



Society of Actuaries



Canadian Institute of Actuaries

Exam MLC Models for Life Contingencies

Friday,
October 30, 2015
8:30 a.m. – 12:45 p.m.

MLC

INSTRUCTIONS TO CANDIDATES

General Instructions

- Write your candidate number here _____. Your name must not appear.
- Do not break the seal of this book until the supervisor tells you to do so.
- Tables and numerical values necessary for solving some of the questions on this examination will be distributed by the Supervisor.
- This examination has a total of 96 points. It consists of:

Section A: 20 multiple-choice questions, each worth 2 points for a total of 40 points, and

Section B: 6 written-answer questions, worth a total of 56 points. The point value for each written-answer question is indicated at the beginning of the question.

You may divide your time between the two sections of the examination (written-answer, and multiple-choice) as you choose. You should keep in mind the relative weight of the two sections.

Your written-answer paper will be graded only if your multiple-choice score is at or above a threshold set after the examination is administered.

- Failure to stop writing after time is called will result in the disqualification of your answers or further disciplinary action.
- While every attempt is made to avoid defective questions, sometimes they do occur. If you believe a question is defective, the supervisor or proctor cannot give you any guidance beyond the instructions on the exam booklet.

Multiple-Choice Instructions

- A separate answer sheet for the multiple-choice questions is inside the front cover of this book. During the time allotted for this examination, record all your answers on the back of the answer sheet. **NO ADDITIONAL TIME WILL BE ALLOWED FOR THIS PURPOSE.**

No credit will be given for anything indicated in the examination book but not transferred to the answer sheet. Failure to stop writing or coding your answer sheet after time is called will result in the disqualification of your answer sheet or further disciplinary action.

- On the front of the answer sheet, space is provided to write and code candidate information. Complete the information requested by printing in the squares and blackening the circles (one in each column) corresponding to the letters or numbers printed. For each empty box blacken the small circle immediately above the "A" circle. Fill out the boxes titled:

- Name
(include last name, first name and middle initial)
- Candidate Number
(Candidate/Eligibility Number, use leading zeros if needed to make it a five digit number)
- Test Site Code
(The supervisor will supply the number.)
- Examination Part
(Code the examination that you are taking by blackening the circle to the left of "Exam MLC")
- Booklet Number
(The booklet number can be found in the upper right-hand corner of this examination book. Use leading zeros if needed to make it a four digit number.)

In box titled "Complete this section only if instructed to do so," fill in the circle to indicate if you are using a calculator and write in the make and model number.

In the box titled "Signature and Date" sign your name and write today's date. **If the answer sheet is not signed, it will not be graded.**

Leave the boxes titled "Test Code" and "Form Code" blank.

On the back of the answer sheet fill in the Booklet Number in the space provided.

CONTINUED ON INSIDE FRONT COVER

3. Your score will be based on the number of questions which you answer correctly. No credit will be given for omitted answers and no credit will be lost for wrong answers; hence, you should answer all questions even those for which you have to guess.
4. Five answer choices are given with each multiple-choice question, each answer choice being identified by a key letter (A to E). Answer choices for some questions have been rounded. For each question, blacken the circle on the answer sheet which corresponds to the key letter of the answer choice that you select.
5. Use a soft-lead pencil to mark the answer sheet. To facilitate correct mechanical scoring, be sure that, for each question, your pencil mark is dark and completely fills only the intended circle. Make no stray marks on the answer sheet. If you have to erase, do so completely.
6. Do not spend too much time on any one question. If a question seems too difficult, leave it and go on.
7. Clearly indicated answer choices in the test book can be an aid in grading examinations in the unlikely event of a lost answer sheet.
8. Use the blank portions of each page for your scratch work. Extra blank pages are provided at the back of the examination book.
9. After the examination, the supervisor will collect this book and the answer sheet separately. **DO NOT ENCLOSE THE ANSWER SHEET IN THE BOOK OR IN THE ESSAY ANSWER ENVELOPE.** All books and answer sheets must be returned. **THE QUESTIONS ARE CONFIDENTIAL AND MAY NOT BE TAKEN FROM THE EXAMINATION ROOM.**

Written-Answer Instructions

1. Write your candidate number at the top of each sheet. Your name must not appear.
2. Write on only one side of a sheet. Start each question on a fresh sheet. On each sheet, write the number of the question you are answering. Do not answer more than one question on a single sheet.
3. The answer should be confined to the question as set.
4. When you are asked to calculate, show all your work including any applicable formulas.
5. When you finish, insert all your written-answer sheets into the Essay Answer Envelope. Be sure to hand in all your answer sheets because they cannot be accepted later. Seal the envelope and write your candidate number in the space provided on the outside of the envelope. Check the appropriate box to indicate Exam MLC.
6. Be sure your essay answer envelope is signed because if it is not, your examination will not be graded.
7. For all parts of all problems, to maximize the credit earned, candidates should show as much work as possible, considering the time allotted for the question. Answers lacking justification will receive no credit. Answers should be organized so that the methods, logic, and formulas used are readily apparent. Candidates should not round their answers excessively; enough precision should be provided so that their answers can be accurately graded.

In some cases, candidates are asked to show that a calculation results in a particular number. Typically the answer given will be rounded; candidates should provide a greater level of accuracy than the number given in the question. This structure of question is intended to assist the candidate by giving an indication when the calculation has been done incorrectly, providing an opportunity to explore an alternative approach. It also allows a candidate who cannot obtain the correct answer to use the answer given to proceed with subsequent parts of the problem. (Candidates who are able to solve the problem should use their exact answer for subsequent parts.)

For questions requiring candidates to derive or write down a formula or equation, the resulting expression should be simplified as far as possible, and where numerical values are provided in the problem, they should be used.

Exam MLC

SECTION A – Multiple-Choice

****BEGINNING OF EXAMINATION****

- 1.** A father-son club has 4000 members, 2000 of which are age 10 and the other 2000 are age 35. In 25 years, the members of the club intend to hold a reunion.

You are given:

- (i) All lives have independent future lifetimes.
- (ii) Mortality follows the Illustrative Life Table.

Using the normal approximation, calculate the 99th percentile of the number of surviving members at the time of the reunion.

- (A) 3680
- (B) 3690
- (C) 3700
- (D) 3710
- (E) 3720

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2. A group of 100 people start a Scissor Usage Support Group. The rate at which members enter and leave the group is dependent on whether they are right-handed or left-handed.

You are given the following:

- (i) The initial membership is made up of 75% left-handed members (L) and 25% right-handed members (R).
- (ii) After the group initially forms, 35 new (L) and 15 new (R) join the group at the start of each subsequent year.
- (iii) Members leave the group only at the end of each year.
- (iv) $q^L = 0.25$ for all years.
- (v) $q^R = 0.50$ for all years.

Calculate the proportion of the Scissor Usage Support Group's expected membership that is left-handed at the start of the group's 6th year, before any new members join for that year.

- (A) 0.76
- (B) 0.81
- (C) 0.86
- (D) 0.91
- (E) 0.96

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3. Johnny Vegas performs motorcycle jumps throughout the year and has injuries in the course of his shows according to the following three-state model:

State 0: No injuries
State 1: Exactly one injury
State 2: At least two injuries

You are given:

- (i) Transition intensities between States are per year.
- (ii) $\mu_t^{01} = 0.03 + 0.06 \times 2^t, \quad t > 0$
- (iii) $\mu_t^{02} = 2.718\mu_t^{01}, \quad t > 0$
- (iv) $\mu_t^{12} = 0.025, \quad t > 0$

Calculate the probability that Johnny, who currently has no injuries, will sustain at least one injury in the next year.

- (A) 0.35
- (B) 0.39
- (C) 0.43
- (D) 0.47
- (E) 0.51

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4. For three fully discrete insurance products on the same (x) , you are given:

- (i) Z_1 is the present value random variable for a 20-year term insurance of 50.
- (ii) Z_2 is the present value random variable for a 20-year deferred whole life insurance of 100.
- (iii) Z_3 is the present value random variable for a whole life insurance of 100.
- (iv) $E[Z_1] = 1.65$ and $E[Z_2] = 10.75$.
- (v) $Var(Z_1) = 46.75$ and $Var(Z_2) = 50.78$.

Calculate $Var(Z_3)$.

- (A) 62
- (B) 109
- (C) 167
- (D) 202
- (E) 238

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5. For two lives, (50) and (60), with independent future lifetimes, you are given:

(i) Mortality follows the Illustrative Life Table.

(ii) $i = 0.06$

Calculate $a_{\overline{50:60:20}|}$.

(A) 8.60

(B) 8.75

(C) 8.90

(D) 9.05

(E) 9.20

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- 6.** For a special life insurance policy on Rochelle and Suzanne, both age 50, with independent future lifetimes, you are given:
- (i) A death benefit of 30,000 is payable at the end of the year of the first death.
 - (ii) A death benefit of 70,000 is payable at the end of the year of the second death.
 - (iii) Mortality follows the Illustrative Life Table.
 - (iv) $i = 0.06$

Calculate the actuarial present value of this insurance.

- (A) 17,550
- (B) 21,250
- (C) 23,850
- (D) 24,900
- (E) 25,900

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7. Cathy purchases a fully discrete whole life insurance policy of 100,000 on her 35th birthday.

You are given:

- (i) The annual gross premium, calculated using the equivalence principle, is 1770.
- (ii) The expenses in policy year 1 are 50% of premium and 200 per policy.
- (iii) The expenses in policy years 2 and later are 10% of premium and 50 per policy.
- (iv) All expenses are incurred at the beginning of the policy year.
- (v) $i = 0.035$

Calculate \ddot{a}_{35} .

- (A) 20.0
- (B) 20.5
- (C) 21.0
- (D) 21.5
- (E) 22.0

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8. For a fully discrete whole life insurance of 100 on (x) , you are given:

- (i) The first year expense is 10% of the gross annual premium.
- (ii) Expenses in subsequent years are 5% of the gross annual premium.
- (iii) The gross premium calculated using the equivalence principle is 2.338.
- (iv) $i = 0.04$
- (v) $\ddot{a}_x = 16.50$
- (vi) ${}^2A_x = 0.17$

Calculate the variance of the loss at issue random variable.

- (A) 900
- (B) 1200
- (C) 1500
- (D) 1800
- (E) 2100

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9. For a 2-year deferred, 2-year term insurance of 2000 on $[65]$, you are given:

(i) The following select and ultimate mortality table with a 3-year select period:

x	$q_{[x]}$	$q_{[x]+1}$	$q_{[x]+2}$	q_{x+3}	$x+3$
65	0.08	0.10	0.12	0.14	68
66	0.09	0.11	0.13	0.15	69
67	0.10	0.12	0.14	0.16	70
68	0.11	0.13	0.15	0.17	71
69	0.12	0.14	0.16	0.18	72

(ii) $i = 0.04$

(iii) The death benefit is payable at the end of the year of death.

Calculate the actuarial present value of this insurance.

- (A) 260
- (B) 290
- (C) 350
- (D) 370
- (E) 410

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10. For a fully continuous whole life insurance policy of 100,000 on (35), you are given:

(i) The density function of the future lifetime of a newborn:

$$f(t) = \begin{cases} 0.01e^{-0.01t}, & 0 \leq t < 70 \\ g(t), & t \geq 70 \end{cases}$$

(ii) $\delta = 0.05$

(iii) $\bar{A}_{70} = 0.51791$

Calculate the annual net premium rate for this policy.

(A) 1000

(B) 1110

(C) 1220

(D) 1330

(E) 1440

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11. For a whole life insurance of 100,000 on (x) , you are given:

- (i) Death benefits are payable at the moment of death.
- (ii) Deaths are uniformly distributed over each year of age.
- (iii) Premiums are payable monthly.
- (iv) $i = 0.06$
- (v) $\ddot{a}_x = 9.19$

Calculate the monthly net premium.

- (A) 450
- (B) 460
- (C) 470
- (D) 480
- (E) 490

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12. For a fully discrete whole life insurance policy on (30) with a death benefit of 150,000, you are given:

- (i) Reserves at the end of years 20 and 21 are 24,496 and 26,261, respectively.
- (ii) The gross premium is 1212.
- (iii) Expected expenses equal 60 plus $W\%$ of the gross premium payable at the beginning of each year.
- (iv) $q_{50} = 0.004736$.
- (v) The expected interest rate is 8%.
- (vi) The expected profit in the 21st policy year for a policy in force at the beginning of that year is 722.

Calculate $W\%$.

- (A) 8%
- (B) 9%
- (C) 10%
- (D) 11%
- (E) 12%

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13. For a universal life insurance policy with death benefit of 100,000 plus the account value, you are given:

(i)

Component	Policy Year	
	1	2
Percent of Premium Charge	25%	$R\%$
Cost of Insurance Rate per Month	0.002	0.003
Monthly Expense Charge	6	6
Surrender Charge	3000	1000

- (ii) The credited interest rate is $i^{(12)} = 0.06$.
- (iii) The actual cash surrender value at the end of month 11 is 10,000 and at the end of month 13 is 13,330.
- (iv) Premiums of 1000 are paid at the beginning of months 12 and 13.
- (v) The policy is in force at the end of month 13.

Calculate $R\%$.

- (A) 3%
- (B) 5%
- (C) 7%
- (D) 9%
- (E) 11%

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- 14.** A life insurance company sells a portfolio of 1000 fully discrete whole life insurance policies of 500, on lives age 45.

You are given:

- (i) There are no expenses.
- (ii) The annual gross premium is 8.80 per policy.
- (iii) At the end of the third policy year:
 - The reserve per policy is 19.90.
 - 980 policies remain in force.
- (iv) During the fourth policy year:
 - The interest earned on the assets that back these policies was $j\%$.
 - There were 7 deaths.
- (v) At the end of the fourth policy year the reserve per policy is 27.77.
- (vi) There was no gain or loss during the fourth policy year.

Calculate $j\%$.

- (A) 6.5%
- (B) 7.0%
- (C) 7.5%
- (D) 8.0%
- (E) 8.5%

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15. For a fully discrete whole life insurance of 1000 on (35), you are given:

- (i) First year expenses are 30% of the gross premium plus 300.
- (ii) Renewal expenses are 4% of the gross premium plus 30.
- (iii) All expenses are incurred at the beginning of the policy year.
- (iv) Gross premiums are calculated using the equivalence principle.
- (v) The gross premium reserve at the end of the first policy year is R .
- (vi) Using the Full Preliminary Term Method, the modified reserve at the end of the first policy year is S .
- (vii) Mortality follows the Illustrative Life Table.
- (viii) $i = 0.06$.

Calculate $R - S$.

- (A) -280
- (B) -140
- (C) 0
- (D) 140
- (E) 280

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16. A special fully discrete 10-payment 10-year deferred whole life annuity-due on (55) of 1000 per year provides for a return of premiums without interest in the event of death within the first 10 years. You are given:

(i) Mortality follows the Illustrative Life Table.

(ii) $i = 0.06$

(iii) $(IA)_{55:\overline{10}|}^1 = 0.51209$

Calculate ${}_9V$, the net premium reserve at the end of year 9.

(A) 8000

(B) 8200

(C) 8400

(D) 8600

(E) 8800

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17. For a universal life insurance policy with a death benefit of 100,000 plus the account value, you are given:

(i)

Policy Year	Annual Premium	Percent of Premium Charge	Annual Expense Charge	Annual Cost of Insurance Rate Per 1000
1	G	30%	100	10
2	G	10%	50	20

(ii) $i^c = 0.04$

Calculate G such that the account value at the end of year 2 will be 10,000.

- (A) 7800
- (B) 7820
- (C) 7840
- (D) 7860
- (E) 7880

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18. For a defined benefit pension plan, you are given:

- (i) The retirement benefit is 25 per month for each year of service.
- (ii) The normal retirement age is 65.
- (iii) Early retirement is allowed beginning at age 63 with the retirement benefit reduced by 7.2% per year prior to age 65.
- (iv) Bob is an employee age 50 who was hired at age 30.
- (v) $i = 0.07$
- (vi) Benefits are valued assuming that retirement (r) is the only decrement from employment.
- (vii) Employees retire only on their birthdays.
- (viii)

Age x	$q_x^{(r)}$	$\ddot{a}_x^{(12)}$
63	0.4	12.0
64	0.2	11.5
65	1.0	11.0

Calculate the actuarial present value of Bob's retirement benefit.

- (A) 16,900
- (B) 23,180
- (C) 29,470
- (D) 35,750
- (E) 42,040

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- 19.** A defined benefit pension plan provides its members, upon retirement at age 65, retirement benefits from one of two options:
- (i) An annual pension equal to 2% of career average salary for each year of service.
 - (ii) An annual pension equal to $R\%$ of final average salary for each year of service. Final average salary is defined to be the average salary earned in the 5 years immediately preceding retirement.

Colton joins the pension plan at age 35 and his salary increases by 2.5% each year on his birthday.

Under either option, Colton would receive the same annual pension benefit.

Calculate $R\%$.

- (A) 1.1%
- (B) 1.3%
- (C) 1.5%
- (D) 1.7%
- (E) 1.9%

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20. A new employee, age 35, has a choice between two pension plans:

Plan I: The employer makes contributions of 15% of salary each year. Contributions are made at the beginning of each year and earn 3% per year. Accumulated contributions at retirement are used to purchase a monthly life annuity due.

Plan II: The annual pension benefit is 1.5% of the two-year final average salary for each year of service. These benefits are payable on a monthly basis, at the beginning of each month.

You are given:

- (i) $\ddot{a}_{65}^{(12)} = 9.44$
- (ii) Annual salary increases by 3% each year on the participant's birthday.
- (iii) Retirement occurs at age 65.

Calculate the ratio of the monthly payments under Plan I to those under Plan II.

- (A) 0.79
- (B) 0.99
- (C) 1.05
- (D) 1.11
- (E) 1.15

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SECTION B – Written-Answer

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1. (7 points) You are given the following survival function for (40):

$$S_{40}(t) = \begin{cases} 1 - (0.02t)^2, & 0 \leq t < 25 \\ 0.75e^{b(t-25)}, & t \geq 25 \end{cases}$$

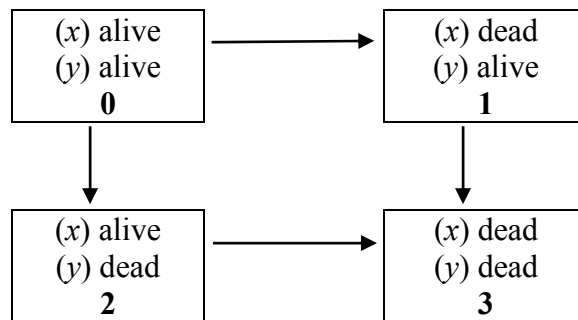
- (a) (3 points)

- (i) List three necessary and sufficient conditions that a survival function for a lifetime distribution must satisfy in order to be valid.
- (ii) State with reasons whether each of the following values for b will result in $S_{40}(t)$ being valid.
- $b = -0.2$
 - $b = 0.0$
 - $b = 0.2$

- (b) (4 points) You are given that $b = -0.1$.

- (i) Calculate μ_{60} .
- (ii) Calculate μ_{70} .
- (iii) Calculate $\overset{\circ}{e}_{40:\overline{35}|}$.

2. (11 points) The mortality of a couple, (x) and (y), is modeled using the Markov multiple-state model described in the following diagram:



You are given:

- $\mu_{x+t:y+t}^{01} = A + Bc^{x+t}$ $\mu_{x+t:y+t}^{02} = A + Bc^{y+t}$ $\mu_{y+t}^{13} = D + Bc^{y+t}$ $\mu_{x+t}^{23} = E + Bc^{x+t}$
- $A = 0.0001$ $B = 10^{-5}$ $c = 1.12$ $D = 0.00015$ $E = 0.0002$

- (a) (1 point) State with reasons whether (x) and (y) have independent future lifetimes under this model.
- (b) (3 points)
- Write down the Kolmogorov forward differential equation for ${}_t p_{xy}^{00}$, and give the associated boundary condition.
 - Starting from the equation in (i), prove that

$${}_t p_{xy}^{00} = \exp \left\{ - \int_0^t (\mu_{x+r:y+r}^{01} + \mu_{x+r:y+r}^{02}) dr \right\}.$$

A couple, who are ages $x = 50$ and $y = 55$, purchases a special single premium, deferred joint and last survivor annuity. The annuity will pay 50,000 per year while both are alive, and will pay 30,000 per year while only one is alive. Payments are continuous, and begin 10 years after the annuity purchase.

If neither life survives the deferred period, then a sum of 3 times the single premium is paid immediately on the second death.

There are no other benefits.

2. Continued

You are given:

$$(i) \quad {}_{10}P_{50:55}^{00} = 0.86041 \quad {}_{10}P_{50:55}^{01} = 0.04835 \quad {}_{10}P_{50:55}^{02} = 0.08628$$

$$(ii) \quad \bar{A}_{50:55:\overline{10}|}^{03} = 0.003421$$

$$(iii) \quad \bar{a}_{60:65}^{00} = 8.8219 \quad \bar{a}_{60:65}^{01} = 1.3768 \quad \bar{a}_{60:65}^{02} = 3.0175$$

$$(iv) \quad \bar{a}_{65}^{11} = 10.1948 \quad \bar{a}_{60}^{22} = 11.8302$$

$$(v) \quad i = 0.05$$

(c) (3 points)

(i) Show that the expected present value of the future benefits at time 10, if both lives survive to time 10, is 573,000 to the nearest 1000. You should calculate the value to the nearest 100.

(ii) Calculate the single net premium for the policy.

(d) (4 points)

(i) Determine ${}_{10}V^{(0)}$, ${}_{10}V^{(1)}$ and ${}_{10}V^{(2)}$ for the policy.

(ii) Write down Thiele's differential equation for the reserve at t , $t \geq 10$, assuming both lives survive to t .

(iii) Using Euler's forward method, with a step size of $h = 0.5$, calculate the reserve required at 10.5 assuming both lives are alive at that time.

3. (10 points) An insurer issues fully discrete whole life policies with a death benefit of 100,000 to independent lives age 50. You are given the following premium assumptions:

- (i) Mortality follows the Illustrative Life Table.
- (ii) $i = 6\%$
- (iii) Pre-contract expenses are 1000.
- (iv) Maintenance expenses of 50 are incurred at the start of each year including the first.
- (v) The gross premium is computed using the equivalence principle.

(a) (2 points) Show that the gross premium is 2000 to the nearest 10. You should calculate the value to the nearest 1.

The insurer performs a profit test of these policies using the same expenses and mortality assumptions as in the premium basis. You are given the following additional information for the profit test:

- (i) Reserves for the policy at $t = 0, 1, 2, 3$ are:

$${}_0V = 0 \quad {}_1V = 750 \quad {}_2V = 1500 \quad {}_3V = 2250$$

- (ii) Cash Values for policyholders surrendering at the end of the t -th year, $t = 1, 2, 3$ are:

$$CV_1 = 0 \quad CV_2 = 600 \quad CV_3 = 1400$$

- (iii) At the end of each year, 5% of policyholders still in force surrender their policies.
- (iv) The insurer earns interest of 5% in the first year and 7% in each subsequent year.
- (v) The hurdle rate is 10% per year.
- (vi) Pr_t denotes the expected profit at time t per policy in force at $t - 1$ for $t = 1, 2, 3, \dots$
- (vii) Pr_0 denotes the pre-contract, time 0 profit.

3. Continued

- (b) (4 points) Calculate Pr_t for $t = 0, 1, 2, 3$.

These policies are participating. Dividends will be distributed at the end of year t only if $NPV(t-1) > 0$ and $Pr_t > 0$. In this case, 85% of the total profit emerging in policy year t is distributed as a cash dividend to policies that are in force at the end of policy year t . Policies that have been surrendered, or for which a death benefit has been paid, do not qualify for the dividend distribution.

- (c) (3 points)

- (i) Show that the first dividend payment is distributed at the end of the third year.
- (ii) Calculate the projected dividend payment at $t = 3$ to a policy in force at that time.

The insurer offers policyholders an option to convert the dividend into an addition to the death benefit with no underwriting.

- (d) (1 point) Describe, from the insurer's perspective, one advantage and one disadvantage of offering policyholders this option.

4. (10 points) For a special fully discrete two-year term insurance on Elizabeth, age 60, you are given:

- (i) The death benefit is 1000 plus the return of gross premiums paid with interest at 6%.
- (ii) The following double decrement table, where decrement (d) is death and decrement (w) is withdrawal:

x	$q_x^{(d)}$	$q_x^{(w)}$
60	0.06	0.04
61	0.12	0.00

- (iii) There are no withdrawal benefits.
- (iv) $i = 0.06$
- (v) G denotes the annual gross premium.
- (vi) L_0 denotes the insurer's loss at issue random variable for Elizabeth's policy.

(a) (4 points) Calculate the values in the following table. Express L_0 in terms of G where appropriate:

Event	Value of L_0 , Given that the Event Occurred	Probability of Event
Death in year 1		
Withdrawal in year 1		
Death in year 2		
Neither death or withdrawal		

4. Continued

(b) (4 points)

(i) Show that $E[L_0] = a - bG$, where a is 150 to the nearest 10 and b is 1.58 to the nearest 0.01. You should calculate a to the nearest 1 and b to the nearest 0.001.

(ii) Show that $Var(L_0) = cG^2 + dG + e$, where c is 0.5 to the nearest 0.1, d is 480 to the nearest 10, and e is 116,000 to the nearest 1000. You should calculate c to the nearest 0.01, d to the nearest 1, and e to the nearest 100.

(c) (2 points) The insurer expects to issue 200 such policies to insureds with independent future lifetimes. The premium for each policy is $G = 130$.

Let L_{agg} denote the insurer's aggregate future loss random variable at issue for these 200 policies.

Calculate $\Pr(L_{agg} > 0)$ using the normal approximation without continuity correction.

5. (10 points) Dana buys a Type B universal life contract of 100,000. You are given:

(i)

Policy Year k	Annual Premium	Annual Cost of Insurance Rate per 1000 of Insurance	Percent of Premium Charge	Annual Expense Charge	Surrender Charge
1	1000	--	60%	--	--
2	P_2	2	10%	10	200
3	P_3	3	10%	10	100
$k \geq 4$	P_k	k	5%	10	0

(ii) The credited interest rate is $i = 0.06$.

(iii) Dana's account value at the end of year 1 is 165.

(iv) Except as indicated, there are no deaths or surrenders.

(a) (2 points) Show that if P_2 were 1000, Dana's account value at the end of year 2 would be 920 to the nearest 10. You should calculate the account value to the nearest 1.

(b) (2 points) Dana's account value at the end of year 3 can be expressed as $aP_2 + bP_3 + c$. Calculate a , b , and c .

5. Continued

- (c) (4 points) In year 2, Dana pays a premium of 1000 with probability 0.6, or 200 with probability 0.4.

If he paid 1000 in year 2, then in year 3 he will pay either 1000 with probability 0.6, or 200 with probability 0.4.

If he paid 200 in year 2, then in year 3 he will pay either 1000 with probability 0.2, or 200 with probability 0.8

- (i) Calculate the expected death benefit payable at the end of year 3, if Dana dies then.
- (ii) Calculate the expected surrender benefit payable at the end of year 3, if Dana surrenders the contract then.
- (d) (2 points) Dana's identical twin, Mark, buys a contract identical to Dana's. If Mark pays 1000 every year, Mark's account value at the end of year 10 will be 5114.

Mark will pay premiums of 1000 in 9 of the first 10 years. Mark will pay no premium in one year, with the year of no premium equally likely to be year 3 or year 10.

Calculate Mark's expected surrender value at the end of year 10.

6. (8 points) ABC Life Insurance Company sells 5-year pure endowment policies of 1 to 100 independent lives age 85. You are given:
- (i) Mortality follows the Illustrative Life Table.
 - (ii) There is a 60% chance that the force of interest over the next five years will be $\delta_t = 0.03\sqrt{t}$, for $0 \leq t \leq 5$, and there is a 40% chance that the force of interest over the next five years will be $\delta_t = 0.02$, for $0 \leq t \leq 5$.
 - (iii) The present value of benefits random variable for the portfolio of policies is denoted by Y .
- (a) (3 points) Show that the mean of Y is 40 to the nearest 10. You should calculate the value to the nearest 1.
- (b) (3 points) Calculate the probability that Y is less than 30 using the normal approximation without continuity correction.
- (c) (2 points)
- (i) Define the term diversifiable risk.
 - (ii) A colleague of yours claims that because the lives are independent, the risk for this portfolio of policies as measured by Y is diversifiable. State with reasons whether your colleague is correct.

****END OF EXAMINATION****

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