Chapter (6)

Probability Distributions

Example (1)

Two balanced dice are rolled. Let X be the sum of the two dice. Obtain the probability distribution of X.

Solution

When the two balanced dice are rolled, there are 36 equally likely possible outcomes as shown below:

$$S = \begin{Bmatrix} (1,1) & (1,2) & (1,3) & (1,4) & (1,5) & (1,6) \\ (2,1) & (2,2) & (2,3) & (2,4) & (2,5) & (2,6) \\ (3,1) & (3,2) & (3,3) & (3,4) & (3,5) & (3,6) \\ (4,1) & (4,2) & (4,3) & (4,4) & (4,5) & (4,6) \\ (5,1) & (5,2) & (5,3) & (5,4) & (5,5) & (5,6) \\ (6,1) & (6,2) & (6,3) & (6,4) & (6,5) & (6,6) \end{Bmatrix}$$

- The possible values of X are: 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 and 12.
- The discrete probability distribution of X is given by

X	P(X)
2	1 / 36
3	1 / 18
4	1 / 12
5	1 / 9
6	5 / 36
7	1 / 6
8	5 / 36
9	1 / 9
10	1 / 12
11	1 / 18
12	1 / 36
Total	36/36 =1

Example (2)

The number of persons *X*, in Al Riyadh family chosen at random has the following probability distribution:

X	1	2	3	4	5	6	7	8	Total
P(X)	0.34	0.44	0.11	0.06	0.02	0.01	0.01	0.01	1

1/ Find the average family size { E(X)} .

2/ The variance of probability distribution

Solution

X	P(X)	XP(X)	$(X-\mu)$	$(X-\mu)^2$	$P(X)*(X-\mu)^2$
1	0.34	0.34	-1.1	1.21	0.4114
2	0.44	0.88	-0.1	0.01	0.0044
3	0.11	0.33	0.9	0.81	0.0891
4	0.06	0.24	1.9	3.61	0.2166
5	0.02	0.1	2.9	8.41	0.1682
6	0.01	0.06	3.9	15.21	0.1521
7	0.01	0.07	4.9	24.01	0.2401
8	0.01	0.08	5.9	34.81	0.3481
		2.1			1.63

$\mu = x\,P(x) = E(x)$ = 2.1 = 3 person

$\sigma^2 = (x-\mu)^2\,P(x)$ = 1.63

Example (3)

John Ragsdale sells new cars for Pelican Ford. John usually sells the largest number of cars on Saturday. He has developed the following probability distribution for the number of cars he expects to sell on a particular Saturday.

x	0	1	2	3	4	**Total**
$p(x)$	0.10	0.20	0.30	0.30	0.10	$\sum p(x) = 1$

Find:

1. Expected value of x(The mean of probability distribution)
2. σ^2 (The variance of probability distribution)

Solution:

$\mu = x\,P(x) = E(x)$

$\sigma^2 = (x-\mu)^2\,P(x)$

x	$p(x)$	$x\,P(x)$	$(x-\mu)$	$(x-\mu)^2$	$(x-\mu)^2\ p(x)$
0	0.10	0	-2.1	4.41	0.441
1	0.20	0.20	-1.1	1.21	0.242
2	0.30	0.60	-0.1	0.01	0.003
3	0.30	0.90	0.9	0.81	0.243
4	0.10	0.40	1.9	3.61	0.361
	1	2.1			1.29

$\mu = \sum\ x\,P(x) = E(x) = 0\times 0.10 + 1\times 0.20 + 2\times 0.30 + 3\times 0.30 + 4\times 0.10 = 2.1$

$(x-\mu)^2\,P(x) = 4.41\times 0.10 + 1.21\times 0.20 + 0.01\times 0.30 + 0.81\times 3 + 3.61\times 4$

$= 0.441 + 0.242 + 0.003 + 0.243 + 0.361 = 1.29$

Example (4)

Which one of these tables is actually a probability distribution

x	P(x)
5	0.3
10	0.3
15	0.2
20	0.4

A

x	P(x)
5	0.1
10	0.3
15	0.2
20	0.4

B

x	P(x)
5	0.5
10	0.3
15	-0.2
20	0.4

C

Binomial distribution

Example (1)

If the experiment is tossing a coin 6 times, what is the probability of:

1. Getting two heads.
2. Getting at least 4 heads.
3. Getting at most one head.
4. Getting at least two heads
5. Find mean, variance and deviation

Solution:

$$n = 6 \qquad \pi = \frac{1}{2} \qquad 1 - \pi = 1 - \frac{1}{2} = \frac{1}{2}$$

$$P(x) = {}_nC_x \pi^x (1-\pi)^{n-x} = \frac{n!}{x!(n-x)!} \pi^x (1-\pi)^{n-x}$$ **, x = 0, 1, … , n**

1. $$P(x=2) = {}_6C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^{6-2} = \frac{6!}{2!(6-2)!}\left(\frac{1}{4}\right)\left(\frac{1}{16}\right) = \frac{(6)(5)4!}{(2)\,4!}\left(\frac{1}{64}\right) = \frac{15}{64} = 0.23$$

2. $$P(x \ge 4) = P(x=4) + P(x=5) + P(x=6)$$

$$P(x \ge 4) = {}_6C_4 \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^{6-4} + {}_6C_5 \left(\frac{1}{2}\right)^5 \left(\frac{1}{2}\right)^{6-5} + {}_6C_6 \left(\frac{1}{2}\right)^6 \left(\frac{1}{2}\right)^{6-6}$$

$$= \frac{6!}{4!(6-4)!}\left(\frac{1}{16}\right)\left(\frac{1}{4}\right) + \frac{6!}{5!(6-5)!}\left(\frac{1}{32}\right)\left(\frac{1}{2}\right) + \frac{6!}{6!(6-6)!}\left(\frac{1}{64}\right)\left(\frac{1}{2}\right)^0$$

$$= 15\left(\frac{1}{64}\right) + 6\left(\frac{1}{64}\right) + \left(\frac{1}{64}\right) = \frac{22}{64} = 0.34$$

3. $$P(x \le 1) = P(x=0) + P(x=1)$$

$$= {}_6C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^{6-0} + {}_6C_1 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^{6-1} = \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^6 + 6\left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^{6-1}$$

$$= \left(\frac{1}{64}\right) + 6\left(\frac{1}{64}\right) = \frac{7}{64} = 0.11$$

4. $$P(x \ge 2) = 1 - p(x \le 1) = 1 - 0.11 = 0.89$$

5. $$\mu = n\pi = 6\left(\frac{1}{2}\right) = \frac{6}{2} = 3$$

$$\mathrm{var}\,iance(\sigma^2) = n\pi(1-\pi) = 6\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{6}{4} = 1.5$$

$$\sigma = \sqrt{1.5} = 1.22$$

Example (2)

Over a long period of time it has been observed that a given marksman can hit a target on a single trial with probability equal to 0.8. Suppose he fires four shots at the target. Answer the following:

1. What is the probability that he will hit the target exactly two times?
2. What is the probability that he will hit the target at least once?
3. Find mean, variance and standard deviation.

Solution:

$n = 4 \qquad \pi = 0.8 \qquad 1-\pi = 1-0.8 = 0.2$

1. $P(x=2) = {}_4C_2\,(0.8)^2(0.2)^{4-2} = \dfrac{4!}{2!(4-2)!}(0.8)^2(0.2)^2$

$= \dfrac{24}{4}(0.64)(0.04) = 6(0.64)(0.04) = 0.15$

2.

$$P(x \ge 1) = 1 - P(x<1) = 1 - P(x=0) = 1 - \left[\,{}_4C_0\,(0.8)^0(0.2)^{4-0}\right]$$

$$= 1 - \left[\frac{4!}{0!(4-0)!}(0.8)^0(0.2)^4\right] = 1 - [(1)(0.0016)] = 1 - 0.0016 = 0.99$$

3. $\mu = n\pi = 4(0.8) = 3.2$

$\sigma^2 = n\pi(1-\pi) = 4(0.8)(0.2) = 0.64$

$\sigma = \sqrt{0.64} = 0.8$

Example (3)

Find the probability of guessing correctly exactly 6 of the 10 answers on a true-false examination.

Solution:

$n = 10 \qquad \pi = \dfrac{1}{2} \qquad 1-\pi = \dfrac{1}{2}$

$$P(x=6) = {}_{10}C_6\left(\frac{1}{2}\right)^6\left(\frac{1}{2}\right)^{10-6} = 210\left(\frac{1}{64}\right)\left(\frac{1}{16}\right) = \frac{210}{1024} = 0.21$$

Poisson distribution

Example (1)

The average number of traffic accidents on a certain section of highway is two per week assume that the number of accidents follows a Poisson distribution with $\mu = 2$.

1. Find the probability of no accidents on this section of highway during a 1-week period.
2. Find the probability of at most three accidents on this section of highway during a 1-week.
3. Find the probability of at least four accidents during a 1-week
4. Find variance and standard deviation.

Solution:

$\mu = 2$

1. $$P(x=0) = \frac{\mu^x}{e^\mu \, x!} = \frac{2^0}{(2.718)^2 \, 0!} = \frac{1}{7.387524} = 0.1353$$

2. $$P(x \le 3) = P(x=0) + P(x=1) + P(x=2) + P(x=3)$$

$$= \frac{2^0}{(2.718)^2 \, 0!} + \frac{2^1}{(2.718)^2 \, 1!} + \frac{2^2}{(2.718)^2 \, 2!} + \frac{2^3}{(2.718)^2 \, 3!}$$

$$\frac{1}{7.387524} + \frac{2}{7.387524} + \frac{4}{14.7750} + \frac{8}{44.3251}$$

$$= 0.1354 + 0.2707 + 0.2707 + 0.1805 = 0.8573$$

3.. $P(x \ge 4) = 1 - P(x \le 3) = 1 - 0.8573 = 0.1427$

4. $Var(x) = \mu = 2$

$\sigma = \sqrt{2} = 1.414$

Example (2)

Suppose a life insurance company insures the lives of 3000 men aged 42. If actuarial studies show the probability that an 42-year-old man will die in a given year to be o.oo1, find the exact probability that the company will have to pay x = 4 claims during a given year.

Solution:

$u = n\pi = 3000(0.001) = 3$

$$P(x=4) = \frac{3^4}{(2.718)^3 \, 4!} = \frac{81}{(20.0792)(24)} = \frac{81}{481.9008} = 0168$$

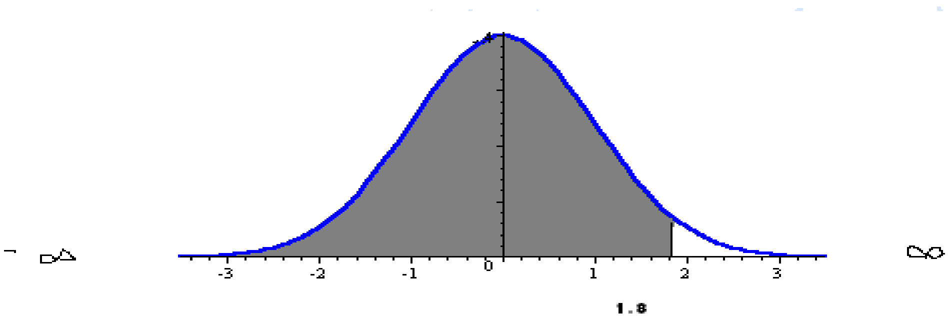
Chapter (7) Normal probability distribution

How to find the area under the normal curve?

$$(1)\, P(Z < a) = 0.5 + p(0 \le Z \le a)$$

$$p(Z < 1.8) = 0.5 + P(0 < Z < 1.8)$$

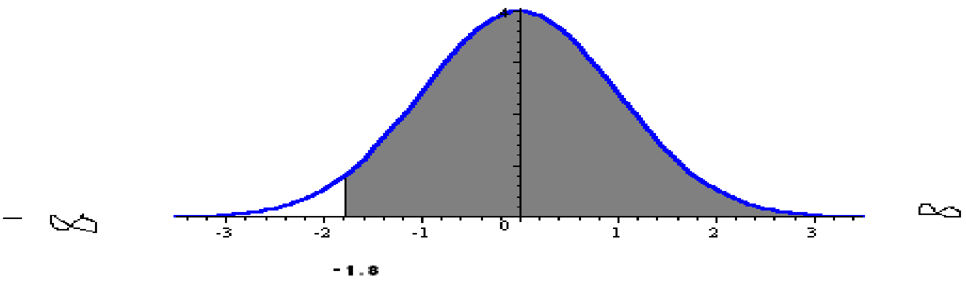
$$= 0.5 + 0.4641 = 0.9641$$



$$(2)\, P(Z > -a) = 0.5 + P(-a \le Z \le 0)$$

$$p(Z > -1.8) = 0.5 + P(-1.8 < Z < 0)$$

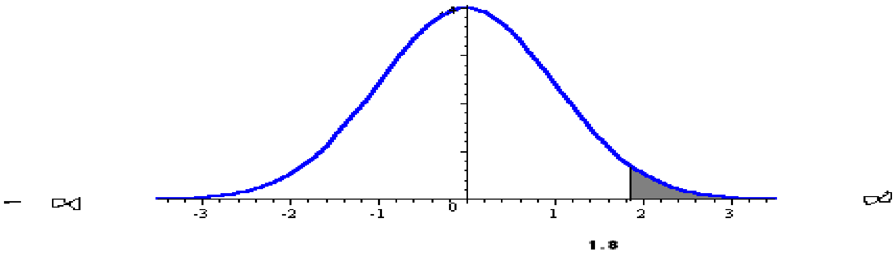
$$= 0.5 + 0.4641 = 0.9641$$



$$(3)\,P(Z > a) = 0.5 - P(0 \le Z \le 0)$$

$$p(Z > 1.8) = 0.5 - P(0 < Z < 1.8)$$

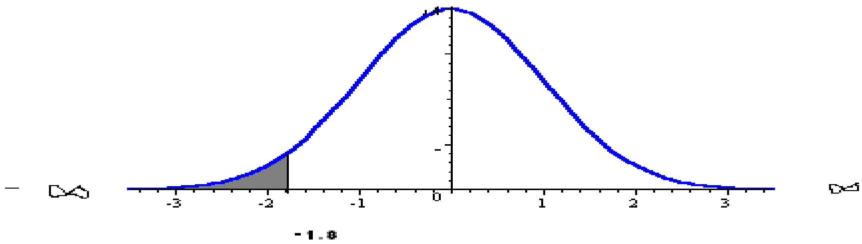
$$= 0.5 - 0.4641 = 0.0359$$



$$(4)\,P(Z < -a) = 0.5 - P(-a \le Z \le 0)$$

$$p(Z < -1.8) = 0.5 - P(-1.8 < Z < 0)$$

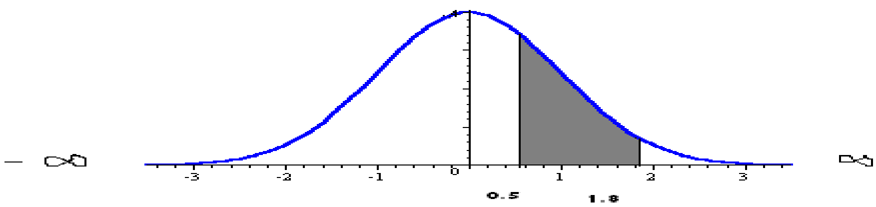
$$= 0.5 - 0.4641 = 0.0359$$



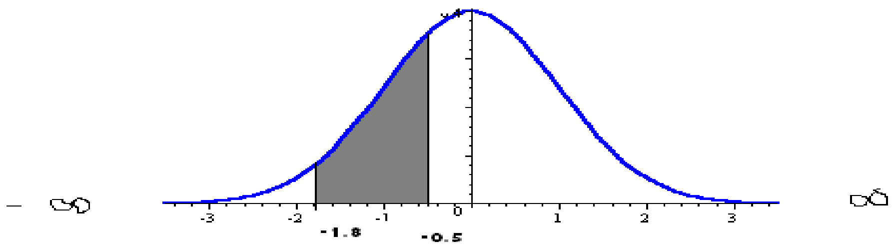
$$(5)\,P(a < Z < b) = P(0 \le Z \le b) - P(0 \le Z \le a)$$

$$p(1.5 < Z < 1.8) = P(0 < Z < 1.8) - P(0 < Z < 1.5)$$

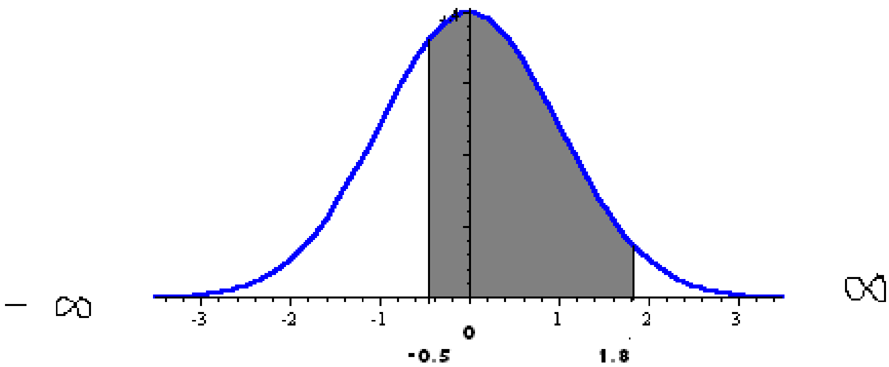
$$= 0.4641 - 0.4332 = 0.0309$$



$$(6)\ P(-b < Z < -a) = P(-b \le Z \le 0) - P(-a \le Z \le 0)$$
$$p(-1.8 < Z < -1.5) = P(-1.8 < Z < 0) - P(-1.5 < Z < 0)$$
$$= 0.4641 - 0.4332 = 0.0309$$



$$(7)\ P(-a < Z < b) = P(0 \le Z \le b) + P(-a \le Z \le 0)$$
$$p(-1.5 < Z < 1.8) = P(-1.5 < Z < 0) + P(0 < Z < 1.8)$$
$$= 0.4332 + 0.4641 = 0.8973$$



Example (1)

Let X be a normally distributed random variable with mean 65 and standard deviation 13. Find the standard normal random variable (z) for P(X>80)

Solution:

$$P(X > 80) = P(Z > \frac{80-65}{13}) = P(Z > \frac{15}{13}) = P(Z > 1.15)$$

$$0.5 - 0.3749 = 0.1251$$

Example (2)

If the mean = 65 and standard deviation =13. Find x from the following:

1. z = 0.6
2. z = -1.93

Solution:

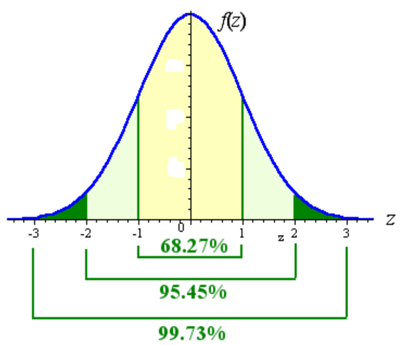
1. $z = 0.6$

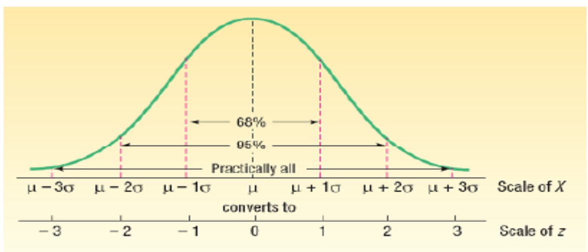
$x = 65 + (0.6)13 = 65 + 7.8 = 72.8$

2. $z = -1.93$

$x = 65 + (-1.93)13 = 65 - 25.09 = 39.91$

The Empirical Rule





Example (3)

A sample of the rental rates at University Park Apartments approximates a systematical, bell- shaped distribution. The sample mean is $500; the standard deviation is $20.Using the Empirical Rule, answer these questions: About 68 percent of the monthly food expenditures are between what two amounts?

1. About 95 percent of the monthly food expenditures are between what two amounts?

1. About all of the monthly (99.7%) food expenditures are between what two amounts?

Solution

1- $\overline{X} \pm 1\sigma = 500 \pm 1(20) = 500 \pm 20$

$\$480, \$520$

About 68 percent are between $480 and $520 .

2- $\overline{X} \pm 2\sigma = 500 \pm 2(20) = 500 \pm 40$

$\$460, \$540$

About 95 percent are between $460 and $540.

3- $\overline{X} \pm 2\sigma = 500 \pm 3(20) = 500 \pm 60$

$\$440, \$560$

About 99.7 percent are between $440 and $560.

Example (4)

The mean of a normal probability distribution is 120; the standard deviation is 10.

a. About 68.26 percent of the observations lie between what two values?
b. About 95.44 percent of the observations lie between what two values?
c. About 99.74 percent of the observations lie between what two values?

Solution:

a. $\mu \pm 1(\sigma) = 120 \pm 1(10\ ) = 120 \pm 10$

$130 \ \textit{and} \ 110$

b. $\mu \pm 2(\sigma) = 120 \pm 2(10) = 120 \pm 20$

$140 \ \textit{and} \ 100$

c. $\mu \pm 3(\sigma) = 120 \pm 3(10) = 120 \pm 30$

$150 \ \textit{and} \ 90$

Example (5)

Studies show that gasoline use for compact cars sold in the United States is normally distributed, with a mean of 25.5 miles per gallon (mpg) and a standard deviation of 4.5 mpg. Find the probability of compact cars that get:

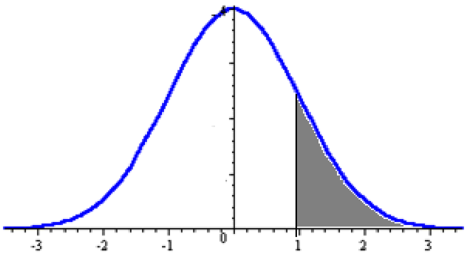
1. 30 mpg or more.
2. 30 mpg or less.
3. Between 30 and 35.
4. Between 30 and 21.

Solution:

$\mu = 25.5 \qquad \sigma = 4.5$

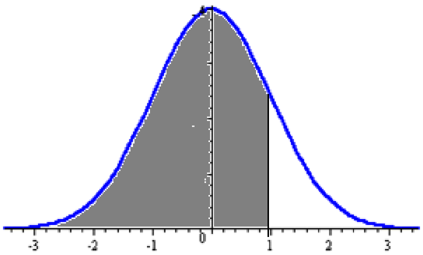
1. $P(x \ge 30) = P(Z \ge \frac{30-25.5}{4.5}) = P(Z \ge \frac{4.5}{4.5}) = P(z \ge 1) = 0.5 - \Phi(1)$

$= 0.5 - 0.3413 = 0.1587$



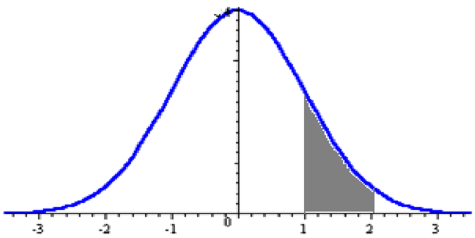
2. $P(x < 30) = P\left(Z < \frac{30-25.5}{4.5}\right) = P\left(Z < \frac{4.5}{4.5}\right) = P(z < 1) = 0.5 + \Phi(1)$

$= 0.5 + 0.3413 = 0.8413$



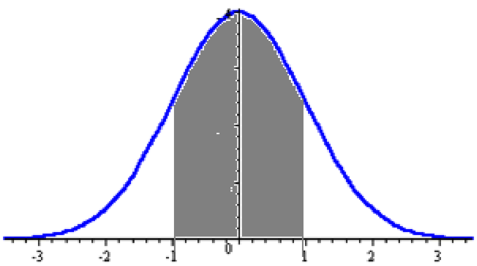
3. $P(30 < x < 35) = P\left(\frac{30-25.5}{4.5} < Z < \frac{35-25.5}{4.5}\right) = P\left(\frac{4.5}{4.5} < Z < \frac{9.5}{4.5}\right)$

$= P(1 < z < 2.11) = \Phi(2.11) - \Phi(1) = 0.4826 - 0.3413 = 0.1413$



4. $P(21 < x < 30) = P\left(\frac{21-25.5}{4.5} < Z < \frac{30-25.5}{4.5}\right) = P\left(\frac{-4.5}{4.5} < Z < \frac{4.5}{4.5}\right)$

$= P(-1 < z < 1) = \Phi(1) + \Phi(1) = 0.3413 + 0.3413 = 0.6826$



Example (6)

Suppose that X ~N (3, o.16). Find the following probability:

1. $P(x \geq 3)$.
2. $P(2.8 < x < 3.1)$.

Solution:

$u = 3 \qquad \sigma = 0.4$

1. $P(x \geq 3) = P\left(Z \geq \frac{3-3}{0.4}\right) = P(z \geq 0) = 0.5000$

2. $P(2.8 < x < 3.1) = P\left(\frac{2.8-3}{0.4} < Z < \frac{3.1-3}{0.4}\right) = P(\frac{-0.2}{0.4} < Z < \frac{0.1}{0.4}) =$

$P(-0.50 < z < 0.25) = \Phi(0.5) + \Phi(0.25) = 0.1915 + 0.0987 = 0.2902$

Example (7)

The grades on a short quiz in math were 0, 1, 2…, 10 point, depending on the number answered correctly out of 10 questions. The mean grade was 6.7 and the standard deviation was 1.2. Assuming the grades to be normally distributed, determine:

1. The percentage of students scoring more than 6 points.
2. The percentage of students scoring less than 8 points.
3. The percentage of students scoring between 5.5and 6 points.
4. The percentage of students scoring between 5.5and 8 points.
5. The percentage of students scoring less than 5.5 points.
6. The percentage of students scoring more than 8 points.
7. The percentage of students scoring equal to 8 points.
8. The maximum grade of the lowest 5 % of the class.
9. The minimum grade of the highest 15 % of the class.

Solution:

$\mu = 6.7 \quad \sigma = 1.2$

1. $P(x > 6) = P\left(Z > \frac{6-6.7}{1.2}\right) = P\left(Z > \frac{-0.7}{1.2}\right) = P(z > -0.58)$

   $= 0.5 + \Phi(0.58) = 0.5 + 0.2190 = 0.7190$

2. $P(x < 8) = P\left(Z < \frac{8-6.7}{1.2}\right) = P\left(Z < \frac{1.3}{1.2}\right) = P(z < 1.08)$

   $= 0.5 + \Phi(1.08) = 0.5 + 0.3599 = 0.8599$

3. $P(5.5 < x < 6) = P\left(\frac{5.5-6.7}{1.2} < Z < \frac{6-6.7}{1.2}\right) = P\left(\frac{-1.2}{1.2} < Z < \frac{-0.7}{1.2}\right)$

   $= P(-1 < z < -.58) = \Phi(1) - \Phi(0.58) = 0.3413 - 0.2190 = 0.1223$

4. $P(5.5 < x < 8) = P\left(\frac{5.5-6.7}{1.2} < Z < \frac{8-6.7}{1.2}\right) = P\left(\frac{-1.2}{1.2} < Z < \frac{1.3}{1.2}\right)$

   $= P(-1 < Z < 1.08) = \Phi(1.08) + \Phi(1) = 0.3599 + 0.3413 = 0.7012$

5. $$P(X < 5.5) = P\left(Z < \frac{5.5 - 6.7}{1.2}\right) = P\left(Z < \frac{-1.2}{1.2}\right)$$
$$= P(z < -1) = 0.5 - \Phi(1) = 0.5 - 0.3413 = 0.1587$$

6. $$P(x > 8) = P\left(Z > \frac{8 - 6.7}{1.7}\right) = P\left(Z > \frac{1.3}{1.2}\right) = P(z > 1.08)$$
$$= 0.5 - \Phi(1.08) = 0.5 - 0.3599 = 0.1401$$

7. $P(x = 8) = 0$

8.
$$0.5 - 0.05 = 0.4500$$
$$z = -1.645$$
$$x = 6.7 - 1.645(1.2) = 6.7 - 1.974 = 4.726 \sim 4.7$$

9.
$$0.5 - 0.15 = 0.3500$$
$$z = 1.04$$
$$x = 6.7 + 1.04(1.2) = 6.7 + 1.248 = 7.948 \sim 7.9$$

Example (8)

If the heights of 300 students are normally distributed, with a mean 172 centimeters and a standard deviation 8 centimeters, how many students have heights?

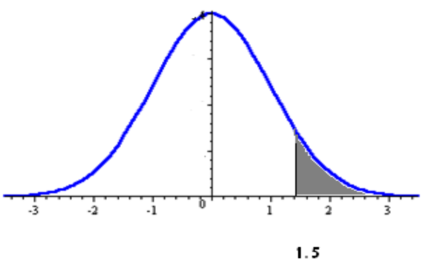
1. Greater than 184 centimeters.
2. Less than or equal to 160 centimeters.
3. Between 164 and 180 centimeters inclusive.
4. Equal to 172 centimeters.

$n = 300 \qquad \mu = 172 \qquad \sigma = 8$

$$P(x > 184) = P\left(Z > \frac{184 - 172}{8}\right) = P\left(Z > \frac{12}{8}\right)$$

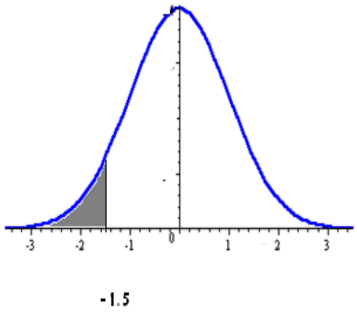
1. $= P(z > 1.5) = 1 - \Phi(1.5) = 0.5 - 0.4332 = 0.0668$

$\textit{number of students have heights greater than } 184 = 300(0.0668) = 20$



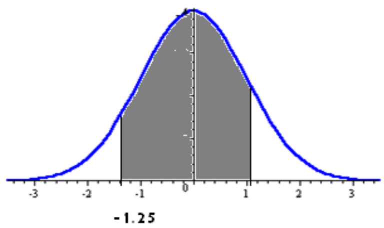
2.

$$P(x<160)=P\left(Z<\frac{160-172}{8}\right)=P\left(Z<\frac{-12}{8}\right)$$
$$=P(z<-1.5)=0.5-\Phi(1.5)=0.5-0.4332=0.0668$$
number of students have heights less than $160=300(0.0668)=20$



3.

$$P(164<x<180)=P\left(\frac{164-172}{8}<Z<\frac{180-172}{8}\right)=P\left(\frac{-8}{8}<Z<\frac{8}{8}\right)$$
$$=P(-1<z<1)=\Phi(1)+\Phi(1)=0.3413+0.3413=0.6826$$
number of students have heights between 164 *and* $180=300(0.6826)=204.78\sim 205$



4.

$$P(x=172)=0$$

Example (9)

- If P(Z $\le z_1$) = 0.9099 , what does the (z_1) equal?

Solution:

$$p(Z \le z_1) = 0.9099$$
$$0.9099 - .5 = 0.4099$$
$$Z_{0.4099} = 1.34$$

- If P(Z $\ge z_1$) = 0.9099 , what does the (z_1) equal?

Solution:

$$p(Z \ge z_1) = 0.9099$$
$$0.9099 - .5 = 0.4099$$
$$Z_{0.4099} = -1.34$$

Example (10)

The mean of a normal probability distribution is 130; the standard deviation is 10.

- **What observation which more than or equal 84.13% percent of the observations?**

Solution:

$$p(Z \ge z_1) = 0.8413$$
$$0.8413 - .5 = 0.3413$$
$$Z_{0.3413} = -1$$
$$X = u + Z\sigma = 130 + 10(-1) = 130 - 10 = 130$$

- **What observation which less than or equal 84.13% percent of the observations?**

Solution:

$$P(Z \le z_1) = 0.8413$$
$$0.8413 - .5 = 0.3413$$
$$Z_{0.3413} = 1$$
$$X = u + Z\sigma = 130 + 10(1) = 130 + 10 = 140$$

Example (11)

The mean of a normal probability distribution is 130; the standard deviation is 10.

- **What observation which more than or equal 8.85 % percent of the observations?**

Solution:

$p(Z \geq z_1) = 0.0885$

$0.5 - 0.0885 = 0.4115$

$Z_{0.4115} = 1.35$

$X = u + Z\sigma = 130 + 10(1.35) = 130 + 13.5 = 143.5$

- **What observation which less than or equal 8.85 % % percent of the observations?**

Solution:

$p(Z \leq z_1) = 0.0885$

$0.5 - 0.0885 = 0.4115$

$Z_{0.4115} = -1.35$

$X = u + Z\sigma = 130 + 10(-1.35) = 130 - 13.5 = 116.5$

Chebyshev's Theorem

The percent=$1 - \frac{1}{K^2}$

Example (12)

According to Chebyshev's theorem,at least what percent of any set of observations will be within 1.8 standard deviations of the mean

Solution

The percent=$1 - \frac{1}{1{,}8^2} = 0.69$

About 69%

Example (13)

The mean income of a group of sample observations is $500;the standard deviation is $40.According to Chebyshev's theorem, at least what percent of income will lie between $400 and $600

Solution

$K = \frac{upper - \overline{X}}{S} = \frac{600 - 500}{40} = 2.5$

$K = \frac{Lower - \overline{X}}{S} = \frac{400 - 500}{40} = -2.5$

The percent=$1 - \frac{1}{2.5^2} = 0.84$

About 84%

The normal approximation to the Binomial

Continuity Correction factor

The normal probability distribution is a good approximation to the binomial probability distribution when $n\pi$ and $n(1-\pi)$ are both at least 5.

Only four cases may arise. These cases are:

1. For the probability at least X occurs, use the area above (X -.5).
2. For the probability that more than X occurs, use the area above (X+.5).
3. For the probability that X or less occurs (at most), use the area below (X +.5).
4. for the probability that less than X occurs, use the area below (X-.5).

Example (14)

Assume a binomial probability distribution with $n = 50$ and $\pi = 0.25$. Compute the following:

1-the mean and the standard deviation of the random variable
2-The probability that x=25
3- The probability that x at least 15
4- The probability that x more than 15
5- The probability that x is 10 or less
6- The probability that x is less 10

Solution

$1-$

$$\mu = n\pi = 50 \times 0.25 = 12.5$$

$$\sigma^2 = n\pi(1-\pi) = 50 \times 0.25(1-0.25) = 9.375$$

$$\sigma = \sqrt{9.375} = 3.0619$$

$2-$

$$p(x = 25) = 50C_{25} \times 0.25^{25} \times (1-0.25)^{50-25} = 0.000084$$

$3-$

$$\because n\pi = 50 \times 0.25 = 12.5 \, and \; n(1-\pi) = 50(1-0.25) = 37.5 \geq 5$$

$$\therefore P(X \geq 15) = P(X \geq 15 - 0,5) = P(X \geq 14.5) = P\left(Z \geq \frac{14.5-12.5}{3.0619}\right) = P(Z \geq 0.65)$$

$$= 0.5 - 0.2422 = 0.2578$$

$4-$

$$\therefore P(X > 15) = P(X > 15 + 0,5) = P(X > 15.5) = P\left(Z > \frac{15.5-12.5}{3.0619}\right) = P(Z > 0.98)$$

$$= 0.5 - 0.3365 = 0.1635$$

$5-$

$$\therefore P(X \leq 10) = P(X \leq 10 + 0,5) = P(X \leq 10.5) = P\left(Z \leq \frac{10.5-12.5}{3.0619}\right) = P(Z \leq -0.65)$$

$$= 0.5 - 0.2422 = 0.2578$$

$6-$

$$\therefore P(X < 10) = P(X < 10 - 0,5) = P(X < 9.5) = P\left(Z < \frac{9.5-12.5}{3.0619}\right) = P(Z < -0.98)$$

$$= 0.5 - 0.3365 = 0.1635$$

Z-Table

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990