

Math 316
First Midterm Exam 1434, 1st semester

Name:

ID:

Q1 Prove or disprove each of the following statements:

- (a) If a set $\{x_1, x_2, \dots, x_n\}$ is orthogonal in an inner product space X , then it is linearly independent.
- (b) If $f(x) = \ln x$ and $\rho(x) = \frac{1}{x}$, then $f \in \mathcal{L}_\rho^2(0, 1)$.

Q2 Consider the sequence of functions

$$f_n(x) = x^n, \quad x \in [0, 1]$$

- (a) Find the limit $f(x)$ of $f_n(x)$ as $n \rightarrow \infty$.
- (b) Does $f_n(x)$ converge to $f(x)$ uniformly? Justify your answer.
- (c) Does $f_n(x)$ converge to $f(x)$ in $\mathcal{L}^2([0, 1])$? Justify your answer.

Q3 Consider the eigenvalue problem

$$\begin{aligned} Lu + \lambda u &= 0, & x \in [a, b], \\ u(a) &= 0, & u(b) = 0 \end{aligned} \tag{1}$$

- (a) Prove that if L is a self-adjoint operator, then $\lambda \in \mathbb{R}$.
- (b) Show that if $L = (1 + 3x^2) \frac{d^2}{dx^2} + 6x \frac{d}{dx}$ in problem (1), then L is a self-adjoint operator.

Q4 Consider the eigenvalue problem

$$\begin{aligned} u'' + 2u' + \lambda u &= 0, & x \in [0, 1], \\ u(0) &= 0, & u(1) = 0 \end{aligned} \tag{2}$$

- (a) Find the eigenvalues and eigenfunctions of problem (2).
- (b) Show that L is not a self-adjoint operator.
- (c) Transform L into a self-adjoint operator.
- (d) Write the orthogonality relation between the eigenfunctions of problem (2).

Good Luck
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