Q1 $\alpha = 0.05$. Test of independence

1) Test $\chi^2 = \sum_{i=1}^{12} \frac{(O_i - E_i)^2}{E_i}$

distribution: chi-squared distribution

2) mathematic expression $\sum_{i} \frac{(O_i - E_i)^2}{E_i}$

3) $E_{23} = \frac{\text{colum total} \times \text{row total}}{\text{grand total}}$

$= \frac{(44+28+21)(30+53+28+32)}{400} = 33.2475$

4) $E_{22} = \frac{(48+53+28)(30+53+28+32)}{400} = 46.1175$

5) $\chi^2 = \frac{(25-32.57)^2}{32.57} + \frac{(48-53.86)^2}{53.86} + \dots + \frac{(18-22.5)^2}{22.5}$

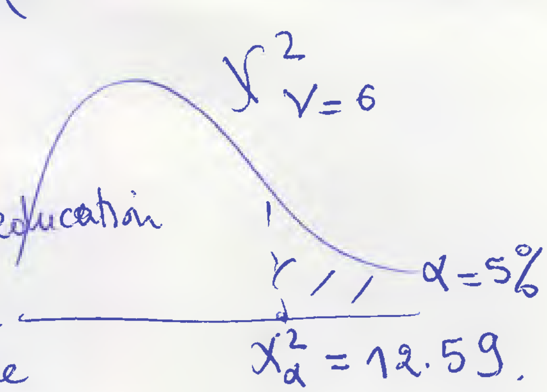
$= 9.75$

6) degree of freedom $= (3-1)(4-1) = 6$

7) $RR =]12.59, +\infty[$

H_0: size independent of education

H_A: size and education not independence



8) $\chi^2 < \chi^2_\alpha \Rightarrow$ not reject H_0.

No sufficient evidence to reject independence

Q2

9) $r_{xy} = \dfrac{S_{xy}}{\sqrt{S_{xx} S_{yy}}} = \dfrac{\sum(x_i-\bar{x})(y_i-\bar{y})/n-1}{\sqrt{\dfrac{\sum(x_i-\bar{x})^2}{n-1} \dfrac{\sum(y_i-\bar{y})^2}{n-1}}}$

$= \dfrac{\sum(x_i-\bar{x})(y_i-\bar{y})}{\sqrt{\sum(x_i-\bar{x})^2 \sum(y_i-\bar{y})^2}}$

$\sum(x_i-\bar{x})(y_i-\bar{y}) = \sum x_i y_i - n\bar{x}\bar{y} = 59648 - 9\frac{742}{9} \times \frac{707}{9}$

$= \cancel{53474.53}\ 1359.778$

$\sum(x_i-\bar{x})^2 = \sum x_i^2 - n\bar{x}^2 = 62240 - 9\left(\frac{742}{9}\right)^2$

$= 1066.222$

$\sum(y_i-\bar{y})^2 = \sum y_i^2 - n\bar{y}^2 = 57557 - 9\left(\frac{707}{9}\right)^2 =$

$= 2018.222$

$r_{xy} = \dfrac{1359.778}{\sqrt{1066.222 \times 2018.222}} = 0.926957.$

$\hat{y} = a + bx.$

10)

$b = \dfrac{S_{xy}}{S_{xx}} = \dfrac{1359.778}{1066.222} = 1.275323$

11)

$a = \bar{y} - b\bar{x} = \frac{707}{9} - 1.275\left(\frac{742}{9}\right) = -26.5611$

12) $\hat{y} = a + b \times (85)$

$= -26.56 + 1.275 \times 85$

$= 81.815$

13) $SSR = \sum_{i=1}^{9} (\hat{y}_i - \bar{y})^2$ $\quad df = 2-1 = 1$

$MSR = \frac{SSR}{df} = SSR$

Then $SSR = MSR = 1733.72$.

14) $SSE = \sum_{i=1}^{9} (y_i - \hat{y}_i)^2$ $\quad df = n-2 = 9-2 = 7$

15) $SSTO = SSR + SSE$.

$SSE = SSTO - SSR$.

$= 2018.22 - 1733.72$

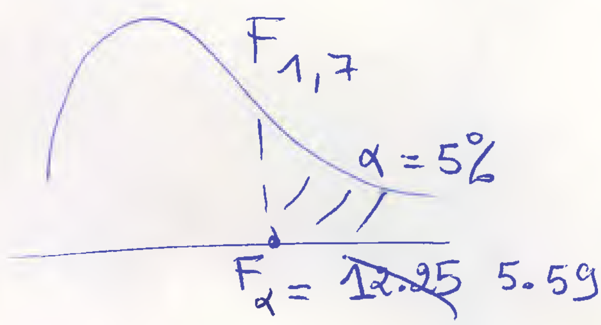
$MSE = \frac{SSE}{7} = 40.64286$

16) Statistic $= f = \frac{MSR}{MSE} = \frac{1733.72}{40.66} = 42.66$

17) F — Fisher $\nu_1 = 1$; $\nu_2 = 7$

RR = reject $H_0 : \beta_1 = 0$

$=]\, 5.59, +\infty [$



18) $f > F_\alpha$

then reject H_0.

slope β_1 is significantly different of 0.

Q3

19) H_0 : The die is fair : $f(1) = f(2) = \dots = f(6) = 1/6$

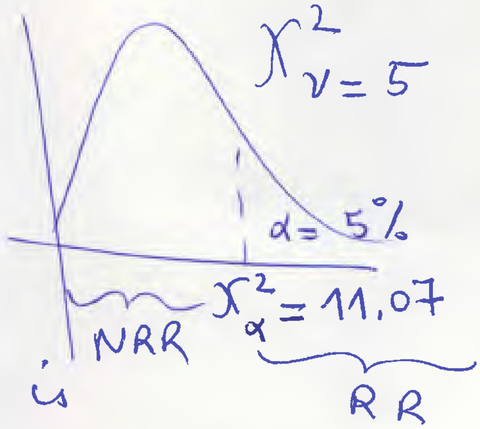
$$E_1 = E_2 = E_3 = E_4 = E_5 = E_6 = 180 \times f(i)$$
$$= 180 \times \frac{1}{6} = 30$$

20) $$\text{Statistic} = \sum_{i=1}^{6} \frac{(O_i - E_i)^2}{E_i} = \chi^2$$

is chi-squared distributed.

21) $df = k - 1 = 6 - 1 = 5$

22) $$\chi^2 = \frac{(15-30)^2}{30} + \frac{(40-30)^2}{30} + \frac{(36-30)^2}{30} + \dots + \frac{(19-30)^2}{30}$$
$$= 24.4$$



23) critical value $= 11.07$

24) $\chi^2 = 24.4 > 11.07$

reject H_0.

There is evidence that the die is not fair.

Q4

$X \sim P(\lambda)$

25) $$\lambda = E(X) = \text{mean of } (X) = \sum_{x=0}^{\infty} x \times P(X = x)$$

$$\hat{\lambda} = 0 \times \hat{P}(X=0) + 1 \times \hat{P}(X=1) + \dots + 4 \times \hat{P}(X=4)$$

$$= 0 \times \frac{75}{200} + \frac{1 \times 66}{200} + \frac{2 \times 42}{200} + \frac{3 \times 13}{200} + \frac{4 \times 4}{200} = 1.025$$

26) $$= P(X=1) = e^{-\hat{\lambda}} \times \frac{\hat{\lambda}^1}{1!} = 0.3677664$$

$$\hat{E}_2 = 200 \times P(X=1) = 73.55328$$

27) $$P(X=3) = e^{-\hat{\lambda}} \cdot \frac{\hat{\lambda}^3}{3!} = 0.04734437$$

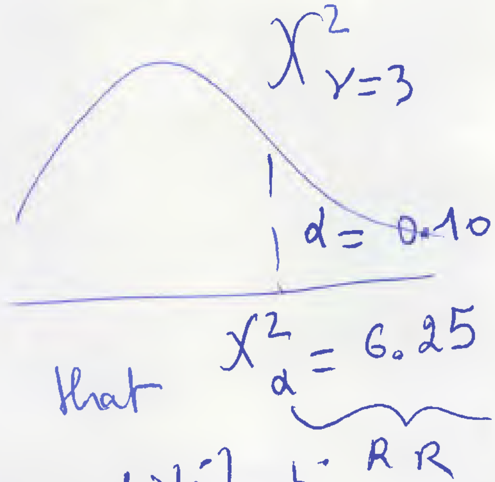
$$E_4 = 200 \times P(X=3) = 9.468874$$

28) $df = n - 1 - 1 = 5 - 2 = 3$

29) $\chi^2 = \sum_{i=1}^{5} \frac{(O_i - E_i)^2}{E_i} = 1.45$

30) $RR =]6.25, +\infty[$

31) $\chi^2 = 1.45 < 6.25$



not reject Ho.
No sufficient evidence that
X has not a Poisson distribution

<u>Q5</u> $n = 4$; $b = 10$

32) $MSB = \frac{SSB}{df} = \frac{227.07}{10-1} = 25.23$

33) $F'_0 = 3.82 = \frac{MSB}{MSE} = \frac{25.23}{MSE}$

$MSE = \frac{25.23}{3.82} = 6.604712$

34) $MSE = \frac{SSE}{df} = \frac{SSE}{(4-1)(10-1)} = 6.6$

$\Rightarrow SSE = 6.6 \times 27 = 178.2$

$SSTO = SSTR + SSB + SSE$

$SSTR = SSTO - SSB - SSE$

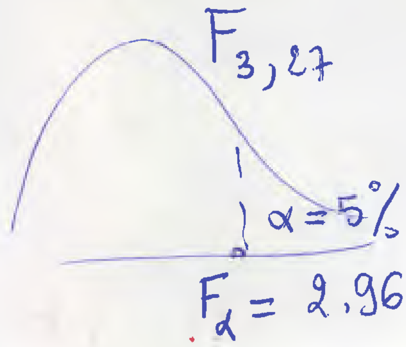
$= 427.1 - 227.07 - 178.2 = 21.83$

35) $MSTR = \frac{SSTR}{df} = \frac{SSTR}{4-1}$

$SSTR = MSTR/3 = 7.276667.$

36) $f_0 = \frac{MSTR}{MSE} = \frac{7.28}{6.6} = 1.10303$

37) $H_0 : \mu_{1\cdot} = \mu_{2\cdot} = \mu_{3\cdot} = \mu_{4\cdot} = 0$



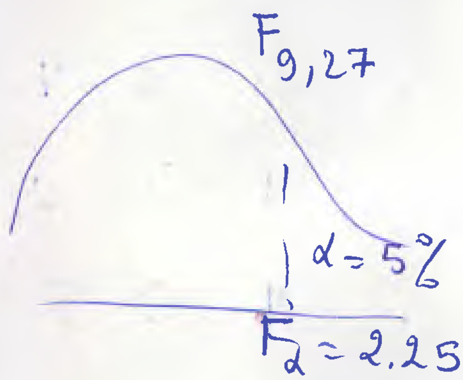
reject H_0 if $F_0 > 2.96$

$RR =]2.96 ; +\infty[$

38) $f_0 = 1.10 < 2.96$

not reject H_0.

39) $f_1 = \frac{MSB}{MSE} = 3.82$



$H_0 : \mu_{\cdot 1} = \mu_{\cdot 2} = \cdots = \mu_{\cdot 9}$

reject H_0 if $f_1 > 2.25$

$RR =]2.25, +\infty[$

40) $f_1 = 3.82 > 2.25$

reject H_0.