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Example 3:

A third example is to test the hypothesis that the frequency distribution of battery lives given in this Table

class	Class Boundaries	Frequency
1	1.45-1.95	2
2	1.95-2.45	1
3	2.45-2.95	4
4	2.95-3.45	15
5	3.45-3.95	10
6	3.95-4.45	5
7	4.45-4.95	3

The frequency may be approximated by a normal distribution with mean $\mu = 3.5$ and standard deviation $\sigma = 0.7$. Test the hypothesis

$$H_0: \quad \text{the data is normal } N(3.5, 0.7),$$
$$H_1: \quad \text{the data is not normal.}$$

at level of significance $\alpha = 0.05$

First : Find the value of Z where Z =

$Z_i = \frac{X_i - \mu}{\sigma} = \frac{X_i - 3.5}{0.7}$ where X_i = lower limit of interval i

Class	Class boundaries	Frequency	$Z = \frac{X_i - 3.5}{0.7}$	Z	
1	1.45-1.95	2	$Z_1 = \frac{1.45-3.5}{0.7}$ = -2.93	-2.93	
2	1.95-2.45	1	$Z_2 = \frac{1.95-3.5}{0.7}$ = -2.21	-2.21	
3	2.45-2.95	4	$Z_3 = \frac{2.45-3.5}{0.7}$ = -1.5	-1.5	
4	2.95-3.45	15	$Z_4 = \frac{2.95-3.5}{0.7}$ = -0.79	-0.79	
5	3.45-3.95	10	$Z_5 = \frac{3.45-3.5}{0.7}$ = -0.07	-0.07	
6	3.95-4.45	5	$Z_6 = \frac{3.95-3.5}{0.7}$ = 0.64	0.64	
7	4.45-4.95	3	$Z_7 = \frac{4.45-3.5}{0.7}$ = 1.36	1.36	
			$Z_8 = \frac{4.95-3.5}{0.7}$ =2.07	2.07	

Second :

Find the value of $P_i = P(Z_i < Z < Z_{i+1}) = From\ Z - table$

$$P_1 = P(Z_1 < Z < Z_2) = P(-2.93 < Z < -2.21) = 0.0136 - 0.0017 = 0.0119$$

$$P_2 = P(Z_2 < Z < Z_3) = P(-2.21 < Z < -1.5) = 0.0668 - 0.0136 = 0.0532$$

$$P_3 = P(Z_3 < Z < Z_4) = P(-1.5 < Z < -0.79) = 0.2148 - 0.0668 = 0.148$$

$$P_4 = P(Z_4 < Z < Z_5) = P(-0.79 < Z < -0.07) = 0.4721 - 0.2148 = 0.2573$$

$$P_5 = P(Z_5 < Z < Z_6) = P(-0.07 < Z < 0.64) = 0.7389 - 0.4721 = 0.2668$$

$$P_6 = P(Z_6 < Z < Z_7) = P(0.64 < Z < 1.36) = 0.9131 - 0.7389 = 0.1742$$

$$P_7 = P(Z_7 < Z < Z_8) = P(1.36 < Z < 2.07) = 0.9808 - 0.9131 = 0.0677$$

Third: Find the expected value $E_i = n\ x\ P_i = 40\ x\ P_i$

+

$E_1 = n\ x\ P_1 = 40\ x0.0119 =$ 0.5 ($E_1 =$ 0.5 < 5)

$E_2 = n\ x\ P_2 = 40\ x0.0532$ =2.1 ($E_2 =$ 2.1 < 5)

$E_3 = n\ x\ P_3 = 40\ x0.148 = 5.9$

$E_4 = n\ x\ P_4 = 40\ x\ 0.2573 = 10.3$

$E_5 = n\ x\ P_5 = 40\ x0.2668 = 10.7$

+

$E_6 = n\ x\ P_6 = 40\ x0.1742 = 7$

$E_7 = n\ x\ P_7 = 40\ x0.0677 = 2.7$ ($E_7 =$ 2.7 < 5)

Forth : The table is

Class	1	2	3	4
Class boundaries	1.45-1.95 1.95-2.45 2.45-2.95	2.95-3.45	3.45-3.95	3.95-4.45 4.45-4.95
Observed(O)	2+1+4	15	10	5+3
Expected (E)	0.5+2.1+5.9	10.3	10.7	7+2.7

Combine intervals and Add Observed and Expected,the table now

Class	1	2	3	4
Class boundaries	1.45-2.95	2.95-3.45	3.45-3.95	3.95-4.95
Observed(O)	7	15	10	8
Expected (E)	8.5	10.3	10.7	9.7

The test is $\chi^2 = \sum_{i=1}^{4} \frac{(O_i - E_i)^2}{E_i}$

$= \frac{(7-8.5)^2}{8.5} + \frac{(15-10.5)^2}{10.5} + \frac{(10-10.7)^2}{10.7} + \frac{(8-9.7)^2}{9.7} =$ 2.54

To find the $\chi^2{}_{\alpha}$: where α =0.05

$$df = v = k - m - 1 = 4 - 0 - 1 = 3$$

v	0.05
3	7.815

Reject H_0 if $\chi^2 > \chi^2{}_{\alpha}$

$2.54 \ngtr 7.815$ (not satisfy)

Accept H_0 :_The data is Normal N(3.5,0.7)