$$n = 33, \sum_{i=1}^{n} x_i \, y_i = 41355, \qquad \sum_{i=1}^{n} x_i = 1104, \qquad \sum_{i=1}^{n} y_i = 1124,$$

$\sum_{i=1}^{n} x_i^2 = 41086$, $\bar{x} = 33.4546$, and $\bar{y} = 34.0606$

$$b_1 = \frac{\sum_{i=1}^{n} x_i y_i - n\bar{x}\bar{y}}{\sum_{i=1}^{n} x_i^2 - n\bar{x}^2} = \frac{(41355) - 33(33.4546)(34.0606)}{(41086) - (33)(33.4546)^2} = 0.9036 \text{ and}$$

$$b_0 = \bar{y} - b_1\bar{x} = 34.0606 - (0.9036)(33.4546) = 3.8296.$$

$$\hat{y}_i = 3.8296 + 0.9036\, x_i$$

From the regression equation we find for x = 20% solids reduction, then the Oxygen Demand Reduction is

$$\hat{y} = 3.8296 + 0.9036(20) = 21.9025\%$$

Let SST=3713.879, and SSE=323.3274

Then $\hat{\sigma}^2 = \frac{SSE}{n-2} = \frac{323.3274}{31} = 10.4299$

$\hat{\sigma} = 3.2295$

$$b_1 - t_{\frac{\alpha}{2},(n-2)} \frac{\hat{\sigma}}{\sqrt{S_{xx}}} < \beta_1 < b_1 + t_{\frac{\alpha}{2},(n-2)} \frac{\hat{\sigma}}{\sqrt{S_{xx}}}$$

Or

$$b_1 - t_{\frac{\alpha}{2},(n-2)} \frac{\hat{\sigma}}{\sqrt{\sum_{i=1}^{n}(x_i - \bar{x})^2}} < \beta_1 < b_1 + t_{\frac{\alpha}{2},(n-2)} \frac{\hat{\sigma}}{\sqrt{\sum_{i=1}^{n}(x_i - \bar{x})^2}}$$

$t_{\frac{\alpha}{2},(n-2)} = t_{0.025,(31)} = 2.045$

Then

$$0.9036 - 2.045 \frac{3.2295}{\sqrt{4152.182}} < \beta_1 < 0.9036 + 2.045 \frac{3.2295}{\sqrt{4152.182}}$$

$$0.8011 < \beta_1 < 1.0061$$

Hypothesis Testing on the Slope

$$H_0: \beta_1 = \beta_{10}$$

The test statistic

$$t = \frac{b_1 - \beta_{10}}{\hat{\sigma}/\sqrt{S_{xx}}}$$

Reject H_0 if $t > t_{\frac{\alpha}{2},(n-2)}$ or $t < -t_{\frac{\alpha}{2},(n-2)}$ (if $H_1: \beta_1 \neq \beta_{10}$)

Reject H_0 if $t > t_{\alpha,(n-2)}$ (if $H_1: \beta_1 > \beta_{10}$)

Reject H_0 if $t < -t_{\alpha,(n-2)}$ (if $H_1: \beta_1 < \beta_{10}$)

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$$H_0: \beta_1 = 1$$

$$H_1: \beta_1 < 1$$

The test statistic

$$t = \frac{b_1 - \beta_{10}}{\hat{\sigma}/\sqrt{S_{xx}}} = \frac{0.9036 - 1}{3.2295/\sqrt{4152.182}} = -1.9235$$

Reject H_0 if $t < -t_{\alpha,(n-2)}$

$t_{\alpha,(n-2)} = t_{0.05,(31)} = 1.696$

$R^2 = 1 - \frac{SSE}{SST} = 1 - \frac{323.3274}{3713.879} = 0.913$