EE310

Solved Problems on MOSFETs

Sedra/Smith 5th/6th ed.

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4.3 With the knowledge that $\mu_p \simeq 0.4\mu_n$, what must be the relative width of n-channel and p-channel devices if they are to have equal drain currents when operated in the saturation mode with overdrive voltages of the same magnitude?

$$I_{D-n} = I_{D-p}$$

$$\frac{k'_n}{\cancel{2}} \times \frac{W_n}{\cancel{L}} \times \cancel{(V_{OV})^2} = \frac{k'_p}{\cancel{2}} \times \frac{W_p}{\cancel{L}} \times \cancel{(V_{OV})^2}$$

$k'_n \times W_n = k'_p \times W_p$, but $k'_n = \mu_n \times C_{ox}$ and $k'_p = \mu_p \times C_{ox}$

$$\Rightarrow \mu_n \times C_{ox} \times W_n = \mu_p \times C_{ox} \times W_p,$$

or $\mu_n \times W_n = \mu_p \times W_p \Rightarrow \dfrac{W_p}{W_n} = \dfrac{\mu_n}{\mu_p} = \dfrac{\cancel{\mu_n}}{0.4\cancel{\mu_n}} = 2.5$

4.4 An n-channel device has $k'_n = 50\ \mu\text{A/V}^2$, $V_t = 0.8$ V, and $W/L = 20$. The device is to operate as a switch for small v_{DS}, utilizing a control voltage v_{GS} in the range 0 V to 5 V. Find the switch closure resistance, r_{DS}, and closure voltage, V_{DS}, obtained when $v_{GS} = 5$ V and $i_D = 1$ mA. Recalling that $\mu_p \simeq 0.4\mu_n$, what must W/L be for a p-channel device that provides the same performance as the n-channel device in this application?

$$r_{DS} = \frac{1}{k'_n \times \left(W_n / L_n\right) \times V_{OV}} = \frac{1}{k'_n \times \left(W_n / L_n\right) \times (V_{GS} - V_t)} = \frac{1}{50 \times 10^{-6} \times (20) \times (5 - 0.8)} = 238\ \Omega$$

$$v_{DS} = i_D \times r_{DS} = 1 \times 10^{-3} \times 238 = 238\ \text{mV}$$

$$\frac{W_p}{L_p} = 2.5 \frac{W_n}{L_n} = 50$$

4.5 An n-channel MOS device in a technology for which oxide thickness is 20 nm, minimum gate length is 1 μm, $k_n' = 100\ \mu A/V^2$, and $V_t = 0.8$ V operates in the triode region, with small v_{DS} and with the gate–source voltage in the range 0 V to +5 V. What device width is needed to ensure that the minimum available resistance is 1 kΩ? W_{max}

$$r_{DS} = \frac{1}{k_n' \times \left(W_n / L_n\right) \times V_{OV}} = \frac{1}{k_n' \times \left(W_n / L_n\right) \times (V_{GS} - V_t)}$$

for $V_{GS} > V_t$ and very close to V_t, r_{DS} will be very large $\Rightarrow$ no worries!

for $V_{GS} = 5$, there is a risk that r_{DS} will fall below 1 kΩ.

$$\frac{1}{k_n' \times \left(W_n / L_n\right) \times (V_{GS} - V_t)} \geq 1\ \text{k}\Omega \Rightarrow \frac{1}{100 \times 10^{-6} \times \left(W_n / 1\times 10^{-6}\right) \times (5 - 0.8)} \geq 1000$$

$$420000 \times W_n \leq 1 \Rightarrow W_n \leq 2.38\ \mu\text{m}$$

4.6 Consider a CMOS process for which $L_{min} = 0.8\ \mu$m, $t_{ox} =$ 15 nm, $\mu_n = 550\ \text{cm}^2/\text{V}\cdot\text{s}$, and $V_t = 0.7$ V.

(a) Find C_{ox} and k_n'.
(b) For an NMOS transistor with $W/L = 16\ \mu\text{m}/0.8\ \mu\text{m}$, calculate the values of V_{OV}, V_{GS}, and V_{DSmin} needed to operate the transistor in the saturation region with a dc current $I_D = 100\ \mu$A.
(c) For the device in (b), find the value of V_{OV} and V_{GS} required to cause the device to operate as a 1000-Ω resistor for very small v_{DS}.

a)

$$C_{ox}=\frac{\varepsilon_{r(ox)}\times\varepsilon_o}{t_{ox}}=\frac{3.9\times 8.854\times 10^{-12}}{15\times 10^{-9}}=2.3\times 10^{-3}\ \text{F/m}^2=2.3\times 10^{-7}\ \text{F/cm}^2$$

$$k_n'=\mu_n\times C_{ox}=550\times 2.3\times 10^{-7}=1.265\times 10^{-4}\ \text{A/V}^2=126.5\ \mu\text{A/V}^2$$

b)

$$i_D=\frac{k_n'}{2}\times\left(\frac{W}{L}\right)\times(v_{GS}-V_t)^2,\ i_D=\frac{k_n'}{2}\times\left(\frac{W}{L}\right)\times(V_{OV})^2\Rightarrow 100=\left(\frac{126.5}{2}\right)\times\left(\frac{16}{0.8}\right)\times(V_{OV})^2$$

$$\Rightarrow (V_{OV})^2=0.07905\Rightarrow V_{OV}=\pm 0.2812\,\text{V, select } V_{OV}=+\ 0.2812\,\text{V}$$

$$V_{OV}=V_{GS}-V_t\ \text{(for NMOS)}\ \Rightarrow V_{GS}=V_{OV}+V_t=0.2812+0.7=0.9812\ \text{V}$$

$$\text{V}_{DS\,(\min)}=V_{DS\,(sat)}=V_{OV}$$

c)

$$r_{DS}=\frac{1}{k_n'\times\left(W_n/L_n\right)\times V_{OV}}\Rightarrow 1000=\frac{1}{126.5\times 10^{-6}\times\left(\frac{16}{0.8}\right)\times V_{OV}}\Rightarrow V_{OV}=0.3953\,\text{V}$$

$$V_{GS}=V_{OV}+V_t=0.3953+0.7=1.0953\,\text{V}$$

4.19 For a particular MOSFET operating in the saturation region at a constant v_{GS}, i_D is found to be 2 mA for $v_{DS}=$ 4 V and 2.2 mA for $v_{DS}=8$ V. What values of r_o, V_A, and λ correspond?

4.20 A particular MOSFET has $V_A=50$ V. For operation at 0.1 mA and 1 mA, what are the expected output resistances? In each case, for a change in v_{DS} of 1 V, what percentage change in drain current would you expect?

{4.19}

$$\boxed{\begin{array}{l} I_{D1}=2\ \text{mA @ } V_{DS1}=4\ \text{V}\\ I_{D2}=2.2\ \text{mA @ } V_{DS2}=8\ \text{V}\end{array}}\quad V_{GS}\ \text{is constant; Find } r_o,\ \lambda,\ \text{and V}_A.$$

$$r_o=\frac{\Delta V_{DS}}{\Delta I_D}=\frac{(8-4)}{(2.2-2)\times 10^{-3}}=20\ \text{k}\Omega$$

$$\frac{I_{D1}}{I_{D2}}=\frac{\frac{k_n'}{2}\times\frac{W}{L}\times(V_{GS}-V_t)^2\times(1+\lambda V_{DS1})}{\frac{k_n'}{2}\times\frac{W}{L}\times(V_{GS}-V_t)^2\times(1+\lambda V_{DS2})}\Rightarrow\frac{I_{D1}}{I_{D2}}=\frac{(1+\lambda V_{DS1})}{(1+\lambda V_{DS2})}\Rightarrow(1+\lambda V_{DS2})I_{D1}=(1+\lambda V_{DS1})I_{D2}\Rightarrow$$

$$\lambda=\frac{I_{D2}-I_{D1}}{V_{DS2}\times I_{D1}-V_{DS1}\times I_{D2}}=0.0278\ \text{V}^{-1}\quad\rightarrow\quad V_A=\frac{1}{\lambda}=36\ \text{V}$$

$\{4.20\}$

$$@I_D = 0.1\text{ mA},\ r_o = \frac{V_A}{I_D} = \frac{50}{0.1\times10^{-3}} = 500\text{ k}\Omega \qquad @I_D = 1\text{ mA},\ r_o = \frac{V_A}{I_D} = \frac{50}{1\times10^{-3}} = 50\text{ k}\Omega$$

$$r_o = \frac{\Delta V_{DS}}{\Delta I_D},\ \Rightarrow \Delta I_D = \frac{\Delta V_{DS}}{r_o} = \frac{1}{500\times10^3},$$

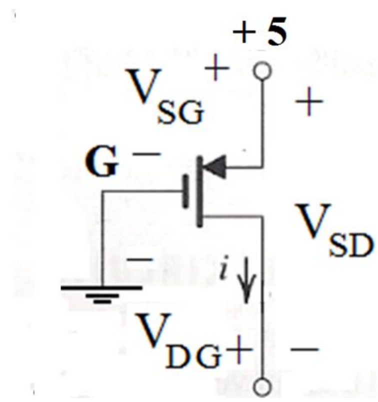
For the I_D=0.1 mA case,

$$\left(\frac{\Delta I_D}{I_D}\right)\times100 = \left[\left(\frac{1}{500\times10^3}\right)\div0.1\times10^{-3}\right]\times100 = 2\%$$

For the I_D=1 mA case,

$$\Delta I_D = \frac{\Delta V_{DS}}{r_o} = \frac{1}{50\times10^3},\ \left(\frac{\Delta I_D}{I_D}\right)\times100 = \left[\left(\frac{1}{50\times10^3}\right)\div1\times10^{-3}\right]\times100 = 2\%$$

4.26 An enhancement PMOS transistor has $k_p'(W/L) = 80\ \mu\text{A/V}^2$, $V_t = -1.5$ V, and $\lambda = -0.02\ \text{V}^{-1}$. The gate is connected to ground and the source to +5 V. Find the drain current for $v_D = +4$ V, +1.5 V, 0 V, and −5 V.



$$k_p' \times \frac{W}{L} = 80\ \mu\text{A/V}^2$$

Case 1: $V_D = 4$ V,

$$V_{SG} = 5-0 = 5\text{ V} > |V_t| \Rightarrow \text{ not cutoff!} \quad V_{DG} = 4-0 = 4\text{ V is not} \le |V_t| \Rightarrow \text{ in Triode!}$$

$$I_D = k_p' \times \frac{W}{L}\left[(V_{SG}-|V_t|)\times V_{SD} - \frac{V_{SD}^{\ 2}}{2}\right] \Rightarrow I_D = 80\times10^{-6}\left[(5-|-1.5|)(5-4) - \frac{(5-4)^2}{2}\right]$$

$$I_D = 80\times10^{-6}\left[3.5-\frac{1}{2}\right] = 240\ \mu\text{A}$$

Case 3: $V_D = 0$ V,

$$V_{SG} = 5-0 = 5\text{ V} > |V_t| \Rightarrow \text{ not cutoff!} \quad V_{DG} = 0-0 = 0\text{ V} \le |V_t| \Rightarrow \text{ in Saturation!}$$

$$I_D = \frac{k_p'}{2}\times\frac{W}{L}(V_{SG}-|V_t|)^2(1+|\lambda|V_{SD})$$

$$I_D = \frac{80\times10^{-6}}{2}(5-|-1.5|)^2\left[1+|-0.02|\times(5-0)\right] \quad \rightarrow I_D = 40\times10^{-6}\times3.5^2\times1.1 = 539\ \mu\text{A}$$

4.44 For each of the circuits shown in Fig. P4.44, find the labeled node voltages. The NMOS transistors have $V_t = 1$ V and $k_n' W/L = 2$ mA/V^2. Assume $\lambda = 0$.

Assuming that the lower MOSFET is in saturation:

$$\frac{k_n' W}{2 \times L}(-V_2 - 1)^2 = \frac{V_2 - (-5)}{1} \quad \rightarrow \quad V_2^2 + 2V_2 + 1 = V_2 + 5$$

$$\rightarrow V_2^2 + V_2 - 4 = 0 \rightarrow V_2 = \frac{-1 \pm \sqrt{1 - [4 \times 1 \times (-4)]}}{2 \times 1} = \frac{-1 \pm \sqrt{17}}{2} = 1.56 \text{ V or } \boxed{-2.56 \text{ V}}$$

We choose -2.56 V for which $V_{GS} > V_t$ (to avoid cutoff).

$$\rightarrow I_D = \frac{k_n' W}{2 \times L}(-V_2 - 1)^2 = 1 \times 10^{-3} \times (2.56 - 1)^2 = 2.433 \text{ mA.}$$

$$\text{OR } I_D = \frac{-2.56 - (-5)}{1} = 2.44 \text{ mA (difference due to rounding!)}$$

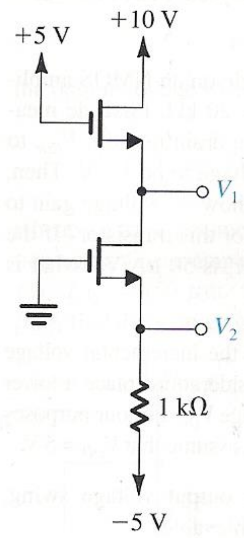
The upper MOSFET is in saturation because $V_{GD} \le V_t$ $(-5 \le 1) \rightarrow$

$$\frac{k_n' W}{2 \times L}(5 - V_1 - 1)^2 = 2.44 \rightarrow V_1^2 - 8V_1 + 13.56 = 0$$

$V_1 = \boxed{2.44 \text{ V}}$ or 5.562 V, we choose 2.44 V for which $V_{GS} > V_t$ (to avoid cutoff).

To verify that the lower MOSFET is in saturation:

$V_{GD} = V_G - V_D = 0 - V_1 = -2.44$ V which is indeed $\le V_t \rightarrow$ saturated as assumed.



In the circuit shown, transistors are characterized by $|V_t| = 2$ V, $k'W/L = 1$ mA/V^2, and $\lambda = 0$. Find V_6 and V_7.

Both MOSFET's are in saturation because $V_{DG} = 0 \le |V_t|$;

Considering the upper MOSFET:

$$I_D = \frac{k_n^{'}W}{2\times L}(10 - V_6 - 2)^2 \;\rightarrow\; 2 = \frac{1}{2}(10 - V_6 - 2)^2 \rightarrow \; V_6^2 - 16V_6 + 64 = 4$$

Solving for V_6 by any mehod:

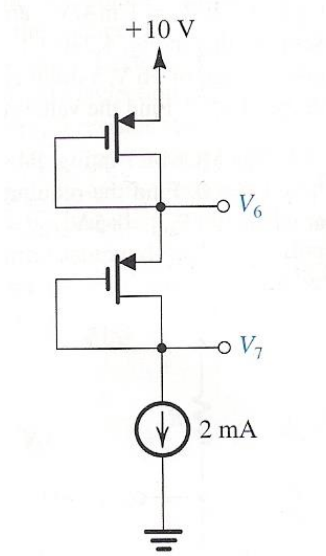
$V_6 = \boxed{6\text{ V}}$ or 10 V, we choose 6 V for which $V_{SG} > |V_t|$ (to avoid cutoff).

For the lower MOSFET:

$$I_D = \frac{k_n^{'}W}{2\times L}(6 - V_7 - 2)^2 \;\rightarrow\; 2 = \frac{1}{2}(6 - V_7 - 2)^2 \rightarrow \; V_7^2 - 8V_7 + 16 = 4$$

Solving for V_7 by any mehod:

$V_7 = \boxed{2\text{ V}}$ or 6 V, we choose 2 V for which $V_{SG} > |V_t|$ (to avoid cutoff).



4.48 In the circuit of Fig. P4.48, transistors Q_1 and Q_2 have $V_t = 1$ V, and the process transconductance parameter $k_n' = 100\ \mu\text{A/V}^2$. Assuming $\lambda = 0$, find V_1, V_2, and V_3 for each of the following cases:

(a) $(W/L)_1 = (W/L)_2 = 20$

The 200-μA current will divide equally between Q_1 and Q_2 because their gates are at the same potential ($V_{GS1} = V_{GS2}$, $\lambda = 0$) $\rightarrow$ $I_{D1} = I_{D2} = 100\ \mu$A.

Assuming both MOSFET's are in saturation:

$$I_{D1} = \frac{k_n' W}{2 \times L}(0 - V_3 - 1)^2 \rightarrow V_3 + 1 = \pm\sqrt{\frac{100 \times 10^{-6}}{\frac{100 \times 10^{-6} \times 20}{2}}} \quad \rightarrow \quad V_3 = -1 \pm \sqrt{0.1} = -1.32 \text{ V or } -0.684 \text{ V}$$

Select the first value because it implies that $V_{GS} > V_t$ and thus $I_D \neq 0$ (to avoid cutoff)

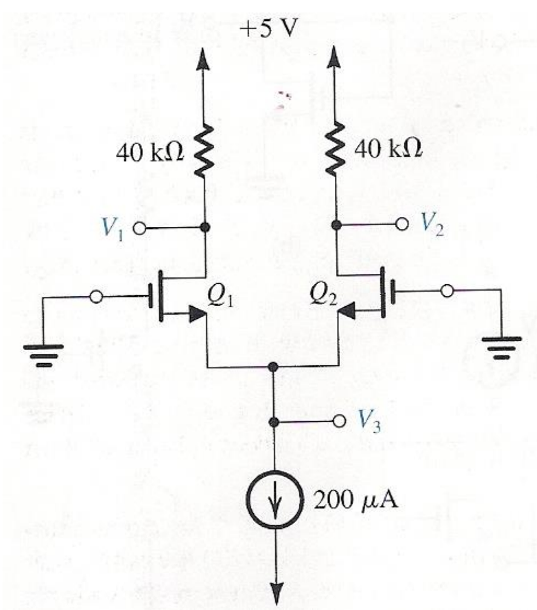
Now,

$$V_1 = V_2 = 5 - 0.1 \times 10^{-3} \times 40 \times 10^3 = 1 \text{ V.}$$

To make sure both MOSFET's are actually in saturation, we find V_{GD1} and V_{GD2}:

$$V_{GD1} = 0 - V_1 = -1 \leq V_t \quad \rightarrow \text{ saturated as assumed!}$$

$$V_{GD1} = 0 - V_2 = -1 \leq V_t \quad \rightarrow \text{ saturated as assumed!}$$

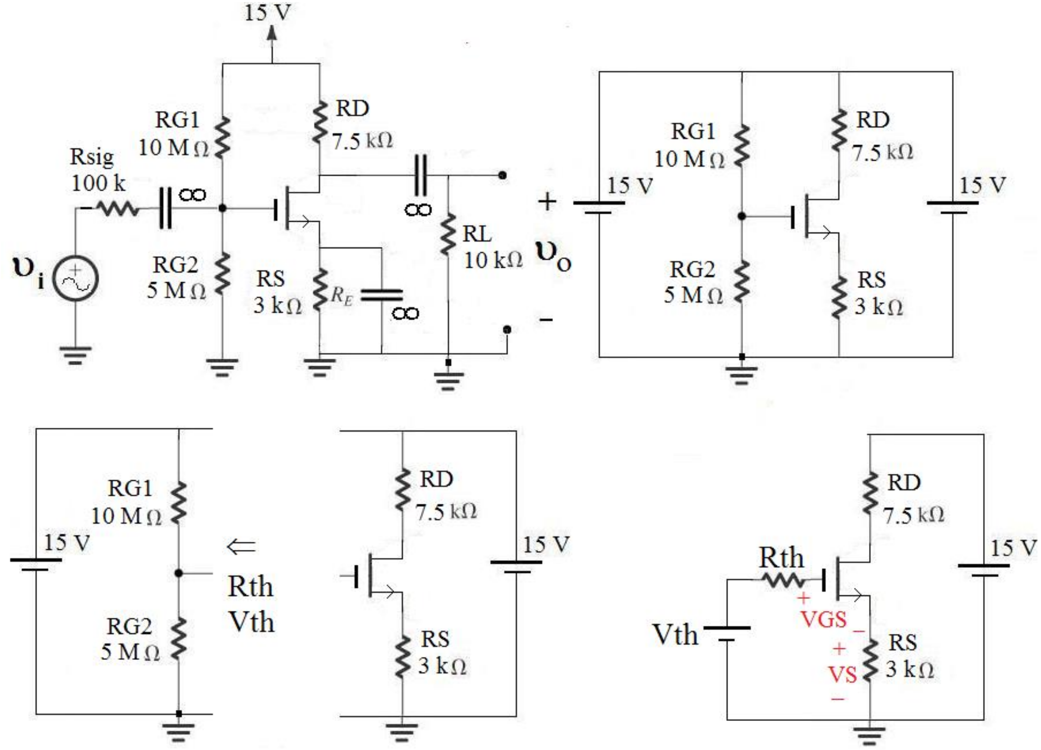


Problem 4.77, page 370, 5th ed. Sedra/Smith

a) If the transistor has $V_t = 1$ V, and $k'_n = 2$ mA/V^2, verify that the bias circuit establishes $V_{GS} = 2$ V, $I_D = 1$ mA and $V_D = 7.5$ V.

b) Find g_m, and r_o if $V_A = 100$ V.

c) Find R_{in}, $\dfrac{v_{gs}}{v_{sig}}$, $\dfrac{v_o}{v_{gs}}$, $\dfrac{v_o}{v_{sig}}$.



a)

$$R_{th} = 10\ \text{M}\Omega \,||\, 5\ \text{M}\Omega = \frac{5\times 10}{5+10} = 3.34\ \text{M}\Omega.$$

$$V_{th} = \left(\frac{5}{5+10}\right)\times 15 = 5\ \text{V}.$$

$$V_G = V_{th}\ (\text{I}_G = 0),\ V_{GS} = 2\ \text{(given)},$$

$$V_{GS} = V_G - V_S \Rightarrow V_S = V_G - V_{GS} = V_{th} - V_{GS} = 3\ \text{V}$$

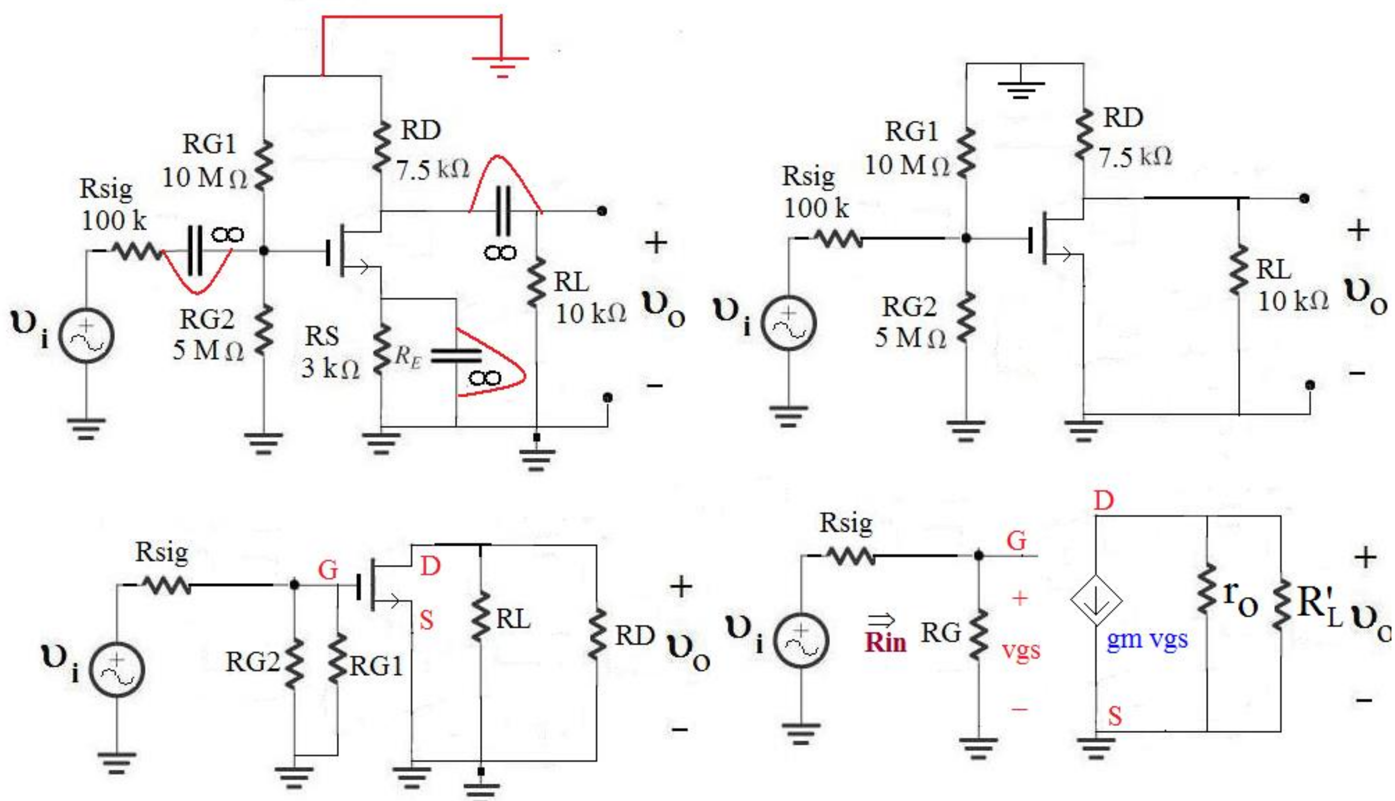
$$I_D = \frac{V_S}{R_S} = \frac{3}{3\times 10^3} = 1\ \text{mA}$$

$$V_D = V_{DD} - I_D \times R_D = 15 - 1\times 7.5 = 7.5\ \text{V}$$

b)

$$g_m = \frac{2I_D}{V_{GS} - V_t} = \frac{2\times 1\times 10^{-3}}{2-1} = 2\times 10^{-3}\ \text{S}$$

$$r_o = \frac{V_A}{I_D} = \frac{100}{1\times 10^{-3}} = 100\ \text{k}\Omega$$



c)

Let's call $(R_{G1} \| R_{G1}) \equiv R_G = 3.34$ MΩ, and $(\mathrm{R}_D \| R_L) \equiv R'_L = 4.29$ kΩ

$R_{in} = R_{G1} \| R_{G1} = R_G = 3.34$ MΩ

$$v_{gs} = \left(\frac{R_G}{R_G + R_{sig}}\right) \times v_{sig} \text{ (VDR)} \Rightarrow \frac{v_{gs}}{v_{sig}} = \frac{R_G}{R_G + R_{sig}} = \frac{3.34}{3.34 + 0.1} = 0.97 \text{ V/V}$$

$$v_o = -g_m \times v_{gs} \times (R'_L \| r_o) \Rightarrow \frac{v_o}{v_{gs}} = -g_m \times (R'_L \| r_o) = -2 \times 10^{-3} \times (4.29 \| 100) = -8.23 \text{ V/V}$$

$$\frac{v_o}{v_{sig}} = \frac{v_{gs}}{v_{sig}} \times \frac{v_o}{v_{gs}} = 0.97 \times (-8.23) = 7.98 \text{ V/V}$$