A Rock Mechanical Model for Overbalanced, Managed Pressure, and Underbalanced Drilling Applications

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1. OIL & GAS RESERVOIRS
Support provided by the removed rock is replaced by the drilling fluid pressure then by the casing and cement.
2. WELLBORE STABILITY MANAGEMENT

Formation Pore Pressure & Strength

Drilling Fluid Pressure & Type

Non-Adjustable

Adjustable

Mud circulation in the hole
2. WELLBORE INSTABILITY MODES
3. DRILLING TECHNOLOGY

1. Overbalanced Drilling (OBD)
   Drilling fluid pressure > Formation pressure

2. Managed Pressure Drilling (MPD)
   Drilling fluid pressure = Formation pressure

3. Underbalanced Drilling (UBD)
   Drilling fluid pressure < Formation pressure
4. OVERBALANCED DRILLING (OBD)

**ADVANTAGES**
Its technique and safety issues are very well known, requires fewer personnel to operate, more economical, requires less rig space, provides good borehole stability, and there is no need for handling of hydrocarbons during this type of drilling.

**DISADVANTAGES**
Potential formation damage, low rate of penetration through harder formations, potential for differential sticking, and potential for getting a kick in case of a section with unknown pore pressures and thief zones.
5. BALANCED DRILLING (MPD)

ADVANTAGES
Avoids the flow of the formation fluids into the wellbore, reduce drilling cost, increase safety, resolve long-lasting drilling problems that contribute to non-productive time such as well instability, stuck pipe, lost circulation, and well control incidents.

DISADVANTAGES
May not be capable of solving the problems encountered, such as when fracture pressure is too close to pore pressure, may not be capable of solving the problems encountered, when variations occur in pore and fracture pressures in different intervals within the same open hole.
6. UNDER BALANCED DRILLING (UBD)

ADVANTAGES
Reducing formation damage in the reservoir, caused by mud solids and liquids invasion and shale swelling, maximizing hydrocarbon production, minimizing lost circulation, increasing drilling rates, extending bit life, and minimizing the need for well stimulation.

DISADVANTAGES
May Well instability and well control issues are big concerns. Drillstring vibrations are often more pronounced, Higher drag and torque will be experienced. complex mixture of fluids and cuttings. Corrosion and ignition potentials. High cost.
7. MPD & UBD APPLICATIONS

It is clear that both MPD and UBD are candidates for drilling oil and gas bearing formations to avoid permeability damage as well as other instability problems when compared to the conventional OBD method.

MPD is normally used in infill drilling in depleted reservoirs where the wellbore stability is the main concern.

UBD is normally used to drill well in newly developed reservoirs when formation damage is a big concern.
8. STUDY OBJECTIVES

The objective of this work is to use:

1. Rock mechanics principles and
2. Laboratory characterization of representative core samples

To elaborate a mathematical model able to predict the wellbore pressure drop, balance, and overbalance ($\Delta P_w$) required for safe UBD, MPD, or OBD.

“A Rock Mechanical Model for Overbalanced, Managed Pressure, and Underbalanced Drilling Applications”
9. THE ELLABORATED MODEL

Formation rock failure criteria are evaluated using Mohr-Coulomb failure criterion which is one of the most famous and applied rock failure criteria (Fjaer et al., 1992). This criterion is shown below:

\[ \tau_f = \tau_0 + \sigma_f \tan \phi \]
9. THE ELLABORATED MODEL

Formation Wellbore instability can be predicted when these principal in-situ stresses are transformed parallel to the wellbore axis (for inclined or horizontal wells) using the following matrices (Fjaer et al., 1992):

\[
\begin{bmatrix}
\sigma_x \\
\sigma_y \\
\sigma_{zz}
\end{bmatrix} = \begin{bmatrix}
\cos^2\beta \cos^2\alpha & \sin^2\beta \cos^2\alpha & \sin^2\alpha \\
\sin^2\beta & \cos^2\beta & 0 \\
\cos^2\beta \sin^2\alpha & \sin^2\beta \sin^2\alpha & \cos^2\alpha
\end{bmatrix} \begin{bmatrix}
\sigma_H \\
\sigma_h \\
\sigma_v
\end{bmatrix}
\]

\[
\begin{bmatrix}
\tau_{yz} \\
\tau_{zx} \\
\tau_{yx}
\end{bmatrix} = \frac{1}{2} \begin{bmatrix}
\sin 2\beta \sin \alpha & -\sin 2\beta \sin \alpha & 0 \\
\sin 2\alpha \cos \beta & \sin^2\beta \sin 2\alpha & -\sin 2\alpha \\
\cos^2\beta \sin^2\alpha & -\sin 2\beta \cos \alpha & 0
\end{bmatrix} \begin{bmatrix}
\sigma_H \\
\sigma_h \\
\sigma_v
\end{bmatrix}
\]
9. THE ELLABORATED MODEL

The drilling process generated an induced stresses are acting on the wall of a borehole (Jaeger, 1979). These are, the vertical induced stress ($\sigma_z$), the radial induced stress ($\sigma_r$) and the tangential induced stress ($\sigma_\theta$) which can be computed as follows:

\[
\begin{align*}
\overline{\sigma}_r &= P_w - P_p = P_{wc} \\
\overline{\sigma}_\theta &= \left( \sigma_x + \sigma_y - P_p - P_w \right) - 2 \left( \sigma_x - \sigma_y \right) \cos 2\theta - 4 \tau_{xy} \sin 2\theta \\
\overline{\sigma}_z &= \sigma_{zz} - P_p - 2\nu \left( \sigma_x - \sigma_y \right) \cos 2\theta - 4\nu \tau_{xy} \sin 2\theta \\
\tau_{\theta z} &= 2 \left[ - \tau_{zx} \sin \theta + \tau_{yz} \cos \theta \right]
\end{align*}
\]
9. THE ELLABORATED MODEL

By knowing the magnitude of the wellbore (mud) pressure, the induced principal stresses acting on the wall of a borehole can be computed as follows:

\[
\begin{align*}
\sigma_1 &= \sigma_r = P_w - P_p = P_{wc} \\
\sigma_2 &= \frac{1}{2}(\sigma_\theta + \sigma_z) - \frac{1}{2}\sqrt{(\sigma_\theta - \sigma_z)^2 + 4\tau_\theta z} \\
\sigma_3 &= \frac{1}{2}(\sigma_\theta + \sigma_z) + \frac{1}{2}\sqrt{(\sigma_\theta - \sigma_z)^2 + 4\tau_\theta z}
\end{align*}
\]
9. THE ELLABORATED MODEL

Borehole instability using underbalanced drilling can be predicted by comparing the computed drilling induced and the experimentally measured shear stresses (i.e. failure criterion) as follows:

\[
\begin{align*}
\tau_{f,\text{Drilling}}^{\text{Maximum}} &= \left[ \frac{\sigma_1 - \sigma_3}{2} \right] \\
\tau_{f,\text{Drilling}}^{\text{Criterion}} &= \left[ \frac{\sigma_1 - \sigma_3}{2} \right] \sin 2\theta \\
\tau_{f,\text{Criterion}} &= \tau_o + \left[ \left( \frac{\sigma_1 + \sigma_3}{2} \right) + \left( \frac{\sigma_1 - \sigma_3}{2} \right) \cos 2\theta \right] \tan \phi
\end{align*}
\]
9. **THE ELLABORATED MODEL**

\[ \tau_f = \tau_o + \bar{\sigma}_f \tan \phi \]

![Diagram showing the relationship between shear stress and normal stress, with the formula and various stress points labeled.](image)
9. THE ELLABORATED MODEL

Wellbore fracturing pressure limit can be estimated using the following relationship (Brady et al., 1985):

\[
(P_w)_{\text{Fracturing}} = \left[ \frac{2 \tau_0 \cos \phi}{1 + \sin \phi} \right]
\]
9. THE ELLABORATED MODEL

Microsoft Excel Spreadsheets for the Elaborated Model
10. INPUT DATA

Hypothetical data used to validate the mathematical model and to predict the wellbore pressure required for safe UBD, MPD, and OBD operations from wellbore instability prospects. These data are a modification of a real case vertical oil well in China (Qiang, 2015).

Table 1 Hypothetical data used for model verification

<table>
<thead>
<tr>
<th>Rock Type</th>
<th>ISRM Grade</th>
<th>UCS, MPa</th>
<th>Poisson’s Ratio</th>
<th>Friction Angle, (Degree)</th>
<th>Depth, m</th>
<th>Pore Pressure, Pp, MPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>Very Weak</td>
<td>R1</td>
<td>6.9</td>
<td>0.20</td>
<td>21</td>
<td>2730</td>
<td>51</td>
</tr>
<tr>
<td>Weak</td>
<td>R2</td>
<td>20.6</td>
<td>0.23</td>
<td>26</td>
<td>2730</td>
<td>51</td>
</tr>
<tr>
<td>Medium Strong</td>
<td>R3</td>
<td>35.6</td>
<td>0.25</td>
<td>31.4</td>
<td>2730</td>
<td>51</td>
</tr>
</tbody>
</table>

Principal In-Situ Stresses Gradients, psi/ft (MPa/m)

|                | σV       | 1.10     | (0.025)        |
|                | σH       | 1.00     | (0.023)        |
|                | σh       | 0.93     | (0.021)        |

Angular Position around the Wellbore, degree

|                | θ        | Zero     |
| Wellbore Inclination from Vertical, degree
|                | α        | Zero or 90|
| Wellbore Orientation Angle from, degree
|                | β        | Zero or 90|
11. COMPUTED DATA
(R1, R2, and R3 Rock Types)

Table 2 Summary of the results of drilling vertical hypothetical wells in R1, R2 and R3 formations

<table>
<thead>
<tr>
<th>Formation Type</th>
<th>Well Type</th>
<th>UCS, MPa</th>
<th>Poisson’s Ratio</th>
<th>Friction Angle, Degree</th>
<th>Safe Drilling Window, MPa</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>OBD</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>P_w= (P_w-P_p)</td>
</tr>
<tr>
<td>Very Weak (R1)</td>
<td>Vertical</td>
<td>6.9</td>
<td>0.20</td>
<td>21</td>
<td>+3.2</td>
</tr>
<tr>
<td>Weak (R2)</td>
<td>Vertical</td>
<td>20.6</td>
<td>0.23</td>
<td>26</td>
<td>+8.1</td>
</tr>
<tr>
<td>Med. Strong (R3)</td>
<td>Vertical</td>
<td>35.6</td>
<td>0.25</td>
<td>31.4</td>
<td>+11.2</td>
</tr>
</tbody>
</table>
12. RESULTS AND DISCUSSION

Fig. 6 Vertical well in moderately strong formation (R3)
12. RESULTS AND DISCUSSION

Fig. 5 Vertical well in weak formation (R2)
12. RESULTS AND DISCUSSION

![Diagram showing failure and drilling envelopes with pressure and induced stresses axes.]

Fig. 4 Vertical well in very weak formation (R1)
12. RESULTS AND DISCUSSION

Fig. 7 Summary of the studied vertical drilling cases
13. COMPUTED DATA
(Medium Strong Rock – R3)

Table 5 Summary of the results of drilling vertical and horizontal hypothetical wells in R3 formation

<table>
<thead>
<tr>
<th>Formation Type</th>
<th>Well Type</th>
<th>UCS, MPa</th>
<th>Poisson’s Ratio</th>
<th>Friction Angle, Degree</th>
<th>Safe Drilling Window, MPa</th>
<th>OBD</th>
<th>MPD</th>
<th>UBD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Medium Strong (R3)</td>
<td>Vertical</td>
<td>35.6</td>
<td>0.25</td>
<td>31.4</td>
<td>+11.2</td>
<td>62.2</td>
<td>0</td>
<td>51</td>
</tr>
<tr>
<td></td>
<td>Horizontal// ( \sigma_H )</td>
<td>35.6</td>
<td>0.25</td>
<td>31.4</td>
<td>+11.2</td>
<td>62.2</td>
<td>0</td>
<td>51</td>
</tr>
<tr>
<td></td>
<td>Horizontal// ( \sigma_H )</td>
<td>35.6</td>
<td>0.25</td>
<td>31.4</td>
<td>+11.2</td>
<td>62.2</td>
<td>0</td>
<td>51</td>
</tr>
</tbody>
</table>
14. RESULTS AND DISCUSSION

Fig. 9 Horizontal well ($\sigma_r / \sigma_h$) in medium strong formation (R3)
15. RESULTS AND DISCUSSION

![Graph showing Induced Stresses vs. Wellbore Pressure]

Fig. 10 Horizontal well ($/sigma_H$) in medium strong formation (R3)
16. COMPUTED DATA (Weak Rock – R2)

Table 4 Summary of the results of drilling vertical and horizontal hypothetical wells in R2 formation

<table>
<thead>
<tr>
<th>Formation Type</th>
<th>Well Type</th>
<th>UCS, MPa</th>
<th>Poisson’s Ratio</th>
<th>Friction Angle, Degree</th>
<th>Safe Drilling Window, MPa</th>
<th>OBD</th>
<th>MPD</th>
<th>UBD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weak (R2)</td>
<td>Vertical</td>
<td>20.6</td>
<td>0.23</td>
<td>26</td>
<td>+8.1</td>
<td>59.1</td>
<td>0</td>
<td>51</td>
</tr>
<tr>
<td></td>
<td>Horizontal// σh</td>
<td>20.6</td>
<td>0.23</td>
<td>26</td>
<td>+8.1</td>
<td>59.1</td>
<td>0</td>
<td>-27.5</td>
</tr>
<tr>
<td></td>
<td>Horizontal// σH</td>
<td>20.6</td>
<td>0.23</td>
<td>26</td>
<td>+8.1</td>
<td>59.1</td>
<td>0</td>
<td>-16.6</td>
</tr>
</tbody>
</table>
16. COMPUTED DATA (Very Weak Rock – R1)

Table 3: Summary of the results of drilling vertical and horizontal hypothetical wells in R1 formation

<table>
<thead>
<tr>
<th>Formation Type</th>
<th>Well Type</th>
<th>UCS, MPa</th>
<th>Poisson’s Ratio</th>
<th>Friction Angle, Degree</th>
<th>Safe Drilling Window, MPa</th>
<th>OBD</th>
<th>MPD</th>
<th>UBD</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Vertical</td>
<td>6.9</td>
<td>0.20</td>
<td>21</td>
<td>+3.2</td>
<td>54.2</td>
<td>0</td>
<td>51</td>
</tr>
<tr>
<td></td>
<td>Horizontal // σa</td>
<td>6.9</td>
<td>0.20</td>
<td>21</td>
<td>+3.2</td>
<td>54.2</td>
<td>0</td>
<td>51</td>
</tr>
<tr>
<td></td>
<td>Horizontal // σh</td>
<td>6.9</td>
<td>0.20</td>
<td>21</td>
<td>+3.2</td>
<td>54.2</td>
<td>0</td>
<td>51</td>
</tr>
</tbody>
</table>

Vertical Well                Horizontal Well
17. RESULTS SUMMARY

Fig. 10 Safe drilling in three different horizontal well orientations and formations strengths.
18. CONCLUSIONS

Based on the analysis of the output data obtained using the mathematical model and the hypothetical formation data presented in this study, the following conclusions are attained:

- The application of UBD, MPD, or OBD is highly dependent on the formation strength, pore fluid pressure, and the in-situ principal stresses acting on the area.

- In underbalanced drilling, the window of safe wellbore pressure for vertical wells is much wider than the window of the horizontal wells under the same conditions.

- Compression (shear) and tensile failure criteria used in this work can be easily replaced if required.
18. CONCLUSIONS

- It was found that it is extremely difficult to use UBD to drill horizontal wells parallel to the maximum principle horizontal in-situ stress in the very weak rock (R1) under the studied conditions.

- In overbalanced drilling, the same window was applicable in both vertical and horizontal wells under the same studies conditions.

- In underbalanced drilling, the order of stability decrease (based on well configurations): vertical wells, horizontal wells drilled parallel to the minimum horizontal principal in-situ stress, and horizontal wells drilled parallel to the maximum horizontal principal in-situ stress accordingly.
Vielen Dank
شكرا جزيلا