Equivalence factors of a parallel-series system

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Equivalence factors of a parallel-series system

Ammar M. Sarhan, L. Tadj, A. Al-khedhairi and A. Mustafa

Abstract. This paper discusses the reliability equivalences of a parallel-series system. It is assumed that the system components are independent and identical. Each has a constant failure rate. We assumed three different method to improve the system. Both the reliability function and the mean time to failure are used to derive two types of reliability equivalence factors. We also obtain the fractiles of the original and improved systems. We illustrate the results obtained with an application example.

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1 Introduction

Operations Research, in its various fields, is concerned with the problem of having a system perform in the best possible way. In reliability theory, one way to improve the performance of a system is to use the redundancy method. There are two main such methods:

1. Hot duplication method: in this case, it is assumed that some of the system components are duplicated in parallel.

2. Cold duplication method: in this case, it is assumed that some of the system components are duplicated in parallel via a perfect switch.

Unfortunately, for many different reasons, such as space limitation, high cost, etc, it is not always possible to improve a system by duplicating some or all of its components. For example, satellites and space aircrafts have limited space which doesn't allow component duplication. Also, some microchips are so expansive that manufacturers cannot afford to duplicate them. In such cases where duplication is not possible, the engineer turns to another well-known method in reliability theory, the so-called reduction method. In this method, it is assumed that the failure rates of some of the system components are reduced by a factor ρ, $0 < \rho < 1$. Now, once the reduction method is adopted, the main problem facing the engineer is to decide to what degree the failure rate should be decreased in order to improve the system. To solve this problem, one can make an equivalence between the reduction method and the duplication method based on some reliability measures. In other words, the design of

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the system improved by the reduction method should be equivalent to the design of the system improved by one of the the duplication methods. The comparison of the designs produces the so-called *reliability equivalence factors*.

The concept of the reliability equivalence factors was introduced by Råde [5]. Råde [5, 6] applied this concept for the two-component parallel and series systems with independent and identical components whose lifetimes follow the exponential distribution. Sarhan [7] -[10] derived the reliability equivalence factors of other more general systems. The systems studied by Sarhan are the series system [7], a basic series/parallel system [8], a bridge network system [9], the parallel system [10], and a general series-parallel system [12]. All these systems have independent and nonidentical exponential components.

Sarhan and Mustafa [11] introduced different vectors of the reliability equivalence factors of a series system consisting of n independent and non-identical components. The results presented in [11] generalize those given in [7]. Sarhan and Mustafa [11] assumed that the failure rates of the system components are constants and used the reliability function and mean time to failure as performances to derive the reliability equivalences of the system.

In this paper we consider a general parallel-series system and assume that all components are independent and follow the exponential distribution with the same parameter $\lambda > 0$. First, we computed the reliability function and the mean time to failure (MTTF) of the system. Second, we computed these same reliability measures when the system is improved using the hot and cold duplication methods. Third, we computed these same measures when the system is improved using the reduction method. Finally, we equate the reliability function (MTTF) of the system improved by duplication with the reliability function (MTTF) of the system improved by reduction to get the survival (mean) reliability equivalence factors. These factors can be used by the engineer to decide to what degree the failure rate of some of the system components should be decreased in order to improve the performance of the system without duplicating any component.

The rest of the paper is organized as follows. The next section contains the description of the system and the derivation of its reliability function and MTTF. Section 3 computes the reliability function and MTTF of the system when it is improved by reduction. Section 4 is divided into two parts. In the first one we obtain the reliability function and MTTF of the system when it is improved by hot duplication while the second one obtains these same measures when the duplication is cold. In each part the reliability equivalence factors are obtained. Also, in each part, the fractiles are computed for comparison with those of the original system. In Section 5, the results obtained are applied to a specific parallel-series system. Finally, the paper is concluded in Section 6.

2 Parallel-series system

The system of interest to us is depicted in Figure 1. It consists of m modules (subsystems) connected in series, with module i consisting of n_i components in parallel for $i = 1, 2, \cdots, m$. Such a system is called a parallel-series system, see Kue et al. [2].

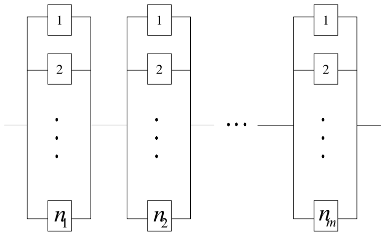

Figure 1. Parallel-series system.

The lives of the components are assumed to be independent and following the exponential distribution with the same failure rate λ. Let $r_{ij}(t)$ be the reliability function of the component j $(1 \leq j \leq n_i)$ in module i $(1 \leq i \leq m)$ and let $R_i(t)$ be the reliability function of module i. That is, $r_{ij}(t) = e^{-\lambda t}$ and

$$\begin{aligned} R_i(t) &= 1 - \prod_{i=1}^{n_i} \left(1 - e^{-\lambda t}\right) \\ &= 1 - \left(1 - e^{-\lambda t}\right)^{n_i}. \end{aligned}$$

Since the modules are connected in series, then the reliability function of the system is

$$\begin{aligned} R(t) &= \prod_{i=1}^{m} R_i(t) \\ &= \prod_{i=1}^{m} \left[1 - \left(1 - e^{-\lambda t}\right)^{n_i}\right] \\ &= \prod_{i=1}^{m} \sum_{j=1}^{n_i} \binom{n_i}{j} (-1)^{j+1} e^{-j\lambda t}. \end{aligned} \tag{2.1}$$

The last equality follows from the binomial expansion. We also derive the MTTF of the system as

$$\text{MTTF} = \int_0^\infty \prod_{i=1}^{m} \sum_{j=1}^{n_i} \binom{n_i}{j} (-1)^{j+1} e^{-j\lambda t} dt. \tag{2.2}$$

This integral can be evaluated explicitly for some specific values of m and n_i but in general it has no closed form.
Also of interest to us is the $\alpha-$fractile, $L(\alpha)$, of the original system, defined by

$$L(\alpha) = \lambda\, R^{-1}(\alpha), \tag{2.3}$$

where R^{-1} denotes the inverse of the reliability function. It can be computed by solving the following equation with respect to (w.r.t.) $L = L(\alpha)$

$$\alpha = \prod_{i=1}^{m} \sum_{j=1}^{n_i} \binom{n_i}{j} (-1)^{j+1} e^{-(n_i - j)L}. \tag{2.4}$$

In the sequel, for any set A, we will denote its cardinality by $|A|$. Also, for any sequence A_i of mutually exclusive sets, we will denote their union by $\sum_{i=1}^{m} A_i$. Furthermore, when r_i components from module i are improved we will use the notation $(1_{r_1}, \cdots i_{r_i}, \cdots m_{r_m})$ to show the number of components improved in each module of the system.

3 Reduction method

We are interested in this section in the system when it is improved by reducing the failure rates of some of its components by a factor $\rho \in (0, 1)$.

Now, let us denote by A the set of system components whose failure rate is reduced and by r their number, so that $|A| = r$ with $r \leq n$. Since these components may be arbitrarily chosen in the system, we will denote by A_i the set of r_i out-of n_i components from module i whose failure rate is reduced, so that $|A_i| = r_i, (i = 1, \cdots, m)$ and $A = \sum_{i=1}^{m} A_i$ with $r = \sum_{i=1}^{m} r_i$.

The reliability function $R_{A,\rho}^{(i)}(t)$ of module $i, (i = 1, \cdots, m)$ is now given by

$$\begin{aligned} R_{A,\rho}^{(i)}(t) &= 1 - \prod_{i=1}^{r_i} \left(1 - e^{-\rho\lambda t}\right) \prod_{i=1}^{n_i - r_i} \left(1 - e^{-\lambda t}\right) \\ &= 1 - \left(1 - e^{-\rho\lambda t}\right)^{r_i} \left(1 - e^{-\lambda t}\right)^{n_i - r_i}, \end{aligned}$$

from which we immediately have the reliability function of the system improved by reduction

$$\begin{aligned} R_{A,\rho}(t) &= \prod_{i=1}^{m} R_{A,\rho}^{(i)}(t) \\ &= \prod_{i=1}^{m} \left[1 - \left(1 - e^{-\rho\lambda t}\right)^{r_i} \left(1 - e^{-\lambda t}\right)^{n_i - r_i}\right] \\ &= \prod_{i=1}^{m} \left[\sum_{j=0}^{n_i - r_i} \sum_{k=1}^{r_i} \binom{n_i - r_i}{j} \binom{r_i}{k} (-1)^{j+k+1} e^{-(j+k\rho)\lambda t}\right. \\ &\qquad \left. + \sum_{j=1}^{n_i - r_i} \binom{n_i - r_i}{j} (-1)^{j+1} e^{-j\lambda t}\right]. \end{aligned} \tag{3.1}$$

We now compute the $\text{MTTF}_{A,\rho}$ of the system improved by improving the set A components by the reduction method as

$$\begin{aligned} \text{MTTF}_{A,\rho} &= \int_0^\infty \prod_{i=1}^{m} \left[\sum_{j=0}^{n_i - r_i} \sum_{k=1}^{r_i} \binom{n_i - r_i}{j} \binom{r_i}{k} (-1)^{j+k+1} e^{-(j+k\rho)\lambda t}\right. \\ &\qquad \left. + \sum_{j=1}^{n_i - r_i} \binom{n_i - r_i}{j} (-1)^{j+1} e^{-j\lambda t}\right] dt. \end{aligned} \tag{3.2}$$

Finally, the α-fractile $L = L(\alpha)$ is found by solving the following equation

$$\begin{aligned} \alpha &= \prod_{i=1}^{m} \left[\sum_{j=0}^{n_i - r_i} \sum_{k=1}^{r_i} \binom{n_i - r_i}{j} \binom{r_i}{k} (-1)^{j+k+1} e^{-(j+k\rho)L} \right. \\ &\qquad \left. + \sum_{j=1}^{n_i - r_i} \binom{n_i - r_i}{j} (-1)^{j+1} e^{-jL} \right]. \end{aligned} \tag{3.3}$$

Of course, the expressions giving the $\mathrm{MTTF}_{A,\rho}$ and the fractiles need to be evaluated numerically since they are both highly nonlinear.

4 Duplication methods

We now obtain the reliability measures of the system when it is improved by duplication. We will successively consider below the hot and then the cold duplication methods.

4.1 Hot duplication

We mentioned earlier that hot duplication means that some of the system components are duplicated in parallel. Therefore, let us assume that the system is improved by hot duplicating each of k components in a set B by a redundant identical standby component, so $|B| = k$. If we assume that k_i out-of n_i components in module i are hot duplicated and if we denote by B_i the set of these k_i components, then we have $|B_i| = k_i, (i = 1, \cdots, m)$ and $B = \sum_{i=1}^{m} B_i$.

The reliability function $R_{B,H}^{(i)}(t)$ of module $i, (i = 1, \cdots, m)$ is now given by

$$\begin{aligned} R_{B,H}^{(i)}(t) &= 1 - \prod_{i=1}^{n_i + r_i} \left(1 - e^{-\lambda t}\right) \\ &= 1 - \left(1 - e^{-\lambda t}\right)^{n_i + r_i}, \end{aligned}$$

from which we immediately have the reliability function of the system improved by hot duplication

$$\begin{aligned} R_B^H(t) &= \prod_{i=1}^{m} R_{B,H}^{(i)}(t) \\ &= \prod_{i=1}^{m} \left[1 - \left(1 - e^{-\lambda t}\right)^{n_i + r_i}\right] \\ &= \prod_{i=1}^{m} \sum_{j=1}^{n_i + r_i} \binom{n_i + r_i}{j} (-1)^{j+1} e^{-j\lambda t}. \end{aligned} \tag{4.1}$$

We now compute the MTTF_B^H of the system improved by improving the set B components by hot duplication method as

$$\text{MTTF}_B^H = \int_0^\infty \prod_{i=1}^{m} \sum_{j=1}^{n_i+r_i} \binom{n_i + r_i}{j} (-1)^{j+1} e^{-j\lambda t} dt, \tag{4.2}$$

and the α-fractile $L = L(\alpha)$ is found by solving the following equation

$$\alpha = \prod_{i=1}^{m} \sum_{j=1}^{n_i+r_i} \binom{n_i + r_i}{j} (-1)^{j+1} e^{-(n_i+r_i-j)L}. \tag{4.3}$$

Finally, to derive the hot reliability equivalence factor, it suffices to solve the set of two equations $R_{A,\rho}(t) = \alpha$ and $R_B^H(t) = \alpha$.

4.2 Cold duplication

We mentioned earlier that cold duplication means that some of the system components are duplicated in parallel via a perfect switch. Therefore, let us assume that the system is improved by cold duplicating each of k components in a set B by a redundant identical standby component via a perfect switch, so $|B| = k$. If we assume that k_i out-of n_i components in module i are cold duplicated and if we denote by B_i the set of these k_i components, then we have $|B_i| = k_i, (i = 1, \cdots, m)$ and $B = \sum_{i=1}^{m} B_i$.

The reliability function $R_{B,C}^{(i)}(t)$ of module $i, (i = 1, \cdots, m)$ is now given by

$$R_{B,C}^{(i)}(t) \quad = \quad 1 - \left[1 - (1 + \lambda t)\, e^{-\lambda t}\right]^{r_i} \left(1 - e^{-\lambda t}\right)^{n_i - r_i},$$

from which we immediately have the reliability function of the system improved by cold duplication

$$\begin{aligned}
R_B^C(t) &= \prod_{i=1}^{m} R_{B,C}^{(i)}(t) \\
&= \prod_{i=1}^{m} \left\{1 - \left[1 - (1 + \lambda t)\, e^{-\lambda t}\right]^{r_i} \left(1 - e^{-\lambda t}\right)^{n_i - r_i}\right\} \\
&= \prod_{i=1}^{m} \left[\sum_{j=0}^{n_i - r_i} \sum_{k=1}^{r_i} \sum_{\ell=0}^{k} \binom{n_i - r_i}{j} \binom{r_i}{k} \binom{k}{\ell} (-1)^{j+k+1} (\lambda t)^{\ell} e^{-(j+k)\lambda t} \right. \\
&\qquad \left. + \sum_{j=1}^{n_i - r_i} \binom{n_i - r_i}{j} (-1)^{j+1} e^{-j\lambda t} \right].
\end{aligned} \tag{4.4}$$

We now compute the MTTF_B^C of the system improved by improving the set B components according to the cold duplication method as

$$\begin{aligned}
\text{MTTF}_B^C &= \int_0^\infty \prod_{i=1}^{m} \left[\sum_{j=0}^{n_i - r_i} \sum_{k=1}^{r_i} \sum_{\ell=0}^{k} \binom{n_i - r_i}{j} \binom{r_i}{k} \binom{k}{\ell} (-1)^{j+k+1} (\lambda t)^{\ell} e^{-(j+k)\lambda t} \right. \\
&\qquad \left. + \sum_{j=1}^{n_i - r_i} \binom{n_i - r_i}{j} (-1)^{j+1} e^{-j\lambda t} \right] dt,
\end{aligned} \tag{4.5}$$

and the α-fractile $L = L(\alpha)$ is found by solving the following equation

$$\begin{aligned}\alpha &= \prod_{i=1}^{m}\left[\sum_{j=0}^{n_i-r_i}\sum_{k=1}^{r_i}\sum_{\ell=0}^{k}\binom{n_i-r_i}{j}\binom{r_i}{k}\binom{k}{\ell}(-1)^{j+k+1}(\lambda t)^{\ell}e^{-(j+k)L}\right.\\ &\qquad \left. + \sum_{j=1}^{n_i-r_i}\binom{n_i-r_i}{j}(-1)^{j+1}e^{-jL}\right]. \qquad (4.6)\end{aligned}$$

Finally, to derive the cold reliability equivalence factor, it suffices to solve the set of two equations $R_{A,\rho}(t) = \alpha$ and $R_B^C(t) = \alpha$.

5 Application

Let us consider in this section the parallel-series system consisting of two modules in series ($m = 2$) and assume that the first module has two components in parallel ($n_1 = 2$) while the second module has three components in parallel ($n_2 = 3$). The total number of components is $n = 5$. The MTTF of the system is 1.05. Figures 2 and 3 show the MTTF of the system improved by improving some sets of components according to the reduction method by the factor ρ, $0 < \rho < 1$. It seems from these two figures that:

1. $\text{MTTF}_{A,\rho}$ decreases with increasing ρ for all possible sets A.

2. Reducing the failure rate of one component from the module 1 gives a better system than that system improved by reducing the failure rate of one component from the module 2, see figure 1.

3. Reducing the failure rates of two components, one from each module, gives a better system than that system improved by reducing the failure rate of two components from the same module, see figures 1 and 2.

4. Reducing the failure rates of all components gives the best system, see figure 2.

One could conclude, for the system considered in this section which consists of two modules, that: (1) improving one component from the module with a smaller number of components gives a better system than that system improved when the component improved belongs to the module with a larger number of components; (2) improving an even number of components selected from the two modules, with equal numbers, produces a better system than the system improved by improving the number of components selected from the same module or selected from the two modules with unequal numbers.

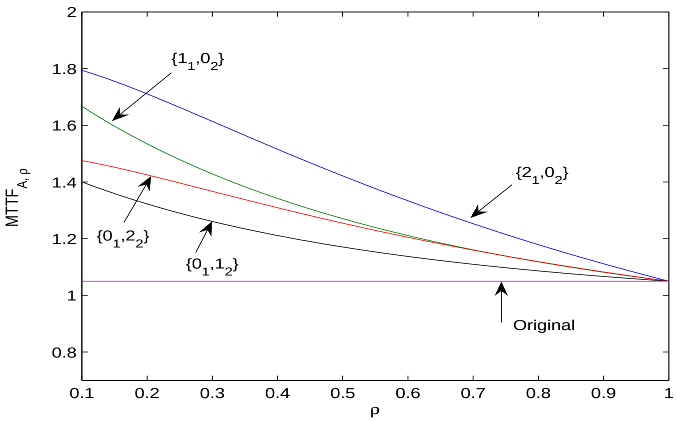

Figure 2. The MTTF of the system improved according to reduction method.

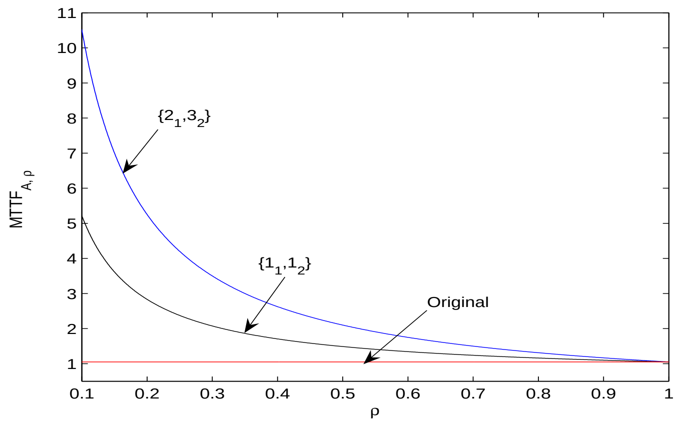

Figure 3. The MTTF of the system improved according to reduction method.

Table 1: The MTTF of the improved systems.

	$\{1_1, 0_2\}$	$\{0_1, 1_2\}$	$\{2_1, 0_2\}$	$\{1_1, 1_2\}$	$\{0_1, 2_2\}$	$\{2_1, 3_2\}$
Hot	1.2167	1.1333	1.3238	1.3238	1.1905	1.6044
Cold	1.3433	1.2044	1.6033	1.4922	1.2885	2.1547

It seems from the results given in Table 1 that

$$MTTF^D_{\{0_1,1_2\}} < MTTF^D_{\{0_1,2_2\}} < MTTF^D_{\{1_1,0_2\}} <$$

$$MTTF^D_{\{2_1,0_2\}} \leq MTTF^D_{\{1_1,1_2\}} < MTTF^D_{\{2_1,3_2\}},$$

where $D = H$(for hot), C(for cold), and

$$MTTF^H_B < MTTF^C_B, \quad \forall\, B.$$

This means that: (1) as it was expected, the cold duplication method provides a better improved system than the hot duplication method; (2) improving components from the module with smaller number of components gives a better improved system than the system improved by improving the same number of components belonging to the module containing a larger number of components.

Tables 2 and 3 present the hot and cold reliability equivalence factors when one component from module 1 is improved by the reduction method and hot and cold duplications of different possible components.

Table 2: The values of $\rho^H_{i_1,j_2}$, $i \in \{0,1,2\}$ and $j \in \{0,1,2,3\}$.

α	$\rho^H_{1_1,0_2}$	$\rho^H_{0_1,1_2}$	$\rho^H_{2_1,0_2}$	$\rho^H_{1_1,1_2}$	$\rho^H_{0_1,2_2}$	$\rho^H_{2_1,3_2}$
0.1	0.7032	0.7901	0.5612	0.5612	0.6692	0.7472
0.2	0.6514	0.7624	0.4927	0.4927	0.6295	0.7072
0.3	0.6097	0.7440	0.4399	0.4399	0.6047	0.6750
0.4	0.5712	0.7309	0.3931	0.3931	0.5886	0.6453
0.5	0.5332	0.7219	0.3486	0.3486	0.5799	0.6156
0.6	0.4932	0.7176	0.3040	0.3040	0.5798	0.5839
0.7	0.4487	0.7197	0.2570	0.2570	0.5918	0.5480
0.8	0.3951	0.7329	0.2045	0.2045	0.6242	0.5034
0.9	0.0100	0.7717	0.1391	0.1391	0.6996	0.4376

Table 3: The values of $\rho^C_{i_1,j_2}$, $i \in \{0,1,2\}$ and $j \in \{0,1,2,3\}$.

α	$\rho^C_{1_1,0_2}$	$\rho^C_{0_1,1_2}$	$\rho^C_{2_1,0_2}$	$\rho^C_{1_1,1_2}$	$\rho^C_{0_1,2_2}$	$\rho^H_{2_1,3_2}$
0.1	0.4854	0.6067	0.3304	0.2466	0.4562	0.5656
0.2	0.4395	0.5896	0.2746	0.1890	0.4334	0.5299
0.3	0.4045	0.5824	0.2348	0.1473	0.4262	0.5024
0.4	0.3736	0.5813	0.2017	0.1120	0.4290	0.4777
0.5	0.3440	0.5860	0.1718	0.0800	0.4414	0.4535
0.6	0.3139	0.5975	0.1436	0.0494	0.4651	0.4283
0.7	0.2815	0.6185	0.1157	0.0190	0.5043	0.4002
0.8	0.2437	0.6556	0.0867	-0.012	0.5677	0.3658
0.9	0.1928	0.7264	0.0539	-0.046	0.6760	0.3159

We computed the α-fractiles of the original system and the system improved by the hot and cold duplications. Table 4 gives the fractiles of the original system. Tables 5 and 6 give the fractiles of the system improved by the hot and cold duplications, respectively.

Table 4: The fractile of the original system.

α	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
$L(\alpha)$	1.9358	1.5314	1.2770	1.0826	0.9190	0.7721	0.6324	0.4908	0.3310

Table 5: The fractile $L^H_{i_1,j_2}$, $i \in \{0,1,2\}$ and $j \in \{0,1,2,3\}$.

α	$L^H_{1_1,0_2}$	$L^H_{0_1,1_2}$	$L^H_{2_1,0_2}$	$L^H_{1_1,1_2}$	$L^H_{0_1,2_2}$	$L^H_{2_1,3_2}$
0.1	2.1285	2.0619	2.2621	2.2621	2.1575	2.5908
0.2	1.7187	1.6477	1.8466	1.8466	1.7343	2.1655
0.3	1.4593	1.3842	1.5820	1.5820	1.4624	1.8917
0.4	1.2599	1.1807	1.3773	1.3773	1.2503	1.6777
0.5	1.0910	1.0074	1.2027	1.2027	1.0679	1.4930
0.6	0.9380	0.8498	1.0434	1.0434	0.9003	1.3222
0.7	0.7911	0.6979	0.8890	0.8890	0.7370	1.1539
0.8	0.6398	0.5414	0.7281	0.7281	0.5677	0.9749
0.9	0.4646	0.3621	0.5383	0.5383	0.3739	0.7568

Table 6: The fractile $L^C_{i_1,j_2}$, $i \in \{0,1,2\}$ and $j \in \{0,1,2,3\}$.

α	$L^C_{1_1,0_2}$	$L^C_{0_1,1_2}$	$L^C_{2_1,0_2}$	$L^C_{1_1,1_2}$	$L^C_{0_1,2_2}$	$L^H_{2_1,3_2}$
0.1	2.3489	2.2155	2.5716	2.7240	2.3859	3.4226
0.2	1.8985	1.7643	2.0980	2.2317	1.9048	2.8898
0.3	1.6127	1.4767	1.7940	1.9165	1.5936	2.5415
0.4	1.3927	1.2543	1.5577	1.6719	1.3505	2.2662
0.5	1.2061	1.0650	1.3555	1.4626	1.1419	2.0263
0.6	1.0369	0.8932	1.1707	1.2713	0.9515	1.8026
0.7	0.8744	0.7282	0.9914	1.0853	0.7685	1.5803
0.8	0.7070	0.5596	0.8050	0.8912	0.5830	1.3416
0.9	0.5131	0.3694	0.5866	0.6614	0.3789	1.0477

Table 7: MREF when reducing one component from module 1 for hot and cold duplications.

Method	Reduction of the failure rate of 1 comp from module 1.					
	$\xi^H_{1_1,0_2}$	$\xi^H_{0_1,1_2}$	$\xi^H_{2_1,0_2}$	$\xi^H_{1_1,1_2}$	$\xi^H_{0_1,2_2}$	$\xi^H_{2_1,3_2}$
Hot	0.5897	0.3876	0.7477	0.7264	0.5754	0.8630
Cold	0.3984	0.1709	0.5886	0.6000	0.3406	0.7817
	Reduction of the failure rate of 5 comp. and cold duplication.					
	$\xi^H_{1_1,0_2}$	$\xi_{0_1,1_2}$	$\xi^H_{2_1,0_2}$	$\xi^H_{1_1,1_2}$	$\xi^H_{0_1,2_2}$	$\xi^H_{2_1,3_2}$
Hot	0.1443	-0.0823	0.3095	0.4411	-1.01752e-006	0.6545
Cold	-0.1409	-0.3367	7.45927e-008	0.2847	1.90735e-007	0.4873

According to the results shown in Tables 2-7, one can see that:

1. Hot duplication of one component belonging to module 1 increases $L(0.1)$ from 1.9358 to 2.1285. The same increase can be reached by reducing the failure rate of one component belonging to the same module by the factor $\rho^H_{1_1,0_2}(0.1) = 0.7032$.

2. Cold duplication of one component belonging to module 1 increases $L(0.1)$ from 1.9358 to 2.3489. The same increase can be reached by reducing the failure rate of one component belonging to the same module by the factor $\rho^C_{1_1,0_2}(0.1) = 0.4854$.

3. Hot duplication of one component belonging to module 1 increases the system mean time to failure from 1.05 to 1.2167. The same increase can be reached by reducing the failure rate of one component belonging to the same module by the factor $\xi^{H}_{1_1,0_2} = 0.5897$.

4. Cold duplication of one component belonging to module 1 increases the system mean time to failure from 1.05 to 1.3433. The same increase can be reached by reducing the failure rate of one component belonging to the same module by the factor $\xi^{C}_{1_1,0_2} = 0.429$.

5. The negative values in Table 3 mean that the reliability function of the system improved by reducing the failure rate of one component can not be increased to be equal to the reliability function of that design obtained by cold duplication method. As an example, $\rho^{C}_{\{1_1,1_2\}}(0.8) = -0.012$ means that at the level $\alpha = 0.8$, it is not possible to reduce the failure rate of one component belonging to module 1 in order to obtain an improved system which can be obtained by cold duplicating two components one of them belonging to module 1 and the second one belonging to module 2.

6. The negative values in Table 7 mean that it is not possible to make equivalence between the system improved by the reduction method and the system improved by the duplication methods. For example, $\xi^{H}_{1_1,0_2} = -0.1409$ means that it is not possible to reduce the failure rate of one component belonging to module 1 to get an improved system which has the mean time to failure equal to the mean time to failure of the system improved by cold duplicating one component belonging to module 1.

7. In a similar way, one can read the rest of the results shown in Tables 2-7.

6 Conclusion

In this paper, we discussed the reliability equivalences of a parallel-series system with independent and identical components. We assumed that each component had a constant failure rate. We also considered three ways, namely the reduction, hod duplication and cold duplication methods, to improve the system. We used both the reliability function and the mean time to failure to derive two types of reliability equivalence factors. The fractiles of the original and improved systems are also obtained. For illustrative purpose, a numerical example is presented. The results discussed in this paper can be extended to include the following cases: (1) more complicated systems with independent and identical or non-identical components; (2) simple systems with non-independent and identical components; (3) systems with non-constant failure rate components. We believe that the cases when the components have non-constant failure rates will be more complicated. Another problem of interest to OR researchers would be to determine the optimal number of components to duplicate in the duplication methods and the optimal number of components whose failure rate is to be reduced in the reduction method.

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