater than 16%, and for A is less than 16%.

That is:

$$IRR(B) < IRR(A) < IRR(C)$$

Comments:

Many candidates evaluated the IRR for all three cases, presumably using the financial functions on the BA2 calculator. This was awarded full credit if correct. However, it was not necessary to determine the IRR to answer the question.

Candidates who demonstrated understanding of the relationship between the release of surplus and the return to the insurer gained partial credit, even if the justification was incomplete.

No credit was awarded for the IRR ordering if there was no accompanying explanation or justification

MLC Fall 2014 Question 6 Model Solution

Learning Outcomes: 4(e)

Textbook References: 13.4.2, 13.4.6, 13.4.7

- (a) Universal Life is regulated as an insurance product. The corridor factor ensures that the insurance benefit is significant throughout the term of the contract, so that the policy is correctly classified as insurance, not pure investment.

Comment: Many candidates answered this part well. For full marks, candidates were required to explain, coherently, that a significant insurance benefit has to be maintained throughout the term of the contract for regulatory purposes. No credit was given in the small number of cases where candidates explained how the corridor factor works instead of explaining why the corridor factor requirement exists.

(b)(i)

$$\begin{aligned}AV_1 &= (15000 (0.99) - CoI) (1.05) \\CoI &= 1.2 q_{50} (100\,000 - AV_1) v_{4\%} = 683.08 - 0.00683 AV_1 \\ \Rightarrow AV_1 &= \frac{(15000 (0.99) - 683.08) (1.05)}{1 - 0.00683 (1.05)} = 14\,983 \\ \Rightarrow CoI &= 580.75\end{aligned}$$

Alternatively, calculating CoI directly,

$$\begin{aligned}COI &= \frac{(F - 0.99P(1.05)) v_{4\%} 1.2q_{50}}{1 - 1.05 v_{4\%} 1.2q_{50}} \\ &= \frac{(100\,000 - (0.99)(15\,000)(1.05)) 0.00683}{1 - 0.00683(1.05)} = 580.67\end{aligned}$$

(b)(ii)

$$\begin{aligned}AV_1 &= (15000 (0.99) - CoI) (1.05) \\CoI &= 1.2 q_{50} (2.2AV_1 - AV_1) v_{4\%} = 0.00820 AV_1 \\ \Rightarrow AV_1 &= \frac{(15000(0.99)) (1.05)}{1 + 0.00820(1.05)} = 15\,459 \\ \Rightarrow CoI &= 126.77\end{aligned}$$

Or, calculating CoI directly,

$$COI = \frac{1.2 q_{50} v_q (1.2 * 1500 * 0.99 * 1.05)}{1 + 1.2^2 q_{50} v_q (1.05)} = \frac{127.80}{1.0082} = 126.76$$

(b)(iii)

$$CoI = \max(580.75, 126.77) = 580.75$$

Comments:

- *Candidates were not expected to memorize the formulas, but to derive them from the account value recursions. Most candidates used the derivation approach, but some used memorization. If the formulas were correctly memorized, candidates received full credit. Incorrect memorization, with no derivation to demonstrate understanding, received little credit.*
- *Some candidates provided answers for a Type B universal life policy instead of Type A. Because the difference between Type A and Type B policies is important, little credit was awarded for using Type B formulas here.*
- *Other common mistakes included failing to apply the COI discount rate (4%) and the crediting rate (5%) properly, and/or not reflecting properly the 120% mortality rate in the calculation of the COI. Partial credit was awarded if the answers were otherwise correct. If the same mistake was made in parts (i) and (ii), it was only penalized once.*
- *For part (ii), a number of candidates calculated the COI based on the corridor factor using the account value (AV) obtained in part (i) instead of recognizing that the AV is different in this case.*
- *Most candidates received full marks for (iii). Choosing the maximum COI as calculated in part (i) and (ii) is an acceptable justification (even if the answers to (i) and (ii) were incorrect). Alternatively, one could justify the choice of the COI obtained in part (i) by comparing 2.2 AV with the face amount.*

| | |
|--------------------|---------------------|
| (c) Account Value: | 14983 |
| ADB: | 85017 |
| CV: | 14983 – 750 = 14233 |

Comments:

- *Some candidates did not understand the term ‘Additional Death Benefit’. It is defined in the textbook as the excess of the total death benefit over the account value.*

- A small number of candidates confused surrender charges with cash surrender value.

(d) Let V denote the reserve for the no-lapse guarantee

$$V = \max \left(100\,000 A_{60:\overline{10}|}^1 - 10\,000, 0 \right)$$

$$A_{60:\overline{10}|}^1 = A_{60} - {}_{10}E_{60}A_{70} = 0.13678$$

$$\Rightarrow V = 13\,678 - 10\,000 = 3\,678$$

Comments: This part was generally done well by those who attempted it.