

1. The probability of being fully functional after two years for a single television is:

$$(0.82 \quad 0.10 \quad 0.08) \begin{pmatrix} 0.82 \\ 0.60 \\ 0.00 \end{pmatrix} = 0.82 * 0.82 + 0.10 * 0.60 + 0.08 * 0.00 = 0.7324$$

The number of the five televisions being fully functional has a binomial distribution with parameters of $n = 5$ and $p = 0.7324$. The probability that there will be exactly two televisions that are fully functioning is therefore:

$$\binom{5}{2} 0.7324^2 (1 - 0.7324)^3 = 10 * 0.53641 * 0.019163 = 0.10279$$

- 2.

$$\begin{aligned} \mu_x &= -\frac{d}{d_x} \ln S_0(x) = -\frac{1}{3} \frac{d}{d_x} \ln \left(1 - \frac{x}{60} \right) \\ &= \frac{1}{180} \left(1 - \frac{x}{60} \right)^{-1} = \frac{1}{3(60-x)} \end{aligned}$$

$$\text{Therefore, } \mu_{35} = \frac{1}{3(25)} = \frac{1}{75} = 0.0133.$$

3. Out of 400 lives initially, we expect $400 * {}_{40}P_{25} = 400 * \frac{l_{65}}{l_{40}} = 400 * \frac{7,533,964}{9,565,017} = 315.0633$

Survivors with standard deviation of $\sqrt{400 * {}_{40}P_{25} (1 - {}_{40}P_{25})} = 8.1793$

To ensure 86% funding, using the normal distribution table, we plan for $315.0633 + 1.08(8.1793) = 323.8969$.

The initial fund must therefore be $F = 324 * 15200 * \left(\frac{1}{1.06} \right)^{40} = 478,799.80$.

$$\begin{aligned}
4. \quad \text{Probability} &= \int_0^5 {}_t p^{\overline{00}} \mu^{01} {}_{5-t} p^{\overline{11}} dt \\
&= \int_0^5 e^{-0.06t} 0.05 e^{-0.08(5-t)} dt \\
&= e^{-0.40} (0.05) \int_0^5 e^{+0.02t} dt \\
&= e^{-0.40} \left(\frac{5}{2} \right) (e^{0.10} - 1) = 0.1762
\end{aligned}$$

5.

$$\ddot{a}_{[x:\overline{n}]} = 1 + v p_{[x]} \ddot{a}_{x+1:\overline{n-1}} = 1 + (1+k) v p_x \ddot{a}_{x+1:\overline{n-1}} = 1 + (1+k) (\ddot{a}_{x:\overline{n}} - 1)$$

Therefore, we have

$$k = \frac{\ddot{a}_{x:[n]} - 1}{\ddot{a}_{x:\overline{n}} - 1} - 1 = \frac{21.167}{20.854} - 1 = 0.015$$

6.

$$\begin{aligned}
100,000 A \frac{1}{50:60:\overline{10}} &= 100,000 \left[A_{50:\overline{10}}^1 + A_{60:\overline{10}}^1 - A_{50:60:\overline{10}}^1 \right] \\
&= 100,000 [0.060495 + 0.136785 - 0.186751] = 1,052.89
\end{aligned}$$

where

$$\begin{aligned}
A_{50:\overline{10}}^1 &= A_{50} - {}_{10}E_{50} A_{60} = 0.24905 - (0.51081)(0.36913) = 0.060495 \\
A_{60:\overline{10}}^1 &= A_{60} - {}_{10}E_{60} A_{70} = 0.36913 - (0.45120)(0.51495) = 0.136785 \\
A_{50:60:\overline{10}}^1 &= A_{50:60} - (1.06)^{10} {}_{10}E_{50} {}_{10}E_{60} A_{60:70} \\
&= 0.42296 - 1.79085(0.51081)(0.45120)(0.57228) = 0.186751
\end{aligned}$$

7. Let G be the annual gross premium. By the equivalence principle, we have
 $G\ddot{a}_{35} = 100,000A_{35} + 0.15G + 0.04G\ddot{a}_{35}$

so that

$$G = \frac{100,000A_{35}}{0.96\ddot{a}_{35} - 0.15} = \frac{100,000(0.12872)}{0.96(15.3926) - 0.15} = 880.023$$

8. By the equivalence principle,

$$4500\bar{a}_{x:\overline{20}|} = 100,000\bar{A}_{x:\overline{20}|}^1 + R\bar{a}_{x:\overline{20}|}$$

where

$$\bar{A}_{x:\overline{20}|}^1 = \frac{\mu}{\mu + \delta} (1 - e^{-20(\mu + \delta)}) = \frac{0.04}{0.12} (1 - e^{-20(0.12)}) = 0.3031$$

$$\bar{a}_{x:\overline{20}|} = \frac{1 - e^{-20(\mu + \delta)}}{\mu + \delta} = \frac{1 - e^{-20(0.12)}}{0.12} = 7.5774$$

Solving for R , we have

$$R = 4500 - 100,000 \left(\frac{0.3031}{7.5774} \right) = 500$$

9. By the equivalence principle, we have

$$G\ddot{a}_{35:\overline{10}|} = 50,000A_{35} + 100a_{35} + 100A_{35}$$

so that

$$\begin{aligned} G &= \frac{50,100A_{35} + 100(\ddot{a}_{35} - 1)}{\ddot{a}_{35} - {}_{10}E_{35}\ddot{a}_{45}} \\ &= \frac{50,100(0.12872) + 100(14.3926)}{15.3926 - 0.54318(14.1121)} \\ &= 1020.828 \end{aligned}$$

10. Let P be the annual net premium

$$P = \frac{1000\bar{A}_{x:\overline{n}|}}{\ddot{a}_{x:\overline{n}|}} = \frac{1000(0.192)}{\ddot{a}_{x:\overline{n}|}}$$

where

$$\ddot{a}_{x:\overline{n}|} = \frac{1 - A_{x:\overline{n}|}}{d} = \frac{(1.05)}{(0.05)}(1 - A_{x:\overline{n}|}^1 - A_{x:\overline{n}|}^{\overline{1}|})$$

$$A_{x:\overline{n}|} = \frac{i}{\delta}(A_{x:\overline{n}|}^1) + {}_nE_x$$

$$\Rightarrow 0.192 = \frac{0.05}{0.0488}(A_{x:\overline{n}|}^1) + 0.172$$

$$\Rightarrow A_{x:\overline{n}|}^1 = 0.01952$$

$$\Rightarrow \ddot{a}_{x:\overline{n}|} = \frac{1.05}{0.05}(1 - 0.01952 - 0.172) = 16.97808$$

Therefore, we have

$$P = \frac{1000(0.192)}{16.97808} = 11.31$$

11. Premium at issue for (20): $65.28/16.5133 = 3.9531$

Premium at issue for (50): $249.05/13.2668 = 18.7724$

Lives in force after ten years:

$$\text{Issued at age 20: } 10,000 {}_{10}p_{20} = 10,000 \times \frac{9,501,381}{9,617,802} = 10,000 \times 0.9878953 = 9878.953$$

$$\text{Issued at age 50: } 10,000 {}_{10}p_{50} = 10,000 \times \frac{8,188,074}{8,950,901} = 10,000 \times 0.9147765 = 9147.765$$

The total number of lives after ten years is therefore: $9878.953 + 9147.765 = 19,026.718$

The average premium after ten years is therefore:

$$\frac{(3.9531 \times 9878.953) + (18.7724 \times 9147.765)}{19,026.718} = 11.078$$

12.

$$V[L_0 \#1] = \left(B_1 + \frac{P_1}{d} \right)^2 \underbrace{({}^2A_x - A_x^2)}_w = 20.55$$

$$= \left(8 + \frac{1.25(1.06)}{(0.06)} \right)^2 \times w = 20.55$$

$$V[L_0 \#2] = \left(12 + \frac{1.875}{0.06}(1.06) \right)^2 \times w$$

$$\frac{V[L\#2]}{V[L\#1]} = \frac{\left[12 + \frac{1.875}{0.06}(1.06) \right]^2}{\left[8 + \frac{1.25}{0.06}(1.06) \right]^2} = 2.25$$

$$\Rightarrow V[L\#2] = 2.25 \times 20.55 = 46.24$$

Or:

$$W = \left(\frac{12}{8} \right)^2 \times V[L_0 \#1] = (1.5)^2 \times (20.55) = 46.24 \quad (\text{because both premium and benefit are scaled by 1.5})$$

13. Calculating the reserve, ${}_{15}V = A_{50:\overline{15}|} - \frac{A_{35:\overline{30}|}}{\ddot{a}_{35:\overline{30}|}} \ddot{a}_{50:\overline{15}|}$

$$\text{Where } \ddot{a}_{35:\overline{30}|} = \frac{1 - A_{35:\overline{30}|}}{d} = \frac{1 - 0.255}{\frac{0.05}{1.05}} = 15.645$$

$$\text{And } \ddot{a}_{50:\overline{15}|} = \frac{1 - A_{50:\overline{15}|}}{d} = \frac{1 - 0.506}{\frac{0.05}{1.05}} = 10.374$$

$$\text{So that } {}_{15}V = 0.506 - \frac{0.255}{15.645} 10.374 = 0.3369128$$

SC = surrender charge

$$\begin{aligned} {}_{15}V - SC &= 0.40 A_{50:\overline{15}|} \Rightarrow SC = {}_{15}V - 0.40 A_{50:\overline{15}|} \\ &= 0.3369128 - 0.40(0.506) \\ &= 0.1345128 \end{aligned}$$

For insurance of 2000, SC = 269.0256

14.

$$AV_0 = 0$$

$$P_1 = 4,450$$

$$EC_1 = 56 + 2\% * 4,450 = 145.00$$

$$\text{COI rate} = q_{36^*} = 1.2 * 0.00214 = 0.002568$$

$$\text{COI}_1 = 200,000 * 0.002568 * (1/1.06) = 484.53$$

$$\text{Credited Interest: } 6\% * (\$4,450 - 142 - 484.53) = 229.41$$

$$AV_1 = 4,450 - 145 - 484.53 + 229.41 = 4,049.88$$

15. We have

$${}_v q_x + \beta (\ddot{a}_{25:\overline{20}|} - 1) + P {}_{20}E_{25} \ddot{a}_{45:\overline{20}|} = P \ddot{a}_{25:\overline{40}|}$$
$$\Rightarrow \beta = \frac{P (\ddot{a}_{25:\overline{20}|}) + P {}_{20}E_{25} \ddot{a}_{45:\overline{20}|} - {}_v q_x}{\ddot{a}_{25:\overline{20}|} - 1} = \frac{P \ddot{a}_{25:\overline{20}|} - {}_v q_x}{\ddot{a}_{25:\overline{20}|} - 1}$$

$$\text{Where } P = \frac{A_{25:\overline{40}|}}{\ddot{a}_{25:\overline{40}|}} = \frac{1}{\ddot{a}_{25:\overline{40}|}} - d = 0.02161656$$

$$\Rightarrow \beta = \frac{0.02161656(11.087) - \frac{1}{1.04}(0.005)}{11.087 - 1} = 0.02328295$$

For insurance of 10,000, $\beta = 233$.

16.

$$q_{50} = 0.00592, \quad q_{51} = 0.00642$$

$$\Rightarrow AV_1 = 1369.895$$

$$AV_2 = \left(AV_1 + 5000(1 - 0.035) - 75 - (500,000 - AV_2) \left(\frac{1.20 q_{51}}{1.03} \right) \right) (1.045)$$

$$\Rightarrow AV_2 = 2506.787$$

17. Let P be the annual net premium at $x+1$.

$$P\ddot{a}_{x+1} = 1000 \sum_{k=0}^{\infty} (1.03)^{k+1} v^{k+1} {}_k|q_{x+1} = 1000A_{x+1}^*$$

We are given

$$110\ddot{a}_{x+1} = 1000 \sum_{k=0}^{\infty} (1.03)^{k+1} v^{k+1} {}_k|q_{x+1} = 1000A_x^*$$

Which implies that

$$110(1 + {}_v p_x \ddot{a}_{x+1}) = 1000(1.03 {}_v q_x + 1.03 {}_v p_x A_{x+1}^*)$$

Solving for A_{x+1}^* , we get

$$A_{x+1}^* = \frac{\frac{110}{1000} [1 + v(0.95)(7)] - 1.03v(0.05)}{1.03v(0.95)} = 0.8141032$$

Thus, we have

$$P = \frac{1000(0.8141032)}{7} = 116.3005$$

18. Under PUC:

${}_{30}V =$ accrual rate \times years of past service \times survival to retirement \times discount to retirement \times retirement benefit

$${}_{35}V = \frac{\text{years of service} + 1}{\text{years of service}} {}_{35}V$$

$${}_{35}V + C = {}_{36}V = \frac{26}{35} {}_{35}V$$

$$C = \frac{26}{35} {}_{35}V - {}_{35}V \Rightarrow C = \frac{{}_{35}V}{35}$$

- 19.** By age 65, member would have served total of 35 years in which case, benefit would be $35 \times 0.02 = 70\%$. Thus set it at 60%.

$$\begin{aligned} \text{EPV}(\text{benefits}) &= 0.60 \times 50,000 \times (1.03)^{19} \times \frac{1}{1.05^{20}} \frac{l_{65}^{(\tau)}}{l_{45}^{(\tau)}} \ddot{a}_{65}^{(12)} \\ &= 0.60 \times 50,000 \times \left(\frac{1}{1.05}\right)^{20} \left(\frac{3}{5}\right) (7.8) (1.03)^{19} \\ &= 92,787.29 \end{aligned}$$

- 20.** Replacement ratios

$$\text{Plan 1: } R = \frac{1250 * 25}{S_0 (1.04)^{24}}$$

$$\text{Plan 2: } R = \frac{S_0 * 0.02 * 25 * \frac{1.04^{25} - 1}{0.04} * \frac{1}{25}}{S_0 (1.04)^{24}}$$

The two are equal, so that

$$S_0 = \frac{1250 \times 25}{0.02 \left(\frac{1.04^{25} - 1}{0.04} \right)} = 37,518.69$$